Real-Parameter Optimization with Differential Evolution

Jani Rönkkönen
Lappeenranta U. of Technology,
Dept. of Information Technology
P.O. Box 20, FIN 53851
ronkkone@lut.fi

Saku Kukkonen
Lappeenranta U. of Technology,
Dept. of Information Technology
P.O. Box 20, FIN 53851
saku.kukkonen@lut.fi

Kenneth V. Price
836 Owl Circle
Vacaville, CA, USA 95687
kvprice@jcis.net

Abstract – This study reports how the Differential Evolution (DE) algorithm performed on the test bed developed for the CEC05 contest for real parameter optimization. The test bed includes 25 scalable functions, many of which are both non-separable and highly multi-modal. Results include DE's performance on the 10 and 30-dimensional versions of each function.

1 Overview

This paper looks at how the Differential Evolution (DE) algorithm performs on the functions developed for the CEC05 contest for real-parameter optimizers (Suganthan 2005). The general problem is to find a set of parameter values, $(x_1, x_2,\ldots, x_D) = \mathbf{x}$, that minimizes a function, $f(\mathbf{x})$, of D real variables, i.e.,

$$\text{Find}: \mathbf{x}^* \middle| f(\mathbf{x}^*) \le f(\mathbf{x}),\ \ \forall \mathbf{x} \in \Re^D .$$

The next section of this paper describes the version of DE used for this competition. The third section discusses how to choose the algorithm's three control parameters. Section 4 briefly discusses the characteristics that make the contest's 25 test functions useful probes of algorithmic performance, while Sect. 5 summarizes the contest's performance criteria. Next, Sect. 6 presents results that show how DE performed on the ten and thirty-dimensional versions of each function. In addition, Sect. 6 provides a measure of DE's algorithmic efficiency and it's scaling characteristics. Section 7 briefly discusses DE's performance on individual functions, while Sect. 8 puts DE's performance into perspective.

2 The Differential Evolution Algorithm

The version of DE used for this contest is known as DE/rand/1/bin, or "classic DE" (Price and Storn 1997; Storn and Price 1997; Price et al. 2005). Classic DE begins by initializing a population of Np, D-dimensional vectors with parameter values that are distributed with random uniformity between pre-specified lower and upper initial parameter bounds, $x_{j,low}$ and $x_{j,high}$, respectively.

$$x_{j,i,g} = x_{j.low} + \text{rand}(0,1)\cdot\left(x_{j,high} - x_{j,low}\right),$$
$$j = (1,2,\ldots,D),\quad i = (1,2,\ldots,Np),\quad g = 0.$$

The subscript, g, is the generation index, while j and i are the parameter and population indices, respectively. Hence, $x_{j,i,g}$ is the j^{th} parameter of the i^{th} population vector in generation g. The random number generator rand(0,1) returns a uniformly distributed value in the range [0, 1). In DE, parameter values are encoded as ordinary floating-point numbers and are manipulated with standard floating-point operators like those available in high level languages like C and FORTRAN.

To generate a trial solution, DE first mutates a vector from the current population by adding to it the scaled difference of two other vectors from the current population:

$$\mathbf{v}_{i,g} = \mathbf{x}_{r1,g} + F\cdot\left(\mathbf{x}_{r2,g} - \mathbf{x}_{r3,g}\right)$$
$$r1,r2,r3 \in \{1,2,\ldots,Np\}.$$

Vector indices $r1$, $r2$ and $r3$ are randomly selected except that all are distinct and different from the population index, i, i.e., $r1 \ne r2 \ne r3 \ne i$. The *mutation scale factor*, F, is a positive real number that is typically less than 1.0.

Next, one or more parameter values of this *mutant vector*, $\mathbf{v}_{i,g}$, are uniformly crossed with those belonging to the i^{th} population vector, $\mathbf{x}_{i,g}$, (a.k.a., the *target vector*). The result is the *trial vector*, $\mathbf{u}_{i,g}$.

$$u_{j,i,g} = \begin{cases} v_{j,i,g} & \text{if rand}(0,1) \le Cr \text{ or } j = j_{\text{rand}}; \\ x_{j,i,g} & \text{otherwise} \end{cases}$$
$$j_{\text{rand}} \in \{1,2,\ldots,D\}.$$

The *crossover constant*, $0.0 \le Cr \le 1.0$ controls the fraction of parameters that the mutant vector contributes to the trial vector. In addition, the trial vector always inherits the mutant vector parameter with the randomly chosen index j_{rand} to ensure that the trial vector differs by at least one parameter from the vector with which it will be compared (i.e., the target vector, $\mathbf{x}_{i,g}$).

If the trial vector's function value is less than or equal to that of the target vector, the trial vector replaces the target vector in the next generation. Otherwise, the target vector remains in the population for at least one more generation.

$$\mathbf{x}_{i,g+1} = \begin{cases} \mathbf{u}_{i,g} & \text{if } f(\mathbf{u}_{i,g}) \le f(\mathbf{x}_{i,g}) \\ \mathbf{x}_{i,g} & \text{otherwise.} \end{cases}$$

To keep solutions feasible when problems are bound-constrained, trial parameters that violate boundary

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constraints are reflected back from the bound by the amount of the violation.

$$u_{j,i,g} = \begin{cases} 2 \cdot x_{j,low} - u_{j,i,g} & \text{if } u_{j,i,g} < x_{j,low} \\ 2 \cdot x_{j,high} - u_{j,i,g} & \text{if } u_{j,i,g} > x_{j,high} \end{cases}$$

Some of the contest functions have added evaluation noise. Our approach did not recalculate the fitness function for each evaluation, so for these noisy functions, each population member received a fixed fitness that includes random noise.

3 Choosing Control Parameters

Classic DE has three control parameters that must be set by the user: *Np*, *F* and *Cr*. The population size, *Np*, typically ranges from $2{\cdot}D$ to $40{\cdot}D$. Separable and uni-modal functions require the smallest population sizes, while parameter-dependent, multi-modal functions require the largest populations. Typically, *Np* must be larger than a critical value to obtain a globally optimal result, but making *Np* too large, while improving convergence probability, may needlessly increase the number of function evaluations.

The scale factor, *F*, is strictly greater than zero. While $F > 1$ can solve many problems, $F \leq 1$ is usually both faster and more reliable. Like *Np*, *F* must be above a certain critical value to avoid premature convergence to a sub-optimal solution, but if *F* becomes too large, the number of function evaluations to find the optimum grows very quickly. Typically, $0.4 < F \leq 0.95$, with $F = 0.9$ being a good compromise between speed and probability of convergence.

Several studies have explored making *F* a random variable. By transforming *F* into a Gaussian random variable, Zaharie (2002) proved that DE will converge to the global optimum in the long time limit, i.e., DE becomes "provably convergent" (Rudolph 1996). Despite this theoretical advantage over classic DE, both Zaharie (2002) and Rönkkönen and Lampinen (2003) found that transforming *F* into a Gaussian distributed random variable did not significantly enhance DE's performance. In addition, Price et al. (2005) demonstrated that unless the variance of the randomizing distribution is very small, DE will suffer a significant performance loss on highly conditioned non-separable functions. Because randomizing *F* fails to significantly enhance DE's performance and because it involves additional assumptions (e.g., the type of distribution and its defining characteristics), only *F* = constant (= 0.9) was considered for this contest.

Since *Cr* is a probability, $0 \leq Cr \leq 1$. When the objective function is separable, a value for *Cr* from the range (0.0, 0.2) is best because then each trial vector frequently competes with a target vector from which it differs by a single parameter. In effect, this "change-one-parameter-at-a-time" strategy optimizes each parameter independently as a series of *D*, randomly interleaved, one-dimensional optimizations (Salomon 1997). While effective on separable functions, using low values of *Cr* when the objective function is multi-modal and non-separable can cause the search to take longer than a simple random search (Salomon 1996). For such functions, all parameters may have to be adjusted simultaneously for the search to remain efficient. In DE, this occurs when $Cr = 1$ and each parameter of an existing vector is modified by the addition of a scaled vector difference. When $Cr = 1$, however, the population may stagnate because the size of the pool of potential trial vectors is limited. At the setting chosen for this contest ($Cr = 0.9$), trial vectors having all new parameter values are common enough that the search remains efficient when a function is both non-separable and multi-modal. Additionally, the occasional uniform crossover with the target vector when $Cr = 0.9$ inflates the size of the pool of potential trial vectors and minimizes the risk of stagnation.

For this contest, control parameters were $F = 0.9$, $Cr = 0.9$, $Np = 30$ for all functions. Only the population size, *Np*, was tuned to yield DE's best performance. A population of only thirty vectors is much too small to produce regular convergence to the global optimum for any but the simplest (e.g. uni-modal) ten-dimensional functions. Ordinarily, population sizes should be between $Np = 200$ and $Np = 600$ to optimize difficult ten and thirty-dimensional functions. For this contest, however, *Np* was chosen smaller than usual to emphasize speed over convergence probability because the maximum allowed number of function evaluations per trial (Max_FES) was typically too brief to allow DE to reach the termination error. In other words, populations large enough to produce regular convergence would converge too slowly for DE to perform well at the specified function error checkpoints.

4 Test Functions

Functions f_1–f_5 are uni-modal while the remaining twenty functions are multi-modal. Seven of these are simple test functions, while two others are expanded functions (Whitley et al. 1996). The remaining eleven functions are hybrid composition functions. Only f_1 and f_9 are separable. Test bed functions can be scaled to arbitrarily high dimension, but this contest uses only their ten and thirty-dimensional versions.

Test functions were designed to test an optimizer's ability to locate a global optimum under a variety of circumstances:

- Function landscape is highly conditioned
- Function landscape is translated
- Function landscape is rotated
- Optimum lies in a narrow basin
- Optimum lies on a bound
- Optimum lies beyond the initial bounds
- Function is not continuous everywhere
- Gaussian noise is added to the function evaluation
- Bias is added to the function evaluation

Test function definitions can be found in Suganthan et al. (2005) and are also available on-line at:

http://www.ntu.edu.sg/home/epnsugan

5 Performance Criteria

The function error, $f(\mathbf{x}) - f(\mathbf{x}^*)$, is recorded at three checkpoints (1.0e3, 1.0e4 and 1.0e5 function evaluations (FES)). The function error is also measured upon termination when either the error is less than Ter_Err (1.0e–8), or the number of function evaluations equals Max_FES (D·1.0e4). Error data is collected for 25 runs after which the trials are ordered from best to worst. The trial mean and standard deviation as well as the results of trials 1, 7, 13, 19 and 25 are tallied for each of the three checkpoints and upon termination.

In addition, trials that reach a preset accuracy level before exceeding Max_FES are deemed successes. Both the fraction of successful trials (*success rate*) and the ratio of the average number of evaluations per success and the success rate (*success performance*) are reported for each function's ten and thirty-dimensional versions. In addition, convergence plots show how the *median* thirty-dimensional function value changes with time.

Finally, the complexity and scaling behavior of DE are estimated by comparing the actual time taken to compute a sum of transcendental functions with the time taken to optimize function f_3 (the high conditioned, rotated elliptic function). Further details about the contest criteria can be found in Suganthan et al. (2005).

6 Results

All results appear on the subsequent 5 pages. Tables 1–3 plot function error versus FES for the ten-dimensional test bed. Table 4 then summarizes DE's success performance on the ten-dimensional test bed. Next, Tables 5–7 report function error values for the thirty-dimensional problems, while Table 8 presents DE's corresponding success performance. Figures 1–5 follow, showing the median trial's function value versus FES. Table 9 summarizes DE's algorithmic efficiency and scaling performance.

7 Discussion of Results

None of the five, ten-dimensional uni-modal functions were a challenge for DE. The shifted sphere (f_1) was solved with regularity, but run times would have been better if Cr had been lowered to exploit this function's separable nature. The shifted Schwefel problem proved to be easy to solve both with (f_2) and without (f_4) the addition of function evaluation noise. In addition, relocating the optimum of this function to the search space boundary did not trick DE. Only in the case of the rotated, high conditioned elliptic function (f_3) was DE unable to successfully optimize all trials in less than Max_FES trials (f_3 success rate = 0.8). A slight adjustment of DE's control variables, or a small increase in the allowed Max_FES would show that DE can easily solve f_3 with regularity.

Once $D = 30$, f_1 is the only uni-modal function that DE can solve within the Max_FES set by the contest. Experience has shown, however, that given a little more time, DE can easily optimize functions f_1–f_5 to within the specified error.

DE also solved f_6 with regularity, failing to reach the optimum before Max_FES only once in 25 trials, but other multi-modal functions were more challenging. Of the other ten-dimensional basic and expanded functions (f_6–f_{14}), DE was able to find solutions not only for f_6 but also for f_7, f_9, f_{11} and f_{12}, albeit not with 100% reliability. With a larger population and higher Max_FES than were used for this contest, all of these functions can be solved with regularity. In addition, experience has shown that DE can also optimize f_{10} given more time, even though none of the trials conducted at $Np = 30$, $F = 0.9$ and $Cr = 0.9$ converged within the Max_FES set by this contest. Except for f_7, the relatively low Max_FES also prevented DE from resolving the optima of the thirty-dimensional versions of these functions quickly enough to qualify any trials as successes. In experiments not reported here, these thirty-dimensional functions have been solved by setting both Cr and F closer to 1.0 (e.g., 0.98) and by increasing both Np and Max_FES. Among the basic and expanded functions, only f_8 proved to be intractable for DE. This function is easy to solve when the optimum is centrally located in the search space. Consequently, DE's failure to locate the optimum would seem to be due either to the location of the optimum at the edge of the search space, or to the additional distance from the optimum at which some points are initialized.

Except for ten-dimensional version of f_{15}, the hybrid composition functions were too hard for DE to solve within the limits imposed by this contest. Without knowing how other contest optimizers performed, it is difficult to assess DE's performance on these newly designed functions based on error measurements at checkpoints. If, however, results for other functions are any indication, increasing both Np and Max_FES for these functions would likely give a better final result than is reported here.

8 Summary

While DE's success performance on hybrid composition functions seems poor, we believe that this is in large part due to both the contest's low Max_FES and its restriction on parameter tuning, especially in the case of Np. Experience has shown that ten-dimensional functions characteristically take anywhere from hundreds of thousands to several millions of functions evaluations to solve when DE is well tuned. In addition, the number of function evaluations that DE takes to optimize parameter-dependent, multi-modal functions tends to scale with D^2, so the linear dependence of the contest Max_FES on dimension places a significant restriction on DE.

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6.1 Results for 10-dimensional Functions

Table 1. Error values for the ten-dimensional functions $f_1 - f_8$, sampled at 1e3, 1e4 and 1e5 FES

FES	Function # :	1	2	3	4	5	6	7	8
1.00E+03	1st (Best)	8,92E+002	1,63E+003	9,76E+006	3,01E+003	3,98E+003	5,12E+006	3,09E+002	2,06E+001
	7th	2,03E+003	3,86E+003	2,35E+007	5,37E+003	6,78E+003	3,23E+007	5,44E+002	2,07E+001
	13th (Median)	2,52E+003	4,98E+003	2,90E+007	5,79E+003	7,42E+003	5,26E+007	6,34E+002	2,08E+001
	19th	2,85E+003	6,95E+003	3,60E+007	8,72E+003	7,94E+003	7,72E+007	6,84E+002	2,08E+001
	25th (Worst)	4,37E+003	7,67E+003	6,86E+007	1,46E+004	8,96E+003	2,49E+008	1,09E+003	2,10E+001
	Mean	2,50E+003	5,14E+003	3,10E+007	6,90E+003	7,21E+003	7,06E+007	6,27E+002	2,08E+001
	Std	8,60E+002	1,70E+003	1,42E+007	2,82E+003	1,19E+003	6,32E+007	1,74E+002	9,75E-002
1.00E+04	1st (Best)	4,89E-001	1,59E+001	1,08E+005	1,19E+001	1,65E+001	3,28E+002	2,25E+000	2,03E+001
	7th	1,03E+000	2,58E+001	1,91E+005	3,94E+001	2,99E+001	9,94E+002	3,47E+000	2,05E+001
	13th (Median)	1,66E+000	3,82E+001	3,08E+005	5,88E+001	3,82E+001	1,37E+003	4,05E+000	2,06E+001
	19th	2,15E+000	4,41E+001	4,36E+005	8,72E+001	6,31E+001	2,24E+003	4,60E+000	2,06E+001
	25th (Worst)	4,94E+000	8,60E+001	8,88E+005	2,33E+002	1,09E+002	6,17E+003	7,30E+000	2,07E+001
	Mean	1,80E+000	3,79E+001	3,45E+005	7,22E+001	4,85E+001	1,80E+003	4,13E+000	2,06E+001
	Std	1,12E+000	1,65E+001	2,04E+005	4,87E+001	2,64E+001	1,32E+003	1,11E+000	8,47E-002
1.00E+05	1st (Best)	0,00E+000	0,00E+000	9,67E-011	0,00E+000	0,00E+000	0,00E+000	7,40E-003	2,03E+001
	7th	0,00E+000	0,00E+000	2,80E-008	0,00E+000	0,00E+000	0,00E+000	8,12E-002	2,04E+001
	13th (Median)	0,00E+000	0,00E+000	1,57E-007	0,00E+000	0,00E+000	0,00E+000	1,08E-001	2,04E+001
	19th	0,00E+000	0,00E+000	8,31E-007	0,00E+000	0,00E+000	1,14E-013	1,60E-001	2,05E+001
	25th (Worst)	0,00E+000	0,00E+000	2,01E-005	1,14E-013	0,00E+000	3,99E+000	6,14E-001	2,05E+001
	Mean	0,00E+000	0,00E+000	1,94E-006	9,09E-015	0,00E+000	1,59E-001	1,46E-001	2,04E+001
	Std	0,00E+000	0,00E+000	4,63E-006	3,15E-014	0,00E+000	7,97E-001	1,38E-001	7,58E-002

Table 2. Error values for the ten-dimensional functions $f_9 - f_{17}$, sampled at 1e3, 1e4 and 1e5 FES

FES	Function # :	9	10	11	12	13	14	15	16	17
1.00E+03	1st (Best)	4,76E+001	6,63E+001	8,99E+000	1,56E+004	3,53E+001	3,98E+000	4,75E+002	2,53E+002	2,63E+002
	7th	6,08E+001	8,04E+001	1,12E+001	3,11E+004	2,40E+002	4,21E+000	6,64E+002	2,92E+002	3,28E+002
	13th (Median)	6,73E+001	8,53E+001	1,21E+001	4,66E+004	6,69E+002	4,31E+000	6,79E+002	3,01E+002	3,65E+002
	19th	7,13E+001	8,90E+001	1,26E+001	5,28E+004	1,55E+003	4,38E+000	6,93E+002	3,22E+002	3,80E+002
	25th (Worst)	8,00E+001	1,02E+002	1,32E+001	6,92E+004	7,64E+003	4,52E+000	7,45E+002	3,67E+002	3,99E+002
	Mean	6,62E+001	8,51E+001	1,18E+001	4,27E+004	1,24E+003	4,30E+000	6,66E+002	3,03E+002	3,54E+002
	Std	8,11E+000	8,75E+000	9,93E-001	1,40E+004	1,65E+003	1,44E-001	6,47E+001	3,01E+001	3,69E+001
1.00E+04	1st (Best)	1,89E+001	3,06E+001	6,48E+000	2,92E+002	3,64E+000	3,58E+000	2,21E+002	1,56E+002	1,65E+002
	7th	2,59E+001	4,08E+001	9,81E+000	1,41E+003	4,74E+000	3,89E+000	3,78E+002	1,85E+002	2,16E+002
	13th (Median)	3,12E+001	4,90E+001	1,03E+001	2,25E+003	5,27E+000	3,98E+000	5,07E+002	2,02E+002	2,38E+002
	19th	3,61E+001	5,23E+001	1,05E+001	4,02E+003	5,59E+000	4,02E+000	5,39E+002	2,12E+002	2,50E+002
	25th (Worst)	5,12E+001	6,31E+001	1,11E+001	6,55E+003	7,40E+000	4,13E+000	5,58E+002	2,26E+002	2,66E+002
	Mean	3,12E+001	4,69E+001	1,00E+001	2,59E+003	5,20E+000	3,95E+000	4,56E+002	1,98E+002	2,31E+002
	Std	7,87E+000	8,85E+000	9,46E-001	1,53E+003	7,94E-001	1,24E-001	1,03E+002	1,82E+001	2,56E+001

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1.00E+05	1st (Best)	0,00E+000	3,98E+000	2,32E-004	0,00E+000	1,39E-001	2,10E+000	0,00E+000	6,14E+001	9,77E+001
	7th	0,00E+000	5,97E+000	1,70E-003	0,00E+000	6,96E-001	3,24E+000	6,84E+001	1,01E+002	1,04E+002
	13th (Median)	9,95E-001	9,95E+000	1,46E-002	2,27E-013	9,63E-001	3,60E+000	4,05E+002	1,16E+002	1,12E+002
	19th	1,99E+000	1,49E+001	1,50E+000	1,44E-011	1,22E+000	3,77E+000	4,18E+002	1,25E+002	1,20E+002
	25th (Worst)	2,98E+000	3,83E+001	5,95E+000	7,12E+002	2,44E+000	3,89E+000	4,34E+002	1,45E+002	1,98E+002
	Mean	9,55E-001	1,25E+001	8,47E-001	3,17E+001	9,77E-001	3,45E+000	2,59E+002	1,13E+002	1,15E+002
	Std	9,73E-001	7,96E+000	1,40E+000	1,42E+002	4,67E-001	4,40E-001	1,83E+002	1,80E+001	2,01E+001

Table 3. Error values for the ten-dimensional functions $f_{18} - f_{25}$, sampled at 1e3, 1e4 and 1e5 FES

FES	Function # :	18	19	20	21	22	23	24	25
1.00E+03	1st (Best)	8,47E+002	1,01E+003	1,02E+003	1,06E+003	8,86E+002	1,03E+003	7,59E+002	9,97E+002
	7th	1,05E+003	1,06E+003	1,07E+003	1,20E+003	9,25E+002	1,27E+003	9,35E+002	1,04E+003
	13th (Median)	1,08E+003	1,10E+003	1,08E+003	1,26E+003	9,66E+002	1,30E+003	9,99E+002	1,08E+003
	19th	1,11E+003	1,11E+003	1,10E+003	1,30E+003	9,76E+002	1,32E+003	1,10E+003	1,13E+003
	25th (Worst)	1,15E+003	1,13E+003	1,17E+003	1,34E+003	1,06E+003	1,36E+003	1,21E+003	1,48E+003
	Mean	1,07E+003	1,08E+003	1,09E+003	1,24E+003	9,56E+002	1,27E+003	9,99E+002	1,10E+003
	Std	6,13E+001	3,53E+001	3,58E+001	7,57E+001	4,37E+001	1,01E+002	1,13E+002	9,81E+001
1.00E+04	1st (Best)	3,25E+002	3,18E+002	3,29E+002	5,01E+002	4,50E+002	5,59E+002	2,00E+002	9,25E+002
	7th	3,97E+002	3,86E+002	3,97E+002	5,02E+002	7,86E+002	5,59E+002	2,01E+002	9,28E+002
	13th (Median)	4,26E+002	4,28E+002	4,48E+002	5,03E+002	7,91E+002	5,60E+002	2,02E+002	9,29E+002
	19th	5,14E+002	4,92E+002	8,01E+002	5,04E+002	7,95E+002	5,60E+002	2,02E+002	9,30E+002
	25th (Worst)	8,07E+002	8,03E+002	8,04E+002	5,18E+002	8,06E+002	1,01E+003	2,04E+002	9,34E+002
	Mean	4,98E+002	5,00E+002	5,43E+002	5,04E+002	7,77E+002	5,90E+002	2,02E+002	9,29E+002
	Std	1,64E+002	1,78E+002	1,86E+002	4,63E+000	6,85E+001	1,07E+002	1,07E+000	1,94E+000
1.00E+05	1st (Best)	3,00E+002	3,00E+002	3,00E+002	3,00E+002	3,00E+002	5,59E+002	2,00E+002	9,22E+002
	7th	3,00E+002	3,00E+002	3,00E+002	5,00E+002	7,68E+002	5,59E+002	2,00E+002	9,23E+002
	13th (Median)	3,00E+002	3,00E+002	3,00E+002	5,00E+002	7,75E+002	5,59E+002	2,00E+002	9,23E+002
	19th	3,00E+002	3,00E+002	8,00E+002	5,00E+002	7,80E+002	5,59E+002	2,00E+002	9,23E+002
	25th (Worst)	8,00E+002	8,00E+002	8,00E+002	5,00E+002	7,89E+002	7,21E+002	2,00E+002	9,23E+002
	Mean	4,00E+002	4,20E+002	4,60E+002	4,92E+002	7,18E+002	5,72E+002	2,00E+002	9,23E+002
	Std	2,04E+002	2,18E+002	2,38E+002	4,00E+001	1,58E+002	4,48E+001	0,00E+000	3,40E-001

Table 4. Ten-dimensional functions: the FES to reach the specified level of accuracy and the success performance

Func.	1st (Best)	7th	13th (Median)	19th	25th (Worst)	Mean	Std.	Success rate	Success Performance
1	2,67E+004	2,84E+004	2,96E+004	3,03E+004	3,30E+004	2,94E+004	1,67E+003	1	29410
2	4,03E+004	4,47E+004	4,60E+004	4,84E+004	5,03E+004	4,63E+004	2,48E+003	1	46308,8
3	7,73E+004	8,72E+004	9,16E+004	9,74E+004	1,00E+005	9,20E+004	7,01E+003	0,8	115018
4	4,74E+004	4,99E+004	5,21E+004	5,48E+004	5,78E+004	5,24E+004	3,25E+003	1	52372
5	3,71E+004	3,95E+004	4,09E+004	4,19E+004	4,36E+004	4,07E+004	1,67E+003	1	40746
6	3,87E+004	4,19E+004	4,31E+004	4,47E+004	1,00E+005	4,55E+004	1,16E+004	0,96	47398,3
7	4,39E+004	1,00E+005	1,00E+005	1,00E+005	1,00E+005	9,62E+004	1,33E+004	0,08	1,20E+006
8	-	-	-	-	-	-	-	0	-
9	3,36E+004	4,90E+004	1,00E+005	1,00E+005	1,00E+005	7,78E+004	2,69E+004	0,44	176805
10	-	-	-	-	-	-	-	0	-
11	6,66E+004	8,20E+004	1,00E+005	1,00E+005	1,00E+005	9,05E+004	1,21E+004	0,48	188522
12	3,39E+004	3,90E+004	4,13E+004	5,00E+004	1,00E+005	5,46E+004	2,62E+004	0,76	71903,7
13	-	-	-	-	-	-	-	0	-
14	-	-	-	-	-	-	-	0	-
15	5,88E+004	1,00E+005	1,00E+005	1,00E+005	1,00E+005	9,84E+004	8,24E+003	0,04	2,46E+006
16–25	-	-	-	-	-	-	-	0	-

6.2 Results for Thirty-dimensional Functions

Table 5. Error values for thirty-dimensional problems $f_1 - f_8$, measured at 1.0e3, 1.0e4, 1.0e5 and 3.0e5 FES

FES	Function # :	1	2	3	4	5	6	7	8
1.00E+03	1st (Best)	3,24E+004	6,07E+004	3,76E+008	5,82E+004	2,50E+004	1,06E+010	4,91E+003	2,11E+001
	7th	4,03E+004	8,12E+004	7,46E+008	8,53E+004	2,83E+004	1,60E+010	6,22E+003	2,12E+001
	13th (Median)	4,45E+004	8,52E+004	8,31E+008	1,01E+005	3,04E+004	1,87E+010	6,51E+003	2,12E+001
	19th	5,08E+004	9,61E+004	9,27E+008	1,10E+005	3,13E+004	2,70E+010	7,70E+003	2,13E+001
	25th (Worst)	5,66E+004	1,02E+005	1,10E+009	1,28E+005	3,29E+004	3,33E+010	8,66E+003	2,13E+001
	Mean	4,53E+004	8,66E+004	8,18E+008	9,90E+004	2,98E+004	2,09E+010	6,83E+003	2,12E+001
	Std	6,99E+003	1,12E+004	1,90E+008	1,83E+004	2,30E+003	6,85E+009	1,04E+003	6,45E-002

continued on next page

1.00E+04	1st (Best)	3,31E+003	2,74E+004	1,11E+008	3,62E+004	1,12E+004	1,73E+008	8,53E+002	2,10E+001
	7th	4,68E+003	3,90E+004	1,96E+008	4,78E+004	1,21E+004	3,25E+008	1,49E+003	2,11E+001
	13th (Median)	5,68E+003	4,09E+004	2,37E+008	5,22E+004	1,38E+004	4,32E+008	1,80E+003	2,11E+001
	19th	6,56E+003	4,42E+004	2,53E+008	5,67E+004	1,45E+004	5,10E+008	2,35E+003	2,11E+001
	25th (Worst)	9,66E+003	6,10E+004	3,27E+008	6,59E+004	1,75E+004	8,30E+008	3,11E+003	2,12E+001
	Mean	5,74E+003	4,19E+004	2,29E+008	5,27E+004	1,36E+004	4,37E+008	1,91E+003	2,11E+001
	Std	1,66E+003	8,15E+003	5,10E+007	7,44E+003	1,82E+003	1,58E+008	6,30E+002	5,21E-002
1.00E+05	1st (Best)	1,27E-004	7,29E+001	1,61E+006	7,45E+002	8,41E+002	2,91E+001	3,28E-001	2,09E+001
	7th	4,91E-004	3,04E+002	3,71E+006	1,27E+003	1,28E+003	5,86E+001	7,16E-001	2,10E+001
	13th (Median)	9,59E-004	4,00E+002	4,94E+006	1,66E+003	1,65E+003	1,16E+002	9,59E-001	2,10E+001
	19th	1,99E-003	5,08E+002	6,07E+006	2,76E+003	2,22E+003	1,64E+002	1,01E+000	2,10E+001
	25th (Worst)	7,59E-003	9,15E+002	1,01E+007	5,00E+003	2,67E+003	5,82E+002	1,05E+000	2,11E+001
	Mean	1,41E-003	4,13E+002	5,23E+006	2,10E+003	1,72E+003	1,42E+002	8,45E-001	2,10E+001
	Std	1,57E-003	1,95E+002	2,23E+006	1,17E+003	5,37E+002	1,25E+002	2,30E-001	4,94E-002
3.00E+05	1st (Best)	0,00E+000	9,84E-004	2,85E+005	8,87E-001	2,08E+001	4,47E-001	3,54E-010	2,08E+001
	7th	0,00E+000	7,98E-003	4,91E+005	2,99E+000	8,13E+001	9,96E+000	2,60E-009	2,09E+001
	13th (Median)	0,00E+000	2,22E-002	7,29E+005	6,68E+000	9,35E+001	1,29E+001	1,20E-008	2,10E+001
	19th	0,00E+000	3,00E-002	8,46E+005	1,87E+001	1,77E+002	1,83E+001	6,69E-008	2,10E+001
	25th (Worst)	0,00E+000	2,42E-001	9,48E+005	7,25E+001	8,06E+002	1,06E+002	1,72E-002	2,10E+001
	Mean	0,00E+000	3,33E-002	6,92E+005	1,52E+001	1,70E+002	2,51E+001	2,96E-003	2,10E+001
	Std	0,00E+000	4,90E-002	2,04E+005	1,81E+001	1,84E+002	2,90E+001	5,55E-003	5,11E-002

Table 6. Error values for thirty-dimensional problems f_9 – f_{17}, measured at 1.0e3, 1.0e4, 1.0e5 and 3.0e5 FES

FES	Function # :	9	10	11	12	13	14	15	16	17
1.00E+03	1st (Best)	3,29E+002	4,69E+002	4,31E+001	9,43E+005	4,05E+004	1,37E+001	7,60E+002	5,25E+002	5,57E+002
	7th	3,89E+002	5,43E+002	4,47E+001	1,33E+006	1,93E+005	1,41E+001	8,62E+002	6,04E+002	6,73E+002
	13th (Median)	4,00E+002	5,54E+002	4,61E+001	1,53E+006	2,70E+005	1,42E+001	9,53E+002	6,67E+002	7,49E+002
	19th	4,05E+002	5,92E+002	4,68E+001	1,63E+006	3,35E+005	1,43E+001	9,80E+002	7,39E+002	7,90E+002
	25th (Worst)	4,37E+002	6,75E+002	4,80E+001	1,91E+006	6,34E+005	1,45E+001	1,09E+003	8,40E+002	8,97E+002
	Mean	3,94E+002	5,66E+002	4,58E+001	1,49E+006	2,92E+005	1,42E+001	9,28E+002	6,65E+002	7,38E+002
	Std	2,39E+001	5,06E+001	1,34E+000	2,45E+005	1,47E+005	1,59E-001	9,46E+001	8,46E+001	8,60E+001
1.00E+04	1st (Best)	2,16E+002	2,47E+002	4,01E+001	4,83E+005	6,21E+002	1,34E+001	5,13E+002	3,13E+002	2,85E+002
	7th	2,45E+002	2,88E+002	4,25E+001	7,09E+005	1,60E+003	1,38E+001	5,24E+002	3,40E+002	3,74E+002
	13th (Median)	2,53E+002	3,12E+002	4,30E+001	8,03E+005	3,94E+003	1,39E+001	5,35E+002	3,51E+002	3,96E+002
	19th	2,61E+002	3,41E+002	4,34E+001	8,81E+005	7,35E+003	1,40E+001	6,39E+002	3,75E+002	4,30E+002
	25th (Worst)	2,81E+002	3,65E+002	4,44E+001	1,07E+006	2,07E+004	1,41E+001	7,27E+002	4,86E+002	6,41E+002
	Mean	2,52E+002	3,14E+002	4,29E+001	8,06E+005	5,62E+003	1,39E+001	5,84E+002	3,67E+002	4,12E+002
	Std	1,65E+001	3,06E+001	9,66E-001	1,37E+005	5,47E+003	1,91E-001	7,06E+001	4,46E+001	7,58E+001
1.00E+05	1st (Best)	1,52E+001	1,82E+002	3,58E+001	2,09E+003	4,22E+000	1,31E+001	2,02E+002	2,29E+002	2,29E+002
	7th	2,07E+001	2,16E+002	4,02E+001	4,09E+003	9,52E+000	1,34E+001	2,06E+002	2,57E+002	2,66E+002
	13th (Median)	2,86E+001	2,20E+002	4,05E+001	5,40E+003	1,13E+001	1,35E+001	4,00E+002	2,67E+002	2,80E+002
	19th	4,72E+001	2,39E+002	4,09E+001	8,04E+003	1,50E+001	1,37E+001	4,00E+002	2,80E+002	3,00E+002
	25th (Worst)	8,04E+001	2,62E+002	4,22E+001	1,76E+004	1,94E+001	1,38E+001	5,03E+002	4,00E+002	5,73E+002
	Mean	3,36E+001	2,27E+002	4,05E+001	6,83E+003	1,20E+001	1,35E+001	3,61E+002	2,84E+002	3,06E+002
	Std	1,58E+001	1,95E+001	1,25E+000	4,27E+003	4,54E+000	1,48E-001	1,07E+002	4,73E+001	7,87E+001
3.00E+05	1st (Best)	9,95E+000	2,15E+001	7,39E+000	9,56E+000	1,83E+000	1,30E+001	2,00E+002	4,77E+001	5,14E+001
	7th	1,49E+001	2,98E+001	3,76E+001	4,02E+002	2,63E+000	1,34E+001	2,01E+002	7,27E+001	1,16E+002
	13th (Median)	1,81E+001	4,88E+001	3,95E+001	1,78E+003	3,18E+000	1,34E+001	4,00E+002	2,28E+002	2,53E+002
	19th	2,09E+001	2,07E+002	4,00E+001	3,79E+003	3,68E+000	1,35E+001	4,00E+002	2,58E+002	2,65E+002
	25th (Worst)	3,28E+001	2,19E+002	4,13E+001	1,13E+004	4,97E+000	1,36E+001	5,03E+002	4,00E+002	5,47E+002
	Mean	1,85E+001	9,69E+001	3,42E+001	2,75E+003	3,23E+000	1,34E+001	3,60E+002	2,12E+002	2,37E+002
	Std	5,20E+000	8,23E+001	1,03E+001	3,22E+003	8,23E-001	1,41E-001	1,08E+002	1,10E+002	1,22E+002

Table 7. Error values for thirty-dimensional problems f_{18} – f_{25}, measured at 1.0e3, 1.0e4, 1.0e5 and 3.0e5 FES

FES	Function # :	18	19	20	21	22	23	24	25
1.00E+03	1st (Best)	1,14E+003	1,15E+003	1,12E+003	1,30E+003	1,29E+003	1,30E+003	1,31E+003	1,51E+003
	7th	1,19E+003	1,17E+003	1,16E+003	1,32E+003	1,38E+003	1,32E+003	1,35E+003	1,98E+003
	13th (Median)	1,20E+003	1,20E+003	1,21E+003	1,35E+003	1,43E+003	1,35E+003	1,38E+003	2,03E+003
	19th	1,23E+003	1,24E+003	1,23E+003	1,39E+003	1,51E+003	1,40E+003	1,40E+003	2,07E+003
	25th (Worst)	1,28E+003	1,30E+003	1,26E+003	1,41E+003	1,64E+003	1,43E+003	1,46E+003	2,13E+003
	Mean	1,21E+003	1,21E+003	1,20E+003	1,36E+003	1,43E+003	1,35E+003	1,38E+003	2,00E+003
	Std	3,75E+001	4,54E+001	4,26E+001	3,71E+001	8,90E+001	3,83E+001	4,17E+001	1,31E+002
1.00E+04	1st (Best)	9,51E+002	9,44E+002	9,36E+002	8,99E+002	1,05E+003	8,67E+002	7,89E+002	8,34E+002
	7th	9,64E+002	9,57E+002	9,59E+002	1,01E+003	1,06E+003	9,78E+002	8,91E+002	8,83E+002
	13th (Median)	9,77E+002	9,65E+002	9,68E+002	1,04E+003	1,10E+003	1,04E+003	9,45E+002	9,26E+002
	19th	9,86E+002	9,75E+002	9,83E+002	1,08E+003	1,12E+003	1,07E+003	9,80E+002	9,54E+002
	25th (Worst)	1,02E+003	9,97E+002	9,99E+002	1,13E+003	1,21E+003	1,11E+003	1,04E+003	1,06E+003
	Mean	9,78E+002	9,66E+002	9,69E+002	1,04E+003	1,10E+003	1,03E+003	9,35E+002	9,22E+002
	Std	2,02E+001	1,26E+001	1,67E+001	5,23E+001	4,39E+001	6,75E+001	6,45E+001	5,49E+001

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1.00E+05	1st (Best)	9,04E+002	9,04E+002	9,04E+002	5,00E+002	8,97E+002	5,34E+002	2,00E+002	7,30E+002
	7th	9,06E+002	9,06E+002	9,06E+002	5,00E+002	9,14E+002	5,34E+002	2,00E+002	7,32E+002
	13th (Median)	9,07E+002	9,07E+002	9,07E+002	5,00E+002	9,20E+002	5,34E+002	2,00E+002	7,33E+002
	19th	9,07E+002	9,07E+002	9,07E+002	5,00E+002	9,30E+002	5,34E+002	2,00E+002	7,34E+002
	25th (Worst)	9,08E+002	9,10E+002	9,08E+002	5,00E+002	9,48E+002	5,34E+002	2,00E+002	7,37E+002
	Mean	9,06E+002	9,06E+002	9,06E+002	5,00E+002	9,21E+002	5,34E+002	2,00E+002	7,33E+002
	Std	9,58E-001	1,25E+000	1,01E+000	5,02E-004	1,38E+001	4,22E-004	6,65E-004	1,56E+000
3.00E+05	1st (Best)	9,03E+002	9,03E+002	9,03E+002	5,00E+002	8,74E+002	5,34E+002	2,00E+002	7,29E+002
	7th	9,04E+002	9,04E+002	9,04E+002	5,00E+002	8,88E+002	5,34E+002	2,00E+002	7,29E+002
	13th (Median)	9,04E+002	9,04E+002	9,04E+002	5,00E+002	8,97E+002	5,34E+002	2,00E+002	7,30E+002
	19th	9,04E+002	9,04E+002	9,04E+002	5,00E+002	9,08E+002	5,34E+002	2,00E+002	7,30E+002
	25th (Worst)	9,05E+002	9,06E+002	9,07E+002	5,00E+002	9,24E+002	5,34E+002	2,00E+002	7,30E+002
	Mean	9,04E+002	9,04E+002	9,04E+002	5,00E+002	8,97E+002	5,34E+002	2,00E+002	7,30E+002
	Std	3,13E-001	6,25E-001	6,15E-001	1,66E-013	1,33E+001	4,26E-004	1,35E-012	3,74E-001

Table 8. Thirty-dimensional functions: the FES to reach the specified level of accuracy and the success performance

Func.	1st (Best)	7th	13th (Median)	19th	25th (Worst)	Mean	Std	Success rate	Success Performance
1	1,28E+005	1,33E+005	1,40E+005	1,43E+005	1,51E+005	1,39E+005	6,08E+003	1	138549
2	-	-	-	-	-	-	-	0	-
3	-	-	-	-	-	-	-	0	-
4	-	-	-	-	-	-	-	0	-
5	-	-	-	-	-	-	-	0	-
6	-	-	-	-	-	-	-	0	-
7	1,33E+005	1,50E+005	1,56E+005	1,60E+005	3,00E+005	1,76E+005	5,07E+004	0,88	199521
8	-	-	-	-	-	-	-	0	-
9	-	-	-	-	-	-	-	0	-
10–25	-	-	-	-	-	-	-	0	-

6.3 Convergence Graphs for Thirty-dimensional Functions

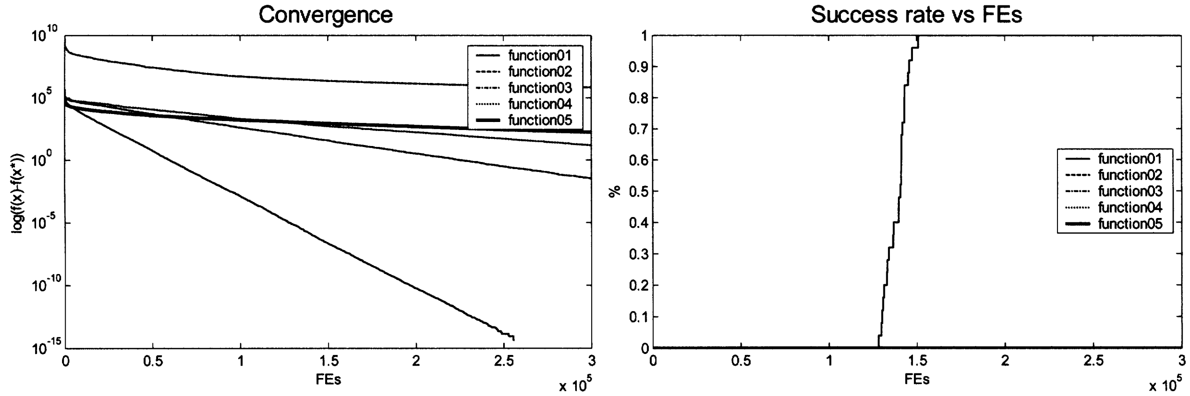

Fig. 1. Error and success rate versus FES for functions $f_1 - f_5$

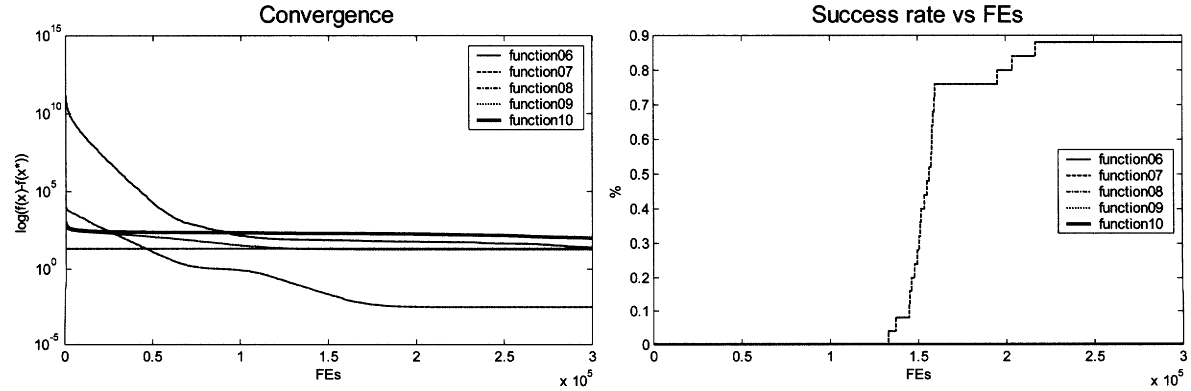

Fig. 2. Error and success rate versus FES for functions $f_6 - f_{10}$

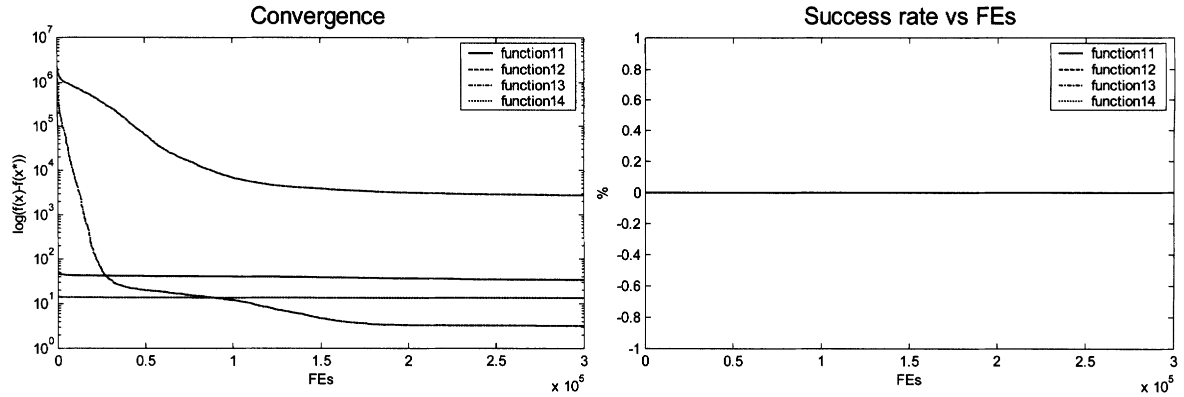

Fig. 3. Error and success rate versus FES for functions $f_{11} - f_{14}$

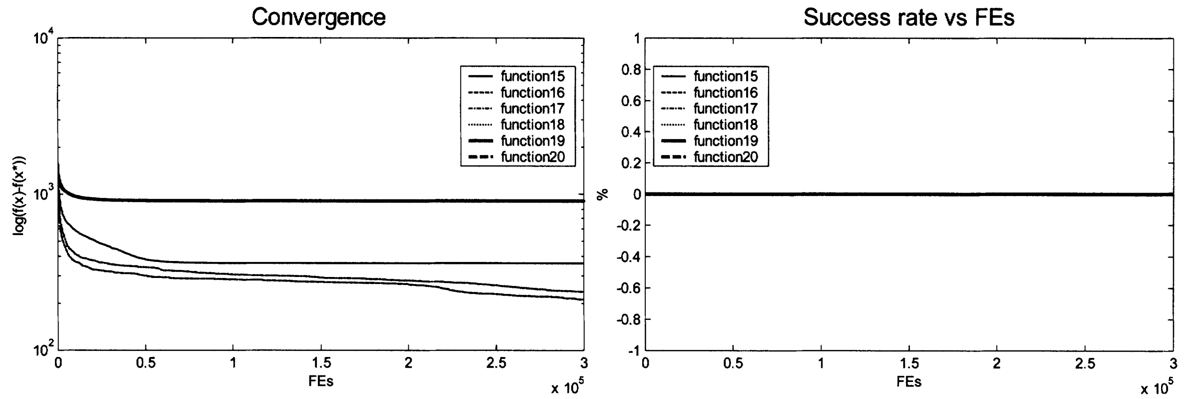

Fig. 4. Error and success rate versus FES for functions $f_{15} - f_{20}$

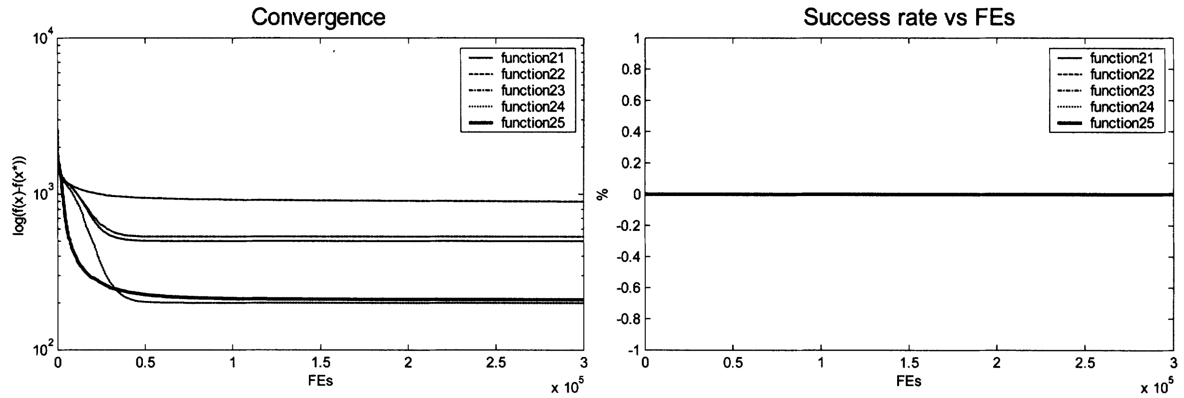

Fig. 5. Error and success rate versus FES for functions $f_{21} - f_{25}$

6.4 Algorithmic complexity

The system used for the computations presented in this paper consists of an AMD Sempron 2800+ CPU with 1 GB of RAM, running Mandrake Linux 10.1. Functions were written in C and compiled using gcc with optimization -O3.

Table 9. Run times in seconds for contest measures T0, T1, T2 and the mean of T2 (i.e., ⟨T2⟩)

T0=0.29	T1	T2					⟨T2⟩	(⟨T2⟩-T1)/T0
D=10	1.2	1.51	1.51	1.48	1.51	1.5	1.502	1.041379
D=30	7.64	8.48	8.53	8.43	8.49	8.53	8.492	2.937931
D=50	19.35	20.99	20.95	21.04	20.97	21.14	21.018	5.751724