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Linguistic Decision-Making Methods in Multicriterial Selection of Models

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Questions are considered of choosing the best model in GMDH algorithms using several mutually opposing external criteria. A linguistic external criterion is proposed based on the theory of fuzzy sets and fuzzy measures. The best model is chosen according to the maximum of the measure of closeness to the best "Utopian" model.

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To ensure high noise immunity in the synthesis of models by the methods of inductive self-organization [1], one uses simultaneously several external criteria, which, in formalized form, reflect the requirements imposed on the model being synthesized. Since these requirements may prove contradictory, the problem arises of multicriterial choice in the field of efficient decisions. The choice of the unique best model can be made only by the decision maker on the basis of doubly pragmatic considerations. The decision maker may use his own experience, intuition, or expert estimates to give to each competing model an integral estimate: the model is good, not very good, bad, etc. Having a system of preferences of the decision maker in the form of a linguistic model [2], one can formalize the process of the decision maker and write it in the form of an algorithm that is suitable for realization on a computer. The purpose of the present article is to introduce a linguistic external criterion characterizing integrally the quality of the models.

Statement of the problem. Suppose that we have a set of models $A=\{A_1, A_2, \dots, A_m\}$ and that the quality of each model has been estimated for external criteria. Consequently, we are given a set of external criteria $\Xi=(\xi_1, \xi_2, \dots, \xi_k)$ and their basis sets X_j, $j=\overline{1,k}$, on which the corresponding sets of linguistic variables Λ_j, $j=\overline{1,k}$ are defined. Corresponding to the values of the linguistic variables $\dot{a}_{kj}\in\Lambda_j$, characterizing the value of the j-th criterion for the k-th model are fuzzy subsets $\tilde{a}_{kj}\in\mathfrak{F}(X_j)$, where $\mathfrak{F}(X_j)$ is the set of fuzzy subsets of the basis set X_j [2]. The linguo-logical model of [2], characterizing the preferences of the decision maker, is given in the form of a scheme of fuzzy reasonings of the form

$$\text{If } \dot{c}_{11}, \dot{c}_{12}, \dots, \dot{c}_{1k}, \text{ then } \dot{u}_1 \text{ otherwise}$$

$$\cdots\cdots\cdots\cdots$$

$$\text{if } \dot{c}_{n1}, \dot{c}_{n2}, \dots, \dot{c}_{nk}, \text{ then } \dot{u}_n,$$

where $\dot{c}_{ij}\in\Lambda_j$, $\dot{u}_i\in Y$, Y being the set of values of the linguistic variable characterizing the utility.

Corresponding to the value of the linguistic variables $\dot{c}_{ij}$ and $\dot{u}_i$ are the fuzzy subsets $\tilde{c}_{ij}\in\mathfrak{F}(X_j)$ and $\tilde{u}_i\in\mathfrak{F}(Y)$, where $\mathfrak{F}(Y)$ is the set of fuzzy subsets of the basis set Y characterizing the utility. The problem consists in determining the fuzzy utility for all models A_k, $k=\overline{1,m}$, in accordance with the chosen scheme of fuzzy reasonings, and choosing the best model according to the maximum of the measure of closeness to the best "Utopian" utility [2]. The fuzzy subset $\tilde{u}_k'\in\mathfrak{F}(Y)$ characterizing the utility of the k-th model can be determined using the compositional rule of derivation [3]

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$$\tilde{u}'_k = (\tilde{a}_{k1} \times \tilde{a}_{k2} \times \ldots \times \tilde{a}_{km}) \cdot \bigcup_{i \in I} ((\tilde{c}_{i1} \times \ldots \times \tilde{c}_{im}) \times \tilde{u}_i).$$

Using the properties of a fuzzy subset, we can show that

$$\tilde{u}'_k = \bigcup_{i \in I} \left(\bigcap_{j \in J} \tilde{a}_{kj} \circ \tilde{c}_{ij} \right) \times \tilde{u}_i; \; I \triangleq \{\overline{1, n}\}; \; J \triangleq \{\overline{1, m}\}.$$

In this expression, the maximum composition is simply the measure of the extent to which $\dot{a}_{kj}$ is $\dot{c}_{ij}$ [2]. Thus,

$$\tilde{u}'_k = \bigcup_{i \in I} \tilde{u}^{\pi}_{ik},$$

where $\tilde{u}^{\pi}_{ik} = \{(y, \mu_{u_i} \wedge \pi_{ik})\}$ is the fuzzy set Π_i of probabilities, $\mu_{u_i}: Y \to [0, 1]$ is the function of membership in the fuzzy subset, $\pi_{ik} = \bigwedge_{j \in J} \mathrm{Poss}(\dot{a}_{kj} \mid \dot{c}_{ij})$; and $\mathrm{Poss}(\dot{a}_{kj} \mid \dot{c}_{ij})$ is the measure of the possibility that «$\dot{a}_{kj}$ is $\dot{c}_{ij}$».

In the case in which the model quality is precisely described by linguistic variables, that is, when $\dot{a}_{kj} \triangleq x^{\bullet}_{kj} \in X_j$, we have

$$\sigma_k = f\mu_{\omega}(y) \cdot g(F_{\alpha}(\omega)) = \sup_{\alpha \in [0,1]} [\alpha \wedge g(F^k_{\alpha})],$$

where $\mu_{c_{ij}}(x_j): X_j \to [0, 1]$ is the function of membership of $\tilde{c}_{ij}$ in the fuzzy set.

The final choice of best model should be made on the basis of the maximum fuzzy measure of closeness of u'_k to the Utopian utility characterized by the fuzzy subset $\omega \in \mathfrak{F}(Y)$. In this case, we can use the fuzzy integral [4]

$$\mathrm{Poss}(\dot{a}_{kj} \mid \dot{c}_{ij}) = \mu_{c_{ij}}(x^{\bullet}_{kj}),$$

where $F^k_{\alpha} = \{y \mid \mu_{u'_k}(y) \geqslant \alpha\}$; $g(\cdot)$ is a fuzzy measure, and $\mu_{u'_k}(y)$ is the membership function of Utopian utility.

As $g(\cdot)$ we can take one of the following measures: Sugeno's measure [4], probability measure, and g_{ν}-measure. When as g_{ν} we take the probability measure, the measure of closeness takes the form

$$\sigma_k = \sup_{y \in Y} \{\mu_{u'_k} \wedge \mu_{\omega}\}.$$

The measure g_{ν} is an extension of Tsukamoto's measure [5]. The normalization condition for it is

$$g^k_{\nu}(Y) = (1 - \nu) \sup_{y \in Y} \mu_{u'_k}(y) + \nu \int_Y \mu_{u'_k}(y)\, dy = 1$$

(where $\nu \geqslant 0$).

For a denumerable set Y, the normalization condition in the case of Sugeno's measure is

$$g^k_{\lambda}(Y) = \frac{1}{\lambda} \left[\prod_{y \in Y} \lambda \mu_{u'_k}(y) + 1) - 1 \right],$$

where $\lambda \in (-1, +\infty)$.

The formally final solution takes the form $s = \operatorname{argsup}_{k \in K} \sigma_k$, where s is the index of the best model.

The above procedure for choosing models on the basis of the maximum of the fuzzy measure of closeness to a hypothetical or Utopian model can be used to obtain the external linguistic criteria also in the case of self-organization of control systems.

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