# Test Suite for the Special Issue of Soft Computing on Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems

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**Abstract**

In this document, we provide the description of the 19 test functions (F1-F19*) that should be used for the experimental study for the Special Issue of Soft Computing on Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems.

In Section 1 (page 2), we report the main features and properties of the functions F1-F11 and, in Section 2 (page 10), we explain the way hybrid composition functions F12-F19* are obtained combining two functions belonging to the set F1-F11.

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# 1 Functions F1-F11

The F1-F11 functions are the following:

- Shifted Unimodal Functions:
  - F1: Shifted Sphere Function
  - F2: Shifted Schwefel's Problem 2.21
- Shifted Multimodal Functions:
  - F3: Shifted Rosenbrock's Function
  - F4: Shifted Rastrigin's Function
  - F5: Shifted Griewank's Function
  - F6: Shifted Ackley's Function
- Not Shifted Unimodal Functions
  - F7: Schwefel's Problem 2.22
  - F8: Schwefel's Problem 1.2
  - F9: Extended f10
  - F10: Bohachevsky
  - F11: Schaffer

The definition of these functions are shown in Table 1 and their features are sketched in Table 2. Figures 1-6 show graphically these functions for the case of $D = 2$. Finally, their properties are described in Table 3.

| Function | Name | Definition |
|---|---|---|
| $F_1$ | Shifted Sphere Function | $\sum_{i=1}^{D} z_i^2 + f\_bias,\ z = x - o$ |
| $F_2$ | Shifted Schwefel Problem 2.21 | $max_i\{\lvert z_i\rvert, 1 \leq i \leq D\} + f\_bias,\ z = x - o$ |
| $F_3$ | Shifted Rosenbrock's Function | $\sum_{i=1}^{D-1} (100(z_i^2 + z_{i+1})^2 + (z_i - 1)^2) + f\_bias,\ z = x - o$ |
| $F_4$ | Shifted Rastrigin's Function | $\sum_{i=1}^{D} (z_i^2 - 10\cos(2\pi z_i) + 10) + f\_bias,\ z = x - o$ |
| $F_5$ | Shifted Griewank's Function | $\sum_{i=1}^{D} \frac{z_i^2}{4000} - \prod_{i=1}^{D} \cos(\frac{z_i}{\sqrt{i}}) + 1 + f\_bias,\ z = x - o$ |
| $F_6$ | Shifted Ackley's Function | $-20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} z_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_i))$ $+20 + e + f\_bias$ |
| $F_7$ | Schwefel's Problem 2.22 | $\sum_{i=1}^{D} \lvert x_i\rvert + \prod_{i=1}^{D} \lvert x_i\rvert$ |
| $F_8$ | Schwefel's Problem 1.2 | $\sum_{i=1}^{D} (\sum_{j=1}^{i} x_j)^2$ |
| $F_9$ | Extended $f_{10}$ | $\left(\sum_{i=1}^{D-1} f_{10}(x_i, x_{i+1})\right) + f_{10}(x_D, x_1)$ $f_{10} = (x^2 + y^2)^{0.25} \cdot \left(\sin^2(50 \cdot (x^2 + y^2)^{0.1}) + 1\right)$ |
| $F_{10}$ | Bohachevsky | $\sum_{i=1}^{D} \left(x_i^2 + 2x_{i+1}^2 - 0.3\cos(3\pi x_i) - 0.4\cos(4\pi x_{i+1}) + 0.7\right)$ |
| $F_{11}$ | Schaffer | $\sum_{i=1}^{D-1} (x_i^2 + x_{i+1}^2)^{0.25} \left(\sin^2(50 \cdot (x_i^2 + x_{i+1}^2)^{0.1}) + 1\right)$ |

Table 1: Functions F1-F11

In Table 3, we can see that there are many non-separable functions that may be easily optimized dimension by dimension. In general, this occurs when the fitness is calculated by the sum operator and:

- Each variable affects to one operator only. Thus, the fitness can be optimized by adjusting each variable. This is the case for functions F1, F4, F6, and F7.
- Each variable affects to two operators of the sum, but it affects to the operators in the same way. Thus, the optimum can be reached making a search dimension by dimension. This is the case of functions F3, F9, and F11.

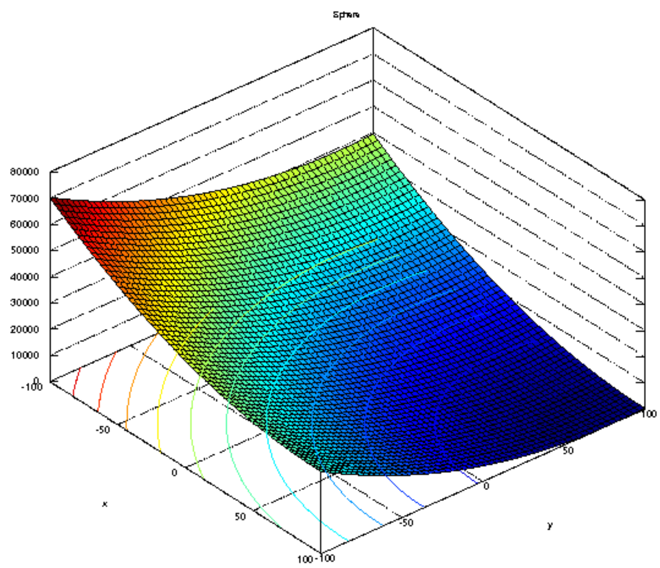
| **Function** | **Name** | **Range** | **Fitness Optimum** |
|---|---|---|---|
| F1 | Shifted Sphere Function | $[-100, 100]^D$ | $-450$ |
| F2 | Shifted Schwefel's Problem 2.21 | $[-100, 100]^D$ | $-450$ |
| F3 | Shifted Rosenbrock's Function | $[-100, 100]^D$ | 390 |
| F4 | Shifted Rastrigin's Function | $[-5, 5]^D$ | $-330$ |
| F5 | Shifted Griewank's Function | $[-600, 600]^D$ | $-180$ |
| F6 | Shifted Ackley's Function | $[-32, 32]^D$ | $-140$ |
| F7 | Schwefel's Problem 2.22 | $[-10, 10]^D$ | 0 |
| F8 | Schwefel's Problem 1.2 | $[-65.536, 65.536]^D$ | 0 |
| F9 | Extended $f_{10}$ | $[-100, 100]^D$ | 0 |
| F10 | Bohachevsky | $[-15, 15]^D$ | 0 |
| F11 | Schaffer | $[-100, 100]^D$ | 0 |

Table 2: Features of F1-F11

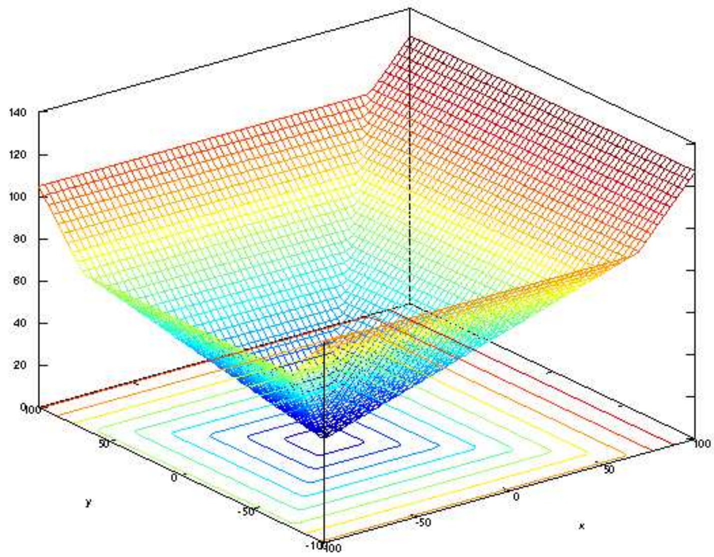
| **Function** | **Unimodal/ Multimodal** | Shifted | **Separable** | **Easily optimized dimension by dimension** |
|---|---|---|---|---|
| F1 | U | Y | Y | Y |
| F2 | U | Y | N | N |
| F3 | M | Y | N | Y |
| F4 | M | Y | Y | Y |
| F5 | M | Y | N | N |
| F6 | M | Y | Y | Y |
| F7 | U | N | Y | Y |
| F8 | U | N | N | N |
| F9 | U | N | N | Y |
| F10 | U | N | N | N |
| F11 | U | N | N | Y |

Table 3: Properties of F1-F11

To sum up, from F1-F11, there are 7 functions that may be easily optimized variable by variable.

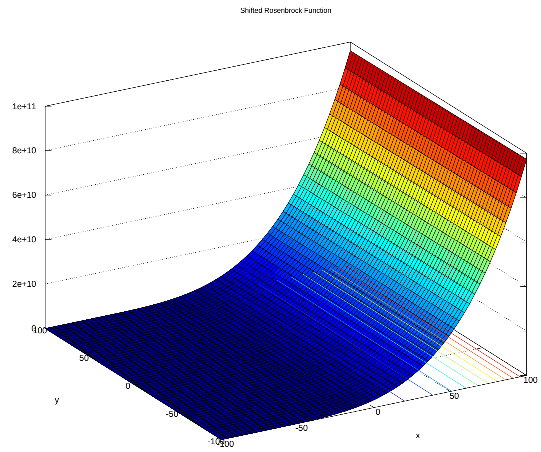



(a) Shifted Sphere Function

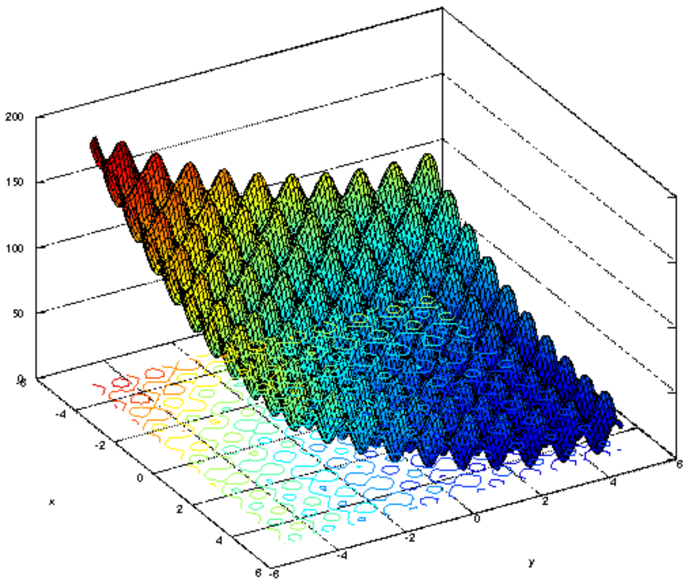



(b) Shifted Schwefel Problem 2.21

Figure 1: Shifted Sphere and Shifted Schwefel

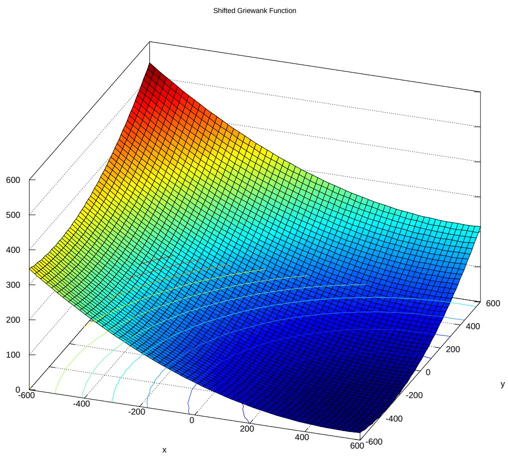



(a) Shifted Rosenbrock Function

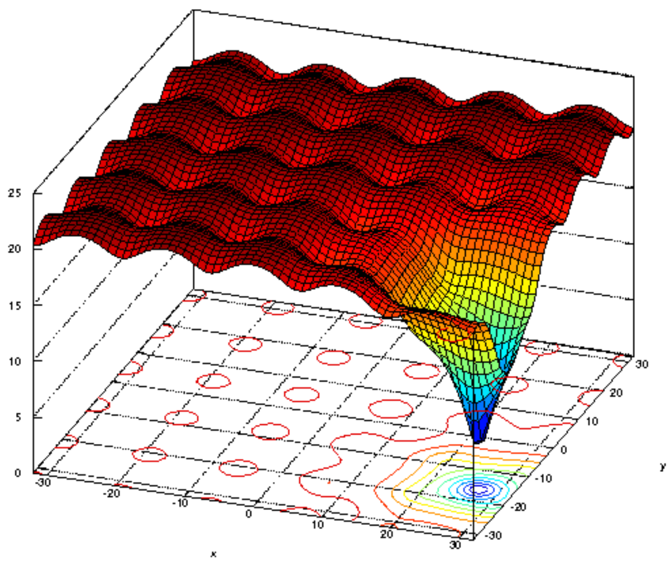



(b) Shifted Rastrigin Function

Figure 2: Shifted Rosenbrock and Shifted Rastrigin

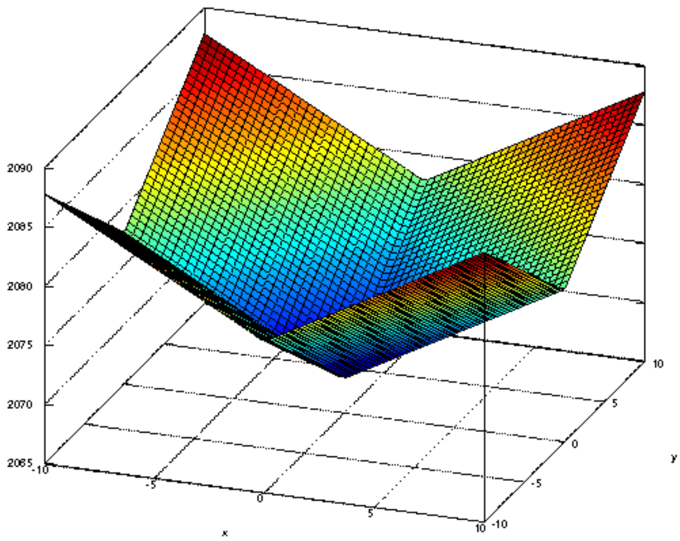



(a) Shifted Griewank Function

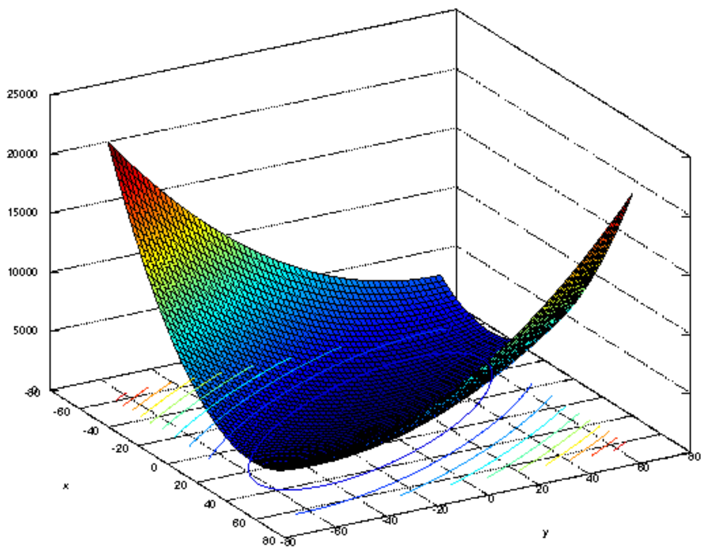



(b) Shifted Ackley Function

Figure 3: Shifted Griewank and Shifted Ackley Figures

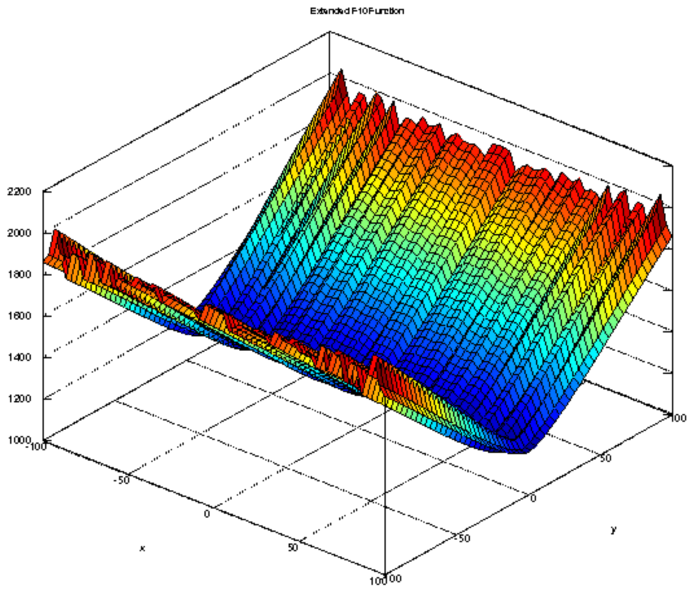



(a) Schwefel Problem 2.22

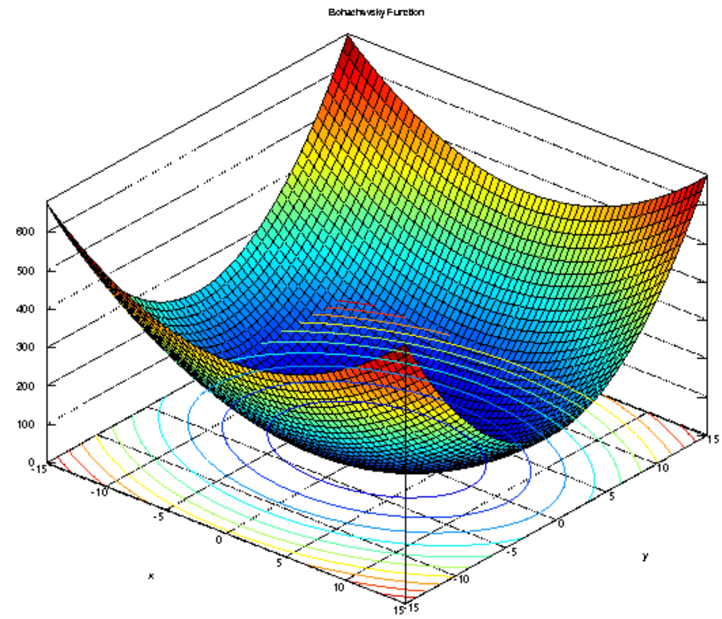



(b) Schwefel Problem 1.2

Figure 4: Schwefel Problems 2.22 and 1.2




(a) Extended Function 10




(b) Bohachevsky Function

Figure 5: Extended Function 10 and Bohachevsky Function

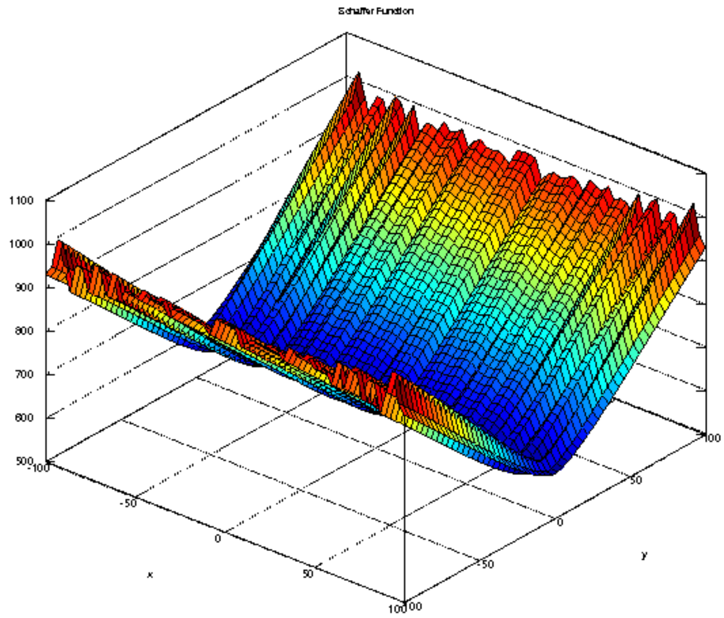



Figure 6: Schaffer Function

## 2 Hybrid Composition Functions (F12-F19*)

The *hybrid composition functions*, F12-F19*, are built combining a non-separable function with other function. The considered functions are:

- **Non-Separable Functions**:
  - F3: Shifted Rosenbrock's Function
  - F5: Shifted Griewank's Function
  - F9: Extended $f10$
  - F10: Bohachevsky
- **Other Component Functions**:
  - F1: Shifted Sphere Function
  - F4: Shifted Rastrigin's Function
  - F7: Schwefel's Problem 2.22

The procedure used to hybridize a non-separable function $F_{ns}$ with other function $F'$ (function $F_{ns} \oplus F'$) is shown in Figure 7. Its main steps are: 1) to divide the solution into two parts, 2) to evaluate each one of them with a different function, and 3) to combine their results. The splitting mechanism uses a parameter, $m_{ns}$, which specifies the ratio of variables that are evaluated by $F_{ns}$. Using a higher value of $m_{ns}$, the hybrid function becomes more difficult to optimize dimension by dimension, because there is a greater interrelation between the variables and the fitness. With this procedure, we have defined the instances of hybrid functions shown in Table 4.

| **Name** | $F_{ns}$ | $F'$ | $m_{ns}$ | **Range** | **Fitness Optimum** |
|---|---|---|---|---|---|
| F12 | F9 | F1 | 0.25 | $[-100, 100]^D$ | 0 |
| F13 | F9 | F3 | 0.25 | $[-100, 100]^D$ | 0 |
| F14 | F9 | F4 | 0.25 | $[-5, 5]^D$ | 0 |
| F15 | F10 | F7 | 0.25 | $[-10, 10]^D$ | 0 |
| F16* | F9 | F1 | 0.5 | $[-100, 100]^D$ | 0 |
| F17* | F9 | F3 | 0.75 | $[-100, 100]^D$ | 0 |
| F18* | F9 | F4 | 0.75 | $[-5, 5]^D$ | 0 |
| F19* | F10 | F7 | 0.75 | $[-10, 10]^D$ | 0 |

Table 4: Hybrid composition functions

**Function** $F_{ns} \oplus F'(S)$

1. $S$ is divided into two parts ($part_1$ and $part_2$):

   - **If** $m_{ns} \leq 0.5$ **then**

     – $part_1$ is composed by the first $D \cdot m_{ns}$ even variables. ($length(part_1) = D \cdot m_{ns}$)

     – $part_2$ is composed by the remaining variables. ($length(part_2) = D - length(part_1)$)

   - **If** $m_{ns} > 0.5$ **then**

     – $part_2$ is composed by the first $D \cdot (1 - m_{ns})$ odd variables. ($length(part_2) = D \cdot (1 - m_{ns})$)

     – $part_1$ is composed by the remaining variables. ($length(part_1) = D - length(part_2)$)

2. **Return** $F_{ns}(part_1) + F'(part_2)$.

Figure 7: Evaluation of a solution $S$ (with $D$ variables) by the hybrid function $F_{ns} \oplus F'$