Group Decision Making: From Consistency to Consensus

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Abstract. In group decision making (GDM) processes, prior to the selection of the best alternative(s), it would be desirable that experts achieve a high degree of consensus or agreement between them. Due to the complexity of most decision making problems, individuals' preferences may not satisfy formal properties. 'Consistency' is one of such properties, and it is associated with the *transitivity property.* Obviously, when carrying out a rational decision making, consistent information, i.e. information which does not imply any kind of contradiction, is more appropriate than information containing some contradictions. Therefore, in a GDM process, consistency should also be sought after.

In this paper we present a consensus model for GDM problems that proceeds from consistency to consensus. This model includes a novel consistency reaching module based on consistency measures. In particular, the model generates advice on how experts should change their preferences in order to reach a solution with high consistency and consensus degrees.

1 Introduction

Any decision making problem includes a selection process which involves, as part of it, the choice between the various alternatives solutions to the problem [14]. In GDM problems, however, it may happen that some experts from the group would not accept the group choice if they consider that their opinions have not been take into account 'properly'. Indeed, group choice should be based on the desires or preferences of 'all' the individuals in the group, a premise on which democratic theory is based on [3].

Preference relations are usually assumed to model experts' preferences in group decision making problems [4,12]. Classically, given two alternatives, an expert judges them in one of the following ways:

V. Torra, Y. Narukawa, and Y. Yoshida (Eds.): MDAI 2007, LNAI 4617, pp. 80–91, 2007.

(i) one alternative is preferred to another;
(ii) the two alternatives are indifferent to him/her;
(iii) he/she is unable to compare them.

However, given three alternatives x_i, x_j, x_k such that x_i is preferred to x_j and x_j to x_k, the question whether the "degree or strength of preference" of x_i over x_j exceeds, equals, or is less than the "degree or strength of preference" of x_j over x_k cannot be answered by the classical preference modelling. The implementation of the degree of preference between alternatives may be essential in many situations, and this can be modelled using fuzzy preference relations [1,2].

The main advantage of pairwise comparison is that of focusing exclusively on two alternatives at a time which facilitates experts when expressing their preferences. However, this way of providing preferences limits experts in their global perception of the alternatives and, as a consequence, the provided preferences could be not rational. Usually, rationality is related to consistency, which is associated with the *transitivity property* [8]. Many properties have been suggested to model transitivity of a fuzzy preference relation and, consequently, consistency may be measured according to which of these different properties is required to be satisfied. One of these properties is the "additive transitivity", which, as shown in [8], can be seen as the parallel concept of Saaty's consistency property in the case of multiplicative preference relations.

In any 'rational' decision making process, consistent information, i.e. information which does not imply any kind of contradiction, is more relevant or important than information containing some contradictions. As a consequence, in GDM processes consistency should also be sought after in order to make rational choices. To do this, experts should know how consistent they are. By letting the experts know their associated consistency measures at any moment, they could judge whether or not it is high enough. Also, with this information, expert would be able to analyse their preferences and make the necessary changes to their most inconsistent preference values to increase their global consistency.

In GDM situations, consensus between experts is usually searched using the basic rationality principles that each expert presents. Thus, consistency criteria should be first applied to fix the rationality of each expert and only afterwards experts' agreement should be obtained. If we were to secure consensus and only thereafter consistency, we could destroy the consensus in favour of the individual consistency and the final solution might not be acceptable for the group of expert.

In [5,6] a consensus model was proposed for GDM problems which used two types of measurements to guide the consensus reaching process [10]: *consensus degrees* to evaluate the agreement of all the experts, and *proximity degrees* to evaluate the distance between the experts' individual preferences and the group or collective ones. In [9] a consensus model which uses a recommendation module to help experts to change their preferences was presented. In [11] a consensus model with an adaptive recommendation module to the current level of agreement in each one of the consensus round was defined. In this paper, we continue improving that consensus model by incorporating a consistency criteria, and, when necessary, to advice experts on how to become more consistent. We define a new adaptive consensus model in which once the experts provide their individual preference relations, consistency measures for each one are computed. These consistency measures are used to generate a consistency feedback mechanism that generates advice to the most inconsistent experts on the necessary changes to their most inconsistent preference values to increase their global consistency. Once the agreed minimum level of consistency has been reached, consensus is sought after.

The rest of the paper is set out as follows. Section 2 presents the structure of the new adaptive consensus model with its consistency control module. Section 3 describes in detail the consistency control module. Finally, Section 4 draws our conclusions.

2 An Adaptive Consensus Model with Consistency Control

In this section we present the structure of a new adaptive consensus model with consistency control. The structure of this new consensus model is depicted in Fig. 1.

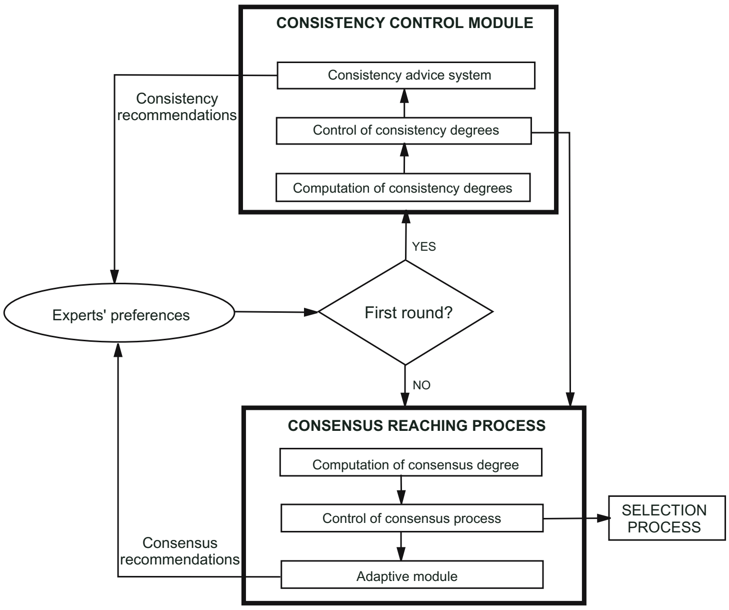

Fig. 1. Adaptive consensus model with consistency control

It is composed of two processes:

i) **Consistency Control Process.** Once experts' preferences are given, their consistency degrees are computed. If an expert is not consistent enough, that expert will receive appropriate changes of preference values in order to increase his/her global consistency to an acceptable/agreed threshold level one. This process is explained in detail in Section 3.

ii) **Adaptive Consensus Reaching Process.** The consensus process is considered adaptive because the search for consensus is adapted to the current level of agreement among experts.

This adaptive process to achieve consensus among the group of experts consists of three steps:

1 *Computation of consensus degrees.* In this step consensus measures are computed for each fuzzy preference relation. In each consensus round, these measures are used to compute the level of agreement (consensus) between the experts of the group at the three different levels of a preference relation: pairs of alternatives, alternatives and preference relation.

2 *Consensus control.* In this step, it is decided whether to stop or to continue the application of the consensus process. This decision is based on the achievement or not of a fixed a priori consensus threshold value, $\gamma \in [0, 1]$, representing the minimum level of global agreement the experts should reach in order to proceed with the selection of the solution alternative to the problem.

3 *Adaptive module.* The consensus reaching process involves a procedure which identifies those preference values experts should change to achieve the desired agreement level. This identification is not fixed but it adapts to the current level (low, medium, high) of consensus computed in step 1:

   3.1 *Low consensus.* Naturally, at the very beginning of the consensus process experts' preferences may differ substantially. In these cases the level of agreement could be quite low and a large number of experts' preferences should change in order to make the opinions closer. At this stage of the consensus process, and while the consensus is considered as 'low', 'all' experts are advised to change 'all' the preference values in which disagreement has been identified.

   3.2 *Medium consensus.* In the 'intermediate' rounds of the consensus reaching process the consensus degree might not be considered as low anymore. In this stage of a consensus process, and while the consensus degree is considered as 'medium', only those experts furthest from the group as a collective will be advised to make changes on the preference values of those alternatives in which disagreement has been identified.

   3.3 *High consensus.* When the level of consensus is approaching the consensus threshold value, only those experts furthest from the group as a collective will be advised to make changes on the preference values in which disagreement has been identified.

For more details on the described adaptive consensus reaching process the reader is referred to [11]. We should point out that the consistency control process is applied only in the first round of the consensus reaching process, because, as we shall show in the following section, when all the individual preference relations have associated a consistency degree above a particular minimum threshold value then any weighted average collective preference relation will also have associated a consistency degree above that threshold value. Adding to this the fact that the consensus process tends to make the individual opinions closer to the collective ones [7], we conclude that individual consistency degrees will tend towards the collective one and therefore above the threshold value. Therefore, it is unnecessary to control the consistency level of each expert in each consensus round.

3 Consistency Control Module

The purpose of the consistency control module is to measure the level of consistency of each individual preference relation (expert) in order to identify the experts, alternatives and preference values most inconsistent within the GDM problem. This inconsistency identification is also used to suggest new 'consistent' preference values. The consistency control module develops its activity by means of three processes as illustrated in Fig. 2, which will be covered in the following subsections.

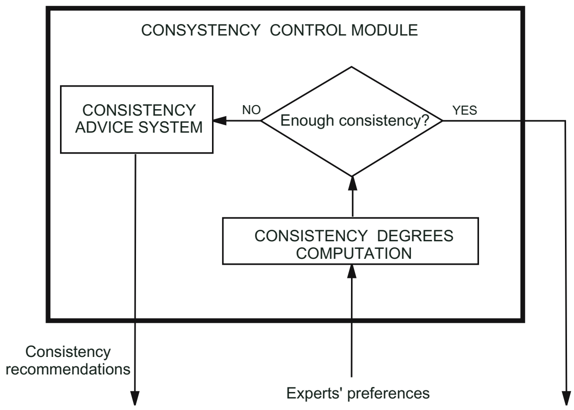

Fig. 2. Consistency control module

3.1 Computation of Consistency Degrees

In GDM problems with fuzzy preference relations some properties about the preferences expressed by the experts are usually assumed and desirable in order to avoid contradictions in their opinions, i.e, inconsistent opinions. One of these properties is associated with the transitivity in the pairwise comparison among any three alternatives. For fuzzy preference relations, transitivity has been modelled in many different ways due to the role the intensities of preference have (see [8]). In this paper, we make use of the *additive transitivity property*.

Being $P = (p_{ij})$ a fuzzy preference relation, the mathematical formulation of the *additive transitivity* was given by Tanino in [15]:

$$(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ik} - 0.5) \;\; \forall i, j, k \in \{1, \ldots, n\} \tag{1}$$

As shown in [8,13] additive transitivity can be used to obtain more consistent fuzzy preference relation from a given one. Additive transitivity implies additive reciprocity. Indeed, because $p_{ii} = 0.5 \; \forall i$, if we make $k = i$ in (1) then we have: $p_{ij} + p_{ji} = 1 \;\; \forall i, j \in \{1, \ldots, n\}$. Then, (1) can be rewritten as:

$$p_{ik} = p_{ij} + p_{jk} - 0.5 \;\; \forall i, j, k \in \{1, \ldots, n\} \tag{2}$$

We will consider a fuzzy preference relation P to be "*additive consistent*" when for every three alternatives in the problem $x_i, x_j, x_k \in X$ their associated preference degrees p_{ij}, p_{jk}, p_{ik} fulfil (2). An additive consistent fuzzy preference relation will be referred as consistent throughout the paper, as this is the only transitivity property we are considering.

Given a reciprocal fuzzy preference relation, (2) can be used to calculate an estimated value of a preference degree using other preference degrees. Indeed, using an intermediate alternative x_j, the following estimated value of p_{ik} $(i \neq k)$ is obtained:

$$ep_{ik}^{j} = p_{ij} + p_{jk} - 0.5 \tag{3}$$

The overall estimated value ep_{ik} of p_{ik} is obtained as the average of all possible values ep_{ik}^{j}, i.e.,

$$ep_{ik} = \sum_{\substack{j=1 \\ j \neq i,k}}^{n} \frac{ep_{ik}^{j}}{n-2}. \tag{4}$$

The value $|ep_{ik} - p_{ik}|$ can be used as a measure of the error between a preference value and its estimated one.

It is easy to obtain the expression of the estimated value of an estimated value, $e^2 p_{ik}$, which is:

$$e^2 p_{ik} = ep_{ik} + \frac{2}{n-2} \cdot (p_{ik} - ep_{ik})$$

This expression implies that the process of estimating preference values converges toward perfect consistency, which is expressed in the following proposition:

Proposition 1. *Let P be a reciprocal fuzzy preference relation. The following holds:*

$$\left| e^r p_{ik} - e^{r-1} p_{ik} \right| = \left(\frac{2}{n-2} \right)^{r-1} |ep_{ik} - p_{ik}| \,, \;\; r > 1.$$

When the information provided is completely consistent then $ep_{ik}^{j} = p_{ik} \; \forall j$. However, because experts are not always fully consistent, the information given by an expert may not verify (2) and some of the estimated preference degree values ep_{ik}^j may not belong to the unit interval $[0,1]$. We note, from (3), that the maximum value of any of the preference degrees ep_{ik}^j is 1.5 while the minimum one is -0.5. In order to normalize the expression domains in the decision model the final estimated value of p_{ik} $(i \neq k)$, cp_{ik}, is defined as the median of the values 0, 1 and ep_{ik}:

$$cp_{ik} = med\{0, 1, ep_{ik}\}. \tag{5}$$

The error in $[0,1]$ between a preference value p_{ik} and its final estimated cp_{ik} is:

$$\varepsilon p_{ik} = |cp_{ik} - p_{ik}|. \tag{6}$$

Given a preference value $p_{ik} \in [0,1]$, the following holds $|ep_{ik} - p_{ik}| = |ep_{ik} - cp_{ik}| + |cp_{ik} - p_{ik}|$ and therefore $\varepsilon p_{ik} \leq |ep_{ik} - p_{ik}|$ $\forall i,k$.

For being $P = (p_{ij})$ reciprocal, it is obvious that the preference relation $CP = (cp_{ik})$ is also reciprocal and $\varepsilon p_{ik} = \varepsilon p_{ki}$. We interpret $\varepsilon p_{ik} = 0$ as a situation of total consistency between p_{ik} (p_{ki}) and the rest of information in P. Obviously, the higher the value of εp_{ik} the more inconsistent is p_{ik} (p_{ki}) with respect to the rest of information in P.

This interpretation allows us to evaluate the consistency in each one of the three different levels of a reciprocal fuzzy preference relation P:

Level 1. *Consistency degree associated to a pair of alternatives* p_{ik} (p_{ki}),

$$cd_{ik} = 1 - \varepsilon p_{ik} \tag{7}$$

Level 2. *Consistency degree associated to an alternative* x_i,

$$cd_i = \sum_{\substack{k=1\\k\neq i}}^{n} \frac{cd_{ik}}{n-1} \tag{8}$$

When $cd_i = 1$ all the preference values involving the alternative x_i are fully consistent, otherwise, the lower cd_i the more inconsistent these preference values are with respect to the rest of information in P.

Level 3. *Consistency degree of the reciprocal fuzzy preference relation*,

$$cd = \sum_{i=1}^{n} \frac{cd_i}{n} \tag{9}$$

When $cd = 1$ the reciprocal fuzzy preference relation P is fully consistent, otherwise, the lower cd the more inconsistent P.

The computation of the estimated values and consistency degrees for a reciprocal preference relation are illustrated in the following example:

Example 1. *The following are a reciprocal fuzzy preference relation and its reciprocal estimated fuzzy preference relation*

$$P = \begin{pmatrix} - & 0.7 & 0.9 & 0.5 \\ 0.3 & - & 0.6 & 0.7 \\ 0.1 & 0.4 & - & 0.8 \\ 0.5 & 0.3 & 0.2 & - \end{pmatrix} \longrightarrow CP = \begin{pmatrix} - & 0.55 & 0.5 & 1.0 \\ 0.45 & - & 0.55 & 0.6 \\ 0.5 & 0.45 & - & 0.35 \\ 0.0 & 0.4 & 0.65 & - \end{pmatrix}$$

The value $cp_{14} = 1$ has been obtained as follows:

$$ep_{14} = \frac{ep_{14}^2 + ep_{14}^3}{2} = \frac{0.9 + 1.2}{2} = 1.05 \Rightarrow cp_{14} = med\{0, 1, 1.05\} = 1.$$

The consistency degrees at the three levels of the preference relation are:

Level 1. *Consistency degrees at the level of pairs of alternatives*

$$CD = \begin{pmatrix} - & 0.85 & 0.6 & 0.5 \\ 0.85 & - & 0.95 & 0.9 \\ 0.6 & 0.95 & - & 0.55 \\ 0.5 & 0.9 & 0.55 & - \end{pmatrix}$$

Level 2. *Consistency degree of each alternative:*

$$cd_1 = 0.65 \quad cd_2 = 0.9 \quad cd_3 = 0.7 \quad cd_4 = 0.65$$

Level 3. *Consistency degree of the relation:*

$$cd = 0.73.$$

Let $P^c = (p_{ij}^c)$ be a weighted mean collective preference relation obtained from a set of reciprocal fuzzy preference relations $\{P^1, \ldots, P^m\}$. The estimated value of the collective preference value $p_{ij}^c = \sum_{l=1}^n w_l \cdot p_{ij}^l$, with $w_l \geq 0\ \forall l$ and $\sum_l w_l = 1$, is $ep_{ij}^c = \sum_{\substack{k=1 \\ j\neq i,k}}^n \frac{(ep_{ij}^c)^k}{n-2}$, with $(ep_{ij}^c)^k = p_{ik}^c + p_{kj}^c - 0.5$. Putting these expressions together we get:

$$ep_{ij}^c = \sum_{\substack{k=1 \\ j\neq i,k}}^n \frac{\sum_{l=1}^n w_l \cdot p_{ik}^l + \sum_{l=1}^n w_l \cdot p_{kj}^l - 0.5}{n-2} = \sum_{\substack{k=1 \\ j\neq i,k}}^n \frac{\sum_{l=1}^m w_l \cdot (p_{ik}^l + p_{kj}^l - 0.5)}{n-2}$$
$$= \sum_{l=1}^m w_l \frac{\sum_{\substack{k=1 \\ j\neq i,k}}^n p_{ik}^l + p_{kj}^l - 0.5}{n-2} = \sum_{l=1}^m w_l \cdot ep_{ij}^l$$

This result is summarised in the following proposition:

Proposition 2. *The estimated collective preference relation of a weighted mean collective preference obtained from a set of reciprocal fuzzy preference relations is also the weighted mean of the estimated individual preference relations.*

The error between a collective preference value and its estimated one is

$$\left|ep_{ij}^c - p_{ij}^c\right| = \left|\sum_{l=1}^m w_l \cdot ep_{ij}^l - \sum_{l=1}^n w_l \cdot p_{ij}^l\right| = \left|\sum_{l=1}^m w_l \cdot (ep_{ij}^l - p_{ij}^l)\right|$$

Using the well known properties $|a+b| \leq |a| + |b|$ and $|a \cdot b| = |a| \cdot |b|$ we have

$$\left|\sum_{l=1}^m w_l \cdot (ep_{ij}^l - p_{ij}^l)\right| \leq \sum_{l=1}^m w_l \cdot \left|ep_{ij}^l - p_{ij}^l\right| \leq \max_l \left|ep_{ij}^l - p_{ij}^l\right|$$

Therefore, we have proved that

Proposition 3. *The error between a collective preference value and its estimated one is lower or equal to the maximum error between the individual preference values and their estimated ones.*

From $\varepsilon p_{ij} \leq |ep_{ij} - p_{ij}|$ and proposition 3 we can easily prove that when all estimated values of each one of the individual preference relations of the set $\{P^1, \ldots, P^m\}$ are in $[0,1]$ the consistency degree of a weighted mean collective preference relation will be greater or equal than the minimum of the individual consistency degrees, $\min_l cd^l$. When one or more individual estimated values are not in $[0,1]$ then this limit is reduced by the quantity $\sum_{i,j;j\neq i} \frac{\max_l |ep_{ij}^l - cp_{ij}^l|}{n(n-1)}$. In a situation of high individual consistency degrees the distance between $cp_{ij}^l = med\{0, 1, ep_{ik}^l\}$ and p_{ij}^l will be small (or zero) and as a consequence the distance between ep_{ij}^l and cp_{ij}^l will also be small (or zero). All this together can be used to claim that in GDM problems in which all experts provide highly consistent preferences the (weighted mean) collective preference will also be highly consistent.

3.2 Consistency Control

We assume that before providing any preferences the group of experts agree on a threshold consistency degree value (β) for an expert to be considered as consistent. After providing preferences, experts' associated consistency degrees are obtained, $cd^i \forall i$. If all experts are consistent, i.e. $cd^i \geq \beta \; \forall i$, then the consensus reaching process is applied. Otherwise, a consistency advice system is applied i) to identify the inconsistent experts, alternatives, and preference values; and ii) to generate an alternative consistent value for each one of the inconsistent preference values.

3.3 Consistency Advice System

This system suggests experts some changes on the most inconsistent preference values. To do so, the following three steps are carried out:

1. To identify those experts (l) in the group with a global consistency level (cd^l) lower than the minimum threshold consistency value (β).
2. To identify for each one of these experts those alternatives (i) with a consistency degree (cd_i^l) lower than β.
3. To identify for each one of these alternatives the preference values whose consistency level (cd_{ij}^l) is lower than β.
   The set of preference values to be recommended for change will be:

$$\{(l,i,j) | \max\{cd^l, cd_i^l, cd_{ij}^l\} < \beta\}.$$

Based on proposition 1, a preference value of the above set (p_{ij}^l) will be recommended to be changed to a value closer to its final estimated value (cp_{ij}^l). This change will bring the original individual preference relation (P^l) closer to its estimated one (CP^l) and therefore it will become more consistent globally. Thus, if $cd^l_{ij} < \beta$, in order to reach the minimum threshold value, p^h_{ij} will be recommended to be changed to

$$\bar{p}^l_{ij} = p^l_{ij} + sign(cp^l_{ij} - p^l_{ij}) \cdot (\beta - cd^l_{ij}),$$

where $sign(X)$ returns the sign of X. Finally, in order to maintain reciprocity, the value p^l_{ji} will be recommended to be changed to $\bar{p}^l_{ji} = 1 - \bar{p}^l_{ij}$.

Example 2. *Suppose we have a set of four experts providing the following fuzzy preference relations on a set of four alternatives:*

$$P^1 = \begin{pmatrix} - & 0.2 & 0.6 & 0.4 \\ 0.8 & - & 0.9 & 0.7 \\ 0.4 & 0.1 & - & 0.3 \\ 0.6 & 0.3 & 0.7 & - \end{pmatrix} \qquad P^2 = \begin{pmatrix} - & 0.7 & 0.9 & 0.5 \\ 0.3 & - & 0.6 & 0.7 \\ 0.1 & 0.4 & - & 0.8 \\ 0.5 & 0.3 & 0.2 & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & 0.3 & 0.5 & 0.7 \\ 0.7 & - & 0.1 & 0.3 \\ 0.5 & 0.9 & - & 0.25 \\ 0.3 & 0.7 & 0.75 & - \end{pmatrix} \qquad P^4 = \begin{pmatrix} - & 0.25 & 0.15 & 0.65 \\ 0.75 & - & 0.6 & 0.8 \\ 0.85 & 0.4 & - & 0.5 \\ 0.35 & 0.2 & 0.5 & - \end{pmatrix}$$

Let the threshold value be $\beta = 0.8$. We have the following global consistency values:

$$cd^1 = 1,\ cd^2 = 0.73,\ cd^3 = 0.63;\ cd^4 = 0.82.$$

This means that recommendations of change will be given to experts e_2, e_3. For these two experts we have the following consistency degree matrices and consistency degree of alternatives:

$$CD^2 = \begin{pmatrix} - & 0.85 & 0.6 & 0.5 \\ 0.85 & - & 0.95 & 0.9 \\ 0.6 & 0.95 & - & 0.55 \\ 0.5 & 0.9 & 0.55 & - \end{pmatrix};\quad cd^2_1 = 0.65,\ cd^2_2 = 0.9,\ cd^2_3 = 0.7,\ cd^2_4 = 0.65$$

$$CD^3 = \begin{pmatrix} - & 0.4 & 0.93 & 0.48 \\ 0.4 & - & 0.48 & 0.93 \\ 0.93 & 0.48 & - & 0.55 \\ 0.48 & 0.93 & 0.55 & - \end{pmatrix};\quad cd^3_1 = 0.6,\ cd^3_2 = 0.6,\ cd^3_3 = 0.65,\ cd^3_4 = 0.65$$

The recommended new preference values would be:

e_2**:** $\bar{p}_{13} = 0.7$ ($\bar{p}_{31} = 0.3$); $\bar{p}_{14} = 0.8$ ($\bar{p}_{41} = 0.2$); $\bar{p}_{34} = 0.55$ ($\bar{p}_{43} = 0.45$).
e_3**:** $\bar{p}_{12} = 0.7$ ($\bar{p}_{21} = 0.3$); $\bar{p}_{14} = 0.38$ ($\bar{p}_{41} = 0.62$); $\bar{p}_{23} = 0.43$ ($\bar{p}_{32} = 0.57$); $\bar{p}_{34} = 0.5$ ($\bar{p}_{43} = 0.5$).

If these recommended values were assumed by these experts, their new global consistency values would become $cd^2 = 0.94$ and $cd^3 = 0.92$, which represent a considerable improvement regarding their previous global consistency levels. Afterwards the consensus and selection process are carried out.

4 Conclusions

In any rational GDM process, both consensus and consistency should be sought after. In this paper we have addressed the issues of measuring consistency and of achieving a high level of consistency within an adaptive consensus reaching process. For doing that, we have developed a consistency advice module, based on theoretical results, for recommending 'consistent' changes to experts for the most inconsistent preference values. We have argued that consistency is needed to be checked just once before the application of the consensus process, because (a) when all individual experts provide highly consistent preferences the (weighted mean) collective preference will also be highly consistent, and (b) the consensus process tends to make the individual opinions closer to the collective ones. Also, if we were to secure consensus and only thereafter consistency, we could destroy the consensus in favour of the individual consistency and the final solution could not be acceptable for the group of expert.

Acknowledgements

This work was prepared during Dr Francisco Mata's research visit to the Centre for Computational Intelligence at De Montfort University (Leicester, UK) in the summer 2006. This work has been supported by the Research Projects TIN2006-02121 and SAINFOWEB-PAI00602.

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