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METHODOLOGIES AND APPLICATION

Soft consensus measures in group decision making using unbalanced fuzzy linguistic information

F. J. Cabrerizo[1] · R. Al-Hmouz[2] · A. Morfeq[2] · A. S. Balamash[2] · M. A. Martínez[3] · E. Herrera-Viedma[2,4]

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Abstract An important question in group decision-making situations is how to estimate the consensus achieved within the group of decision makers. Dictionary meaning of consensus is a general and unanimous agreement among a group of individuals. However, most of the approaches deal with a more realistic situation of partial agreement. Defining a partial agreement of decision makers as a consensus up to some degree, the following question is how to obtain that soft degree of consensus. To do so, different approaches, in which the decision makers express their opinions by using symmetrical and uniformly distributed linguistic term sets, have been proposed. However, there exist situations in which the opinions are represented using unbalanced fuzzy linguistic term sets, in which the linguistic terms are not uniform and symmetrically distributed around the midterm. The aim of this paper was to study how to adapt the existing approaches obtaining soft consensus measures to handle group decision-making situations in which unbalanced fuzzy linguistic information is used. In addition, the advantages and drawbacks of these approaches are analyzed.

Keywords Group decision making · Consensus · Unbalanced fuzzy linguistic information

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✉ F. J. Cabrerizo
cabrerizo@issi.uned.es

R. Al-Hmouz
ralhmouz@kau.edu.sa

A. Morfeq
morfeq@kau.edu.sa

A. S. Balamash
asbalamesh@kau.edu.sa

M. A. Martínez
mundodesilencio@ugr.es

E. Herrera-Viedma
viedma@decsai.ugr.es

1 Department of Software Engineering and Computer Systems, Universidad Nacional de Educación a Distancia, 28040 Madrid, Spain

2 Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

3 Department of Social Services and Work, University of Granada, 18071 Granada, Spain

4 Department of Computer Science and Artificial Intelligence, University of Granada, 18071 Granada, Spain

1 Introduction

A group decision-making (GDM) situation is that in which there is a problem to solve, a set of possible alternatives, and a group of decision makers who convey their preferences about the alternatives. In such a situation, it is usual that the decision makers have unique goals and motivations, and therefore, the decision process may be approached from various angles. However, it is also usual that the decision makers have a joint concern in achieving agreement on choosing the "best" alternative (Chen and Hwang 1992; Fodor and Roubens 1994).

To express their opinions, the decision makers have usually used precise numerical values (Kacprzyk et al. 1992; Pérez et al. 2014a). Nevertheless, it seems natural that a decision maker utilizes linguistic terms (words) instead of precise numerical values to convey his/her assessments (Zadeh 1975a, b, c). It is due to the fact that the standard representation of the concept that humans utilize for interaction is the natural language. Therefore, a decision maker should express his/her opinions by using a fuzzy linguistic modeling

(Alonso et al. 2013; Cabrerizo et al. 2013, 2015b; Massanet et al. 2014; Pérez-Asurmendi and Chiclana 2014; Tejada-Lorente et al. 2014). For instance, to evaluate the "speed" of a car, linguistic terms like "very low," "low," or "fast" could be utilized.

Usually, the linguistic terms are symmetrically and uniformly distributed in the set, that is it is assumed the identical discrimination levels on both sides of the mid-linguistic term (Dong et al. 2013a; Herrera et al. 1997a; Tapia-García et al. 2012; Wang et al. 2015b). However, in some GDM situations, the alternatives have to be assessed by using linguistic term sets which are not symmetrical and uniformly distributed (Cabrerizo et al. 2010c; Dong et al. 2015a, c; Herrera et al. 2008; Jiang et al. 2015). For example, when a company carries out a consumer test to study the satisfaction of its product, the company is focused on obtaining the degree of satisfaction consumer: "completely satisfied," "very satisfied," and "slightly satisfied" (Estrella et al. 2014). However, if consumers are dissatisfied, generally, the company is not interested in knowing at what level. This particular type of linguistic term set is named unbalanced linguistic term set (Herrera et al. 2008).

Once the way in which the decision makers express their opinions is established, the next question is how to obtain the solution for the problem. To accomplish it, two processes are applied (Kacprzyk et al. 1992; Wu and Chiclana 2015): a consensus process and a selection process. On the one hand, the consensus process refers to how to get the maximum degree of agreement or consensus among the group of decision makers on the set of possible alternatives to solve the problem. On the other hand, the selection process consists in how to get the solution from the assessments given by the decision makers. Obviously, it is more desirable that the decision makers achieve a consensus before carrying out the selection process.

With the aim of measuring the level of consensus achieved within the group of decision makers, the similarity among the decision makers' opinions has to be obtained. Several approaches can be found in the literature to accomplish it (Cabrerizo et al. 2010b). Among them, those based on soft consensus measures are the most used because they depict better the human understanding of the basic nature of consensus (Herrera-Viedma et al. 2014; Kacprzyk and Fedrizzi 1986). According to it, consensus tries to reach the approval, not necessarily the agreement, of the decision makers by adapting opinions of all individuals implicated to achieve a decision which will yield. This decision will be advantageous to all the decision makers within the group, not necessarily to the individual decision maker that may give consent to what will not necessarily be his/her first option but because, for example, he/she wants to collaborate with the group. However, this consent does not signify that each decision maker is in full agreement (Butler and Rothstein 2006). In such a way, it makes more sense to speak about a degree of consensus, and here, the fuzzy set theory introduced by Zadeh (1965) has offered new instruments for the analysis of such imprecise phenomena like consensus.

This soft concept of consensus is based on the coincidence concept (Herrera et al. 1997a), measured by means of similarity criteria defined among the decision makers' preferences. A statistical comparative study of several similarity measures of consensus in GDM may be found in Chiclana et al. (2013). However, in some situations it is not viable to calculate directly the similarity among preferences and then, we can find some problems. This is the case, for example, when the decision makers utilize: (i) different representation formats to provide their opinions (Pérez et al. 2010), (ii) multi-granular fuzzy linguistic information (Morente-Molinera et al. 2015), (iii) unbalanced fuzzy linguistic information (Cabrerizo et al. 2010c; Herrera et al. 2008), or (iv) decision making under incomplete information (Alonso et al. 2008; Ureña et al. 2015).

The aim of this paper is to show how to adapt the existing approaches computing soft consensus measures to handle GDM situations defined in unbalanced fuzzy linguistic contexts. On the one hand, three coincidence criteria are identified to compute soft consensus measures: (i) strict coincidence among preferences, (ii) soft coincidence among preferences, and (iii) coincidence among solutions. On the other hand, we study their use in GDM situations in which the decision makers make use of unbalanced fuzzy linguistic information to provide their preferences. Finally, we analyze their drawbacks and advantages.

The rest of the paper is set out as follows. In Sect. 2, we present some considerations about GDM situations and describe the existing approaches to compute soft consensus measures. In Sect. 3, we describe GDM situations with unbalanced fuzzy linguistic information and show how to apply the existing approaches to compute soft consensus measures. An example of application of each one of the above approaches is illustrated in Sect. 4, and their advantages and drawbacks are discussed in Sect. 5. Finally, some conclusions are pointed out in Sect. 6.

2 Preliminaries

In this section, some important considerations about GDM situations are described, and the existing approaches computing soft consensus measures are introduced.

2.1 GDM situations

A standard GDM situation may be defined as a decision situation where (Fodor and Roubens 1994): there exists a group of decision makers, $E = \{e_1, \ldots, e_m\}$ $(m \geq 2)$, there is a problem to solve in which a solution must be selected among a set of feasible alternatives, $X = \{x_1, \ldots, x_n\}$ $(n \geq 2)$, and the decision makers try to reach a joint solution. The goal is to rank the possible alternatives from best to worst, providing them some degrees of preference.

There are several preference representation formats which may be utilized by the decision makers to verbalize their preferences (Herrera-Viedma et al. 2002): preference ordering of the alternatives, preference relations, utility functions, and so on. The most used ones are the preference relations because a decision maker has much more freedom when providing his/her opinions, and in addition, he/she may gain in expressivity. According to the domain which is being studied to assess the intensity of the preference, different types of preference relations may be used. The following definition expresses it:

Definition 1 A preference relation P on a set of alternatives X is characterized by a function $\mu_P : X \times X \to D$, where D is the domain of representation of preference degrees.

A preference relation P may be depicted by the $n \times n$ matrix $P = (p_{ik})$, being $p_{ik} = \mu_P(x_i, x_k)$ $(\forall i, k \in \{1, \ldots, n\})$ interpreted as the preference degree or intensity of the alternative x_i over x_k. In this case, if D is a linguistic domain, then linguistic terms as "low," "medium," or "high" could be utilized.

A way of solving a GDM problem is by carrying out a selection process consisting in choosing a solution set of alternatives according to the opinions expressed by the decision makers (Fodor and Roubens 1994), without taking into account the consensus achieved within the group of decision makers. It involves two steps (Cabrerizo et al. 2010a):

1. *Aggregation* In order to obtain a collective opinion, in this step of the selection process, all the opinions given by the decision makers are combined into only one preference structure that reflects or summarizes the properties contained in all the individual opinions. It may be carried out by means of aggregation operators defined for this purpose (Yager 1988).
2. *Exploitation* At this point, with the aim of identifying the solution set of alternatives, this step utilizes the information generated in the above step. Here, some mechanism must be applied to obtain a partial order of the alternatives and, in this way, selecting the best one(s). Among the different ways to do it, a usual one is to provide a utility value, based on the aggregated information, to each alternative, generating a natural order of the alternatives. To do so, two quantifier-guided choice degrees of alternatives may be utilized: a dominance and a non-dominance degree (Cabrerizo et al. 2010a).

However, this way of solving a GDM problem may lead solutions which are not well admitted by some decision makers (Butler and Rothstein 2006). It is because the decision makers could think that their opinions have not been considered correctly to obtain the solution. In such a way, these decision makers might refuse it. Therefore, it is recommendable that the decision makers conduct a consensus process in which they discuss and change their opinions gradually to reach a consensus before applying the selection process (Cabrerizo et al. 2014). Consequently, in a GDM situation, a consensus process and a selection process are usually applied before a final solution is obtained (Cabrerizo et al. 2015a; Herrera-Viedma et al. 2014).

With the aim of obtaining the consensus achieved among the group of decision makers, it is necessary to measure coincidence existing among them. To do so, GDM approaches determine soft consensus degrees, which are employed to obtain the level of agreement achieved among the group of decision makers during the decision process, given in three different levels of a preference relation (Herrera et al. 1996a): pairs of alternatives, alternatives, and relation. The computation of these soft consensus measures is as follows:

1. For each pair of decision makers, e_h and e_l, $(h = 1, \ldots, m-1,\ l = h+1, \ldots, m)$ a similarity matrix, $\mathrm{SM}^{hl} = (\mathrm{sm}^{hl}_{ik})$, is defined as:

$$\mathrm{sm}^{hl}_{ik} = 1 - d\left(p^h_{ik}, p^l_{ik}\right). \tag{1}$$

   where $d : D \times D \to [0, 1]$ is a distance function (Deza and Deza 2009). The closer to 1 sm^{hl}_{ik} is, the more similar p^h_{ik} and p^l_{ik} are.
2. A consensus matrix, $\mathrm{CM} = (\mathrm{cm}_{ik})$, is computed by means of the aggregation of all the $(m-1) \times (m-2)$ similarity matrices. This aggregation is carried out by means of an aggregation function, ϕ:

$$\mathrm{cm}_{ik} = \phi\left(\mathrm{sm}^{hl}_{ik}\right),\ h = 1, \ldots, m-1,\ l = h+1, \ldots, m. \tag{2}$$

3. Once CM is calculated, the consensus degrees are obtained at the three different levels:

   (a) *Consensus degree on pairs of alternatives*, cp_{ik}, It measures the consensus degree among all the decision makers on the pair of alternatives (x_i, x_k). This is expressed by the element of the consensus matrix CM:

$$\mathrm{cp}_{ik} = \mathrm{cm}_{ik}. \tag{3}$$

   (b) *Consensus degree on alternatives*, ca_i It measures the consensus degree among all the decision makers on the alternative x_i. In this case, this consensus degree is computed by aggregating the consensus degrees of all the pairs of alternatives implicating it:

$$\mathrm{ca}_i = \phi(\mathrm{cp}_{ik}), \quad k = 1, \ldots, n \wedge k \neq i. \tag{4}$$

(c) *Consensus degree on the relation, cr* It measures the global consensus degree among all the decision makers' assessments. It is calculated by means of the aggregation of all the consensus degrees at the level of alternatives:

$$\mathrm{cr} = \phi(\mathrm{ca}_i), \quad i = 1, \ldots, n. \tag{5}$$

The last consensus degree is the value utilized to control the consensus state. The closer to 1 cr is, the greater the consensus achieved among the group of decision makers is.

2.2 Approaches based on the coincidence concept to calculate soft consensus measures

To measure the current consensus state, soft consensus measures have to be computed. To do so, the similarity or the closeness among the preferences expressed by the decision makers on the alternatives is calculated. Using the concept of coincidence to obtain these soft consensus measures (Herrera et al. 1997a), three different approaches may be identified (Cabrerizo et al. 2010b):

- *Strict coincidence among preferences* Here, the coincidence is measured by means of similarity calculated among decision makers' preferences. There are two possible evaluations: value 1 meaning a total coincidence and value 0 meaning non-existent coincidence. Kacprzyk (1987) presented a first approach based on this strict concept of coincidence: given a pair of alternatives, if their values are equal, then they are in agreement (value 1). Otherwise, they are in disagreement (value 0). This approach was defined assuming fuzzy preference relations for representing the preferences provided by the decision makers. See Herrera et al. (1996a) and Herrera et al. (1997b) for other examples on how this strict concept of coincidence is utilized to define soft consensus measures in the case in which fuzzy linguistic preference relations are used.
- *Soft coincidence among preferences* In this situation, the coincidence is again measured by means of similarity obtained among decision makers' preferences. However, different partial coincidence degrees are here considered: it is assumed a gradual conception of the coincidence that is assessed in [0, 1]. Kacprzyk (1987) also presented a first consensus approach based on this gradual coincidence concept. See in Alonso et al. (2013), Bordogna et al. (1997), Chiclana et al. (2013), Herrera et al. (1997a), Herrera et al. (1997b) some examples of this soft concept of coincidence. It should be pointed out that this soft concept of coincidence is very used in GDM situations.
- *Coincidence among solutions* Here, the coincidence is measured by means of similarity criteria obtained among the individual solutions acquired from the decision makers' preferences. The coincidence is also a gradual concept evaluated in the unit interval, but here, we work in the locations of the alternatives observed in the individual solutions and the collective solution. Herrera-Viedma et al. (2002) proposed the first approach based on this concept of coincidence. Another example can be found in Ben-Arieh and Chen (2006). It should be pointed out that this concept of coincidence offers a more realistic consensus degree among the group of decision makers.

3 Soft consensus measures in GDM situations defined in unbalanced fuzzy linguistic contexts

In GDM situations defined in fuzzy linguistic contexts, decision makers have usually conveyed their testimonies by using linguistic variables assessed in linguistic term sets whose linguistic terms are uniformly and symmetrically distributed, that is, assuming the same discrimination levels on both sides of the mid-linguistic term (Alonso et al. 2013; Cabrerizo et al. 2015b; Dong et al. 2013a). For example, the following linguistic term set of nine linguistic terms could be used: {None = N, Quite Low = QL, Very Low = VL, Low = L, Medium = M, High = H, Quite High = QH, Very High = VH, Total = T} (see Fig. 1a). However, there exist problems that need to assess their variables with linguistic term sets that are not uniformly and symmetrically distributed (Cabrerizo et al. 2009; Herrera et al. 2008). The unbalanced fuzzy linguistic information could appear as a consequence of the nature of the linguistic variables that participate in the problem as it happens, for example, in the grading system (see Fig. 1b). In other cases, it could appear because decision makers need to deal with scales for assessing their preferences with a number of terms in a side of reference domain higher than in the other one (see Fig. 1c).

In this context, a decision maker e_h provides his/her preferences about the set of alternatives X by means of an unbalanced fuzzy linguistic preference relation, $P^h = (p^h_{ik})$. Therefore, in this case, the domain of representation of preference degrees, D, is an unbalanced fuzzy linguistic term set $S_{\mathrm{un}} = \{s_0, s_1, \ldots, s_g\}$, which has a minimum linguistic term, a maximum linguistic term, and a central linguistic term, and the remaining linguistic terms are non-uniformly and non-symmetrically distributed around the central one.

In the following subsections, we show how to apply the above coincidence concepts in GDM situations with unbal-

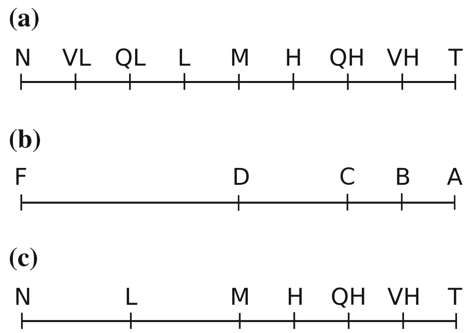

Fig. 1 Fuzzy linguistic information. **a** Linguistic term set, **b** grading system evaluations, **c** unbalanced linguistic term set

anced fuzzy linguistic information to compute soft consensus measures.

3.1 Soft consensus measures based on strict coincidence

A strict coincidence concept may easily be applied to obtain soft consensus measures in a GDM situation defined in an unbalanced fuzzy linguistic context. Here, it is not necessary to develop a computational linguistic model to obtain the similarity among the opinions given by the decision makers.

Following the scheme shown in Sect. 2.1 to compute soft consensus measures, the following distance function d is defined in order to obtain the similarity among the opinions given by the decision makers e_h and e_l:

$$d\left(p_{ik}^h, p_{ik}^l\right) = \begin{cases} 0 & \text{if } p_{ik}^h = p_{ik}^l \\ 1 & \text{otherwise} \end{cases} \tag{6}$$

Using the above distance function, the similarity matrices for each pair of decision makers are obtained. Then, using these similarity matrices, the consensus degree on pairs of alternatives, the consensus degree on alternatives, and the consensus degree on the relation are computed according to Eqs. (3)–(5), respectively.

3.2 Soft consensus measures based on soft coincidence

In this situation, we cannot apply directly the approaches to compute soft consensus measures defined in Alonso et al. (2013), Bordogna et al. (1997) and Herrera et al. (1997a, b), because it is not possible to obtain the coincidence values without previously to define a computational methodology to compare unbalanced fuzzy linguistic information. Then, we could act by developing a computational methodology in order to define a similarity function among unbalanced fuzzy linguistic assessments, or by defining a closeness table expressing the coincidence values among all possible unbalanced fuzzy linguistic assessments. In what follows, both possibilities are described.

3.2.1 Similarity functions

To define similarity functions among unbalanced fuzzy linguistic information, a methodology to manage this type of linguistic information need to be developed. To do so, different methodologies have been proposed in the literature (Cabrerizo et al. 2009; Dong et al. 2015c; Herrera et al. 2008; Herrera-Viedma and López-Herrera 2007; Wang et al. 2015a).

Here, with the aim of showing how to apply a soft coincidence concept to obtain soft consensus measures in GDM situations defined in an unbalanced fuzzy linguistic context using a similarity function, the methodology proposed by Herrera-Viedma and López-Herrera (2007) to manage unbalanced fuzzy linguistic information is used. This methodology is based on the transformation of the unbalanced fuzzy linguistic information in a Linguistic Hierarchy (LH) (Herrera and Martínez 2001) which is the linguistic representation domain that allows us to develop comparison and combination processes of unbalanced fuzzy linguistic information. However, it should be pointed out that any other approach dealing with unbalanced fuzzy linguistic information could be used, as, for example, the approach based on numerical scales (Dong et al. 2013b, 2015b; Dong and Herrera-Viedma 2015).

A LH is a set of levels, where each level represents a linguistic term set with different granularity from the remaining levels of the hierarchy. Each level is denoted as $l(t, n(t))$, where t is a number indicating the level of the hierarchy, and $n(t)$ is the granularity of the linguistic term set of t. Then, a LH can be defined as the union of all levels t:

$$\mathrm{LH} = \bigcup_t l(t, n(t)) \tag{7}$$

Given a LH, we denote as $S^{n(t)}$ the linguistic term set of LH corresponding to the level t of LH characterized by a cardinality $n(t)$: $S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$. Furthermore, the linguistic term set of the level $t+1$ is obtained from its predecessor as:

$$l(t, n(t)) \rightarrow l(t+1, 2 \cdot n(t) - 1) \tag{8}$$

Previously to define the similarity function among unbalanced fuzzy linguistic labels assessed on S_{un}, the following elements are applied:

1. The representation model defined in Herrera-Viedma and López-Herrera (2007) is used to represent S_{un} in LH:
   (a) Choose a level t^- of LH with an adequate granularity to represent the subset of linguistic terms of S_{un} on the left of the mid-linguistic term, and

(b) choose a level t^+ of LH with an adequate granularity to represent the subset of linguistic terms of S_{un} on the right of the mid-linguistic term.

2. To operate with the linguistic information in LH, the 2-tuple fuzzy linguistic model is used (Herrera and Martínez 2000).

Definition 2 Let S be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation; then, the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\begin{aligned} &\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5) \\ &\quad \Delta(\beta) = (s_i, \alpha) \\ &\quad i = round(\beta) \\ &\quad \alpha = \beta - i \end{aligned} \tag{9}$$

where "round" is the usual round operation, s_i has the closest index label to "β", and "α" is the value of the symbolic translation. Furthermore, there is always a function Δ^{-1}, such that from a 2-tuple value, it returns its equivalent numerical value $\beta \in [0, g] \subset \mathbb{R}$:

$$\begin{aligned} &\Delta^{-1} : S \times [-0.5, 0.5) \longrightarrow [0, g] \\ &\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \end{aligned} \tag{10}$$

3. Transformation functions among levels of LH (Herrera and Martínez 2001):

Definition 3 Let $\text{LH} = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$, and let us consider the 2-tuple fuzzy linguistic representation. The transformation function from a linguistic label in level t to a label in level t' is defined as $\text{TF}_{t'}^t : l(t, n(t)) \longrightarrow l(t', n(t'))$ such that:

$$\text{TF}_{t'}^t \left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta_{t'} \left(\frac{\Delta_t^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right) \cdot (n(t') - 1)}{n(t) - 1} \right) \tag{11}$$

Using these elements, we define the distance function $f : S_{\text{un}} \times S_{\text{un}} \to [0, 1]$ such that:

$$d(a, b) = \frac{\left|\Delta_{t'}^{-1}\left(\text{TF}_{t'}^t(a)\right) - \Delta_{t'}^{-1}\left(\text{TF}_{t'}^t(b)\right)\right|}{n(t') - 1} \tag{12}$$

where $a = (s_v^{n(t)}, \alpha_1)$, $b = (s_w^{n(t)}, \alpha_2)$, $t \in \{t^-, t^+\}$ and $t' \in \{t^-, t^+\}$, such that $n(t') = \max\{n(t^-), n(t^+)\}$.

This function allows us to compute the similarity matrix between each pair of decision makers. Again, using these similarity matrices, the consensus degree on pairs of alternatives, the consensus degree on alternatives, and the consensus degree on the relation are computed according to Eqs. (3)–(5), respectively.

It should be pointed out that the 2-tuple fuzzy linguistic modeling has been used here to manage the fuzzy linguistic information. However, this approach is pure ordinal based on the 2-tuple representation, and it cannot be considered fuzzy as no membership functions are used. Therefore, a cardinal approach based on the use of fuzzy sets could be also used. In such a case, a direct approach to aggregating information using the type-1 OWA operator is possible (Chiclana and Zhou 2013; Mata et al. 2014; Zhou et al. 2008, 2011). In fact, Pérez-Asurmendi and Chiclana (2014) proved that the two main representation methodologies of linguistic preferences, that is, the cardinal, based on the use of fuzzy sets, and the ordinal, based on the use of the 2-tuples, are indeed mathematically isomorphic when fuzzy numbers are represented using their respective centroids, and therefore, it can be concluded that the cardinal approach constitutes a more general framework to model linguistic information, and it preserves the original vagueness that is claimed to be useful in this context.

Table 1 Similarity table

Ω	N	L	M	H	QH	VH	T
N	1.0	0.7	0.5	0.3	0.2	0.1	0.0
L	0.7	1.0	0.7	0.5	0.3	0.2	0.1
M	0.5	0.7	1.0	0.8	0.7	0.6	0.5
H	0.3	0.5	0.8	1.0	0.8	0.7	0.6
QH	0.2	0.3	0.7	0.8	1.0	0.8	0.7
VH	0.1	0.2	0.6	0.7	0.8	1.0	0.8
T	0.0	0.1	0.5	0.6	0.7	0.8	1.0

3.2.2 Closeness table

On the other hand, a table may be used as in Herrera et al. (1997a). In this case, we can establish a closeness table, $\Omega : S_{\text{un}} \times S_{\text{un}} \to [0, 1]$, according to the decision makers' feeling. For example, if the linguistic term set shown in Fig. 1c is assumed, Table 1 could be defined.

Here, the similarity function is as follows:

$$\text{sm}_{ik}^{hl} = \Omega\left(p_{ik}^h, p_{ik}^l\right). \tag{13}$$

where p_{ik}^h indicates the preference value of the decision maker e_h over the alternatives (x_i, x_k) and p_{ik}^l indicates the preference value of the decision maker e_l over the alternatives (x_i, x_k).

Table 2 Similarity table representing a stricter concept of coincidence

Ω	N	L	M	H	QH	VH	T
N	1.0	0.5	0.1	0.0	0.0	0.0	0.0
L	0.5	1.0	0.5	0.2	0.0	0.0	0.0
M	0.1	0.5	1.0	0.7	0.5	0.1	0.0
H	0.0	0.2	0.7	1.0	0.7	0.3	0.1
QH	0.0	0.0	0.5	0.7	1.0	0.7	0.5
VH	0.0	0.0	0.1	0.3	0.7	1.0	0.7
T	0.0	0.0	0.0	0.1	0.5	0.7	1.0

We should point out that we can change the values of the table in order to have a concept of coincidence more or less strict. Therefore, a similarity table representing a concept of coincidence stricter could be as shown in Table 2.

3.3 Soft consensus measures based on coincidence among solutions

Here, we show how to apply the approach using a concept of coincidence based on solutions, i.e., comparing the positions of the alternatives between the individual solutions and the collective solution, when dealing with unbalanced fuzzy linguistic information. In this case, we use again the methodology presented in Herrera-Viedma and López-Herrera (2007) in order to manage the unbalanced fuzzy linguistic information.

In this approach, in order to compute the soft consensus measures, firstly, a selection process is applied to obtain a temporary collective solution. Then, the closeness between the individual solutions and the collective solution is measured. The steps are as follows:

1. To obtain the collective ordered vector of alternatives (temporary collective solution) V^c. To do so, a selection process is applied:

   (a) *Aggregation* To obtain the collective preference relation $P^c = (p^c_{ik})$, all the individual preference relations $\{P^1, P^2, \ldots, P^m\}$ have to be aggregated. Here, the LOWA$_{\text{un}}$ operator, which is an extension of the linguistic ordered weighted averaging (LOWA) operator (Herrera et al. 1996b), is used. It is defined as follows:

Definition 4 Let $\{(a_1, \alpha_1), \ldots, (a_m, \alpha_m)\}$ be a set of unbalanced assessments to aggregate; then, the LOWA$_{\text{un}}$ operator ϕ_{un} is defined as:

$$\begin{aligned}\phi_{\text{un}}\{(a_1, \alpha_1), \ldots, (a_m, \alpha_m)\} &= W \cdot B^{\mathrm{T}}\\ &= C^m_{\text{un}}\{w_k, b_k,\ k = 1, \ldots, m\}\\ &= w_1 \otimes b_1 \oplus (1 - w_1) \otimes C^{m-1}_{\text{un}}\{\beta_h, b_h,\ h = 2, \ldots, m\}\end{aligned} \tag{14}$$

where $b_i = (a_i, \alpha_i) \in (S \times [-0.5, 0.5))$, $W = [w_1, \ldots, w_m]$, is a weighting vector, such that, $w_i \in [0, 1]$ and $\sum_i w_i = 1$, $\beta_h = \frac{w_h}{\sum_2^m w_k}$, $h = 2, \ldots, m$, and B is the associated ordered unbalanced 2-tuple vector. Each element $b_i \in B$ is the ith largest unbalanced 2-tuple in the collection $\{(a_1, \alpha_1), \ldots, (a_m, \alpha_m)\}$, and C^m_{un} is the convex combination operator of m unbalanced 2-tuples. If $w_j = 1$ and $w_i = 0$ with $i \neq j\ \forall i, j$ the convex combination is defined as: $C^m_{\text{un}}\{w_i, b_i,\ i = 1, \ldots, m\} = b_j$. And if $m = 2$ then it is defined as:

$$\begin{aligned}&C^2_{\text{un}}\{w_l, b_l,\ l = 1, 2\}\\ &\quad= w_1 \otimes b_j \oplus (1 - w_1) \otimes b_i = \mathrm{TF}^{t'}_t\left(s^{n(t')}_k, \alpha\right)\end{aligned} \tag{15}$$

where $(s^{n(t')}_k, \alpha) = \Delta(\lambda)$ and $\lambda = \Delta^{-1}(TF^t_{t'}(b_i)) + w_1 \cdot (\Delta^{-1}(TF^t_{t'}(b_j)) - \Delta^{-1}(\mathrm{TF}^t_{t'}(b_i)))$, $b_j, b_i \in (\mathcal{S} \times [-0.5, 0.5))$, $(b_j \geq b_i)$, $\lambda \in [0, n(t') - 1]$.

Yager (1988) defined an expression to obtain W by means of a fuzzy linguistic non-decreasing quantifier Q (Zadeh 1983):

$$w_i = Q(i/m) - Q((i-1)/m), \quad i = 1, \ldots, m \tag{16}$$

   (b) *Exploitation* To obtain the global ranking of the alternatives, the quantifier-guided dominance degree QGDD is used. For the alternative x_i, the QGDD$_i$, used to quantify the dominance that alternative x_i has over all the others in a fuzzy majority sense, is calculated as follows:

$$\mathrm{QGDD}_i = \phi_{\text{un}}\left(p^c_{ik},\ k = 1, \ldots, n\right) \tag{17}$$

2. Calculating the individual ordered vector of alternatives (individual solution) V^h for every decision maker e_h. To do so, we apply directly the exploitation step on each individual unbalanced fuzzy linguistic preference relation P^h.
3. Calculating the proximity of each decision maker e_h for each alternative x_i, called $p^h(x_i)$, by comparing the ranking positions of that alternative in the decision makers' individual solution V^h (symbolized by V^h_i) and in the collective solution V^c (symbolized by V^c_i) as $p^h(x_i) = p(V^h, V^c)(x_i) = f(|V^c_i - V^h_i|)$. As a general dissimilarity function, $f(x) = (a \cdot x)^b$, $1 \geq b \geq 0$

may be considered, and in particular, the function taking $a = 1/(n-1)$ may be used, and then:

$$p^h(x_i) = p\left(V^h, V^c\right)(x_i) = f\left(\left|V_i^c - V_i^h\right|\right) = \left(\frac{\left|V_j^c - V_i^h\right|}{n-1}\right)^b \in [0,1] \tag{18}$$

The parameter b controls the rigorousness of the consensus process, in such a way that values of b close to one decrease the rigorousness and therefore the number of rounds to develop in the group discussion process, and values of b close to zero increase the rigorousness and therefore the number of rounds. Appropriate values for b are: 0.5, 0.7, 0.9, 1.

4. Calculating the consensus degree of all decision makers on each alternative x_i using the following expression:

$$C(x_i) = 1 - \sum_{h=1}^{m} \frac{p^h(x_i)}{m} \tag{19}$$

5. The consensus measure over the set of alternatives, called C_X, will be calculated by the aggregation of the above consensus degrees on the alternatives. It is considered that the consensus degrees about the solution set of alternatives has to take a more important weight in this aggregation. To do so, the S-OWA OR-LIKE operator defined by Yager and Filev (1994) is used:

$$\begin{aligned} C_X &= S_{\text{OWA OR-LIKE}}(\{C(x_s);\ x_s \in X_{\text{sol}}\},\\ &\{C(x_t);\ x_t \in X - X_{\text{sol}}\}) =\\ &= (1-\beta)\cdot\sum_{t=1}^{\nu}\frac{C(x_t)}{\nu} + \beta\cdot\sum_{s=1}^{\gamma}\frac{C(x_s)}{\gamma} \end{aligned} \tag{20}$$

where γ is the cardinal of the set X_{sol}; ν is the cardinal of the set $X - X_{\text{sol}}$; $\beta \in [0,1]$. β is a parameter to control the OR-LIKE behavior of the aggregation operator. The higher the value of β, the higher the influence of the consensus degrees of the solution alternatives on the global consensus degree.

4 Example of application

Let us suppose four decision makers $E = \{e_1, e_2, e_3, e_4\}$ providing the following preference relations on a set of four alternatives $X = \{x_1, x_2, x_3, x_4\}$ using the unbalanced fuzzy linguistic term set given in Fig. 1c.

$$P^1 = \begin{pmatrix} - & H & \text{QH} & L \\ L & - & M & H \\ L & M & - & L \\ \text{VH} & L & \text{VH} & - \end{pmatrix}; \quad P^2 = \begin{pmatrix} - & H & H & \text{VH} \\ L & - & \text{QH} & \text{T} \\ L & L & - & H \\ L & N & L & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & H & M & \text{VH} \\ L & - & \text{QH} & L \\ L & L & - & T \\ L & H & N & - \end{pmatrix}; \quad P^4 = \begin{pmatrix} - & L & \text{QH} & M \\ \text{QH} & - & M & \text{VH} \\ L & M & - & L \\ M & L & \text{QH} & - \end{pmatrix}$$

In the following subsections, we are going to show how to apply the above three concepts of coincidence in order to obtain the consensus degree achieved among the above group of decision makers.

4.1 Soft consensus measures based on strict coincidence

In this case, to obtain the soft consensus measures, firstly, we compute the similarity matrices for each pair of decision makers are calculated using Eq. (6). As we have $\frac{m^2-m}{2}$ pairs of decision makers, then we have to compute six similarity matrices:

$$\text{SM}^{12} = \begin{pmatrix} - & 1.00 & 0.00 & 0.00 \\ 1.00 & - & 0.00 & 0.00 \\ 1.00 & 0.00 & - & 0.00 \\ 0.00 & 0.00 & 0.00 & - \end{pmatrix};$$

$$\text{SM}^{13} = \begin{pmatrix} - & 1.00 & 0.00 & 0.00 \\ 1.00 & - & 0.00 & 0.00 \\ 1.00 & 0.00 & - & 0.00 \\ 0.00 & 0.00 & 0.00 & - \end{pmatrix}$$

$$\text{SM}^{14} = \begin{pmatrix} - & 0.00 & 1.00 & 0.00 \\ 0.00 & - & 1.00 & 0.00 \\ 1.00 & 1.00 & - & 1.00 \\ 0.00 & 1.00 & 0.00 & - \end{pmatrix};$$

$$\text{SM}^{23} = \begin{pmatrix} - & 1.00 & 0.00 & 1.00 \\ 1.00 & - & 1.00 & 0.00 \\ 1.00 & 1.00 & - & 0.00 \\ 1.00 & 0.00 & 0.00 & - \end{pmatrix}$$

$$\text{SM}^{24} = \begin{pmatrix} - & 0.00 & 0.00 & 0.00 \\ 0.00 & - & 0.00 & 0.00 \\ 1.00 & 0.00 & - & 0.00 \\ 0.00 & 0.00 & 0.00 & - \end{pmatrix};$$

$$\text{SM}^{34} = \begin{pmatrix} - & 0.00 & 0.00 & 0.00 \\ 0.00 & - & 0.00 & 0.00 \\ 1.00 & 0.00 & - & 0.00 \\ 0.00 & 0.00 & 0.00 & - \end{pmatrix}$$

Then, the consensus matrix is computed using Eq. (2) and the arithmetic mean as aggregation operator ϕ:

$$\mathrm{CM} = \begin{pmatrix} - & 0.50 & 0.17 & 0.17 \\ 0.50 & - & 0.33 & 0.00 \\ 1.00 & 0.33 & - & 0.17 \\ 0.17 & 0.17 & 0.00 & - \end{pmatrix}$$

It should be point out that different aggregation operators could be used depending on the nature of the GDM problem to solve (Chiclana et al. 2013). Finally, the soft consensus measures are obtained using Eqs. (3)–(5), respectively, and the arithmetic mean as aggregation operator ϕ:

1. *Consensus degrees on pairs of alternatives* The element (i, k) of CM represents the consensus degrees on the pair of alternatives (x_i, x_k).
2. *Consensus degrees on alternatives:*

$$\mathrm{ca}_1 = 0.42, \quad \mathrm{ca}_2 = 0.30, \quad \mathrm{ca}_3 = 0.33, \quad \mathrm{ca}_4 = 0.11.$$

3. *Consensus degree on the relation:*

$$\mathrm{cr} = 0.29.$$

4.2 Soft consensus measures based on soft coincidence: similarity function

In this case, to compute the soft consensus measures, we use the methodology to manage unbalanced fuzzy linguistic information defined using the hierarchical linguistic contexts based on the linguistic 2-tuple computational model described in Sect. 3.2.1. Then, the similarity matrices are firstly computed:

$$\mathrm{SM}^{12} = \begin{pmatrix} - & 1.00 & 0.87 & 0.37 \\ 1.00 & - & 0.75 & 0.62 \\ 1.00 & 0.75 & - & 0.62 \\ 0.37 & 0.75 & 0.37 & - \end{pmatrix};$$

$$\mathrm{SM}^{13} = \begin{pmatrix} - & 1.00 & 0.75 & 0.37 \\ 1.00 & - & 0.75 & 0.62 \\ 1.00 & 0.75 & - & 0.00 \\ 0.37 & 0.62 & 0.12 & - \end{pmatrix}$$

$$\mathrm{SM}^{14} = \begin{pmatrix} - & 0.62 & 1.00 & 0.75 \\ 0.50 & - & 1.00 & 0.75 \\ 1.00 & 1.00 & - & 1.00 \\ 0.62 & 1.00 & 0.87 & - \end{pmatrix};$$

$$\mathrm{SM}^{23} = \begin{pmatrix} - & 1.00 & 0.87 & 1.00 \\ 1.00 & - & 1.00 & 0.25 \\ 1.00 & 1.00 & - & 0.62 \\ 1.00 & 0.62 & 0.75 & - \end{pmatrix}$$

$$\mathrm{SM}^{24} = \begin{pmatrix} - & 0.62 & 0.87 & 0.62 \\ 0.50 & - & 0.75 & 0.87 \\ 1.00 & 0.75 & - & 0.62 \\ 0.75 & 0.75 & 0.50 & - \end{pmatrix};$$

$$\mathrm{SM}^{34} = \begin{pmatrix} - & 0.62 & 0.75 & 0.62 \\ 0.50 & - & 0.75 & 0.37 \\ 1.00 & 0.75 & - & 0.25 \\ 0.75 & 0.62 & 0.25 & - \end{pmatrix}$$

Then, the consensus matrix is computed using Eq. (2) and the arithmetic mean as aggregation operator ϕ:

$$\mathrm{CM} = \begin{pmatrix} - & 0.81 & 0.85 & 0.62 \\ 0.75 & - & 0.83 & 0.58 \\ 1.00 & 0.83 & - & 0.68 \\ 0.64 & 0.72 & 0.47 & - \end{pmatrix}$$

Finally, the soft consensus measures are obtained using Eqs. (3)–(5), respectively, and the arithmetic mean as aggregation operator ϕ:

1. *Consensus degrees on pairs of alternatives* The element (i, k) of CM represents the consensus degrees on the pair of alternatives (x_i, x_k).
2. *Consensus degrees on alternatives:*

$$\mathrm{ca}_1 = 0.77, \quad \mathrm{ca}_2 = 0.75, \quad \mathrm{ca}_3 = 0.77, \quad \mathrm{ca}_4 = 0.61.$$

3. *Consensus degree on the relation:*

$$\mathrm{cr} = 0.72.$$

4.3 Soft consensus measures based on soft coincidence: closeness table

To compute the similarity matrix for each pair of decision makers, we use the values of Table 1. According to it, the following similarity matrices are obtained:

$$\mathrm{SM}^{12} = \begin{pmatrix} - & 1.00 & 0.80 & 0.20 \\ 1.00 & - & 0.80 & 0.60 \\ 1.00 & 0.70 & - & 0.50 \\ 0.20 & 0.70 & 0.20 & - \end{pmatrix};$$

$$\mathrm{SM}^{13}=\begin{pmatrix} - & 1.00 & 0.70 & 0.20\\ 1.00 & - & 0.70 & 0.50\\ 1.00 & 0.70 & - & 0.10\\ 0.20 & 0.50 & 0.10 & - \end{pmatrix}$$

$$\mathrm{SM}^{14}=\begin{pmatrix} - & 0.50 & 1.00 & 0.70\\ 0.30 & - & 1.00 & 0.60\\ 1.00 & 1.00 & - & 1.00\\ 0.60 & 1.00 & 0.80 & - \end{pmatrix};$$

$$\mathrm{SM}^{23}=\begin{pmatrix} - & 1.00 & 0.80 & 1.00\\ 1.00 & - & 1.00 & 0.10\\ 1.00 & 1.00 & - & 0.50\\ 1.00 & 0.30 & 0.70 & - \end{pmatrix}$$

$$\mathrm{SM}^{24}=\begin{pmatrix} - & 0.50 & 0.80 & 0.60\\ 0.30 & - & 0.70 & 0.80\\ 1.00 & 0.70 & - & 0.50\\ 0.70 & 0.70 & 0.30 & - \end{pmatrix};$$

$$\mathrm{SM}^{34}=\begin{pmatrix} - & 0.50 & 0.70 & 0.60\\ 0.30 & - & 0.70 & 0.20\\ 1.00 & 0.70 & - & 0.10\\ 0.70 & 0.50 & 0.20 & - \end{pmatrix}$$

Then, the consensus matrix is computed using Eq. (2) and the arithmetic mean as aggregation operator ϕ:

$$\mathrm{CM}=\begin{pmatrix} - & 0.75 & 0.81 & 0.55\\ 0.65 & - & 0.81 & 0.47\\ 1.00 & 0.80 & - & 0.45\\ 0.57 & 0.62 & 0.38 & - \end{pmatrix}$$

Finally, the soft consensus measures are obtained using Eqs. (3)–(5), respectively, and the arithmetic mean as aggregation operator ϕ:

1. *Consensus degrees on pairs of alternatives* The element (i, k) of CM represents the consensus degrees on the pair of alternatives (x_i, x_k).
2. *Consensus degrees on alternatives:*

$$\mathrm{ca}_1 = 0.72,\quad \mathrm{ca}_2 = 0.68,\quad \mathrm{ca}_3 = 0.71,\quad \mathrm{ca}_4 = 0.51.$$

3. *Consensus degree on the relation:*

$$\mathrm{cr} = 0.66.$$

4.4 Soft consensus measures based on coincidence among solutions

In this case, the soft consensus degrees are obtained as follows:

1. Obtaining the collective ordered vector of alternatives V^c:

   (a) *Aggregation* Firstly, we obtain the collective unbalanced fuzzy linguistic preference relation by aggregating all individual preference relations. We use the $\mathrm{LOWA}_{\mathrm{un}}$ operator and the linguistic quantifier *most of* defined as $Q(r) = r^{1/2}$, which applying Eq. (16), generates the following weighting vector $W = \{0.5, 0.20, 0.16, 0.14\}$.

$$\mathrm{CM}=\begin{pmatrix} - & (M,0.00) & (H,0.26) & (H,0.13)\\ (M,0.00) & - & (H,0.20) & (\mathrm{QH},-0.30)\\ (L,0.00) & (M,-0.42) & - & (H,0.30)\\ (M,-0.30) & (L,0.42) & (M,0.23) & - \end{pmatrix}$$

   (b) *Exploitation* The quantifier-guided dominance degree, QGDD, is applied to obtain the ordered vector of alternatives. We use again the same fuzzy quantifier *most of* and the corresponding weighting vector $W = \{0.5, 0.20, 0.16, 0.14\}$. Then, the following QGDD_i are obtained.

$$\mathrm{QGDD}_1 = (\mathrm{H}, -0.25)$$
$$\mathrm{QGDD}_2 = (\mathrm{H}, 0.00)$$
$$\mathrm{QGDD}_3 = (\mathrm{M}, -0.20)$$
$$\mathrm{QGDD}_4 = (\mathrm{M}, -0.25)$$

   The collective ordered vector of alternatives is $\{x_2, x_1, x_3, x_4\}$.

2. Calculating the ordered vector of alternatives (individual solution) for every decision maker $\{V^h;\ h = 1, \ldots, m\}$:

$$e_1 : \{x_4, x_1, x_2, x_3\}$$
$$e_2 : \{x_1, x_2, x_3, x_4\}$$
$$e_3 : \{x_1, x_3, x_2, x_4\}$$
$$e_4 : \{x_2, x_4, x_1, x_3\}$$

3. The differences between the ranking of causes in the temporary collective solution and the individual solution are as follows:
4. Consensus degrees on alternatives calculated for $b = 1$:

$$(C(x_1), C(x_2), C(x_3), C(x_4)) = (0.75, 0.58, 0.75, 0.58).$$

$V_j^c - V_j^h$	x_1	x_2	x_3	x_4
e_1	0	−2	−1	3
e_2	1	−1	0	0
e_3	1	−2	1	0
e_4	−1	0	−1	2

5. Consensus measure calculated for $b = 1$ and $\beta = 0.8$ is:

$$C_X = 0.60.$$

5 Discussion

In this section, the advantages and drawbacks of the different approaches used to obtain soft consensus measures in unbalanced fuzzy linguistic contexts are analyzed.

1. *Approach using strict coincidence among preferences* This approach compares the decision makers' opinions about the alternatives and assigns a value of 1 if the opinions are equal and a value of 0 in another case. The advantages of this approach are: (i) the computation of the soft consensus degrees is simple and easy because if $p^l_{ik} = p^h_{ik}$, it assigns a value of 1 and otherwise a value of 0, and therefore, (ii) it does not need a methodology to manage unbalanced fuzzy linguistic information. However, its drawback is that the soft consensus degrees obtained do not reflect the real consensus state within the group of decision makers because it only assigns values of 1 or 0 when comparing the decision makers' opinions about the alternatives and it is not similar a value of 0 if we compare $p^h_{ik} = H$ with $p^l_{ik} =$ VH than if we compare $p^h_{ik} = M$ with $p^l_{ik} =$ VH.
2. *Approach using soft coincidence based on similarity functions among preferences* This approach compares the decision makers' opinions about the alternatives and assigns a value provided by a similarity function. The advantage of this approach is that the soft consensus degrees computed reflect the real consensus state within the group of decision makers because they are obtained using similarity functions that assign values between 0 and 1, which are not so strict as in the above approach. However, the drawback of this approach is that it needs a methodology to manage unbalanced fuzzy linguistic information with the aim of computing the soft consensus degrees, and therefore, the computation of the soft consensus degrees is more difficult than in the above approach.
3. *Approach using soft coincidence based on closeness tables among preferences* This approach compares the decision makers' opinions about the alternatives and assigns a value provided by a similarity or closeness table. The advantages of this approach are: (i) the soft consensus degrees calculated reflect the real consensus state because they are obtained using similarity tables that assign values in between 0 and 1 that are not so strict as in the first approach, (ii) we can establish the values of the similarity table, and it can be more or less strict, (iii) it does not need a methodology to manage unbalanced fuzzy linguistic information because the coincidence between the opinions is obtained from the table, and therefore, (iv) the computation of the soft consensus degrees is simple and easy. However, the drawback of this approach is the way in which the values of the similarity table are obtained because it needs the agreement among the decision makers and an environment where the decision makers can discriminate perfectly the same linguistic term set under a similar conception (Herrera et al. 1997a).
4. *Approach using coincidence among solutions* This approach obtains the soft consensus degrees comparing the positions of the alternatives between the collective solution and the individual solutions. On the one hand, the advantage of this approach is that the soft consensus degrees are computed comparing not the preferences but the position of the alternatives in each solution, allowing us to indicate the real consensus state in each moment of the consensus process. On the other hand, the drawbacks of this approach are: (i) it needs a methodology to manage unbalanced fuzzy linguistic information with the aim of obtaining the consensus degrees and (ii) the computation of the consensus degrees is more difficult than in the above approaches because a selection process has to be applied before obtaining the soft consensus degrees.

Table 3 summarizes all the approaches used to obtain soft consensus measures in unbalanced fuzzy linguistic contexts and shows their respective advantages and drawbacks.

6 Concluding remarks

In many real-world situations, it is important to reach decisions with a high level of consensus as, for example, in digital libraries, in which the decisions have to be made according to the opinions expressed by many users (Pérez et al. 2014b), or in systems enabling the automatic identification of occurring situations, in which solid mechanisms are needed for reaching a shared consensus on the same observations among the agents monitoring the same phenomenon (D'Aniello et al. 2015a, b).

In this paper, we have studied different approaches to obtain soft consensus degrees in GDM situations using unbalanced fuzzy linguistic information to represent the preferences given by the decision makers. We have analyzed the approaches using a strict concept of coincidence, the

Table 3 Advantages and drawbacks of the soft consensus approaches

Approach	Advantages	Drawbacks
Strict coincidence among preferences	The computation of the soft consensus degrees is simple and easy A methodology to manage unbalanced fuzzy linguistic information is not needed	It does not reflect the real consensus state within the group of decision makers
Soft coincidence based on similarity functions	It reflects the real consensus state within the group of decision makers	A methodology to manage unbalanced fuzzy linguistic information is needed The computation of the soft consensus degrees is complex
Soft coincidence based on closeness tables	It reflects the real consensus state within the group of decision makers The computation of the soft consensus degrees is simple and easy A methodology to manage unbalanced fuzzy linguistic information is not needed The values of the closeness table can be established in such a way that they can be more or less strict	The way in which the values of the closeness table are obtained
Coincidence among solutions	It reflects the real consensus state in each moment of the consensus process	A methodology to manage unbalanced fuzzy linguistic information is needed The computation of the consensus degrees is complex A selection process has to be applied before obtaining the soft consensus degrees

approaches using a soft concept of coincidence, and the approaches based on solutions, i.e., comparing the positions of the alternatives between the individual solutions and collective solution. Finally, we have compared these approaches and it may be concluded that the approach using a soft concept of coincidence with similarity tables is the best approach to use when we are dealing with unbalanced fuzzy linguistic information.

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Compliance with ethical standards

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