Information Sciences 207 (2012) 1–18

Contents lists available at SciVerse ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

An overview on the 2-tuple linguistic model for computing with words in decision making: Extensions, applications and challenges

L. Martínez [a,*], F. Herrera [b]

[a] Dept. of Computer Science, University of Jaén, 23071 Jaén, Spain
[b] Dept. of Computer Science and A.I., University of Granada, 18071 Granada, Spain

A R T I C L E I N F O

Article history:
Received 25 July 2011
Received in revised form 5 March 2012
Accepted 10 April 2012
Available online 27 April 2012

Keywords:
Linguistic variable
Computing with words
2-Tuple linguistic representation model
Fuzzy linguistic approach

A B S T R A C T

Many real world problems need to deal with uncertainty, therefore the management of such uncertainty is usually a big challenge. Hence, different proposals to tackle and manage the uncertainty have been developed. Probabilistic models are quite common, but when the uncertainty is not probabilistic in nature other models have arisen such as fuzzy logic and the fuzzy linguistic approach. The use of linguistic information to model and manage uncertainty has given good results and implies the accomplishment of processes of computing with words. A bird's eye view in the recent specialized literature about linguistic decision making, computing with words, linguistic computing models and their applications shows that the 2-tuple linguistic representation model [44] has been widely-used in the topic during the last decade. This use is because of reasons such as, its accuracy, its usefulness for improving linguistic solving processes in different applications, its interpretability, its ease managing of complex frameworks in which linguistic information is included and so forth. Therefore, after a decade of extensive and intensive successful use of this model in computing with words for different fields, it is the right moment to overview the model, its extensions, specific methodologies, applications and discuss challenges in the topic.

1. Introduction

In the real world there are many situations in which problems must deal with vague and imprecise information that usually involves uncertainty in their definition frameworks. The use of numerical based modelling to represent such uncertain information is not always adequate. In those cases in which the uncertainty is not of probabilistic nature, it is hard to provide numerical precise information when the knowledge is vague. Often the experts that take part in this type of problems use linguistic descriptors to express their assessments regarding the uncertain knowledge they have about the problem [83,86]. Therefore, the use of linguistic modelling in problems dealing with non-probabilistic uncertainty seems logic and has produced successful results in different fields such as: situation awareness [74], decision models [11,15,19,33,72,85,124], information retrieval [50–52], risk assessment [38,69,107], engineering evaluation [81,83], sensory evaluation [20,75,77,132], performance appraisal [25,27], recommender systems [78,84,99], data mining [56], and social choice [39]. This success would not have been possible without methodologies to carry out the processes of computing with words (CW) [116,139,140] that implies the use of linguistic information.

* Corresponding author.
E-mail addresses: martin@ujaen.es (L. Martínez), herrera@decsai.ugr.es (F. Herrera).

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http://dx.doi.org/10.1016/j.ins.2012.04.025

Fig. 1. Computing with words scheme [130].

Notwithstanding some methodologies for CW are based on probability [54,61–63], the uncertainty modelled in those problems is rather related to the imprecision and vagueness of the meaning of the linguistic descriptors. Consequently other tools as Fuzzy Logic [135] and the Fuzzy Linguistic Approach [136–138] grounds the basis for different computational models for CW, such as:

- *The Linguistic Computational Model Based on Membership Functions* [28,76,95]. It is based on the fuzzy linguistic approach and makes the computations directly on the membership functions of the linguistic terms by using the Extension Principle [34,58].
- *The Linguistic Symbolic Computational Models Based on Ordinal Scales* [126]. It represents the information according to the fuzzy linguistic approach and uses the ordered structure of the linguistic term set to accomplish symbolic computations in such ordered linguistic scales. Similar approaches based on this way of computing were presented in [29,125]. It is remarkable that this model has been extensively applied to decision making processes because its easy adaptation and simplicity for decision makers [126,128,129].

These models follow the computational scheme depicted by Yager in [130,131] (see Fig. 1), that points out the importance of the translation and retranslation processes in CW, likewise Mendel and Wu highlight similar processes in computing with perceptions [88,89]. Because the former involves taking information linguistically and translates into machine manipulative format. Meanwhile the latter involves taking the results from the manipulation machine format and transforms them into linguistic information that will be understandable by human beings that is one of the main objectives of CW [90].

The previous linguistic computational models present an important weakness, because they performed the retranslation step as an approximation process to express the results in the original expression domain (initial term set) provoking a lack of accuracy [45]. To avoid such inaccuracy in the retranslation step was introduced:

- *The 2-tuple Linguistic Computational Model* [44]. It is a *symbolic* model that extends the use of indexes modifying the fuzzy linguistic approach representation by adding a parameter to the basic linguistic representation in order to improve the accuracy of the linguistic computations after the retranslation step keeping the CW scheme showed in Fig. 1 and the interpretability of the results.

A deep revision of the specialized literature in CW shows the rapid growth and applicability of the 2-tuple linguistic representation model, that has been applied to many different problems (mainly decision analysis) and extended in different ways across the various hundreds of papers that have cited the seminal paper of this model.[1]

Therefore, we consider that after more than ten years time that the 2-tuple linguistic representation model was presented, it would be very interesting to make an overview about this model, paying attention to the necessity and foundations of the model [44,45], methodologies for CW in complex frameworks [36,42,43,46,47] and new linguistic computing models based on it [32,114]. As well it will be revised the different applications and problems in which the model has been successfully applied. Eventually, we concern about the challenges that the 2-tuple linguistic representation model should face to provide satisfactory solutions to other decision analysis problems.

The paper is structured as: Section 2 provides a revision about the foundations of the 2-tuple linguistic representation model both basics and use of the model. Section 3 presents the different methodologies based on the 2-tuple linguistic model for CW in complex frameworks. New linguistic computational models based on the 2-tuple are revised in Section 4. In Section 5 a deep review of the applications in which the 2-tuple linguistic model has been applied is showed. Section 6 points out some challenges of the 2-tuple linguistic model in CW and finally the paper is concluded in Section 7.

2. 2-Tuple linguistic representation model for CW

The aim of this section is to review the foundations of the 2-tuple linguistic representation model both representation and computing models and afterwards to show the use of this model on the main research branches in which it has been applied.

2.1. Foundations of the 2-Tuple linguistic representation model

In the *Introduction* was mentioned that the linguistic computing models based on membership functions and on ordinal scales [28,126] presented a drawback regarding their accuracy due to the loss of information produced in the retranslation

[1] Scholar Google: 656 cites, Scopus: 416 cites and ISI Web of Knowledge: 336 cites. March 2012.

step of the CW scheme (see Fig. 1). Therefore, the 2-tuple linguistic model [44] aimed to improve the accuracy and facilitate the processes of CW by treating the linguistic domain as continuous but keeping the linguistic basis (syntax and semantics).

It extended the use of indexes modifying the fuzzy linguistic approach adding a new parameter, so-called *symbolic translation*, and representing the linguistic information by means of a linguistic *2-tuple* that consisted of a pair of values namely, $(s_i, \alpha) \in \overline{S} \equiv S \times [-0.5, 0.5)$, being $s \in S$ a linguistic term and $\alpha \in [-0.5,0.5)$ a numerical value representing the symbolic translation.

Definition 1 [44]. The symbolic translation is a numerical value assessed in $[-0.5,0.5)$ that supports the *difference of information* between a counting of information β assessed in the interval of granularity $[0,g]$ of the term set S and the closest value in $\{0, \ldots, g\}$ which indicates the index of the closest linguistic term in S.

This representation model defined a set of transformation functions between numeric values and linguistic 2-tuples to facilitate linguistic computational processes.

Definition 2. Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0,g]$ a value supporting the result of a symbolic aggregation operation. A 2-tuple that expresses the equivalent information to β is then obtained as follows:

$$\Delta : [0, g] \to \overline{S}$$

$$\Delta(\beta) = (s_i, \alpha), \quad \text{with} \quad \begin{cases} s_i, & i = \text{round } (\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5), \end{cases}$$

being *round* the usual round operation, i the index of the closest label, s_i, to β, and α the value of the symbolic translation.

It is noteworthy that Δ is a one to one mapping [44] and $\Delta^{-1} colon \overline{S} \to [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$. In this way, the 2-tuple of $\overline{S}$ is identified by the numerical value in the interval $[0,g]$ and the retranslation step is carried out accurately.

Remark 1. The conversion between a linguistic term into a 2-tuple linguistic consists in adding a value 0 as symbolic translation: $s_i \in S \Rightarrow (s_i, 0) \in \overline{S}$.

Let us suppose that $\beta = 3.25$ is a value representing the result of a symbolic aggregation operation on the set of labels, $S = \{s_0$:*Nothing*,s_1:*Very Low*, s_2:*Low*, s_3:*Medium*, s_4:*High*, s_5:*VeryHigh*, s_6:*Perfect*$\}$, the 2-tuple that expresses the equivalent information to β is (*Medium*, .25) (see Fig. 2).

Remark 2. The 2-tuple linguistic model could be used with any membership function that supports the semantics of the linguistic terms, improving the accuracy of classical symbolic methods. However in [44,45] was showed that the use of triangular membership functions produce more accurate results.

Additionally, the 2-tuple linguistic model has associated a computational model where a total order for linguistic 2-tuples is defined together a negation operator and several aggregation functions, it was further detailed in [44]. Such a computational model is based on the previous functions Δ and Δ^{-1}.

2.2. On the use of the 2-tuple linguistic representation model

The 2-tuple linguistic model was inspired by the symbolic models used in decision making [29,126,128,129]. Hence it is reasonable that its main application field has been decision analysis and decision making, however it is remarkable its use in another area as fuzzy systems modelling though its importance in it is much less than in the decision field. A brief revision of its use is presented below:

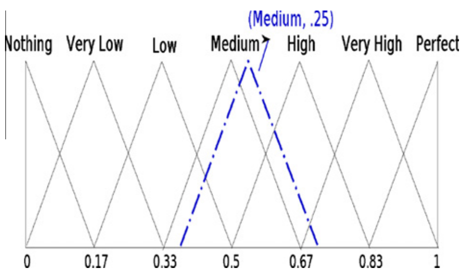

Fig. 2. A 2-tuple linguistic representation.

Table 1
Decision models based on the 2-tuple linguistic model.

TOPICS	PAPERS
Multi-criteria decision making	[9,31] [30,33] [41,64] [82,121]
Group decision making	[7,11] [40,47] [57,64] [119,122] [123,141]
Consensus reaching processes	[12,13] [48,49,53] [87,94]
Evaluation models	[10,27] [71,77]
Aggregation operators	[91,97] [118,120]

1. **Decision making and decision analysis**: Initially the 2-tuple linguistic model was introduced to accomplish processes of CW in linguistic decision making problems, but afterwards it has been widely used as basis for different models in problems based on decision analysis that will be further detailed in Section 5, a brief summary of such applications is showed in Table 1.
2. **Fuzzy Systems Modelling**: The use of linguistic modelling in fuzzy systems tries to increase the interpretability of such systems [56,108], but unfortunately it usually implies a decrease of the accuracy. Consequently, to increase the accuracy of linguistic fuzzy systems two main processes have been introduced, a *tuning process* that aims to modify the membership function of the linguistic variables to obtain a better representation of the data and a *rule selection process* to optimize the rule base [23]. There exist tuning methods of domain, parameters and linguistic modifiers that in a genetic optimization need to codify three parameters for each linguistic term [14]. Even though the 2-tuple linguistic model was not introduced thinking of fuzzy systems, Alcalá et al. [2] have recently used it in a lateral tuning process based on the 2-tuple linguistic model that reduces the information to code in the genetic scheme by just one parameter that indicates the symbolic translation of the linguistic label to increase the accuracy of the fuzzy system (see Fig. 3). This representation is also used for learning membership functions in fuzzy modelling regression [3,4] and fuzzy association rule extraction [5,6].

In spite of its application in linguistic fuzzy systems the main application fields of the 2-tuple linguistic model have been and still are decision analysis and decision making.

3. 2-tuple based methodologies for linguistic complex frameworks

The 2-tuple linguistic representation model aimed to improve the accuracy of processes of CW in linguistic frameworks where the universe of discourse was just one linguistic term set with the terms uniform and symmetrically distributed.

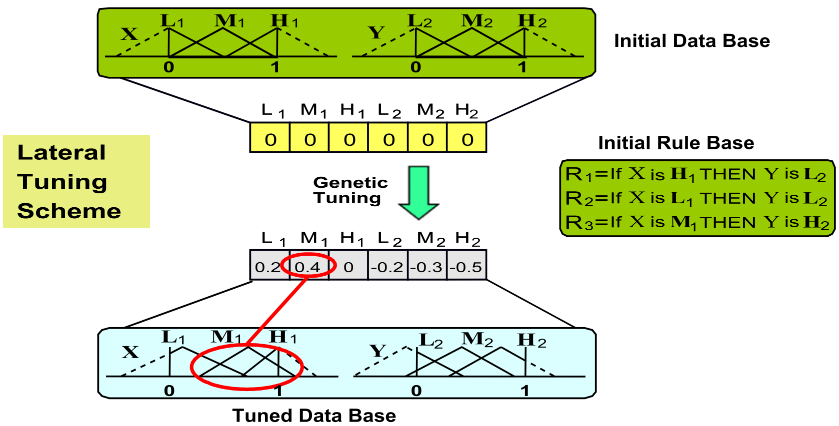

Fig. 3. Coding scheme for genetic tuning based on 2-tuples.

However, it is common that in decision making problems under uncertainty the decision framework would be more complex than a symmetrical linguistic term set, such as:

- *Multi-granular linguistic information*: In problems with multiple experts or multiple criteria [17,27,55] in which appear linguistic information assessed in multiple linguistic term sets with different granularity.
- *Non-homogeneous information*: Problems whose information might be from different nature (linguistic, numerical, interval-valued, etc.) [16,66,106].
- *Linguistic information not uniformly distributed*: Problems whose linguistic information is distributed in a unbalanced scale [79,104,111].

For all the previous situations have been proposed 2-tuple based methodologies to perform processes of CW in such frameworks having obtained satisfactory results. The most important 2-tuple based methodologies for such complex decision frameworks are briefly detailed below.

3.1. 2-tuple based CW models for multi-granular linguistic information

The necessity of dealing with linguistic frameworks in which the linguistic information may belong to various terms sets with different granularity are fairly common [17,21,55,57,65]. The 2-tuple linguistic representation model has been the basis for different CW models [36,42,46] to accomplish processes of CW with multi-granular linguistic information (MGLI) that improve the accuracy and understanding of the results. Based on the CW scheme (Fig. 1), these 2-tuple based models for computing with multiple linguistic scales perform according to the scheme below (see graphically in Fig. 4):

(a) Unification of the MGLI: It consists of a translation process that expresses the MGLI in a unique linguistic domain.
(b) Computational phase: The processes of CW that manipulate the linguistic information are carried out by using linguistic 2-tuples.
(c) Results: A retranslation process is needed to express the results by linguistic 2-tuples in the initial expression domains.

Different CW approaches based on the 2-tuple linguistic model to deal with multiple linguistic scales are reviewed below.

3.1.1. Fusion approach for managing multi-granular linguistic information

This approach introduced in [42,47] deals with information assessed in different linguistic scales by using the extension principle and the 2-tuple linguistic model. Its computing model, graphically in Fig. 5, adapts the scheme showed in Fig. 4 to the fusion approach utilized:

(a) *Unification*: Firstly it unifies the MGLI into a linguistic term set, so called Basic Linguistic Term Set (BLTS) by means of fuzzy sets in a two-step process:
  - Election of the BLTS: In [42] were introduced the rules that lead to the election of the BLTS, S_T, with the aim of keeping as much information as possible.
  - Unification process: Once the BLTS has been chosen the MGLI is transformed into the BLTS by using a transformation function $\tau_{S_iS_T}$ which expresses any linguistic term $s^i_j \in S_i$ as a fuzzy set defined in S_T.

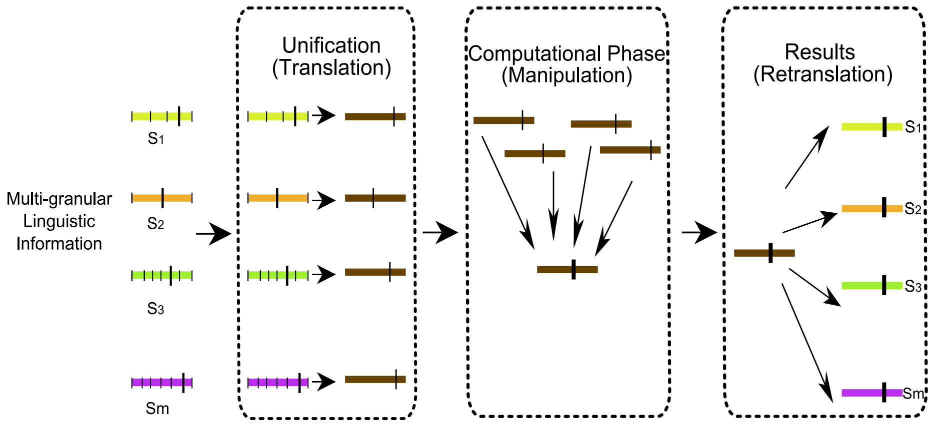

Fig. 4. A CW scheme for multi-granular linguistic information.

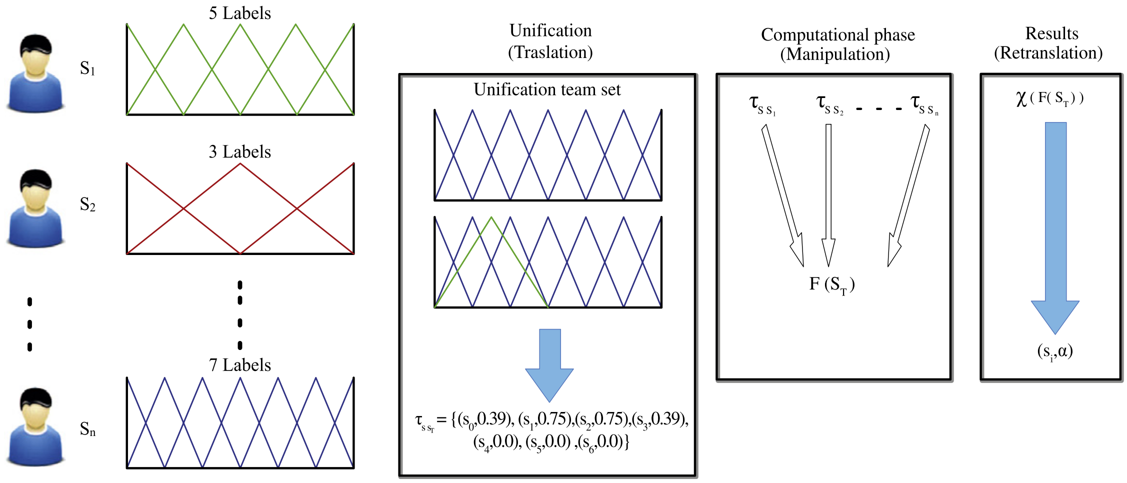

Fig. 5. A 2-tuple fusion scheme for multi-granular linguistic information.

Definition 3 [42]. Let $S_i = \left\{s_0^i, \ldots, s_{g_i}^i\right\}$ and $S_T = \left\{s_0^T, \ldots, s_{g_T}^T\right\}$ be two linguistic term sets, such that, $g_T > g_i$. The multi-granularity transformation function, $\tau_{S_iS_T}$ is then defined as:

$$\begin{aligned} &\tau_{S_iS_T} : S_i \rightarrow F(S_T) \\ &\tau_{S_iS_T}\left(s_j^i\right) = \sum_{k=0}^{g_T} s_k^T / \gamma_k^j \end{aligned} \tag{1}$$

being $F(S_T)$ the set of fuzzy sets defined in S_T, $\mu_{s_j^i}(y)$ and $\mu_{s_k^T}(y)$ the membership functions of the fuzzy sets associated to the terms s_j^i and s_k^T, respectively.

(b) *Computational phase*: Once the MGLI has been unified into just one expression domain, the computations are directly operated on the fuzzy sets by using the fuzzy arithmetic [34]. Therefore, the results obtained will be fuzzy sets in the BLTS.

(c) *Results*: Consequently to obtain the results by means of linguistic 2-tuples, it is necessary a retranslation process. In [47] was presented a function χ that transforms a fuzzy set into a linguistic 2-tuple in $\overline{S_T}$:

$$\chi : F(S_T) \rightarrow \overline{S_T}$$

$$\chi(F(S_T)) = \chi\left(\sum_{j=0}^{g} s_j/\gamma_j\right) = \Delta\left(\frac{\sum_{j=0}^{g} j\gamma_j}{\sum_{j=0}^{g}\gamma_j}\right) = \Delta(\beta) = (s, \alpha) \tag{2}$$

3.1.2. Linguistic hierarchies

Although, the previous approach provides a way to deal with MGLI, it presents lack of accuracy in the retranslation process. In order to overcome this drawback it was introduced a new approach to deal with MGLI in a symbolic and precise way by means of Linguistic Hierarchies (LH) [24,46]. This approach builds a structure, so-called linguistic hierarchy, and over it a computational symbolic model based on the 2-tuple is defined to accomplish processes of CW.

A linguistic hierarchy, LH, is the union of all levels t: $LH = \bigcup_t l(t, n(t))$, where each level t of a LH corresponds to a linguistic term set with a granularity of uncertainty of $n(t)$ denoted as: $S^{n(t)} = \left\{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\right\}$.

The construction of a LH must satisfy several rules, so-called *linguistic hierarchy basic rules* [46], that impose the following restrictions:

- *Rule 1:* to preserve all *former modal points* of the membership functions of each linguistic term from one level to the following one.
- *Rule 2:* to make *smooth transitions between successive levels*. The aim is to build a new linguistic term set, $S^{n(t+1)}$. A new linguistic term will be added between each pair of terms belonging to the term set of the previous level t. To carry out this insertion, it is necessary to reduce the support of the linguistic labels in order to keep place for the new one located in the middle of them.

Table 2
Linguistic hierarchies.

	$l(t,n(t))$	$l(t,n(t))$
Level 1	$l(1,3)$	$l(1,7)$
Level 2	$l(2,5)$	$l(2,13)$
Level 3	$l(3,9)$	

These rules induce a limitation regarding the term sets that can be used in a *LH*. Because, a linguistic term set of level $t + 1$ is obtained from its predecessor as:

$$l(t,n(t)) \rightarrow l(t+1, 2 \cdot n(t) - 1) \tag{3}$$

Table 2 shows the granularity for each linguistic term set of a LH according to the rules (graphically in Fig. 6).

To accomplish processes of CW this approach also follows the scheme showed in Fig. 4, but it introduces some slightly modifications to deal with multiple linguistic scales in a symbolic way (see Fig. 7):

(a) *Unification*: The LH can unify the MGLI in any term set of the LH by using a transformation function, $TF^{t}_{t'}$, between any two linguistic levels t and t':

Definition 4 [46]. Let $LH = \bigcup_t l(t,n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \left\{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\right\}$, and let us consider the 2-tuple linguistic representation. The transformation function, $TF^{t'}_{t} : \overline{S^{n(t)}} \rightarrow \overline{S^{n(t')}}$, from a linguistic label in level t to its correspondent label in level t', satisfying the linguistic hierarchy basic rules, is defined as:

$$TF^{t'}_{t}\left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right) \cdot (n(t') - 1)}{n(t) - 1}\right) \tag{4}$$

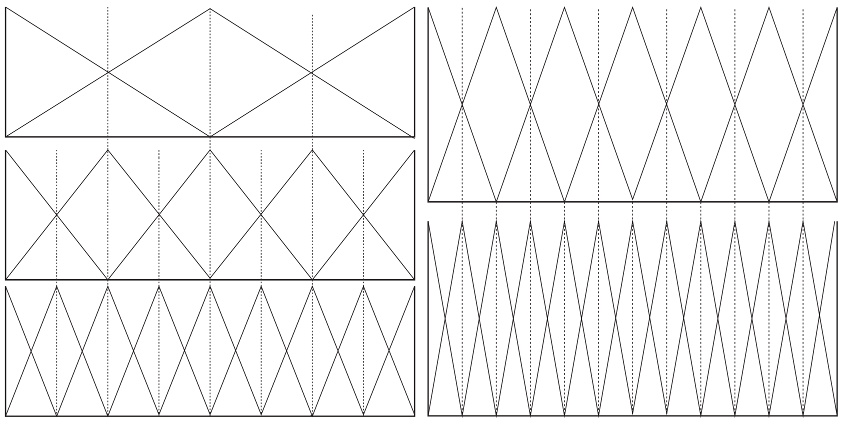

Fig. 6. Linguistic hierarchies of 3, 5 and 9 labels and 7 and 13 labels.

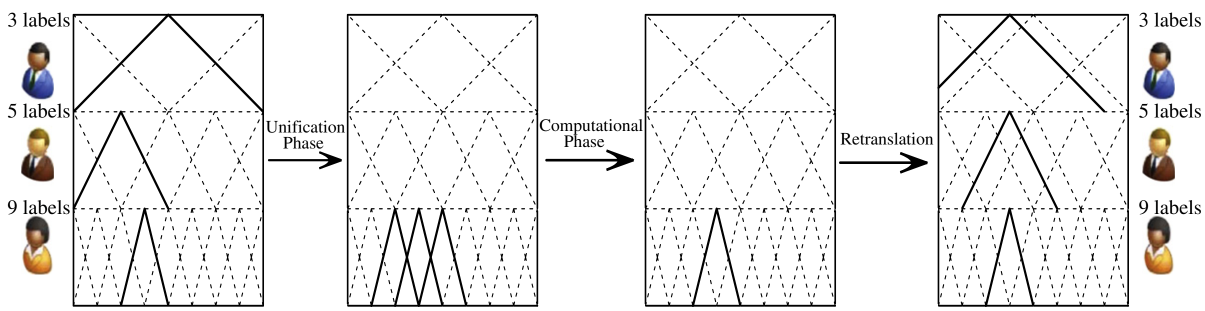

Fig. 7. A CW scheme for linguistic hierarchies.

(b) *Computational phase*: Due to the fact that the representation model used by LH and the results of the unification process are linguistic 2-tuples, its computational model is based on the one presented in [44] for the 2-tuple linguistic model.
(c) *Results*: In [46] it was proved that the transformation function between linguistic terms in different levels of the linguistic hierarchy is a one-to-one mapping, which guarantees the transformations between levels of a linguistic hierarchy are carried out without loss of information. It is then remarkable that the final results of any computational process can be expressed in any linguistic term set of the LH by means of a retranslation process accomplished by $TF^{t}_{t'}$.

3.1.3. Extended linguistic hierarchies

Despite the use of LH has provided successful results, it presents the limitation regarding the term sets that can be used, for example term sets with 5 and 7 labels are incompatible in LH, but sometimes both are necessary in a specific problem. In order to overcome such a limitation recently it has been proposed an extended model that constructs a structure, so-called Extended Linguistic Hierarchy (ELH) [36], that is a set of linguistic terms sets. Being each level, $l(t,n(t))$, a linguistic term set with different granularity, $n(t)$, from the remaining levels of the ELH. The ELH can manage any term set because its construction process redefines the rules to build the hierarchy as follows:

- *Rule 1:* include a finite number of the levels $l(t,n(t))$ with $t = \{1,\ldots,m\}$, that defines the multi-granular linguistic context,
- *Rule 2:* add a final level $l(t',n(t'))$ such that $t' = m + 1$ with the following granularity:

$$n(t') = (lcm(n(1) - 1, \ldots, n(m) - 1)) + 1, \tag{5}$$

being *lcm* the *least common multiple*.

Its computational scheme is similar to the previous one showed in Fig. 7 taking into account that the unification level must be t':

(a) *Unification*: In an ELH the MGLI is unified by means of a transformation function of linguistic terms between different of its levels.

Definition 5. Let $S^{n(a)} = \left\{s_0^{n(a)}, s_1^{n(a)}, \ldots, s_{n(a)-1}^{n(a)}\right\}$ and $S^{n(b)} = \left\{s_0^{n(b)}, s_1^{n(b)}, \ldots, s_{n(b)-1}^{n(b)}\right\}$ be two linguistic term sets, with $a < b$. The linguistic transformation function $TF_a^b : \overline{S^{n(a)}} \to \overline{S^{n(b)}}$ is defined by:

$$TF_a^b\left(s_j^{n(a)}, \alpha_j\right) = \Delta_S\left(\frac{\Delta_S^{-1}\left(s_j^{n(a)}, \alpha_j^{n(a)}\right) \cdot (n(b) - 1)}{n(a) - 1}\right) = \left(s_k^{n(b)}, \alpha_k\right).$$

However to guarantee the accuracy of the results must be satisfied that the linguistic transformation function, TF_a^b, performed in the unification phase, a might be any level in the set $\{t = 1, \ldots, m\}$ and b must be the level t'.

(b) *Computational phase*: The processes of CW are carried out in the level t' by using the 2-tuple linguistic computational model.
(c) *Results*: The results are then retranslated to the initial domains by using TF_a^b where b could be any level in the set $\{t = 1, \ldots, m\}$ and a is always the level t'.

3.2. 2-tuple based models to deal with non-homogeneous information

Another common situation in problems with multiple criteria and/or multiple experts is the appearance of information of different nature that might be modelled with non-homogeneous information [16,60,66,81] such as, numerical, intervals, linguistic and so forth.

For this type of frameworks it was introduced in [47] a 2-tuple based model to manage and operate with such a type of information that performs a computing scheme (see Fig. 8) with similar phases to the one for MGLI.

(a) *Unification*: It unifies the heterogeneous information in one linguistic domain.
(b) *Computational phase:* It accomplishes the processes of CW
c) *Results*: A retranslation process is used to express the results by linguistic 2-tuples

This 2-tuple based computational scheme for non-homogeneous information is further detailed below:

(a) *Unification*: This phase consists of two processes. First, the non-homogeneous information is unified in a unique linguistic expression domain, so-called BLTS, whose selection follows the suggestions provided in [47]. Second, once the BLTS, $S_T = \{s_0, \ldots, s_g\}$, has been chosen the non-homogeneous information is unified by means of transformation functions according to the nature of the information:

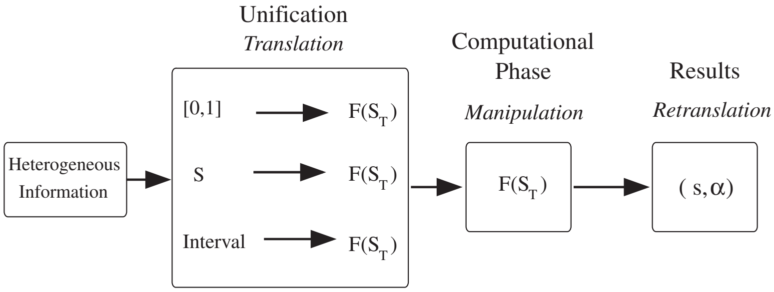

Fig. 8. Operating with heterogeneous information.

(i) ***Numerical values***. A numerical value $\vartheta \in [0,1]$ is transformed into a fuzzy set in $F(S_T)$ by computing the membership value of ϑ in the fuzzy number associated with the linguistic terms of S_T.

Definition 6 [47]. The function τ_{NS_T} transforms a numerical value into a fuzzy set in S_T:

$$\tau_{NS_T} : [0,1] \to F(S_T)$$

$$\tau_{NS_T}(\vartheta) = \sum_{i=0}^{g} s_i/\gamma_i \tag{6}$$

$$\gamma_i = \mu_{s_i}(\vartheta) = \begin{cases} 0, & \text{if } \vartheta \notin Support(\mu_{s_i}(x)) \\ \frac{\vartheta - a_i}{b_i - a_i}, & \text{if } a_i \leqslant \vartheta \leqslant b_i \\ 1, & \text{if } b_i \leqslant \vartheta \leqslant d_i \\ \frac{c_i - \vartheta}{c_i - d_i}, & \text{if } d_i \leqslant \vartheta \leqslant c_i \end{cases}$$

Remark 3. Here it is considered membership functions, $\mu_{s_i}(\cdot)$, for linguistic labels, $s_i \in S_T$, represented by a parametric function (a_i, b_i, d_i, c_i). A particular case are the linguistic assessments whose membership functions are triangular, i.e., $b_i = d_i$.

(ii) ***Linguistic terms***. For linguistic information the transformation function utilized to transform the linguistic terms into fuzzy sets in the BLTS is the same as the one introduced in Definition 3, see Eq. (1).

(iii) ***Intervals***. Let $A = [\underline{a}, \bar{a}]$ be an interval in $I([0,1])$, to unify this type of information we assume that the interval presents a representation inspired in the membership function of fuzzy sets [59], as follows:

$$\mu_A(\vartheta) = \begin{cases} 0, & \text{if } \vartheta < \underline{a} \\ 1, & \text{if } \underline{a} \leqslant \vartheta \leqslant \bar{a} \\ 0, & \text{if } \bar{a} < \vartheta \end{cases}$$

being ϑ a value in $[0,1]$. Therefore the transformation function of the interval into a fuzzy set in the BLTS is computed as:

Definition 7 [47]. Let $S_T = \{s_0, \ldots, s_g\}$ be a BLTS. Then, the function τ_{AS_T} transforms a interval A in $I([0,1])$ into a fuzzy set in S_T.

$$\tau_{AS_T} : A \to F(S_T)$$
$$\tau_{AS_T}(A) = \sum_{i=0}^{g} s_i/\gamma_i \tag{7}$$

$$\gamma_k^i = \max_y \min\{\mu_A(y), \mu_{s_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_A(\cdot)$ and $\mu_{s_k}(\cdot)$ are the membership functions associated with the interval A and the terms $s_k \in S_T$, respectively.

(b) *Computational phase*: Once the information has been unified into just one expression domain, the computations are directly operated on the fuzzy sets by using the fuzzy arithmetic [34] similarly to the fusion approach for MGLI (see Section 3.1.1)

c) *Results*: Hence, to express the results by linguistic 2-tuples a retranslation process that uses the function, χ, see Eq. (2), is performed.

3.3. 2-tuple based models to deal with unbalanced linguistic information

Despite most of problems modelling information with linguistic assessments use linguistic variables assessed in linguistic term sets whose terms are uniform and symmetrically distributed [22,38,77,107]. It is also common, problems whose assessments are better modelled by means of linguistic term sets that are not uniform either symmetrically distributed, i.e., *unbalanced linguistic term sets*. In some cases, the unbalanced linguistic information appears either due to the nature of the linguistic variables that participate in the problem, or in problems dealing with scales in which it is necessary to assess preferences with a greater granularity on a side of the scale than on the another one (see Fig. 9).

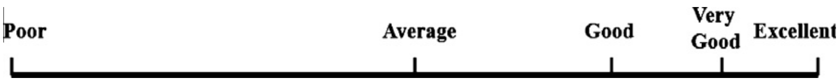

Fig. 9. Unbalanced linguistic term set of five labels.

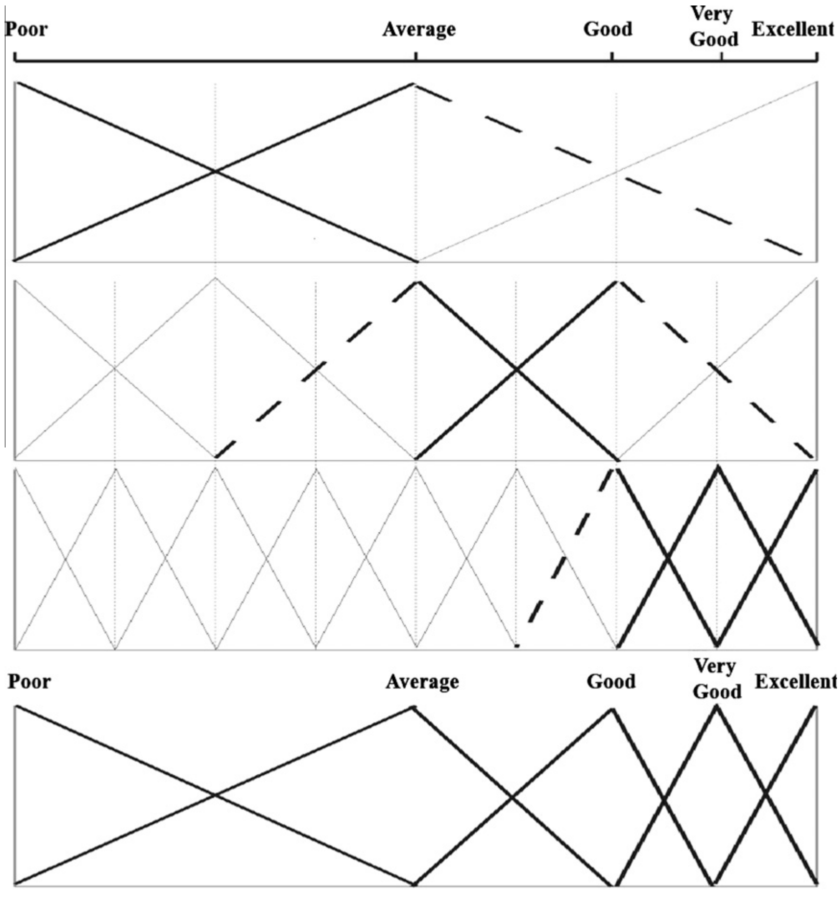

Fig. 10. Semantic representation, *LH*(*S*), of the unbalanced linguistic term set *S* in the linguistic hierarchy, *LH*.

Table 3
LH(*S*) and *Brid*(*S*).

S	*LH*(*S*)	*Brid*(*S*)
s_0 = *Poor*	$s^{G(0)}_{I(0)} = s^3_0$	False
s_1 = *Average*	$s^{G(1)}_{I(1)} = s^3_1$ or $s^{G(1)}_{I(1)} = s^5_2$	True
s_2 = *Good*	$s^{G(2)}_{I(2)} = s^5_3$ or $s^{G(2)}_{I(2)} = s^9_6$	True
s_3 = *Very Good*	$s^{G(3)}_{I(3)} = s^9_7$	False
s_4 = *Excellent*	$s^{G(4)}_{I(4)} = s^9_8$	False

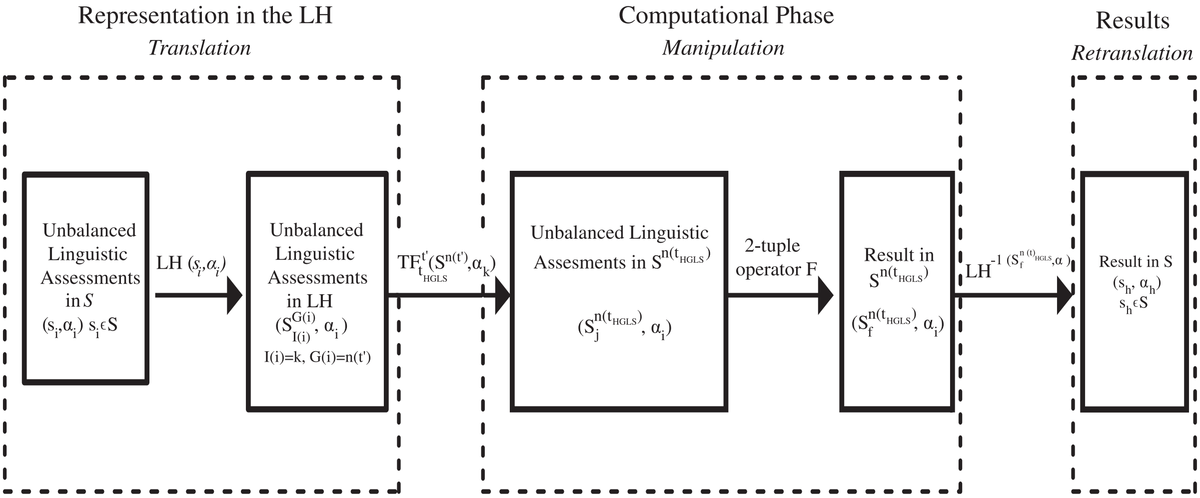

Fig. 11. Operating with unbalanced linguistic information.

In [43] was presented a 2-tuple based methodology to deal with unbalanced linguistic information, that provides an algorithm to represent the linguistic terms and a computational model to accomplish processes of CW based on the 2-tuple linguistic model.

The representation algorithm obtains the semantics, $LH(S)$, for an unbalanced term set, S, by using triangular membership functions computed from a Linguistic Hierarchy, LH (see Fig. 10 and Table 3).

Additionally the algorithm provides a boolean function, $Brid(S)$, that will be used in the processes of CW.

The computational model for unbalanced linguistic information accomplishes in an accurate way the processes of CW following the scheme presented in Fig. 11 based on the 2-tuple linguistic model and the use of linguistic hierarchies.

(a) *Representation in the linguistic hierarchy*: The representation algorithm uses a linguistic hierarchy, LH, to model the unbalanced terms. Therefore, the first step to accomplish processes of CW with this information is to transform it into their correspondent terms in the LH, $s_k^{n(t)} \in LH = \bigcup_t l(t, n(t))$, by using the transformation function, $\mathfrak{LH}$, that associates with each unbalanced linguistic 2-tuple $(s_i, \alpha) \in \overline{S}$ its respective linguistic 2-tuple in $LH(\overline{S})$, $\left(s_k^{n(t)}, \alpha\right), s_k^{n(t)} \in LH(S)$.

$$\begin{aligned} &\mathfrak{LH} : \overline{S} \to LH(\overline{S}),\ \textit{such that} \\ &\forall (s_i, \alpha_i) \in \overline{S} \Rightarrow \mathfrak{LH}(s_i, \alpha_i) = \left(s_{I(i)}^{G(i)}, \alpha_i\right). \end{aligned} \tag{8}$$

(b) *Computational phase*: It accomplishes the processes of CW by using the computational model defined for the LH (see Section 3.1.2).

(c) *Results*: A retranslation process is used to express the results in the original unbalanced term set, S, by linguistic 2-tuples using the transformation function, $\mathfrak{LH}^{-1}$, that associates with each linguistic 2-tuple expressed in $LH(\overline{S})$ its respective unbalanced linguistic 2-tuple in $\overline{S}$.

$$\mathfrak{LH}^{-1} : LH(\overline{S}) \to \overline{S}, \tag{9}$$

$\mathfrak{LH}^{-1}$ it was defined by cases [43] depending on the satisfaction of conditions imposed on $LH(S)$ and $Brid(S)$.

A further detailed description of the methodology to manage for unbalanced linguistic term sets can be found in [43].

4. New linguistic computational models based on 2-tuple representation

In spite of the youth of the 2-tuple representation model, many researchers has paid attention to it both for its application to different problems and for extending the model to improve several aspects in CW. In this section we review new linguistic computational models [32,114] based on extensions of the 2-tuple representation model and/or hybridizing it with other linguistic models.

4.1. Proportional 2-tuple linguistic computational model

Wang and Hao [114] introduced the proportional 2-tuple, that is a new way to represent and operate with the linguistic information, being a generalization and extension of 2-tuple linguistic representation model [44].

This model represents the linguistic information by means of proportional 2-tuples, such as (0.15*A*, 0.85*B*) for the case when someone's grades in the answer scripts of a whole course are distributed as 15%*A* and 85%*B*. The authors pointed out that if *B* were used as the approximative grade then some performance information would be lost. This proportional 2-tuple linguistic model is based on the concept of *symbolic proportion* [114].

Definition 8 [114]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be an ordinal term set, $I = [0,1]$ and

$$IS \equiv I \times S = \{(\alpha, s_i) : \alpha \in [0,1] \text{ and } i = \{0, 1, \ldots, g\} \tag{10}$$

where S is the ordered set of $g + 1$ ordinal terms $\{s_0, \ldots, s_g\}$. Given a pair (s_i, s_{i+1}) of two successive ordinal terms of S, any two elements (α, s_i), (β, s_{i+1}) of IS is so-called a symbolic proportion pair and α, β are a pair of symbolic proportions of the pair (s_i, s_{i+1}) if $\alpha + \beta = 1$. A symbolic proportion pair (α, s_i), $(1-\alpha, s_{i+1})$ is denoted by $(\alpha s_i, (1-\alpha)s_{i+1})$ and the set of all the symbolic proportion pairs is denoted by $\overline{\overline{S}}$, i.e., $\overline{\overline{S}} = \{(\alpha s_i, (1-\alpha)s_{i+1}) : \alpha \in [0,1] and i = \{0, 1, \ldots, g-1\}$.

$\overline{\overline{S}}$ is called the *ordinal proportional 2-tuple set* generated by S and the members of $\overline{\overline{S}}$, *ordinal proportional 2-tuples*, which are used to represent the ordinal information for CW.

In a similar way to the 2-tuple linguistic model, Wang and Hao introduced functions in order to facilitate the computations with this type of representation.

Definition 9 [114]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be an ordinal term set and $\overline{\overline{S}}$ be the ordinal proportional 2-tuple set generated by S. The function $\pi : \overline{\overline{S}} \to [0, g]$ was defined by

$$\pi((\alpha s_i, (1-\alpha)s_{i+1})) = i + (1-\alpha), \tag{11}$$

where $i = \{0, 1, \ldots, g-1\}$, $\alpha \in [0,1]$ and π is called the position index function of ordinal proportional 2-tuples.

Note that, under the identification convention which was remarked after the Eq. (10), the position index function π becomes a one-to-one mapping from $\overline{\overline{S}}$ to $[0,g]$ and its inverse $\pi^{-1} : [0,g] \to \overline{\overline{S}}$ is defined by

$$\pi^{-1}(x) = ((1-\beta)s_i, \beta s_{i+1}), x \in [0, g] \tag{12}$$

where $i = E(x)$, E is the integer part function, $\beta = x - i$.

Wang and Hao claimed that this model can operate in a precise way further of uniformly and symmetrically distributed triangular membership labels as the 2-tuple representation model. To do so, they propose the use of symmetrical trapezoidal fuzzy numbers $s_i = (a_i, b, c_i, d_i)$ and the use of their canonical characteristic values, $CCV(s_i) = (b_i + c_i)/2$ that were extended for proportional 2-tuples in [115] as follows:

Definition 10 [115]. Let S and $\overline{\overline{S}}$ and CCV on S as previously, the CCV for a proportional 2-tuple, $(\alpha s_i, (1-\alpha)s_{i+1}) \in \overline{\overline{S}}$, is defined:

$$CCV((\alpha s_i, (1-\alpha)s_{i+1})) = \alpha CCV(s_i), (1-\alpha)CCV(s_{i+1}). \tag{13}$$

To operate with linguistic information under proportional 2-tuple contexts, Wang and Hao expanded the computational techniques for symbolic information to proportional 2-tuples and underlying definitions of linguistic labels and linguistic variables are taken into account in the process of aggregating linguistic information by assigning CCV of the corresponding linguistic labels [114,115], and they presented an interesting transformation function between $\overline{S}$ and $\overline{\overline{S}}$:

Proposition 1 [114]. *Let S, $\overline{S}$ and $\overline{\overline{S}}$ be as before. The transformation function $h : \overline{\overline{S}} \to \overline{S}$ is defined as:*

$$h((\alpha s_i, (1-\alpha)s_{i+1})) = \begin{cases} (s_{i+1}, -\alpha), 0 \leqslant \alpha \leqslant 1/2 \\ (s_i, 1-\alpha), 1/2 \leqslant \alpha \leqslant 1 \end{cases} \tag{14}$$

Being h a one to one mapping.

4.2. Numerical scale. Extending the 2-tuple representation model

Dong et al. [32] considered that the key task of the 2-tuple based models [44,114] is the definition of a function that establishes a one to one mapping between the linguistic information and numerical values. Keeping this idea in mind, Dong et al. proposed an extension of the 2-tuple based models by means of the concept *numerical scale*.

Definition 11 [32]. Let $S = \{s_i | i = 0, \ldots, g\}$ be a linguistic term set and R be the real number set. The function $NS: S \to R$ is defined as a *numerical scale* of S and $NS(s_i)$ is so-called the numerical index of s_i.

Definition 12 [32]. Let $S, \overline{S}$ and NS on S be as previously. For $(s_i, \alpha) \in \overline{S}$ the function NS on $\overline{S}$ is defined by:

$$NS(s_i,\alpha) = \begin{cases} NS(s_i) + \alpha \times (NS(s_{i+1}) - NS(s_i)), \alpha \geqslant 0 \\ NS(s_i) + \alpha \times (NS(s_i) - NS(s_{i-1})), \alpha < 0 \end{cases} \tag{15}$$

It was also provided the following propositions and their proofs.

Proposition 2 [32]. *If $NS(s_i) = i$, for $i = 0, 1, \ldots, g$ then $NS(s_j, \alpha_j) = \Delta^{-1}(s_j, \alpha_j)$ for any $(s_i, \alpha_i) \in \bar{S}$.*

Remark 4. Proposition 2 is only valid with triangular membership functions. But if the semantics are represented by other membership functions as could be the proposition then it should be reconsidered.

Proposition 3 [32]. *If $NS(s_i) = CCV(s_i)$, for $i = 1, \ldots, g$ then $NS(s_j, \alpha_j) = CCV(h^{-1}((s_j, \alpha_j))$ for any $(s_i, \alpha_i) \in \bar{S}$.*

Therefore, according to *proposition 2* the 2-tuple linguistic model is obtained when setting $NS(s_i) = i$ and according to *proposition 3* the proportional 2-tuple linguistic model is obtained when setting $NS(s_i) = CCV(s_i)$. Dong et al. introduced in [32] different concepts and models such as, transitive calibration matrix, its consistent index and an optimization model. In order to compute the numerical scale of a linguistic term set with the aim to complete the 2-tuple based models for CW and make the information of the experts more consistent in different decision situations. In [31] are presented different scale functions to deal with linguistic 2-tuples in the well known multi-criteria decision making model, Analytic Hierarchy Process (AHP) [105].

5. Applications of the 2-tuple representation linguistic model

Once we have reviewed the preponderant position that the 2-tuple linguistic model plays among the different linguistic computing models for CW and its use as basis for different models in research purposes. In this section our aim is to show different applications (published in the specialized literature) based on this linguistic model.

Table 4
Applications based on the use of the 2-tuple linguistic model.

APPLICATIONS	PAPERS	YEAR
Supply chain	Yeh et al. [133]	2007
	Li and Xie [68]	2009
Nuclear safeguards	Rodríguez et al. [103]	2010
	Liu et al. [70]	2002
Risk evaluation	Chang and Wang [18]	2010
	Yu [134]	2009
	Pei and Shi [96]	2011
	Liu et al. [73]	2011
	Zhou et al. [142]	2011
Selection processes	Halouani et al. [41]	2009
	Balezentis and Balezentis [9]	2011
	Sun et al. [109]	2008
Engineering systems	Alcalá et al. [1]	2009
	Martínez et al. [82]	2006
Recommender systems	Porcel and Herrera-Viedma [98]	2010
	Rodríguez et al. [101]	2010
	Porcel et al. [100]	2012
	Martínez et al. [78]	2008
	Martínez et al. [84]	2007
Information retrieval	Li et al. [67]	2009
	Herrera-Viedma et al. [52]	2007
	Herrera-Viedma and López-Herrera [50]	2007
Sensory evaluation	Martínez et al. [79]	2009
	Martínez et al. [80]	2008
Human resources	Dursun and Karsak [35]	2010
	De Andrés and García-Lapresta [26]	2010
	De Andrés et al. [27]	2010
Cognition evaluation	Fan et al. [37]	2009
	Tai and Chen [110]	2009
Product design and development	Wang [117]	2009
	Martínez et al. [83]	2005
	Dhouib and Elloum [30]	2011
	Ngan [93]	2011

The 2-tuple linguistic model and its extensions have been applied to a wide variety of applications, mainly based on decision making and decision analysis problems. These applications range from evaluation processes for different aims to industrial ones passing by resource management, Internet based purposes, human resources and so on. Table 4 summarizes the previous 2-tuple applications among the whole number of applications developed.

6. Linguistic 2-tuple representation model. Challenges

The management of uncertain and vague information is always hard and complex, across this paper it has been showed that linguistic modelling is a good choice to model and manage such a type of information but it implies the accomplishment of processes of CW. The different views that exist about CW [89,90] open many problems that might be modelled and solved by means of linguistic modelling and CW processes.

Despite the different linguistic computing models introduced in the specialized literature, the symbolic models and mainly the 2-tuple linguistic one has provided tools that have improved significantly the solving processes based on CW, regarding the accuracy and understanding of the results of such processes. However, these achievements do not mean that every problem can be modelled and solved successfully by these models, on the contrary there still exist different challenges that linguistic computing models, hence the linguistic 2-tuple representation model must face to fulfil important needs required in problems dealing with linguistic information.

Currently the results obtained by symbolic computing models are easy to interpret and accurate but they are still quite inflexible because of the nature of the linguistic variables, a main challenge for these models is to enrich the vocabulary for expressing the results and increase the operations that can be applied to the linguistic information. These challenges should be fulfilled on the basis of the fuzzy linguistic approach and keeping the simplicity of the current models as much as possible from the understanding point of view.

Some proposals have been introduced in the literature to achieve the previous challenges [64,125], but they do not fulfil satisfactorily the previous objectives.

One promising direction that must be further researched is the use of free-context grammars together the symbolic models [102] to achieve the objectives aforementioned, such as it is showed in Fig. 12.

This general scheme figures out a basic process that should be further improved to fulfil the challenges of enriching the interpretability of results and increase the linguistic operators in processes of CW. For example, allowing either results or elicitation of linguistic preferences such as *higher than or equal to label "low"*.

The use of the linguistic 2-tuple extensions such as, unbalanced and multi-granular linguistic structures, have permitted the extension and enrichment of linguistic management in different types of problems, but still such extensions could be improved to achieve better results in decision problems. The former should progress in the modelling of any unbalanced structure that is not always possible, and the latter should explore different computing models that keep the accuracy of the results by using any membership function as semantic representation of the linguistic terms.

So far, most of problems dealing with uncertain information have applied a determined technique to model and manage such a uncertainty. However, it is clear that in real world problems the use of only one technique is not realistic, because different attributes, criteria, etc., suits better to different types of modelling. Therefore, another important challenge to deal with linguistic information in uncertain contexts, mainly in decision making problems, is the development of complex aggregation frameworks. Dong et al. [32] and Truck and Malefant [113] have proposed frameworks to integrate different symbolic models based on the 2-tuple linguistic model and its extensions. However our point of view because of the necessity of modelling uncertainty in different ways depending on the problem, the aggregation framework should be able to deal with the previous linguistic modelling approaches and the different extensions of fuzzy sets such as type 2 fuzzy sets [34,92], *type n fuzzy sets* [34], intuitionistic fuzzy sets [8], fuzzy multisets [127], hesitant fuzzy sets [112], the fuzzy linguistic approach [136–138] and its extensions. Not only such a complex aggregation framework would facilitate the hybridizing of the previous techniques that are quite common and useful for dealing with problems defined under uncertainty, but also

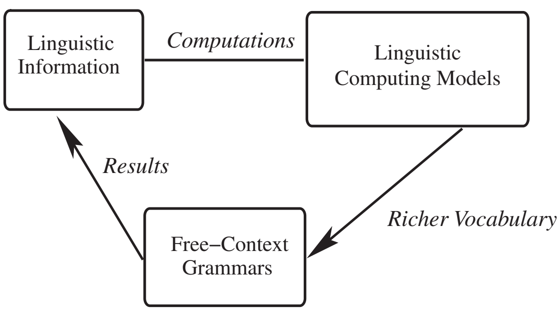

Fig. 12. Improving the results in processes of CW.

it would produce results that can be expressed and treated in the more suitable approach depending on the necessity of the solved problem.

Even though there would be other challenges to point out the previous ones could be the most interesting from a decision making and decision analysis point of view.

7. Concluding remarks

Uncertainty usually appears in many real world problems, the use of probability can cope with it in some situations. However, when such an uncertainty is not probabilistic the use of the linguistic information has provided very successful results. There exist different models to deal with linguistic information and accomplish the processes of CW, we have paid attention to one of them, the 2-tuple linguistic model that has been widely used in many fields and applications due to its accuracy and simplicity. This model not only provides a computational model for linguistic information but also has been extended to deal with complex frameworks. In order to provide a higher flexibility for modelling the uncertainty, decrease the complexity of linguistic computing processes and the hybridizing with other linguistic models. Eventually it has been served as basis for new linguistic computing models.

Notwithstanding the usefulness of the 2-tuple linguistic representation model evidenced by the great number of applications and decision making processes in which it has been applied, it is clear that there are still challenges in the field of CW that should be fulfilled in the coming years, we have pointed out some of these challenges and figured out some possible directions to achieve them by extending the 2-tuple linguistic representation model.

Acknowledgments

This paper has been partially supported by the research Projects TIN2009-08286 and Feder Fonds.

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