1 Hybrid Composition Functions (F12-F19*)

The *hybrid composition functions*, F12-F19*, are built combining a non-separable function with other function. The considered functions are:

- **Non-Separable Functions**:
  - F3: Shifted Rosenbrock's Function
  - F5: Shifted Griewank's Function
  - NS-F9: Non-Shifted Extended $f10$
  - NS-F10: Non-Shifted Bohachevsky
- **Other Component Functions**:
  - F1: Shifted Sphere Function
  - F4: Shifted Rastrigin's Function
  - NS-F7: Non-Shifted Schwefel's Problem 2.22

The procedure used to hybridize a non-separable function F_{ns} with other function F' (function $F_{ns} \oplus F'$) is shown in Figure 1. Its main steps are: 1) to divide the solution into two parts, 2) to evaluate each one of them with a different function, and 3) to combine their results. The splitting mechanism uses a parameter, m_{ns}, which specifies the ratio of variables that are evaluated by F_{ns}. Using a higher value of m_{ns}, the hybrid function becomes more difficult to optimize dimension by dimension, because there is a greater interrelation between the variables and the fitness. With this procedure, we have defined the instances of hybrid functions shown in Table 1.

Name	F_{ns}	F'	m_{ns}	**Range**	**Fitness Optimum**
F12	NS-F9	F1	0.25	$[-100, 100]^D$	0
F13	NS-F9	F3	0.25	$[-100, 100]^D$	0
F14	NS-F9	F4	0.25	$[-5, 5]^D$	0
F15	NS-F10	NS-F7	0.25	$[-10, 10]^D$	0
F16*	NS-F9	F1	0.5	$[-100, 100]^D$	0
F17*	NS-F9	F3	0.75	$[-100, 100]^D$	0
F18*	NS-F9	F4	0.75	$[-5, 5]^D$	0
F19*	NS-F10	NS-F7	0.75	$[-10, 10]^D$	0

Table 1: Hybrid composition functions

We should point out that the hybrid F15 and F19* functions were shifted.

Function $F_{ns} \oplus F'(S)$

1. S is divided into two parts ($part_1$ and $part_2$):

    - **If** $m_{ns} \leq 0.5$ **then**
        - $part_1$ is composed by the first $D \cdot m_{ns}$ even variables. $(length(part_1) = D \cdot m_{ns})$
        - $part_2$ is composed by the remaining variables. $(length(part_2) = D - length(part_1))$
    - **If** $m_{ns} > 0.5$ **then**
        - $part_2$ is composed by the first $D \cdot (1 - m_{ns})$ odd variables. $(length(part_2) = D \cdot (1 - m_{ns}))$
        - $part_1$ is composed by the remaining variables. $(length(part_1) = D - length(part_2))$

2. **Return** $F_{ns}(part_1) + F'(part_2)$.

Figure 1: Evaluation of a solution S (with D variables) by the hybrid function $F_{ns} \oplus F'$