A Fuzzy Group Decision Making Model for Large Groups of Individuals

S. Alonso, I.J. Pérez, F.J. Cabrerizo and E. Herrera-Viedma

***Abstract*— Group Decision Making (GDM) refers to the selection of an alternative from a set of feasible alternatives that better satisfies some criteria according to a group of individuals (experts). There exist several different models to simulate GDM processes, but many of those models do not usually take into account some dynamical aspects of real decision processes. For example, those models normally do not allow the experts set to change during the process (adding or removing experts), the alternatives to change (incorporating or discarding alternatives) or even to change the criteria. In this work we present a new model which allows to undertake GDM situations in which a large number of individuals (for example an on-line community) has to choose among different alternatives. To be able to obtain a good solution of consensus, the group of experts will be firstly simplified into a smaller group (using a simple clustering technique and a kind of trust network) which can then discuss about best solution to be selected.**

I. Introduction

Group Decision Making (GDM) is a very common human activity that refers to the selection of the best option from a set of feasible alternatives according to the opinions of a group of individuals (usually referred as experts). The main goal of any GDM process is to identify the best alternative according to some established criteria, and it is normally assumed that the experts have a common interest in obtaining a final solution for the problem. Examples of typical GDM processes are to vote in an election, to choose a place for family vacations or to select the model of laptop that a firm will buy to its employees.

There have been several efforts in the specialized literature to create different models to correctly address and solve GDM situations. Many of them make use of fuzzy theory as it is a good tool to model and deal with vague or imprecise opinions (which is a quite common situation in any GDM process) [1], [2]. Many of those models are usually focused on solving GDM situations in which a particular issue or difficulty is present. For example, there have been models that allow to use linguistic assessments instead of numerical ones, thus making it easier for the experts to express their preferences about the alternatives [3]. Other models allow experts to use multiple preference structures (and even multi-granular linguistic information) [4], [5] and other different approaches deal with incomplete information situations if experts are not able to provide all their preferences when solving a GDM problem [6] or when a consensus process is carried out [7].

However, there are still many different situations in real GDM problems that have not been addressed. For example, situations in which the group of experts vary over time are quite common in real decision processes: a new expert could incorporate to the process, some experts could leave it or a large group of experts could be simplified in order to minimize communications and to ease the computation of solutions. For instance, in democratic systems it is usual that the individuals delegate into a smaller group of experts to make decisions as it is usually not possible to involve everyone in each decision. There have been some efforts to model this kind of situations: in [8] a recursive procedure allows to select a qualified subgroups of individuals taking into account their own opinions about the group.

In this paper we introduce a new model specially designed to manage GDM situations where the group of experts is large. This kind of situations are nowadays quite common in on-line communities [9]. We can think of a large group of people (hundreds or even thousands) that share a common interest and that form an on-line community about their topic of interest. At a certain point, some decisions should be made, as choosing a better web hosting, or choosing a particular date and place to make a meeting of the community members. In those situations, to carry out a proper GDM process is a difficult task. For example, not every member of the community is willing to participate and contribute to solve the problem [10]. Usual approaches involve using opinion polls and forums, but those methods do not offer methods to control the consensus of the process and to control the discussions (which tend to be very disperse due to the large number of speakers). To solve these issues, we will introduce a preliminary step in the GDM process in which the group of experts is simplified by using a simple clustering algorithm and where the decision process is carried out using a particular kind of trust network [11] which allows to take into account the opinions of all the experts involved in the process (not only the simplified group) in a more efficient way.

To do so, the paper is set as follows: in section II we present our preliminaries, that is, the usual scheme of a GDM model and the basic concepts that we use in our paper. In section III we introduce the new GDM model that allows to carry out decision processes with a large number of individuals, which is divided in three main steps: a preliminary step where the set of experts is simplified, a

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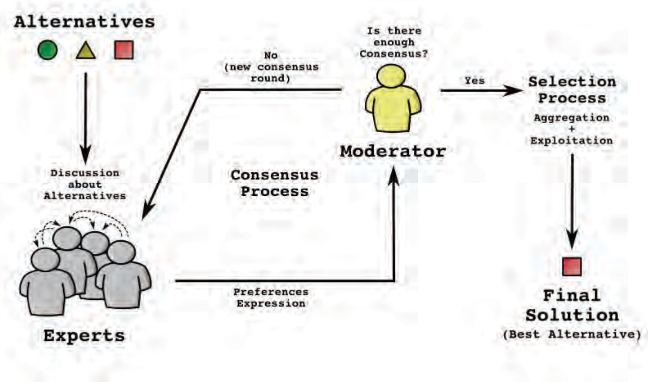

Fig. 1. Typical scheme of GDM models

second step where a consensus process is carried out and a final selection step where the final solution to the problem is obtained. Finally, in section IV we point out our conclusions.

II. Preliminaries

Usual GDM models follow a scheme in which two phases are differentiated (see figure 1): the first one consists in a *consensus process* in which the experts, discuss about the alternatives and express their preferences about them using a particular preference representation format. A special individual (the moderator) checks the different opinions and confirms if there is enough consensus among all the experts. If there is not enough consensus the moderator urges the experts to re-discuss about the alternatives and to provide a new set of opinions to improve the consensus level in a new consensus round. Once the desired consensus have been reached (or a maximum number of consensus rounds has been reached) the second phase (the *selection process*) starts and the best solution is obtained by agreggating the last opinions from the experts and applying an exploitation step which identifies the best alternative from the agreggated information.

In this paper we assume that each one of the experts of the group $E = \{e^1, \ldots, e^m\}$ provide their preferences about the set of alternatives $X = \{x_1, \ldots, x_n\}$ in form of fuzzy preference relations [4]:

Definition: A fuzzy preference relation P^h given by expert e^h on a set of alternatives X is a fuzzy set on the product set $X \times X$, which is characterized by a membership function $\mu_P^h : X \times X \rightarrow [0, 1]$.

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P^h = (p_{ij}^h)$, being $p_{ij}^h = \mu_P^h(x_i, x_j)$ $\forall i, j \in \{1, \ldots, n\}$ interpreted as the preference degree or intensity of the alternative x_i over x_j: $p_{ij}^h = 1/2$ indicates indifference between x_i and x_j ($x_i \sim x_j$), $p_{ij}^h = 1$ indicates that x_i is absolutely preferred to x_j, and $p_{ij}^h > 1/2$ indicates that x_i is preferred to x_j ($x_i \succ x_j$). Based on this interpretation, we have that $p_{ii}^h = 1/2$ $\forall i \in \{1, \ldots, n\}$ $(x_i \sim x_i)$.

For the selection process it is usually necessary to perform an aggregation of the preferences expressed by the experts. There exist many different possible aggregation operators in the literature. One of the most important operators in the GDM field is the IOWA operator [12], a variation of the OWA operator where the arguments are reordered according to a order inducing variable u:

Definition: An IOWA operator of dimension n is a function $\Phi_W : (R \times R) \rightarrow R$, to which a set of weights or weighting vector is associated, $W = \{w_1, \ldots, w_n\}$, such that $w_i \in [0, 1]$ and $\sum_i w_i = 1$, and it is defined to aggregate the set of second arguments of a list of n 2-tuples $\{\langle u_1, p_1 \rangle, \ldots, \langle u_n, p_n \rangle\}$ according to the following expression,

$$\Phi_W(\langle u_1, p_1 \rangle, \ldots, \langle u_n, p_n \rangle) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)}$$

being σ a permutation of $\{1, \ldots, n\}$ such that $\langle u_{\sigma(i)}, p_{\sigma(i)} \rangle$ is the 2-tuple with $u_{\sigma(i)}$ the i-th highest value in the set $\{u_1, \ldots, u_n\}$.

III. A GDM Model for Large Groups of Individuals

The new GDM model that we propose follows an scheme based in the one presented in the previous section, but it incorporates several important differences in order to deal with a large number of experts. The most important difference is the inclussion of a previous step, prior to the consensus phase, in which the large group of experts is simplified into a “selected experts group” or spokespersons, trying to maintain the diversity on the opinions of the whole group. Once this simplication is made the experts that have not been selected will provide information about the trust that the selected experts inspire to them, thus creating a trust network. After this initial step the consensus process begins, but only the selected experts that are allowed to take part in the process. Thus it is possible to carry out a proper consensus process, with discussion among the smaller group of spokespersons. At each consensus round the non-selected experts will be able to change their trust evaluations if their opinion about the selected experts has changed. Once a proper level of consensus is reached, the selection process begins, the opinions of the selected experts are aggregated, and the final solution is obtained. In this section we describe each of the three steps of the model in greater detail.

A. First Step: Initial Grouping and Delegation

This first step of the model precedes the consensus process. Its main purpose is to simplify the large expert group into a smaller one to ease the consensus and selection steps. The simplification of the group of experts is made trying to preserve the diversity of opinions of the group, avoiding that all the selected experts have similar opinions. We will follow the scheme presented in figure 2 during the description of this step. Note that, for simplicity reasons, we have represented only a small amount of experts in figure 2, but in real problems the amount of experts could be much higher.

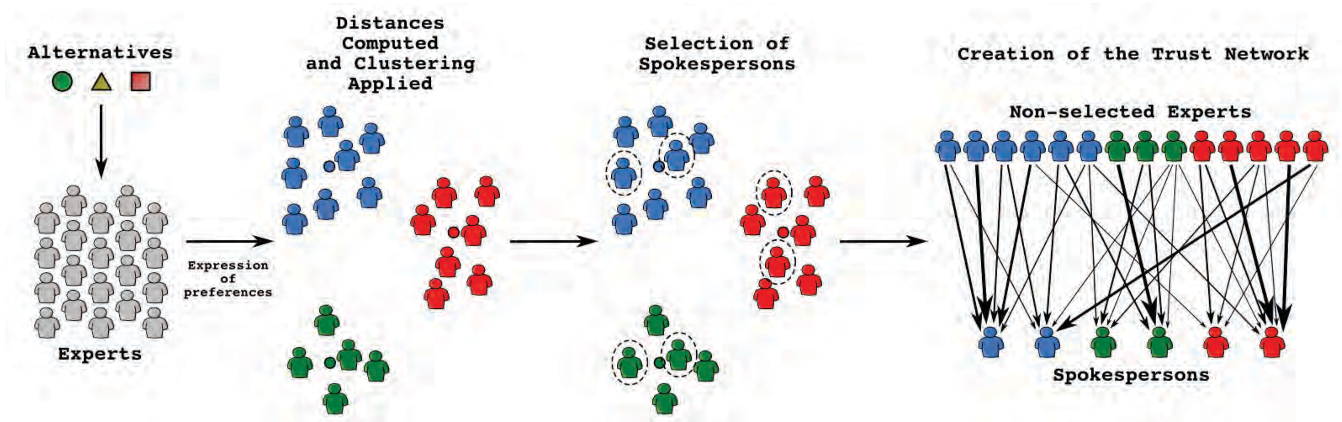

Fig. 2. Scheme of the first step in the GDM model

The first action that all the experts that want to be involved in the decision have to do, once the alternatives have been presented to them, is to provide a fuzzy preference relation about the alternatives in the problem. For sake of simplicity, we assume that all experts $e^h \in E$ provide a preference relation P^h. The distance among each pair of experts e^h and e^g is then computed in the following way:

$$d^{hg} = d^{gh} = \sqrt{\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} (p_{ij}^h - p_{ij}^g)^2}$$

Once the distances among experts have been computed we apply a clustering algorithm to classify them according to their opinions, thus identifying the different opinion trends in the group. The clustering algorithm will use the distance measures in order to distribute the experts in different partitions where experts with similar opinions (low distances among them) will be grouped toghether. Regarding the clustering algorithm to be applied, we propose to use the k-means algorithm for its simplicity of application. The number of partitions k to be used by the algorithm may vary depending on the problem that we are facing, but we recommend to use the number of feasible alternatives in the problem ($k = n$), thus allowing to identify at least one opinion trend for each one of the alternatives. We denote E_c to the group of experts that have been classified in cluster number c. Once each expert has been associated with a particular opinion cluster we compute the centroid opinion of each cluster (P^{E_c}) as an average of the opinions of the experts of the group:

$$p_{ij}^{E_c} = \frac{\sum_{e_h \in E_c} p_{ij}^h}{\#E_c}$$

Once the centroid opinions have calculated (represented as colored circles in figure 2 we have to choose some spokespersons for each cluster. In this way, we assure that in the next steps of the model the opinions will be diverse, thus allowing to find a solution in which every point of view has been considered. The number of spokespersons per cluster n_s should be fixed in advance and will depend on the problem. It is clear that a low n_s will provide less diversity, but a high n_s would not reduce the complexity of the GDM resolution process. If we use $n_s = 3$, that means that for each cluster we will select 3 spokespersons. The selected spokespersons should have an opinion near to the centroid of its cluster, but also the opinions of the selected spokespersons should be different enough to avoid a very fast convergence and to have more diversity of opinions. Then, to select the spokespersons we firstly compute the Farthest opinion of each pair of Experts in each cluster FE_c and the Farthest opinion to the Centroid of each cluster FC_c:

$$\forall c \in 1, \ldots, k \; : \; FE_c = max\{d^{hg}\}, e^h, e^g \in E_c$$

$$\forall c \in 1, \ldots, k \; , \; e^h \in E_c : \; d_c^h = \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} |p_{ij}^h - p_{ij}^{E_c}|$$

$$\forall c \in 1, \ldots, k \; : \; FC_c = max\{d_c^h\}, e^h \in E_c$$

where d_c^h is the distance of expert e^h to the centroid of its cluster.

Now we compute a measure B that balances both the normalized distance between each group of n_s experts and their normalized distances to the centroid. For example, if $n_s = 3$ we compute:

$$\forall c \in 1, \ldots, k \; , \; e_h, e_j, e_i \in E_c, h \neq j \neq i :$$

$$B_{hji} = \alpha \cdot \frac{\frac{d^{hj}}{FE_c} + \frac{d^{ji}}{FE_c} + \frac{d^{hi}}{FE_c}}{3} + (1-\alpha) \cdot \frac{3 - \frac{d_c^h}{FC_c} - \frac{d_c^i}{FC_c} - \frac{d_c^j}{FC_c}}{3}$$

where $\alpha \in [0, 1]$ is a parameter to weight both kinds of distances. We propose to use $\alpha = 0.5$ to select experts with a quite central opinion but also that their opinions differ in a high enough quantity. For each cluster, the subgroup of n_s experts with the *maximum* B_{hji} measure will be incorporated to the spokespersons group S. Note that in S there are experts for each one of the computed clusters, so a variety of different opinion should be reflected in that group. We will denote $s_1, \ldots, s_x \quad S$ to the spokespersons that conform the S subgroup.

Finally, we should establish a trust network among the experts. To do so, the not selected experts $NS = E - S$ will be asked to provide a kinf of utility function which represents the trust that they concede to each one of the chosen experts. Thus, each expert $e^h \quad NS$ will provide a vector of weights

$$T^h = \quad t_1^h, \ldots, t_x^h$$

where $t_i^h \quad [0, 1]$ and $\quad {}_i\, t_i^h = 1$. In figure 2 these weights are represented by arrows between the non-selected experts and the spokespersons group. The thickness of the arrows represent a higher t_i^h, and weights equal to 0 have not been represented for simplicity reasons. Note that each expert in NS can express some trust even in spokespersons which where not from his cluster. It is even possible that some experts give all their trust to a single spokesperson. This trust network establishment can be seen as a kind of delegation that non-selected experts make towards the spokespersons. Usual delegation schemes (like in elections) normally imply that a person delegates all his trust in a single person. We think that our approach offers a more flexible framework as it is possible to distribute the trust into several candidates if the expert opinion does not fully coincide with any of the selected spokespersons.

For each one of the spokespersons we can now compute a Trust Level acquired from the opinions of the non-selected experts:

$$s_i : \; TL_i = \sum_{e^h \quad NS} t_i^h \tag{1}$$

B. Second Step: Consensus Process

The consensus process follows an iterative scheme in which the preferences expressed by the spokespersons are compared and a global consensus measure is obtained. If that consensus measure does not satisfy a particular threshold the moderator sohuld urge the spokespersons to change their opinions and narrow their. They are supposed to discuss about the alternatives and provide new fuzzy preference relations. In addition, in this consensus process the non-selected experts have a different participation role: they will be allowed to change their trust evaluations in each consensus round if they feel that their confidence in each spokesperson has changed. A much more detailed explanation of each one of the substeps that are carried out in this step of the GDM model is presented in what follows and it is graphically represented in figure 3.

At the beginning of each consensus round the spokespersons $s_1, \ldots, s_x$ are required to express their preferences about the alternatives. To properly do so, it is supposed that they firstly discuss and share their points of view about the alternatives and that at a certain point they give their fuzzy preference relations $P^1, \ldots, P^x$ to the moderator. The moderator must check if there is a high enough consensus

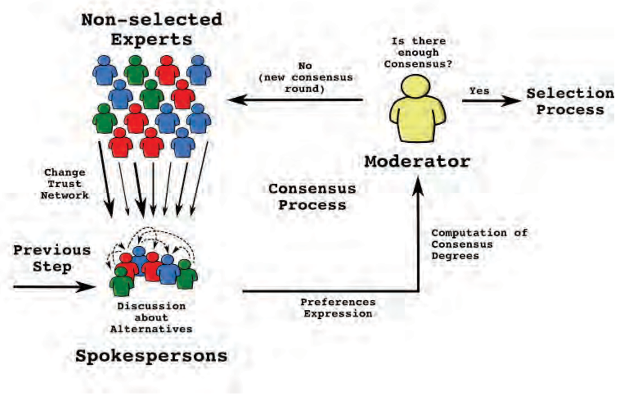

Fig. 3. Scheme of the second step in the GDM model

level in order to go to the third step of the model (the selection process) or to urge the spokespersons to change their opinions and narrow their differences in the next consensus round. To help the moderators task of checking the level of consensus, several consensus degrees are computed at three different levels: level of pair of alternatives, level of alternatives and level of relations:

Firstly, for each pair of spokespersons (s^h, s^l) $(h < l)$ we define a similarity matrix $SM^{hl} = \left(sm_{ik}^{hl}\right)$ where

$$sm_{ik}^{hl} = 1 - \quad p_{ik}^h - p_{ik}^l$$

Then, a collective similarity matrix, $SM = (sm_{ik})$ is obtained by aggregating all the $(x - 1) \times (x - 2)$ similarity matrices using the arithmetic mean as the aggregation function ψ:

$$sm_{ik} = \psi(sm_{ik}^{hl}) \; ; \quad h, l = 1, ..., x \quad h < l.$$

Once the similarity matrices are computed we proceed to calculate the consensus degrees at the three different levels:

L. 1. *Consensus degree on pairs of alternatives*. The consensus degree on a pair of alternatives (x_i, x_k), denoted cop_{ik}, is defined to measure the consensus degree amongst all the experts on that pair of alternatives:

$$cop_{ik} = sm_{ik}$$

L. 2. *Consensus degree on alternatives*. The consensus degree on alternative x_i, denoted ca_i, is defined to measure the consensus degree amongst all the experts on that alternative:

$$ca_i = \frac{\sum_{k=1; k=i}^{n} (cop_{ik} + cop_{ki})}{2(n-1)}$$

L. 3. *Consensus degree on the relation*. The consensus degree on the relation, denoted CR, is defined to measure the global consensus degree amongst all the experts' opinions:

$$CR = \frac{\sum_{i=1}^{n} ca_i}{n}$$

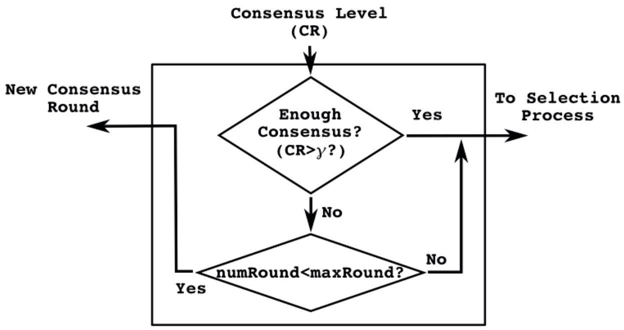

Fig. 4. Scheme of the consensus control substep.

At this point the moderator checks if $CR > \gamma$, being γ a threshold value fixed prior to the beginning of the GDM process. In the case that the consensus level is high enough, the model continues to the selection process and, if not, a new consensus round must begin. Note that in real applications it might be desirable to include a $maximumRounds$ parameter to control the maximum consensus rounds that can be executed in order to avoid stagnation (see figure 4).

Prior to the new discussion of the spokespersons in order to narrow their differences, all the non-selected experts are allowed to change their trust evaluations T^h. This modification of the trust evaluations pretends to reflect the possible changes in the confidence that non-selected experts have in the spokespersons. We must remark that although non-selected experts cannot contribute in the discussion phases, and they do not explicitly provide their preferences about the alternatives, they are allowed to follow the consensus process. Thus, it is possible to discover that one of the spokespersons that they trusted is no longer satisfying their expectations or, on the contrary, a previously non-trusted spokesperson may be expressing a much more appropriate opinion about the alternatives during the consensus rounds, and thus, he might receive more trust. Once the new trust network is established, the trust levels of the spokespersons are computed again (see expression 1).

The information about the trust levels is also provided to the spokespersons. Then, they can check if in the last round of consensus they have lost some trust from the non-selected experts or, on the contrary, they have won the confidence of the rest of experts. This is an interesting "security" mechanism that, in a certain way, tries to avoid situations in which one of the selected spokespersons radically changes his opinion, thus "betraying" the experts that put their trust on him.

At this point a new consensus round starts, the spokespersons begin their discussion and they will again provide their new fuzzy preference relations to the moderator.

C. Third Step: Selection Process

Once a certain level of consensus is reached, the selection process begins and the final solution to de GDM problem is obtained. To do so, the selection process is divided into two different sub-steps: an aggregation step, where a collective fuzzy preference relation is obtained and an exploitation phase, where the final solution is selected from the aggregated information. In the following we describe in more detail this two sub-steps:

1) Aggregation Step: At the beginning of the selection process we have the $P^1, \ldots, P^x$ fuzzy preference relations provided by the spokespersons. We want to obtain a collective fuzzy preference relation P^c that represents the whole group opinion about the alternatives. To do so, we will use the IOWA operator presented in section II, using the trust levels for each spokesperson as the order inducing variable. That is:

$$p^c_{ij} = \Phi_W(\ TL^1, p^1_{ij} \ , \ldots , \ TL^x, p^x_{ij} \ ).$$

The use of the trust levels as order inducing variable allows to give more weight to the more trusted spokespersons. Thus, the final collective fuzzy preference relation will be nearer to the opinion of the majority of the experts. To do so, the IOWA operator is used to implement the concept of fuzzy majority in the aggregation phase by means of a *fuzzy linguistic quantifier* [13] which indicates the proportion of satisfied experts *necessary for a good solution*. This implementation is done by using the quantifier to calculate the OWA weights. In the case of a regular increasing monotone (RIM) quantifier Q, we calculate the OWA weights as follows:

$$w_i = Q(i/n) - Q((i-1)/n), \quad i = 1, \ldots, n.$$

In our particular case, due to its good properties we make use of the linguistic quantifier *most of*, represented by the RIM quantifier $Q(r) = r^{1/2}$ [6].

2) Exploitation Step: At this point, in order to select the alternative *best* acceptable for the majority (Q) of the most trusted spokespersons, we propose two quantifier-guided choice degrees of alternatives, a dominance and a non-dominance degree:

1) $QGDD_i$: The quantifier-guided dominance degree quantifies the dominance that one alternative has over all the others in a fuzzy majority sense and is defined as follows:

$$QGDD_i = \phi_Q(p^c_{i1}, p^c_{i2}, ..., p^c_{i(i-1)}, p^c_{i(i+1)}, ..., p^c_{in})$$

2) $QGNDD_i$: The quantifier-guided non-dominance degree gives the degree in which each alternative is not dominated by a fuzzy majority of the remaining alternatives, its expression being:

$$\begin{aligned} QGNDD_i = \quad & \phi_Q(1 - p^s_{1i}, 1 - p^s_{2i}, ..., 1 - p^s_{(i-1)i}, \\ & 1 - p^s_{(i+1)i}, ..., 1 - p^s_{ni}) \end{aligned}$$

where $p^s_{ji} = max \ \ p^c_{ji} - p^c_{ij}, 0$, represents the degree in which x_i is strictly dominated by x_j.

Both degrees can be applied according to two different selection policies:

1) *Sequential policy:* One of the choice degrees is selected and applied to X according to the preference of the spokespersons, obtaining a selection set of alternatives. If there is more than one alternative in this selection set, then the other choice degree is applied to select the alternative of this set with the best second choice degree.
2) *Conjunctive policy:* Both choice degrees are applied to X, obtaining two selection sets of alternatives. The final selection set of alternatives is obtained as the intersection of these two selection sets of alternatives.

IV. CONCLUSIONS

In this contribution we have presented a novel GDM model which eases the resolution of GDM problems where a large number of experts are implied and where preferences are given in form of fuzzy preference relations. To be able to carry out a proper consensus phase prior to the selection of the best alternative, the group of experts is simplified into a spokespersons group. This simplified group of experts is obtained by means of the application of a simple clustering algorithm which tries to maintain a good diversity in the opinions of its members. In addition, the non-selected experts create a trust network giving some trust evaluations of the spokespersons. The trust network can evolve in each consensus round if the non-selected experts think that the spokespersons better or worse reflect their opinions about the alternatives. Once the consensus process is finished, a selection process is applied. In the selection process the spokespersons' fuzzy preference relations are aggregated using an IOWA operator guided by the trust level obtained by each spokesperson, obtaining a collective fuzzy preference relation that reflects the opinion of all the experts in the problem. Finally, the final solution is obtained from the collective fuzzy preference relation by applying two quantifier-guided choice degrees of alternatives that can be applied according to two different selection policies.

The main properties of this model is that it allows to carry out GDM processes with a large number of experts, that all experts opinions are indirectly taken into account by means of a delegation process and the use of the trust network and that the process offers an implied "security" mechanism that avoids that spokespersons abuse of the trust that experts deposited in them. Moreover, the simplification step in which the number of experts is reduced, allows the system to simplify the computations and information to be given by experts as in successive consensus iterations only a small fraction of experts have to provide new preferences (the spokespersons) and the rest of experts may decide not to change their trust values for the spokespersons (thus maintaining the trust network) unless a drastic change in spokespersons opinions occur.

It is interesting to note that our proposed model may have some similarities to existing group decision processes in which a simplification step of the experts is made. For example, boards of representatives are a very widely used model in which several representatives (which could be considered as the spokespersons) collaborate to reach a solution. However, our model difers to the boards of representatives model because of the creation and use of a trust network (which is highly dynamical as it changes in each consensus round) which determines the weight of each spokesperson. This kind of trust network can be easily exploited in online and web systems where users do interact in real time.

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