Hybrid Real-Coded Genetic Algorithms with Female and Male Differentiation

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Abstract- ***Parent-centric real-parameter crossover operators* create the offspring in the neighborhood of one of the parents, *the female parent,* using a probability distribution. The other parent, *the male one*, defines the range of this probability distribution. The *female and male differentiation* process determines the individuals in the population that may become female or/and male parents. An important property of this process is that it makes possible the design of two kinds of *real-coded genetic algorithms*: ones that promote global search and ones that are effective local searchers. In this paper, we study the performance of a hybridization of these real-coded genetic algorithms when tackling the test problems proposed for the *Special Session on Real-Parameter Optimization of the IEEE Congress on Evolutionary Computation 2005.***

1 Introduction

The crossover operator has always been regarded as one of the main search operator in GAs, because it exploits the available information in previous samples to influence future searches. This is why most real-coded genetic algorithm (RCGA) research has been focused on developing effective real-parameter crossover operators, and as a result, many different possibilities have been proposed ([Deb01, Her98, Her03]). *Parent-centric crossover operators* (PCCOs) is a family of real-parameter crossover operators that has currently received special attention. In general, these operators use a probability distribution for creating offspring in a restricted search space around the region marked by one of the parent, the *female parent*. The range of this probability distribution depends on the distance between the female parent and the other parent involved in the crossover, the *male parent*.

So far, PCCO practitioners have assumed that every chromosome in the population may become either a female parent or a male parent. However, it is very important to emphasize that female and male parents have two differentiated roles:

- Female parents *point* at the search areas that will receive sampling points, whereas,
- Male parents are used to determine the *extent* of these areas.

At this point, it is reasonable to think that some chromosomes may be well-suited to act either as female parents or as male parents. Thus, a promising way to improve the behavior of PCCOs involves the introduction of a *female and male differentiation* (FMD) process for the application of these operators. A such process was proposed in [Gar05]:

- The population of the RCGA contains two different groups: 1) G_F with N_F chromosomes that can be female parents, and 2) G_M with N_M male parents (N_F and N_M are tunable parameters).
- The RCGA uses a specific selection mechanism in order to select the female parents from G_F.
- A different selection mechanism is performed to choose the male parents from G_M.

In [Gar05], it is indicated that adjusting N_F and N_M we may design *local* RCGAs, which offer *accuracy*, and *global* RCGAs, which provide *reliability*. Furthermore, in order to obtain robust behavior, in [Gar05], the authors combined a global RCGA and a local RCGA, producing a *hybrid* RCGA.

In this paper, this hybrid RCGA is tested on the test suite proposed for the *Real-Parameter Optimization Session of the IEEE Congress on Evolutionary Computation* ([Sug05]) (using the *Java* version provided to all participants).

We set up the paper as follows. In Section 2, we describe the pseudo-code of the hybrid RCGA. In Section 3, we presents the results obtained by this algorithm on the test suite when *Dimension=10*. The results with *Dimension=30* appear in Section 4. In Section 5, we study the computational costs of the algorithm, and, in Section 6, we list its associated parameters. In Section 7, we analyze the results obtained. Finally, we draw some conclusions in Section 8.

2 The Hybrid RCGA with the FMD Process

This section is aimed to introduce the hybrid RCGA. It consists on the hybridization of a global RCGA and a local RGGA. They are *steady-state* RCGAs based on a FMD process and that apply the replace worst strategy. In addition, the global RCGA uses the PBX-α crossover operator ([Loz04]) and the local RCGA uses the PCX crossover operator ([Deb02]).

In section 2.1, we describe the two PCCOs used by the algorithm. In section 2.2, the FMD process is introduced. Global and local RCGAs are presented in section 2.3.

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Finally, the final algorithm will be described in section 2.4.

2.1 Parent-Centric Crossover Operators

1) *PBX-α.* Let assume that $X = (x_1 \cdots x_n)$ and $Y = (y_1 \cdots y_n)$ with $x_i, y_i \in [a_i, b_i] \subset \Re, i = 1 \ldots n$ are two chromosomes that have been selected to apply the crossover operator to them. X is the *female parent*, whereas Y is the *male parent*. Then, PBX generates the offspring $Z = (z_1 \cdots z_n)$ where z_i is a randomly (uniformly) chosen number from the interval $[l_i, u_i]$, where $l_i = \max\{a_i, x_i - I \cdot \alpha\}$, $u_i = \min\{b_i, x_i + I \cdot \alpha\}$, $I = |x_i - y_i|$, and α is a parameter associated with this operator.
2) *PCX.* It is a *multiparent* crossover operator because it uses $\mu > 2$ chromosomes for generating the offspring. Initially, it computes the centroid G of the chosen μ parents. Then, one parent, X_F, is chosen with equal probability as *female parent*, and the direction vector $D = X_F - G$ is calculated. Thereafter, from each of the other (μ – 1) parents, the *male ones*, perpendicular distances d_i to the line D are computed and their average $\bar{d}$ is found. Finally, the offspring is created as follows:

$$Z = X_F + w_\varsigma |D| + \sum_{\substack{i=1 \\ i \neq F}}^{\mu} w_\eta \bar{d} E_i$$

where E_i are the (μ – 1) orthonormal bases that span the subspace perpendicular to D. The parameters w_ς and w_η are zero-mean normally distributed variables with variance σ_ς^2 and σ_η^2, respectively.

2.2 The Female and Male Differentiation Process

In this section, we describe the FMD process proposed in [Gar05]. It requires two parameters, N_F and N_M, with $N_F \leq N$ and $N_M \leq N$ (N is the population size) and maintains two groups of chromosomes:

- G_F consists of the N_F best individuals in the population, and
- G_M is made of the N_M best individuals in the population.

Thus, it should be ensured that either $N_F = N$ or $N_M = N$ is fulfilled. Next, we provide two remarks derived form this definition:

- In the case $N_F = N_M$, then, $G_F = G_M$, and,
- $G_F \cap G_M \neq \phi$.

Another important feature of this FMD process is that it uses the *uniform fertility selection* (UFS) to select the female parent from G_F, and a variation of the *negative assortative mating* (NAM) ([Fer01, Mat99]) to choose the male parent from G_M:

- UFS attempts to assign a fair number of offspring to the chromosomes that visit the population, with the aim of providing a widespread search. In order to do this, it selects, as female parent, the individual in G_F with the lowest number of offspring generated (see pseudo-code in Figure 1).

1) p_f ← Find the chromosome with less offsprings in G_F.
2) Increment the number of offsprings of p_f.
3) Return p_f.

Figure 1. Pseudo-code for UFS.

- NAM selects as male parent the most dissimilar chromosome to the female parent from a set of n_{ass} randomly chosen chromosomes (see pseudo-code in Figure 2). We have used $n_{ass} = 5$.

1) L ← Set of n_{ass} randomly chosen chromosomes from G_M.
2) p_M ← Find the chromosome in L that is the most dissimilar from the female parent p_f.
3) Return p_M.

Figure 2. Pseudo-code for NAM.

Figure 3 presents a steady-state RCGA model that performs the FMD process. It uses the *replace worst* (RW) replacement strategy, which replaces the worst individual in the population only if the new individual is better.

1) Initialize P with N randomly chromosomes.
2) Repeat until the stopCriterion is fulfilled.
   a. Select a female parent according to UFS from G_F.
   b. Select (μ - 1) male parents according to NAM from G_M.
   c. Apply the crossover operator to the parents in order to produce λ offspring.
   d. Insert the offspring in P by using the RW strategy.

Figure 3. RCGA model based on the FMD process

2.3 Global RCGAs and Local RCGAs

An important conclusion obtained from [Gar05] is that the FMD process allows us to design two different kinds of specialized RCGAs:

- *Local* RCGAs that reach *accurate* solutions when they deal with unimodal problems. They use low N_F values.
- *Global* RCGAs that offer *reliable* solutions when they attempt to solve multimodal and complex problems. They use high N_F values.

2.4 Combining Global RCGAs and Local RCGAs

Global and local RCGAs may be hybridized in order to achieve a *robust* operation for problems with different characteristics. The hybridization method follows the same ideas exposed in [Che03]. Firstly, we run the global RCGA during the P_G% of the available evaluations, and then, we perform the local RCGA. The best individuals in the final population of the global RCGA become the elements of the initial population of the local RCGA.

We have implemented an instance of the hybrid RCGA, which is called *GL-50*. It considers P_G = 50%. The features of its components are:

- The global RCGA uses the PBX-α crossover operator with α = 0.8, μ=2, and λ =1. In addition, N_F = 100 and N_M = 400.
- The local RCGA apply the PCX crossover operator with σ_ς^2=0.1, σ_η^2 = 0.1, μ = 3, and λ = 2. In addition, N_F = 1 and N_M= 200.

3 Results with Dimension = 10

In this section, we present the empirical results obtained by the GL-50 algorithm when tackling the 25 problems of [Sug05] with Dimension D = 10. The maximum number of fitness evaluations (FEs) for this Dimension is 1e5.

3.1 Achieved Error Values

The error values, $(f(x)-f(x^*))$ with x^* the optimum, are presented in Tables 1-3. Each column corresponds to a problem [Sug05], and the error value has been recorded after 1e3, 1e4, 1e5 FEs for each one of the 25 runs. The values of the 25 runs have been sorted and the tables present the following ones: 1st (Best), 7th, 13th (Median), 19th, 25th (Worst), Mean and Standard Deviation (Std).

Sometimes, the cells present the character 'T' after the fitness value. That indicates that the algorithm stopped the run before reaching the maximum number of FEs because it has obtained an error inferior to 1e-8.

3.2 Number of FEs to achieve the desired accuracy levels

The number of FEs needed in each run to achieve the fixed accuracy level proposed in [Sug05] is presented in Table 4. For each problem and run (25), the numbers of FEs have been sorted and the table presents the following ones: 1st (Best), 7th, 13th (Median), 19th, 25th (Worst), Mean and Standard Deviation (Std). In addition, the table shows the success rate for each problem and the success performance, which is defined as the mean of FEs for the successful runs multiplied by the number of runs and divided by the number of successful runs.

4 Results with Dimension = 30

In this section, we present the empirical results obtained by the GL-50 algorithm when tackling the 25 problems of [Sug05] with Dimension D = 30. The maximum number

		1	2	3	4	5	6	7	8
1e3	1st (Best)	1,5460E+3	4,3087E+3	1,0895E+7	2,9042E+3	3,7378E+3	7,8887E+7	3,9776E+2	2,0286E+1
	7th	3,1248E+3	6,3212E+3	4,0881E+7	6,6412E+3	6,4663E+3	1,7394E+8	5,5991E+2	2,0611E+1
	13th (Median)	3,8836E+3	8,2181E+3	4,9823E+7	8,6263E+3	7,1309E+3	2,3401E+8	6,8789E+2	2,0737E+1
	19th	4,7396E+3	9,7125E+3	5,7347E+7	1,1607E+4	8,0754E+3	3,5873E+8	8,2039E+2	2,0821E+1
	25th (Worst)	6,8764E+3	1,2052E+4	7,9516E+7	1,5056E+4	9,0027E+3	5,1725E+8	1,0403E+3	2,0889E+1
	Mean	4,0250E+3	8,2986E+3	4,9109E+7	8,8377E+3	7,0796E+3	2,6994E+8	6,9915E+2	2,0694E+1
	Std	1,2049E+3	2,3480E+3	1,6323E+7	3,0213E+3	1,2369E+3	1,2828E+8	1,6520E+2	1,7609E-1
1e4	1st (Best)	8,2384E-3	7,7537E+0	1,2078E+5	8,2645E+0	1,8517E+0	1,1702E+1	7,7867E-1	2,0286E+1
	7th	2,9892E-2	1,2033E+1	6,1068E+5	1,4183E+1	2,9902E+0	1,5709E+1	8,8298E-1	2,0436E+1
	13th (Median)	3,8442E-2	1,8028E+1	1,1243E+6	1,8268E+1	3,4640E+0	1,7505E+1	9,3957E-1	2,0521E+1
	19th	4,9787E-2	2,0482E+1	1,8825E+6	2,5042E+1	3,7015E+0	1,9806E+1	1,0595E+0	2,0568E+1
	25th (Worst)	6,8802E-2	3,3093E+1	2,4896E+6	3,7602E+1	5,3034E+0	9,6214E+1	1,4385E+0	2,0676E+1
	Mean	4,0164E-2	1,7086E+1	1,2306E+6	1,9864E+1	3,4043E+0	2,0874E+1	9,9102E-1	2,0504E+1
	Std	1,5428E-2	6,6510E+0	7,0188E+5	7,2360E+0	7,0768E-1	1,6094E+1	1,6493E-1	1,1153E-1
1e5	1st (Best)	4,8228E-9T	5,0940E-9T	1,0812E+1	5,1550E-9T	5,9918E-9T	4,4230E-9T	7,2804E-9T	2,0157E+1
	7th	7,7628E-9T	7,8679E-9T	3,0830E+1	7,3706E-9T	8,7239E-9T	8,3825E-9T	9,8573E-3	2,0313E+1
	13th (Median)	8,4935E-9T	8,2643E-9T	2,7559E+2	8,5682E-9T	9,3005E-9T	9,6229E-9T	1,2316E-2	2,0365E+1
	19th	9,6021E-9T	8,8041E-9T	4,4945E+2	9,2953E-9T	9,5606E-9T	9,8593E-9T	1,7241E-2	2,0410E+1
	25th (Worst)	9,9589E-9T	9,9498E-9T	3,7828E+3	9,9908E-9T	9,8989E-9T	9,9910E-9T	2,2156E-2	2,0477E+1
	Mean	8,3446E-9T	8,2075E-9T	5,7053E+2	8,3177E-9T	8,9375E-9T	8,8737E-9T	1,1723E-2	2,0354E+1
	Std	1,4066E-9T	1,1765E-9T	9,3168E+2	1,4172E-9T	1,0046E-9T	1,4790E-9T	7,6974E-3	8,0547E-2

Table 1: Error Values Achieved for Problems 1-8 with D = 10

		9	10	11	12	13	14	15	16	17
1e3	1st (Best)	5,4306E+1	6,2407E+1	9,9650E+0	1,8289E+4	5,6383E+1	3,7298E+0	4,9883E+2	2,8487E+2	2,9128E+2
	7th	6,6477E+1	8,6668E+1	1,1292E+1	3,8739E+4	3,0925E+2	4,2132E+0	7,0393E+2	3,2690E+2	3,6345E+2
	13th (Median)	7,3870E+1	9,0750E+1	1,1463E+1	4,4290E+4	4,3921E+2	4,3245E+0	7,3466E+2	3,4145E+2	3,9741E+2
	19th	7,7927E+1	9,6683E+1	1,2076E+1	5,5491E+4	7,1486E+2	4,4212E+0	7,5470E+2	3,5019E+2	4,1850E+2
	25th (Worst)	8,8959E+1	1,0868E+2	1,2603E+1	8,3762E+4	3,4260E+3	4,4826E+0	7,9648E+2	3,9924E+2	4,7239E+2
	Mean	7,1658E+1	8,9899E+1	1,1539E+1	4,6266E+4	6,4679E+2	4,2900E+0	7,2335E+2	3,3992E+2	3,9192E+2
	Std	9,3589E+0	1,0229E+1	6,4190E-1	1,5532E+4	7,1429E+2	1,7823E-1	5,8152E+1	3,2185E+1	4,5628E+1
1e4	1st (Best)	2,3915E+0	1,6769E+1	7,9186E+0	9,8155E+0	1,5419E+0	3,3107E+0	4,0087E+2	1,4648E+2	1,6790E+2
	7th	5,6904E+0	3,1203E+1	9,6051E+0	7,5635E+1	2,8455E+0	3,7723E+0	4,0131E+2	1,6602E+2	1,9717E+2
	13th (Median)	9,1165E+0	3,3518E+1	9,9735E+0	1,8199E+2	2,9851E+0	3,8546E+0	4,0152E+2	1,7202E+2	2,0494E+2
	19th	1,5542E+1	3,8804E+1	1,0537E+1	1,2802E+3	3,3067E+0	3,9518E+0	4,0194E+2	1,8227E+2	2,1128E+2
	25th (Worst)	2,9654E+1	4,2588E+1	1,1036E+1	3,0618E+3	3,6937E+0	4,1328E+0	4,3350E+2	1,9834E+2	2,2225E+2
	Mean	1,1008E+1	3,4098E+1	9,9495E+0	7,6142E+2	2,9483E+0	3,8495E+0	4,0308E+2	1,7302E+2	2,0281E+2
	Std	7,1721E+0	5,7953E+0	7,4983E-1	9,6472E+2	5,3507E-1	1,5954E-1	6,4302E+0	1,3255E+1	1,2668E+1
1e5	1st (Best)	6,7539E-9T	9,9496E-1	1,0879E-2	9,3660E-9T	3,6741E-1	1,3779E+0	4,0000E+2	7,2278E+1	9,7154E+1
	7th	9,9496E-1	2,9849E+0	1,0084E+0	9,9010E-9T	5,9359E-1	1,8933E+0	4,0000E+2	9,1099E+1	1,0126E+2
	13th (Median)	9,9496E-1	4,9748E+0	2,5518E+0	9,9936E-9T	7,9050E-1	2,2243E+0	4,0000E+2	9,3617E+1	1,0895E+2
	19th	9,9496E-1	5,9698E+0	3,3545E+0	5,6710E+1	8,6821E-1	2,5162E+0	4,0000E+2	9,6759E+1	1,1573E+2
	25th (Worst)	3,9798E+0	1,1939E+1	6,7778E+0	2,9749E+3	1,1239E+0	3,0307E+0	4,0000E+2	1,0590E+2	1,2114E+2
	Mean	1,1542E+0	4,9748E+0	2,3340E+0	4,0691E+2	7,4985E-1	2,1725E+0	4,0000E+2	9,3493E+1	1,0902E+2
	Std	7,9597E-1	2,7698E+0	1,6064E+0	8,8891E+2	2,1007E-1	4,8722E-1	0,0000E+0	5,8680E+0	7,8033E+0

Table 2: Error Values Achieved for Problems 9-17 with D = 10

		18	19	20	21	22	23	24	25
1e3	1st (Best)	1,0245E+3	1,0341E+3	9,9671E+2	1,2394E+3	8,9933E+2	1,2452E+3	1,0165E+3	8,4492E+2
	7th	1,0979E+3	1,0735E+3	1,1062E+3	1,3195E+3	9,7178E+2	1,3323E+3	1,1477E+3	1,1431E+3
	13th (Median)	1,1240E+3	1,1214E+3	1,1312E+3	1,3357E+3	1,0110E+3	1,3468E+3	1,1787E+3	1,2777E+3
	19th	1,1477E+3	1,1441E+3	1,1410E+3	1,3464E+3	1,0423E+3	1,3551E+3	1,2042E+3	1,3095E+3
	25th (Worst)	1,1615E+3	1,1790E+3	1,1620E+3	1,3661E+3	1,0673E+3	1,3617E+3	1,2668E+3	1,3968E+3
	Mean	1,1181E+3	1,1113E+3	1,1189E+3	1,3296E+3	1,0056E+3	1,3370E+3	1,1716E+3	1,2147E+3
	Std	3,6430E+1	4,0515E+1	3,6807E+1	2,6585E+1	4,4031E+1	2,7115E+1	5,5747E+1	1,4737E+2
1e4	1st (Best)	3,0071E+2	3,0126E+2	3,0121E+2	5,0009E+2	7,7083E+2	5,5947E+2	2,0004E+2	3,9652E+2
	7th	3,0205E+2	3,0342E+2	3,0298E+2	5,0022E+2	7,7743E+2	5,5947E+2	2,0013E+2	4,0719E+2
	13th (Median)	3,0347E+2	3,0515E+2	3,0659E+2	5,0036E+2	7,8060E+2	5,5947E+2	2,0014E+2	4,1013E+2
	19th	3,0684E+2	8,0046E+2	8,0059E+2	8,1224E+2	7,8699E+2	5,5947E+2	2,0021E+2	4,1101E+2
	25th (Worst)	9,9192E+2	1,0279E+3	9,5448E+2	1,2473E+3	8,6520E+2	1,2686E+3	2,0047E+2	4,1312E+2
	Mean	4,3009E+2	4,5260E+2	4,4981E+2	6,9471E+2	7,9608E+2	6,6506E+2	2,0019E+2	4,0806E+2
	Std	2,3356E+2	2,4602E+2	2,3909E+2	3,0614E+2	3,1847E+1	2,4776E+2	1,0743E-1	4,6651E+0
1e5	1st (Best)	3,0000E+2	3,0000E+2	3,0000E+2	5,0000E+2	7,3509E+2	5,5947E+2	2,0000E+2	3,9340E+2
	7th	3,0000E+2	3,0000E+2	3,0000E+2	5,0000E+2	7,4208E+2	5,5947E+2	2,0000E+2	4,0097E+2
	13th (Median)	3,0000E+2	3,0000E+2	3,0000E+2	5,0000E+2	7,4535E+2	5,5947E+2	2,0000E+2	4,0541E+2
	19th	3,0000E+2	8,0000E+2	8,0000E+2	8,0000E+2	7,5650E+2	5,5947E+2	2,0000E+2	4,0719E+2
	25th (Worst)	8,0046E+2	1,0263E+3	9,4983E+2	1,2343E+3	8,2802E+2	1,2686E+3	2,0000E+2	4,0849E+2
	Mean	4,2002E+2	4,4905E+2	4,4599E+2	6,8933E+2	7,5865E+2	6,3887E+2	2,0000E+2	4,0357E+2
	Std	2,1798E+2	2,4767E+2	2,4061E+2	2,9841E+2	2,8614E+1	2,0620E+2	0,0000E+0	4,6781E+0

Table 3: Error Values Achieved for Problems 18-25 with D = 10

Problem	1st	7th	13th	19th	25th	Mean	Std	Success Rate	Success Performance
1	17950	18257	18527	18807	19319	18547,24	391,5847375	100%	18547,24
2	36140	39811	40521	41487	43357	40584,56	1437,267288	100%	40584,56
3	-	-	-	-	-	-	-	0%	-
4	39168	40251	41702	42766	44171	41561,84	1506,329415	100%	41561,84
5	26912	27890	28156	28338	29226	28153,76	514,5927419	100%	28153,76
6	51803	51953	52100	52175	52286	52070,48	136,6473929	100%	52070,48
7	17456	24162	-	-	-	20803,22	3789,685666	36%	57786,7284
8	-	-	-	-	-	-	-	0%	-
9	19371	-	-	-	-	20465,33	976,5420285	12%	170544,4444
10-11	-	-	-	-	-	-	-	0%	-
12	50399	51530	52970	-	-	51536	669,6006272	52%	99107,69231
13-25	-	-	-	-	-	-	-	0%	-

Table 4: Number of FES to achieve the desired accuracy levels (D = 10)

of fitness evaluations (FEs) for this Dimension is 3e5.

4.1 Convergence Graphs

The graphs show the mean performance of 25 runs for each problem. In addition, they present the success rate versus FEs.

The convergence graph and the success rate of the same problem are represented, in the same figure, by the same symbol. In order to differentiate these different graphs, notice that the convergence graph tends to -∞, whereas the success rate tends to 100 from 0.

Finally, the success rate graph will be shown only when it is greater than 0 due to the graphs are represented using logarithmic scale. In this way, most figures do not present this kind of graphs.

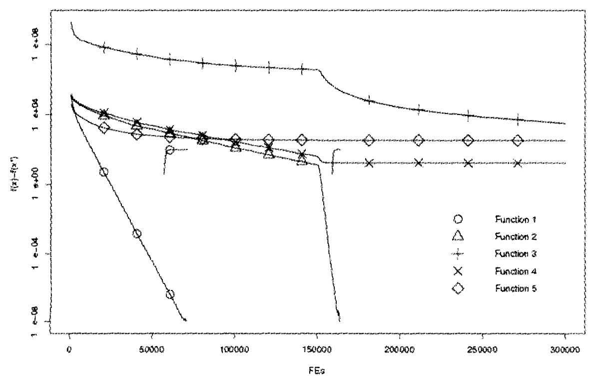

Figure 4: Convergence Graphs for Problems 1-5

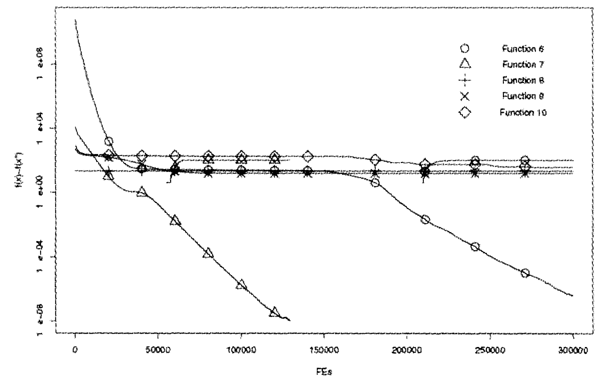

Figure 5: Convergence Graphs for Problems 6-10

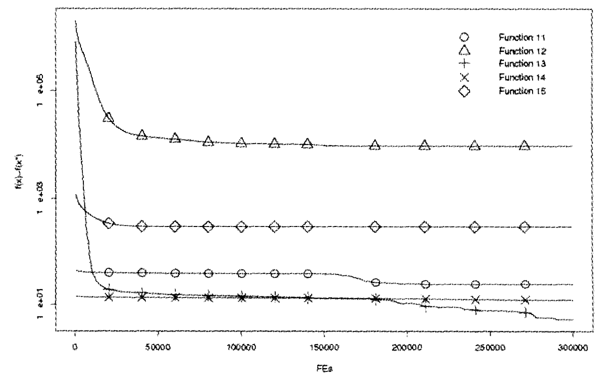

Figure 6: Convergence Graphs for Problems 11-15

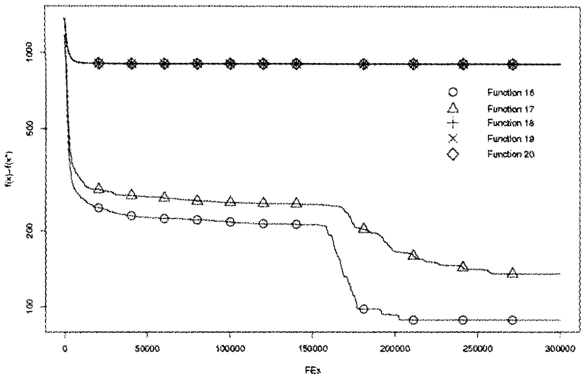

Figure 7: Convergence Graphs for Problems 16-20

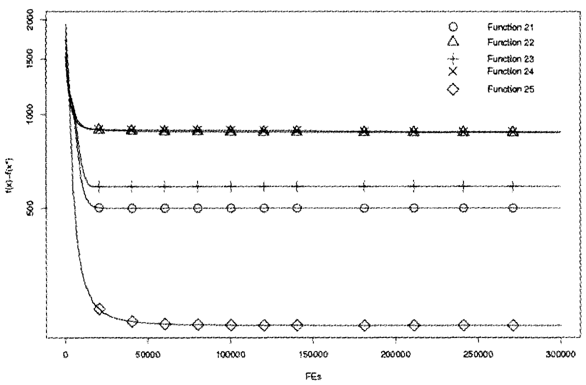

Figure 8: Convergence Graphs for Problems 21-25

4.2 Achieved Error Values

The Tables 5-7 shows the same information than Tables 1-3 but with D = 30. In addition, they record the error values at termination at 3e5 FEs. As before, when the character 'T' is presented next to the value, it indicates that the algorithm stopped the run before reaching the 3e5 FEs.

4.3 Number of FEs to achieve the desired accuracy levels

Table 8 presents the same information than Table 4, but with D = 30.

5 Computational Costs

The Table 9 presents computational costs associated with the GL-50 algorithm, the computing system and the programming language. They have been obtained by using a Pentium® 4, 2.40GHz with 256MB as RAM and Microsoft ® Windows XP. The used programming language was Java 1.5.0_01.

T0 is the time of the test program described in [Sug05]. T0 does not depend on the dimension. T1 corresponds to the computing time for 200000 evaluations of the function 3 of [Sug05]. It is calculated for every dimension. Finally, T2 is the complete computing time of the algorithm with 200000 evaluations as stopping criterion when tackling benchmark function 3. T2 is recorded 5 times and $\hat{T}2$ is the mean of the five values.

		1	2	3	4	5	6	7	8
1e3	1st (Best)	3,0257E+4	6,3291E+4	4,6627E+8	7,9578E+4	1,9577E+4	8,8659E+9	4,6243E+3	2,1124E+1
	7th	4,0830E+4	7,7729E+4	6,1969E+8	9,0174E+4	2,6079E+4	1,6125E+10	5,7717E+3	2,1189E+1
	13th (Median)	4,8394E+4	8,2459E+4	7,5481E+8	1,0273E+5	2,7278E+4	1,9125E+10	6,1206E+3	2,1214E+1
	19th	5,5460E+4	9,2064E+4	8,5731E+8	1,1919E+5	3,0342E+4	2,3049E+10	6,3596E+3	2,1253E+1
	25th (Worst)	6,3160E+4	1,1175E+5	1,0368E+9	1,4672E+5	3,1741E+4	3,1442E+10	7,2251E+3	2,1323E+1
	Mean	4,7991E+4	8,5830E+4	7,4707E+8	1,0667E+5	2,7794E+4	2,0037E+10	6,0673E+3	2,1223E+1
	Std	9,6471E+3	1,3656E+4	1,6578E+8	1,8261E+4	2,9202E+3	5,6230E+9	6,8086E+2	4,9099E-2
1e4	1st (Best)	2,3109E+2	1,5213E+4	7,8047E+7	1,7161E+4	3,2455E+3	7,8207E+5	1,4677E+2	2,0920E+1
	7th	3,4515E+2	1,8434E+4	1,1146E+8	2,2439E+4	3,9103E+3	1,0130E+6	2,3737E+2	2,1060E+1
	13th (Median)	3,7022E+2	2,1727E+4	1,2833E+8	2,5307E+4	4,3413E+3	1,3947E+6	2,6428E+2	2,1108E+1
	19th	4,0729E+2	2,4065E+4	1,5241E+8	2,9367E+4	5,0650E+3	1,5264E+6	2,9268E+2	2,1138E+1
	25th (Worst)	5,5559E+2	2,7088E+4	2,4202E+8	4,6392E+4	6,3446E+3	2,5006E+6	3,4889E+2	2,1195E+1
	Mean	3,8227E+2	2,1513E+4	1,3615E+8	2,6619E+4	4,4801E+3	1,3875E+6	2,6450E+2	2,1093E+1
	Std	6,5406E+1	3,5654E+3	3,8237E+7	6,3904E+3	7,9494E+2	4,7416E+5	4,7198E+1	7,2922E-2
1e5	1st (Best)	6,2906E-9T	6,7936E+1	2,2516E+6	1,2212E+2	7,0536E+1	2,3785E+1	2,5040E-7	2,0832E+1
	7th	8,4591E-9T	8,0411E+1	3,3235E+6	2,1829E+2	2,1125E+2	2,3925E+1	1,0026E-6	2,0961E+1
	13th (Median)	9,0668E-9T	1,2650E+2	5,4922E+6	2,6196E+2	2,7657E+2	2,3971E+1	1,6991E-6	2,0999E+1
	19th	9,6491E-9T	1,4456E+2	7,7759E+6	3,3684E+2	5,4295E+2	2,4029E+1	2,6141E-6	2,1042E+1
	25th (Worst)	9,9909E-9T	2,0777E+2	1,5160E+7	4,0837E+2	1,1600E+3	2,4228E+1	7,2652E-6	2,1099E+1
	Mean	8,8775E-9T	1,2138E+2	6,0390E+6	2,7300E+2	3,7911E+2	2,3980E+1	1,8589E-6	2,0996E+1
	Std	9,8507E-10	3,7976E+1	3,3411E+6	8,3686E+1	2,4864E+2	9,0809E-2	1,4339E-6	6,4292E-2
3e5	1st (Best)	6,2906E-9T	9,3444E-9T	5,8348E+2	5,3470E+0	5,2094E+1	9,4041E-9T	6,8700E-9T	2,0815E+1
	7th	8,4591E-9T	9,8014E-9T	1,8738E+3	1,1357E+1	1,5475E+2	9,9895E-9T	8,8605E-9T	2,0920E+1
	13th (Median)	9,0668E-9T	9,9146E-9T	2,2072E+3	1,4229E+1	2,3747E+2	4,5148E-8	9,2913E-9T	2,0961E+1
	19th	9,6491E-9T	9,9441E-9T	4,2402E+3	2,1453E+1	5,0078E+2	2,6251E-7	9,6609E-9T	2,0979E+1
	25th (Worst)	9,9909E-9T	9,9939E-9T	1,2467E+4	5,5222E+1	1,0975E+3	1,5521E-6	9,8770E-9T	2,1036E+1
	Mean	8,8775E-9T	9,8351E-9T	3,1117E+3	1,6844E+1	3,3256E+2	2,5982E-7	9,0730E-9T	2,0946E+1
	Std	9,8507E-10	1,7465E-10	2,3263E+3	9,9601E+0	2,4743E+2	4,3227E-7	7,8752E-10	5,7488E-2

Table 5: Error Values Achieved for Problems 1-8 with D = 30

		9	10	11	12	13	14	15	16	17
1e3	1st (Best)	3,4731E+2	4,8688E+2	4,2496E+1	1,0610E+6	9,2820E+4	1,3941E+1	7,7218E+2	5,3425E+2	5,8864E+2
	7th	3,6612E+2	5,1978E+2	4,5369E+1	1,1820E+6	1,2007E+5	1,4162E+1	9,7651E+2	6,7377E+2	7,3336E+2
	13th (Median)	3,8552E+2	5,4583E+2	4,5907E+1	1,3540E+6	2,0601E+5	1,4191E+1	1,0322E+3	7,2084E+2	7,8887E+2
	19th	3,9753E+2	5,7443E+2	4,6599E+1	1,4133E+6	2,8608E+5	1,4306E+1	1,0617E+3	7,6996E+2	8,6142E+2
	25th (Worst)	4,1857E+2	6,3429E+2	4,7913E+1	1,6737E+6	4,6700E+5	1,4362E+1	1,1065E+3	8,6347E+2	9,8758E+2
	Mean	3,8372E+2	5,5056E+2	4,5750E+1	1,3245E+6	2,2942E+5	1,4213E+1	1,0017E+3	7,0630E+2	7,9761E+2
	Std	2,0306E+1	3,9332E+1	1,2190E+0	1,6041E+5	1,1196E+5	1,0666E-1	8,7600E+1	9,0711E+1	8,6351E+1
1e4	1st (Best)	1,5871E+2	1,9835E+2	4,0603E+1	9,8644E+4	3,0639E+1	1,3497E+1	3,8121E+2	2,3359E+2	2,4349E+2
	7th	1,8462E+2	2,0940E+2	4,1815E+1	1,4823E+5	4,0191E+1	1,3782E+1	4,7876E+2	2,4309E+2	2,7654E+2
	13th (Median)	1,9505E+2	2,2194E+2	4,2496E+1	1,7556E+5	5,0749E+1	1,3850E+1	4,9112E+2	2,4836E+2	2,8824E+2
	19th	2,0322E+2	2,3016E+2	4,3243E+1	2,1255E+5	5,9355E+1	1,3936E+1	5,1209E+2	2,5782E+2	3,4046E+2
	25th (Worst)	2,1375E+2	2,4188E+2	4,4331E+1	3,6658E+5	6,9898E+1	1,4035E+1	5,8552E+2	4,3609E+2	4,7621E+2
	Mean	1,9126E+2	2,1989E+2	4,2489E+1	1,8526E+5	4,9496E+1	1,3852E+1	4,8381E+2	2,6874E+2	3,1445E+2
	Std	1,4628E+1	1,2789E+1	1,1096E+0	5,6659E+4	1,1074E+1	1,1802E-1	5,1166E+1	5,5234E+1	6,8314E+1
1e5	1st (Best)	6,9647E+0	1,5116E+2	3,8959E+1	2,3632E+2	1,2504E+1	1,2983E+1	2,0000E+2	1,7040E+2	1,8384E+2
	7th	1,0945E+1	1,6835E+2	4,0060E+1	4,5085E+3	1,3841E+1	1,3324E+1	3,0000E+2	1,9110E+2	2,1834E+2
	13th (Median)	1,5242E+1	1,7635E+2	4,0570E+1	1,0731E+4	1,4347E+1	1,3393E+1	3,0000E+2	1,9853E+2	2,2639E+2
	19th	1,8011E+1	1,7957E+2	4,0935E+1	1,5811E+4	1,5170E+1	1,3529E+1	3,0000E+2	2,0650E+2	2,9627E+2
	25th (Worst)	2,6769E+1	1,8890E+2	4,2179E+1	3,4880E+4	1,5838E+1	1,3698E+1	5,0000E+2	4,0000E+2	4,3533E+2
	Mean	1,5105E+1	1,7403E+2	4,0575E+1	1,0664E+4	1,4382E+1	1,3393E+1	3,0400E+2	2,1760E+2	2,5872E+2
	Std	5,0385E+0	9,2439E+0	7,5258E-1	7,8890E+3	9,9250E-1	1,6105E-1	7,3484E+1	6,0457E+1	7,2964E+1
3e5	1st (Best)	6,9647E+0	1,7909E+1	1,8917E+1	5,9142E+1	1,8666E+0	1,0463E+1	2,0000E+2	3,7547E+1	4,7562E+1
	7th	1,0945E+1	2,6864E+1	2,1734E+1	2,9718E+3	2,8623E+0	1,1711E+1	3,0000E+2	4,5912E+1	7,2139E+1
	13th (Median)	1,5242E+1	3,2834E+1	2,4105E+1	8,0675E+3	3,1779E+0	1,2320E+1	3,0000E+2	5,8650E+1	8,3368E+1
	19th	1,8011E+1	4,2245E+1	2,7982E+1	1,4577E+4	4,6446E+0	1,2589E+1	3,0000E+2	7,0040E+1	1,7988E+2
	25th (Worst)	2,6769E+1	5,5150E+1	3,2275E+1	3,4880E+4	1,3949E+1	1,3339E+1	5,0000E+2	4,0000E+2	4,1194E+2
	Mean	1,5105E+1	3,5199E+1	2,4741E+1	9,5208E+3	5,1515E+0	1,2120E+1	3,0400E+2	8,8672E+1	1,3496E+2
	Std	5,0385E+0	1,0321E+1	3,5303E+0	8,0700E+3	4,0209E+0	6,6871E-1	7,3485E+1	9,6921E+1	1,1247E+2

Table 6: Error Values Achieved for Problems 9-17 with D = 30

		18	19	20	21	22	23	24	25
1e3	1st (Best)	1,1226E+3	1,1283E+3	1,1255E+3	1,3136E+3	1,2858E+3	1,3023E+3	1,2882E+3	1,5292E+3
	7th	1,1710E+3	1,1688E+3	1,1752E+3	1,3538E+3	1,3657E+3	1,3391E+3	1,3824E+3	1,5867E+3
	13th (Median)	1,1982E+3	1,1986E+3	1,2103E+3	1,3683E+3	1,3988E+3	1,3553E+3	1,4020E+3	1,6121E+3
	19th	1,2305E+3	1,2239E+3	1,2273E+3	1,3884E+3	1,4640E+3	1,3706E+3	1,4237E+3	1,6343E+3
	25th (Worst)	1,2676E+3	1,2594E+3	1,2820E+3	1,4164E+3	1,6724E+3	1,3993E+3	1,4626E+3	1,7158E+3
	Mean	1,2006E+3	1,1954E+3	1,1990E+3	1,3703E+3	1,4266E+3	1,3545E+3	1,3992E+3	1,6143E+3
	Std	4,0812E+1	3,7198E+1	3,9444E+1	2,7620E+1	9,1509E+1	2,4530E+1	4,0270E+1	4,5252E+1
1e4	1st (Best)	9,0983E+2	9,0936E+2	9,0926E+2	5,8036E+2	8,9281E+2	6,1033E+2	3,4404E+2	2,6776E+2
	7th	9,1025E+2	9,1041E+2	9,0992E+2	5,9902E+2	9,0928E+2	6,6125E+2	9,8674E+2	2,9462E+2
	13th (Median)	9,1075E+2	9,1071E+2	9,1076E+2	6,0945E+2	9,2088E+2	6,7554E+2	9,9461E+2	3,0809E+2
	19th	9,1097E+2	9,1130E+2	9,1102E+2	6,2356E+2	9,2636E+2	7,0343E+2	9,9821E+2	3,2438E+2
	25th (Worst)	9,1337E+2	9,1251E+2	9,1339E+2	6,5976E+2	9,3631E+2	9,6286E+2	1,0083E+3	3,6609E+2
	Mean	9,1089E+2	9,1080E+2	9,1077E+2	6,1318E+2	9,1754E+2	6,9375E+2	9,2665E+2	3,1090E+2
	Std	8,8917E-1	7,4509E-1	1,0854E+0	2,0060E+1	1,1312E+1	6,8259E+1	1,9021E+2	2,4108E+1
1e5	1st (Best)	9,0320E+2	9,0309E+2	9,0312E+2	5,0000E+2	8,5135E+2	5,3416E+2	2,0000E+2	2,1032E+2
	7th	9,0336E+2	9,0343E+2	9,0330E+2	5,0000E+2	8,6108E+2	5,5281E+2	9,7516E+2	2,1106E+2
	13th (Median)	9,0351E+2	9,0350E+2	9,0338E+2	5,0000E+2	8,7893E+2	5,7040E+2	9,8054E+2	2,1123E+2
	19th	9,0368E+2	9,0365E+2	9,0360E+2	5,0000E+2	8,9395E+2	5,8865E+2	9,8797E+2	2,1140E+2
	25th (Worst)	9,0627E+2	9,0584E+2	9,0428E+2	5,0000E+2	9,0736E+2	9,1715E+2	9,9404E+2	2,1204E+2
	Mean	9,0364E+2	9,0364E+2	9,0347E+2	5,0000E+2	8,7828E+2	5,8714E+2	8,8850E+2	2,1125E+2
	Std	5,9490E-1	5,1214E-1	2,4422E-1	8,1222E-14	1,7457E+1	7,7016E+1	2,5955E+2	3,3803E-1
3e5	1st (Best)	9,0320E+2	9,0309E+2	9,0312E+2	5,0000E+2	8,4201E+2	5,3416E+2	2,0000E+2	2,0998E+2
	7th	9,0336E+2	9,0342E+2	9,0330E+2	5,0000E+2	8,5677E+2	5,5281E+2	9,5971E+2	2,1049E+2
	13th (Median)	9,0350E+2	9,0349E+2	9,0337E+2	5,0000E+2	8,7703E+2	5,7040E+2	9,6838E+2	2,1060E+2
	19th	9,0368E+2	9,0364E+2	9,0359E+2	5,0000E+2	8,9136E+2	5,8865E+2	9,7395E+2	2,1065E+2
	25th (Worst)	9,0626E+2	9,0584E+2	9,0427E+2	5,0000E+2	9,0098E+2	9,1715E+2	9,8885E+2	2,1111E+2
	Mean	9,0363E+2	9,0363E+2	9,0346E+2	5,0000E+2	8,7350E+2	5,8714E+2	8,7694E+2	2,1060E+2
	Std	5,9543E-1	5,1263E-1	2,4387E-1	8,7602E-14	1,8248E+1	7,7015E+1	2,5528E+2	2,1493E-1

Table 7: Error Values Achieved for Problems 18-25 with D = 30

Problem	1st	7th	13th	19th	25th	Mean	Std	Success rate	Success Performance
1	56877	57524	58024	58587	60735	58161,48	936,4059002	100%	58161,48
2	158448	159129	159381	159840	161355	159556,92	636,1756125	100%	159556,92
3-5	-	-	-	-	-	-	-	0%	-
6	209763	211425	213630	216948	229524	215126,4	4921,165487	100%	215126,4
7	55310	60353	61643	62726	66109	61543,24	2357,956465	100%	61543,24
8-25	-	-	-	-	-	-	-	0%	-

Table 8: Number of FES to achieve the fixed accuracy (D = 30)

	T0	**T1**	$\hat{T}2$	**($\hat{T}2$ -T1)/T0**
D = 10	909193766,2 ns	2231674587 ns	11950379113 ns	10,68936555
D = 30		8298123266 ns	20646053674 ns	13,58118684

Table 9: Algorithm Complexity

Parameter	Dynamic Range	Cost of Parameter Tuning (FEs)	Actual Parameter Value
N_F^G	[100, 200]	1e4 · D	100
N_M^G	[300, 450]	1e4 · D	400
N_F^L	[1, 5]	1e4 · D	1
N_M^L	[100, 250]	2e4 · D	200
P_G	[0.25, 0.80]	6e4 · D	0.5

Table 10: Parameters

6 Parameters

Table 10 presents the different parameters that modify the behavior of the algorithm, their ranges, estimated cost of the parameter tuning and the value used in the experiments. The algorithm has other parameters that have taken the suggested values from other authors (see section 2.4).

It is important to remark that the parameters in Table 10 use the values suggested in [Gar05]. We have tried other values, however the results were poorer. Thus, the used values seem to perform well on most problems and it is not necessary to expend so much effort tuning them.

At the other hand, we introduce some guidelines in order to adjust the parameter values to other problems:

- N_F^G: 100 will usually perform well on most problems. However, if the problem is so much complex, you can try with 150 or 200.
- N_M^G: 400 produce enough diversity in order to perform adequately. You can try other values, but the behavior of the algorithm will be similar.
- N_F^L: 1 is good in order to reach the local optimum near the population, which have been optimized with previous parameters. However, if the fitness landscape is simplex (unimodal), it could be preferable 5 (with a low value for P_G).
- N_M^L: 200 is good when the population have been optimized with the previous values (N_F^G and N_M^G). However, when the fitness landscape is simplex, you can try with 100 (with a low value of P_G).
- P_G: This parameter is the most preferable in order to be modified. You can try with a higher value when the fitness function is extremely complex. At the other hand, if the fitness function is very simplex, you can choose a smaller value.

7 Analysis of the results

These experiments let us to draw some characteristics of the GL-50 algorithm. Firstly, the algorithm does not usually reach the desired accuracy level proposed in [Sug05]. It only achieves this level for test functions 1, 2, 4, 5 and 6 with D = 10 in every execution. When D = 30, it achieves the fixed level for test functions 1, 2, 6 and 7. It is known that the high performance on function 7 with D = 30, when compared with D = 10, is due to that this function becomes easier when the dimension grows up.

Another important characteristic is that the algorithm gets trapped rapidly in a local optimum. This is easy to see looking to the convergence graphs. This aspect incites us to study the introduction of reinitialization techniques in future works. However, we do not think that the local optimal solutions, where the algorithm is trapped, were very bad. This is due to we have tried other values for the parameters in order to change the behavior of the algorithm, and the results were poorer.

Finally, the different functions can be related according to some characteristics. These relations let us to analyze the behavior of the algorithm when tackling problems with the same characteristics:

- *Different Condition Number* (Functions 1, 2 and 3): The algorithm performs more or less well on functions 1 and 2. However, when the functions is highly conditioned, like function 3, GL-50 presents poorer results. This problem becomes a little more important when the dimension grows up.
- *Function with Noise Vs without Noise* (Functions 2, 4, and 16, 17): When D = 10, the algorithm does not present any problems when using function 4, but some problems appear when tackling function 17. However, when D = 30, the situation is the contrary. GL-50 has problems with function 4 and none with 17. It could be due to that the ranges of possible values of the fitness function 2 and 4 are wider than those of functions 16 and 17, and not by the noise.
- *Function without Rotation Vs with Rotation* (Functions 9, 10 and 15, 16): It seems that the rotation of functions does not affect to the performance of the algorithm. The results for function 10 are worse than those for function 9, however they are similar. On the other hand, the algorithm returns better results for function 16 than for function 15. In addition, when D = 30, the differences become smaller.
- *Continuous Vs Non-continuous* (Functions 21 and 23): The algorithm returns very similar results for both functions.
- *Global Optimum on Bounds Vs Narrow Global Optimum Basin* (Functions 18 and 19): It seems that these properties do not affect to the performance of the algorithm. However, the results of both functions deteriorate when D = 30.
- *Orthogonal Matrix Vs High Condition Number Matrix* (Functions 21 and 22): The results for function 22 are a little worse than for function 21. On the other hand, when the dimension grows up, the results, of

each function, seem similar to those ones when D = 10.

- *Global Optimum in the Initialization Range Vs Global Optimum outside of the Initialization Range* (Functions 24 and 25): When D = 10, the results for both functions are similar. Curiously, when D = 30, the results for function 24 deteriorate and those for function 25 are improved. We think that the algorithm is not affected by this characteristic.
- *Unimodal Functions* (Functions 1-5): When D = 10, the performance of the algorithm seems good in every function except for function 3. However, when D = 30, the performance deteriorates except for functions 1 and 2. It could be due to that the range of values of the fitness function 3, 4 and 5 is very wide.
- *Multi-modal Functions* (Functions 6-25): We think that the algorithm perform well on this type of problems, taking into account that other instances of the algorithm returned poorer results. In addition, the algorithm is not usually so much affect by the use of higher dimensions.
- *Functions with Global Optimum outside of the Initialization Range* (Functions 7 and 25): We think that the algorithm is able to go through the fitness landscape looking for the best regions. In [Gar05] the algorithm shows good results when using this kind of functions. In addition, it seems to perform well in the functions 7 and 25.
- *Functions with Global Optimum on Bounds* (Functions 5, 8 and 20): We think that function 8 is not a good one in order to measure the quality of an Evolutionary Algorithm, because, looking at the fitness landscape, there is no relation between the location of the global optimum and the information of the location of other solutions and their fitness values. So we will concentrate our comments on functions 5 and 20.

  The algorithm seems to be not affected by the location of the global optimum on bounds. The results for function 5 when D = 10 are very good. And it seems to perform well on function 20 when D = 10. However, when D = 30, the results for both functions deteriorate.

8 Conclusions

We have applied the hybrid RCGA presented in [Gar05] to the test suite for the Special Session on Real-Parameter Optimization of the IEEE Congress on Evolutionary Computation 2005. The results have let us to draw some characteristics of this algorithm when tackling problems with different properties.

For future works, we will be interested on improving this algorithm in order to increase the quality of its results for some type of functions, such as high conditioned functions, and to compare it with other algorithms on this test suite.

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