A Consensus Support System Model for Group Decision-Making Problems With Multigranular Linguistic Preference Relations

E. Herrera-Viedma, L. Martínez, F. Mata, and F. Chiclana

***Abstract*—The group decision-making framework with linguistic preference relations is studied. In this context, we assume that there exist several experts who may have different background and knowledge to solve a particular problem and, therefore, different linguistic term sets (multigranular linguistic information) could be used to express their opinions. The aim of this paper is to present a model of consensus support system to assist the experts in all phases of the consensus reaching process of group decision-making problems with multigranular linguistic preference relations. This consensus support system model is based on i) a multigranular linguistic methodology, ii) two consensus criteria, consensus degrees and proximity measures, and iii) a guidance advice system. The multigranular linguistic methodology permits the unification of the different linguistic domains to facilitate the calculus of consensus degrees and proximity measures on the basis of experts' opinions. The consensus degrees assess the agreement amongst all the experts' opinions, while the proximity measures are used to find out how far the individual opinions are from the group opinion. The guidance advice system integrated in the consensus support system model acts as a feedback mechanism, and it is based on a set of advice rules to help the experts change their opinions and to find out which direction that change should follow in order to obtain the highest degree of consensus possible. There are two main advantages provided by this model of consensus support system. Firstly, its ability to cope with group decision-making problems with multigranular linguistic preference relations, and, secondly, the figure of the moderator, traditionally presents in the consensus reaching process, is replaced by the guidance advice system, and in such a way, the whole group decision-making process is automated.**

***Index Terms*—Consensus, fuzzy preference relation, group decision-making (GDM), linguistic modeling.**

I. Introduction

GROUP decision-making (GDM) problems may be defined as decision situations where: i) there are two or more experts who are characterized by their own ideas, attitudes, motivations and knowledge, ii) there is a problem to be solved, and iii) they try to achieve a common solution.

The ideal situation would be one where all the experts could express their opinions on the problem in a precise way by means of numerical values. Unfortunately, in many cases, experts deal with vague or imprecise information or have to express their opinions on qualitative aspects that cannot be assessed by means of quantitative values. In these cases, the use of linguistic terms instead of precise numerical values seems to be more adequate. This is the case, for example, when experts try to evaluate the "comfort" or "design" of a car, where linguistic terms like "good,""fair," "poor" are normally used; while "fast," "very fast," "slow" are used when assessing the "speed" [6].

Fuzzy sets theory has proven successful in handling fuzziness and modeling qualitative information [11], [15], [29], [36], [38], [39]. In this theory, the qualitative aspects of the problem, such as the linguistic labels in the above examples, are represented by means of "linguistic variables" [40], [43], [44], i.e., variables whose values are not numbers but words or sentences in a natural or an appropriate artificial language.

An important parameter to determine in a linguistic approach is the "granularity of uncertainty," i.e., the cardinality of the linguistic term set that will be used to express the information. In GDM problems, when experts come from different research areas, and thus have different background and levels of knowledge, it is natural to assume that linguistic term sets of different cardinality and/or semantics could be used to express their opinions on the set of alternatives. In these cases, we say that we are working in a multigranular linguistic context [10], [35], and we will call this type of problem a multigranular linguistic GDM problem.

In GDM problems there, are two processes to carry out before obtaining a final solution [9], [12], [17], [23]: *the consensus process* and *the selection process* (see Fig. 1). The first one refers to how to obtain the maximum degree of consensus or agreement between the set of experts on the solution set of alternatives. Normally, this process is guided by the figure of a moderator [7], [12]–[14], [19]–[23]. The second one consists in how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts. Clearly, it is preferable that the set of experts reach a high degree of consensus before applying the selection process. In [10], the selection process for multigranular linguistic GDM problem was studied. Therefore, in this paper, we focus on the consensus process.

Consensus has become a major area of research in GDM [2]–[4], [7], [8], [12], [17]–[26], [31], [33], [41]. A consensus process is defined as a dynamic and iterative group discussion process, coordinated by a moderator, who helps the experts to bring their opinions closer. In each step of this process, the mod-

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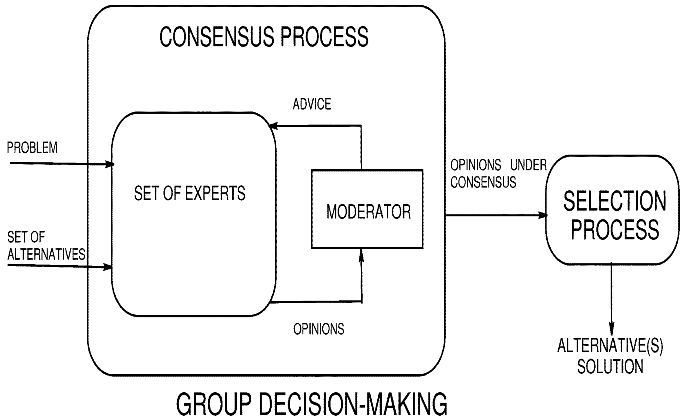

Fig. 1. Resolution process of a GDM problem.

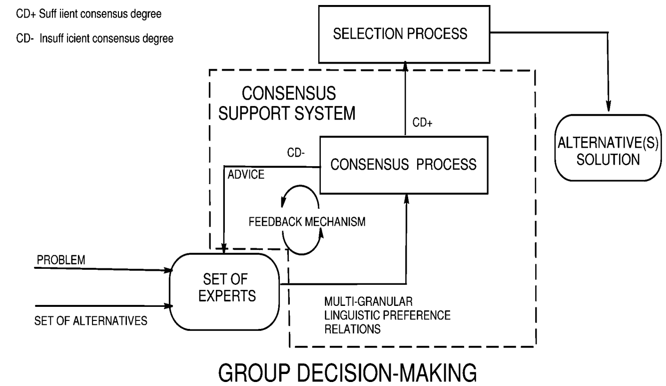

Fig. 2. Resolution process of a multigranular linguistic GDM problem based on a CSS model.

erator, by means of a consensus measure, knows the actual level of consensus between the experts which establishes the distance to the ideal state of consensus. If the consensus level is not acceptable, i.e., if it is lower than a specified threshold, then the moderator would urge the experts to discuss their opinions further in an effort to bring them closer. On the contrary, when the consensus level is acceptable, the moderator would apply the selection process in order to obtain the final consensus solution to the GDM problem. In this framework, a question that needs to be solved is how to substitute the actions of the moderator in the group discussion process in order to automatically model the whole consensus process.

Real important decisions are often difficult to make. To alleviate such difficulty it would be necessary and desirable to use some kind of decision support. The aim of this paper is to present a model of consensus support system (CSS) to automate the consensus reaching process in GDM where the experts provide their opinions by means of multigranular linguistic preference relations. In this CSS model, the figure of the moderator is substituted by a feedback mechanism that uses a guidance advice system based on a set of advice rules to help the experts change their opinions and know the direction of that change in order to obtain the highest degree of consensus possible. In this way, we design a consensus process that is controlled automatically without using any human moderator. This CSS model is based on two types of consensus criteria [12].

a) *Consensus degrees* to identify the level of agreement amongst all the experts and to decide when the consensus process should stop.
b) *Proximity measures* to evaluate the distance between the experts' individual opinions and the group or collective opinion. The proximity values are used in the feedback process to guide the direction of the changes in the experts' opinions in order to increase the consensus degrees.

These consensus criteria are computed at the three different levels of representation of information of a preference relation: pair of alternatives, alternative, and relation. The CSS model that we propose develops its activity in four phases.

1) *Making the linguistic information uniform.* In this phase, all experts' multigranular linguistic preferences are unified in a same linguistic domain. We design a methodology based on transformation functions to unify the multigranular linguistic information. This phase is necessary to make the computation of both consensus degrees and proximity measures easier.
2) *Computation of consensus degrees.* In this phase consensus degrees amongst the experts are computed. To do this, a similarity measure is defined to calculate the coincidence amongst experts' opinions.
3) *Consensus control.* In this phase the CSS controls the level of consensus and the number of rounds of discussion to be carried out. Thus, if the agreement amongst the experts is greater than a specified consensus threshold (γ) then the consensus process will stop and the selection process will be applied to obtain the solution of consensus. If that is not the case, the fourth phase is applied, i.e., the experts' opinions must be modified. In order to avoid that the consensus process does not converge after several rounds of discussion, we incorporate a maximum number of rounds to be developed in the CSS model, *Maxcycles*, as was done in the consensus model proposed in [3], [17].
4) *Production of advice.* To help experts change their opinions, the CSS generates a set of recommendations or advice. To do this, proximity measures are used in conjunction with the consensus degrees to build a guidance advice system, which acts as a feedback mechanism that generates advice so that experts can change their opinions.

A description of the resolution process of a multigranular GDM problem that uses the proposed CSS model is shown in Fig. 2 The CSS receives the experts' opinions expressed by means of multigranular linguistic preference relations. Once the multigranular linguistic preference relations are uniform, the CSS checks consensus by computing the consensus degrees at the three levels of representation of information. If the global consensus degree does not reach a specified consensus threshold γ, then the feedback mechanism is applied to generate appropriate advice on the changes experts should do in their opinions in order to increase the level of consensus. These phases are applied until γ or *Maxcycles* are reached. This CSS model is, therefore, developed using an evolutionary and iterative process [28].

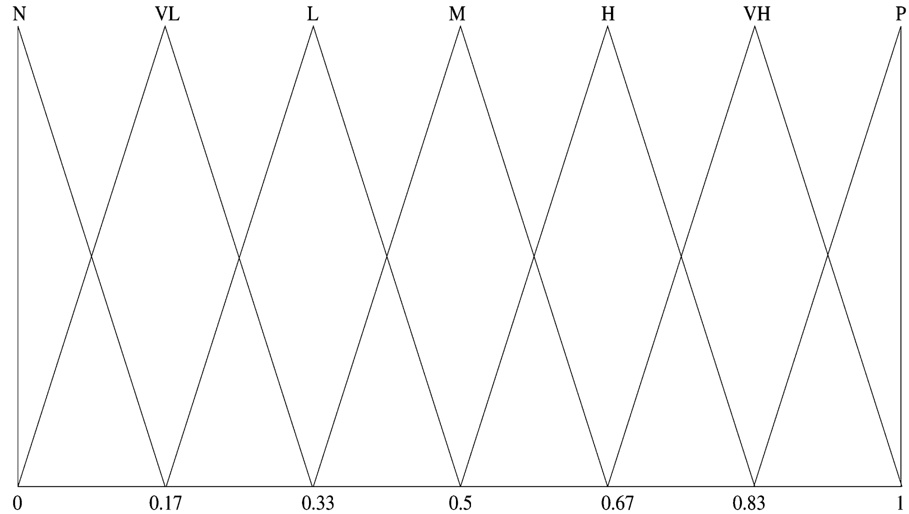

Fig. 3. Set of seven linguistic terms with its semantics.

The rest of this paper is set out as follows. The multigranular linguistic GDM problem is described in Section II. The CSS model for multigranular linguistic GDM problems is detailed in Section III. In Section IV, a practical example is given to illustrate the application of the CSS model. Finally, in Section 5 we draw our conclusions.

II. Multigranular Linguistic GDM Problems

We focus on GDM problems in which two or more experts express their preferences on a set of alternatives to obtain a solution. A classical way to express preferences in GDM problems is by means of preference relations [9]. A GDM problem based on preference relations may be defined as follows: There are a finite set of alternatives, $X = \{x_1, x_2, \dots, x_n\}$ $(n \geq 2)$, and a group of experts, $E = \{e_1, e_2, \dots, e_m\}$ $(m \geq 2)$; each expert e_i provides his/her preferences about X by means of a preference relation, $\mathbf{P}_{\mathbf{e_i}} \subset X \times X$, where the value $\mu_{\mathbf{P}_{\mathbf{e_i}}}(x_j, x_k) = p_i^{jk}$ is interpreted as the preference degree of the alternative x_j over x_k for e_i.

Traditionally, in fuzzy GDM problems, experts express their opinions about X by means of fuzzy preference relations with numerical values [6], [9], [23], [42], i.e., $\mu_{\mathbf{P}_{\mathbf{e_i}}} : X \times X \rightarrow [0,1]$. However, there are situations where it could be very difficult for the experts to provide their opinions using precise numerical values, as is the case, for example, when the knowledge about the alternatives is vague and/or imprecise. In such cases, the alternative use of a fuzzy linguistic approach [40], [43], [44] has provided good results [11], [27], [29], [34], [38], [39]. In this approach, linguistic assessments are used instead of numerical values to represent preferences, i.e., preferences on alternatives are assessed using linguistic terms or labels [40], [43], [44], i.e., $\mu_{\mathbf{P}_{\mathbf{e_i}}} : X \times X \rightarrow S$, where $S = \{s_0, s_1, \dots, s_g\}$ is a linguistic term set characterized by its cardinality or *granularity*, $\#(S) = g+1$. The granularity of S should be small enough so as not to impose useless precision levels on the users but big enough to allow a discrimination of the assessments in a limited number of degrees. Additionally, the following properties are assumed.

1) The set S is ordered: $s_i \geq s_j$, if $i \geq j$.
2) There is the negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = g - i$.
3) There is the min operator: $\mathrm{Min}(s_i, s_j) = s_i$ if $s_i \leq s_j$.
4) There is the max operator: $\mathrm{Max}(s_i, s_j) = s_i$ if $s_i \geq s_j$.

The semantics of the terms is given by fuzzy numbers defined on the [0,1] interval. One way to characterize a fuzzy number is using a representation based on parameters of its membership function [1]. For example, the following semantics, represented in Fig. 3, can be assigned to a set of seven terms via triangular fuzzy numbers:

$$
\begin{aligned}
P &= \text{Perfect} = (0.83, 1, 1) \\
VH &= \text{Very\_High} = (0.67, 0.83, 1) \\
H &= \text{High} = (0.5, 0.67, 0.83) \\
M &= \text{Medium} = (0.33, 0.5, 0.67) \\
L &= \text{Low} = (0.17, 0.33, 0.5) \\
VL &= \text{Very\_Low} = (0, 0.17, 0.33) \\
N &= \text{None} = (0, 0, 0.17).
\end{aligned}
$$

The ideal situation in GDM problems in a linguistic context would be one where all the experts use the same linguistic term set S to express their preferences about the alternatives. However, in some cases, experts may belong to distinct research areas and will, therefore, have different background and levels of knowledge. A consequence of this is that the expression of preferences will be based on linguistic term sets with different granularity, which means that adequate tools to manage and model multigranular linguistic information become essential [10], [16], [35].

In this paper, we deal with multigranular linguistic GDM problems, i.e., GDM problems where the experts e_i may express their multigranular linguistic preference relations $\mathbf{P}_{\mathbf{e_i}} = (p_i^{jk})$ on the set of alternatives X, using different linguistic term sets with different cardinality and/or semantics $S_i = \{s_0^i, \dots, s_g^i\}$. Therefore, $p_i^{jk} \in S_i$ represents the preference of alternative x_j over alternative x_k for the expert e_i assessed on the label set S_i.

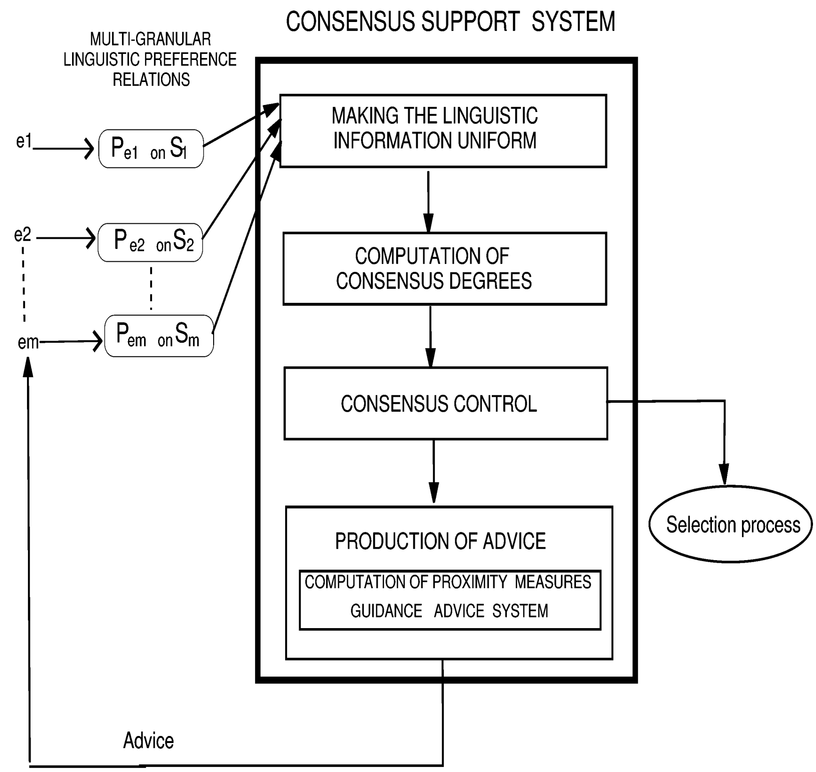

Fig. 4. CSS model in a multigranular linguistic context.

III. A Consensus Support System Model for Multigranular GDM Problems

Decision support systems (DSSs) are becoming an essential tool nowadays for making decisions, mainly due to the large amount of diverse, and frequently uncertain, information that has to be processed in most important decision situations [4], [5], [12]–[14], [20]–[22], [24], [28], [32], [36], [37]. In this section, we design a model of consensus support system (CSS), i.e., the component of a group DSS that provides support to the experts to reach consensus during the process of making a decision. This CSS model has the following two main characteristics.

1) It is designed to guide the consensus process of multigranular linguistic GDM problems.
2) A guidance advice system is included to substitute the moderator's actions and to give advice to the experts to find out the changes they need to make in their opinions in order to obtain the highest degree of consensus possible.

Although the main purpose of this CSS model is to support the decision makers throughout the consensus process, they are responsible for the final decision. In fact, it is the decision makers who decide whether or not to follow the advice generated by the CSS. In any case, because the CSS considerably reduces the time associated with making the decision, it extends the decision makers ability to analyze the information involved in the decision-making process. The advice produced by the CSS will provide the decision makers with a clear picture of their actual position within the group, which they can then use to decide upon their actual position or subsequent action.

The CSS model that we propose is built up using the following.

1) A multigranular linguistic methodology to unify all the different linguistic preferences into a single domain.
2) Two consensus criteria: *consensus degrees* and *proximity measures*. The first ones are used to measure the agreement amongst all the experts, while the second ones are used to learn how close the collective and individual expert's preferences are. Both consensus criteria are calculated at three levels: pairs of alternatives, alternatives, and relation.
3) A set of advice rules and a feedback mechanism, based on the above consensus criteria, to guide the direction of change in the experts' opinions.

The CSS model consists of four consecutive steps as illustrated in Fig. 4, which will be described in detail in the following subsections.

A. Making the Linguistic Information Uniform

To manage multigranular information we need to make it uniform [10], i.e., experts' preferences must be transformed (using a transformation function) into a single domain or linguistic term set that we call *basic linguistic term set* (BLTS) denoted by S_T. To do this, it seems reasonable to impose a granularity high enough to maintain the uncertainty degrees associated to each one of the possible domains to be unified. This means that the granularity of the BLTS has to be as high as possible. Therefore, in a general multigranular linguistic context, to select S_T we proceed as follows.

1) If there is only one linguistic term set, from the set of different domains to be unified, with maximum granularity, then we choose that one as the BLTS, S_T.
2) If there are two or more linguistic term sets with maximum granularity, then the election of S_T will depend on the semantics associated to them.
a) If all of them have the same semantics (with different labels), then any one of them can be selected as S_T.
b) If two or more of them have different semantics, then S_T is defined as a generic linguistic term set with a number of terms greater than the number of terms a person is able to discriminate, which is normally 7 or 9 [30]. For example, we can use a BLTS with 15 terms symmetrically distributed [10].

Once S_T has been selected, the following multigranular transformation function is applied to transform every linguistic value into a fuzzy set defined on S_T.

Definition 1 [10]: If $A = \{l_0, \ldots, l_p\}$ and $S_T = \{c_0, \ldots, c_g\}$ are two linguistic term sets, with $g \geq p$, then a multigranular transformation function, τ_{AS_T}, is defined as

$$\begin{aligned}&\tau_{AS_T} : A \rightarrow F(S_T)\\ &\tau_{AS_T}(l_i) = \{(c_h, \alpha_{ih}) | h \in \{0, \ldots, g\} \qquad \forall\, l_i \in A\\ &\qquad \alpha_{ih} = \max_y \min\{\mu_{l_i}(y), \mu_{c_h}(y)\}\end{aligned}$$

where $F(S_T)$ is the set of fuzzy sets defined on S_T, and $\mu_{l_i}(y)$ and $\mu_{c_h}(y)$ are the membership functions of the fuzzy sets associated to the linguistic terms l_i and c_h, respectively.

The composition of the linguistic preference relations provided by the experts, $\mu_{\mathbf{P}_{\mathbf{e_i}}}$, with the *multigranular transformation functions*, $\tau_{S_i S_T}$, will result in a unification of the preferences for the whole group of experts. In particular, the linguistic preference p_i^{lk} will be transformed into the fuzzy set, defined on $S_T = \{c_0, \ldots, c_g\}$

$$\begin{aligned}\tau_{S_i S_T}(p_i^{lk}) &= \{(c_h, \alpha_{ih}^{lk}) | h = 0, \ldots, g\}\\ \alpha_{ih}^{lk} &= \max_y \min\{\mu_{p_i^{lk}}(y), \mu_{c_h}(y)\}.\end{aligned}$$

To simplify, we will continue to denote $\tau_{S_i S_T}(p_i^{lk})$ by p_i^{lk}, and we will use only the membership degrees to denote the uniformed linguistic preference relation

$$\mathbf{P}_{\mathbf{e_i}} = \begin{pmatrix} p_i^{11} = (\alpha_{i0}^{11}, \ldots, \alpha_{ig}^{11}) & \cdots & p_i^{1n} = (\alpha_{i0}^{1n}, \ldots, \alpha_{ig}^{1n}) \\ \vdots & \ddots & \vdots \\ p_i^{n1} = (\alpha_{i0}^{n1}, \ldots, \alpha_{ig}^{n1}) & \cdots & p_i^{nn} = (\alpha_{i0}^{nn}, \ldots, \alpha_{ig}^{nn}) \end{pmatrix}.$$

B. Computation of Consensus Degrees

The computation of consensus degrees requires the use of some similarity or coincidence function to obtain the level of agreement amongst all the experts [12]–[14]. These similarity or coincidence functions detect how far each individual expert is from the rest. If the experts' preferences are represented as preference vectors, then we can define a similarity function using anyone of the traditional distance measures between vectors, as, for example, the Euclidean distance or the cosine of their vector-angle.

As we said in the previous subsection, in the linguistic preference relation $\mathbf{P}_{\mathbf{e_i}}$ each preference value p_i^{lk} is represented as a fuzzy subset defined on S_T, and therefore, each preference value is a vector of membership degrees. This is why to calculate the proximity between the linguistic preferences p_i^{lk}, p_j^{lk} given by the experts e_i, e_j, we initially applied these traditional distance measures to their associated membership degrees vectors. However, after checking the results of some trials, we discovered cases in which unexpected results were obtained, as is shown in the following example, which implied that these distance measures were not suitable to define a similarity function in our case.

Example 1: If $p_1^{12} = (1,0,0,0,0,0), p_2^{12} = (0,0,0,1,0,0)$, and $p_3^{12} = (0,0,0,0,0,1)$ are three experts' assessments on the pair of alternatives (x_1, x_2), the following values are obtained using the Euclidean distance:

$$\begin{aligned}d\left(p_1^{12}, p_2^{12}\right) &= \sqrt{\sum_{h=0}^{g} \left(\alpha_{1h}^{12} - \alpha_{2h}^{12}\right)^2} = \sqrt{2}\\ d\left(p_1^{12}, p_3^{12}\right) &= \sqrt{\sum_{h=0}^{g} \left(\alpha_{1h}^{12} - \alpha_{3h}^{12}\right)^2} = \sqrt{2}.\end{aligned}$$

With the Euclidean distance, both preference values p_3^{12} and p_2^{12} are at the same distance from the preference value p_1^{12}, although, it is clear, however, that the first one is further from p_1^{12} than the second one. The problem, in this case, is the way the information of these fuzzy sets is interpreted, as a vector of membership degrees without having taken into account their positions in it. To take into account both the membership values and positions, a different similarity function able to represent the distribution of the information in the fuzzy set p_i^{lk} is necessary.

To overcome the previous problem we define a similarity function based on the central value of a fuzzy set, cv_i^{lk}

$$\mathrm{cv}_i^{lk} = \frac{\sum_{h=0}^{g} \mathrm{index}\left(s_h^i\right) \cdot \alpha_{ih}^{lk}}{\sum_{h=0}^{g} \alpha_{ih}^{lk}} \tag{1}$$

which represents the average position or centre of gravity of the information contained in the fuzzy set $p_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$, being $\mathrm{index}(s_h^i) = h$. The range of this central value is the closed interval $[0, g]$. Indeed, from the obvious inequalities

$$0 \leq \mathrm{index}\left(s_h^i\right) \leq g \quad \text{and} \quad 0 \leq \alpha_{ih}^{lk} \qquad \forall\, i, h, k, l$$

we have that

$$0 \leq \mathrm{index}\left(s_h^i\right) \cdot \alpha_{ih}^{lk} \leq g \cdot \alpha_{ih}^{lk} \qquad \forall\, i, h, k, l$$

and

$$0 \leq \sum_{h=0}^{g} \mathrm{index}\left(s_h^i\right) \cdot \alpha_{ih}^{lk} \leq g \cdot \sum_{h=0}^{g} \alpha_{ih}^{lk} \qquad \forall\, i, k, l.$$

Finally, dividing all sides of the inequality by $\sum_{h=0}^{g} \alpha_{ih}^{lk}$, we obtain

$$0 \leq \frac{\sum_{h=0}^{g} \text{index}\left(s_h^i\right) \cdot \alpha_{ih}^{lk}}{\sum_{h=0}^{g} \alpha_{ih}^{lk}} \leq g \; \forall \, i, k, l$$

that is $\text{cv}_i^{lk} \in [0, g] \; \forall i, l, k$.

Example 2: The application of (1) to the assessments of example 1 gives the following central values:

$$\text{cv}_1^{12} = 0 \quad \text{cv}_2^{12} = 3 \quad \text{cv}_3^{12} = 5.$$

For $p_1^{14} = (0.3, 0.8, 0.6, 0, 0, 0), p_1^{24} = (0, 0.3, 0.8, 0.6, 0, 0)$, and $p_1^{34} = (0, 0, 0, 0.3, 0.8, 0.6)$, the central values are

$$\text{cv}_1^{14} = 1.18 \quad \text{cv}_1^{24} = 2.18 \quad \text{and} \quad \text{cv}_1^{34} = 4.18.$$

As expected, when the information (membership values) moves from the left part of the fuzzy set to the right part, the central value increases.

The value $|\text{cv}_i^{lk} - \text{cv}_j^{lk}|$ can be used as a measure of distance between the preference values p_i^{lk} and p_j^{lk}. Thus, we define a similarity function s between these two preference values, measured in the unit interval $[0, 1]$, as follows:

$$s\left(p_i^{lk}, p_j^{lk}\right) = 1 - \left| \frac{\text{cv}_i^{lk} - \text{cv}_j^{lk}}{g} \right|. \tag{2}$$

The closer $s(p_i^{lk}, p_j^{lk})$ to 1 the more similar p_i^{lk} and p_j^{lk} are, while the closer $s(p_i^{lk}, p_j^{lk})$ to 0 the more distant p_i^{lk} and p_j^{lk} are.

Example 3: The values of similarity between the assessments of example 1 are

$$s\left(p_1^{12}, p_2^{12}\right) = 0.4 \quad s\left(p_1^{12}, p_3^{12}\right) = 0.$$

Using the previous similarity function (2), the computation of the consensus degrees is carried out in the following steps.

1) After the experts' preferences are uniform, the *central values* are calculated:

$$\text{cv}_i^{lk}; \; \forall \, i = 1, \ldots, m; \quad l, k = 1, \ldots, n \; \wedge \; l \neq k. \tag{3}$$

2) For each pair of experts $e_i, e_j (i < j)$, a *similarity matrix* $\text{SM}_{ij} = (\text{sm}_{ij}^{lk})$ is calculated, where

$$\text{sm}_{ij}^{lk} = s\left(p_i^{lk}, p_j^{lk}\right). \tag{4}$$

3) A *consensus matrix*, $\text{CM} = (\text{cm}^{lk})$, is obtained by aggregating all the similarity matrices. This aggregation is carried out at the level of pairs of alternatives:

$$\text{cm}^{lk} = \phi\left(\text{sm}_{ij}^{lk}\right);$$
$$i, j = 1, \ldots, m \; \wedge \forall \, l, k = 1, \ldots, n \; \wedge \; i < j.$$

In our case, we propose the use of the arithmetic mean as the aggregation function ϕ, although, different aggregation operators could be used according to the particular properties we want to implement.

4) Computation of consensus degrees. As we said in Section 1, the consensus degrees are computed at the three different levels: pairs of alternatives, alternatives and relation.

Level 1) *Consensus on pairs of alternatives.* The consensus degree on a pair of alternatives (x_l, x_k), called cp^{lk}, is defined to measure the consensus degree amongst all the experts on that pair of alternatives. In our case, this is expressed by the element (l, k) of the consensus matrix CM, i.e.,

$$\text{cp}^{lk} = \text{cm}^{lk}, \; \forall l, k = 1, \ldots, n \; \wedge \; l \neq k.$$

The closer cp^{lk} to 1, the greater the agreement amongst all the experts on the pair of alternatives (x_l, x_k). This measure will allow the identification of those pairs of alternatives with a poor level of consensus.

Level 2) *Consensus on alternatives.* The consensus degree on an alternative x_l, called ca^l, is defined to measure the consensus degree amongst all the experts on that alternative. For this, we take the average of the row l of the consensus matrix CM, i.e.,

$$ca^l = \frac{\sum_{k=1}^{n} cm^{lk}}{n}. \tag{5}$$

These values can be used to propose the modification of preferences associated to those alternatives with a consensus degree lower than a minimal consensus threshold γ, i.e., $ca^l < \gamma$.

Level 3) *Consensus on the relation.* The consensus degree on the relation, called cr, is defined to measure the global consensus degree amongst the experts' opinions. It is computed as the average of all the consensus degrees on the alternatives, i.e.,

$$\text{cr} = \frac{\sum_{l=1}^{n} \text{ca}^l}{n}. \tag{6}$$

This is the value that the CSS model uses to control the consensus situation.

C. Consensus Control

How the CSS controls the consensus level in each discussion round is addressed. Before applying the CSS model, a minimum consensus threshold, $\gamma \in [0, 1]$, is fixed, which will obviously depend on the particular problem we are dealing with. When the consequences of the decision to be made are of a transcendent importance, the minimum level of consensus required to make that decision should be logically as high as possible, and it is not unusual if a minimum value of 0.8 or higher is imposed. At the other extreme, we have cases where the consequences are not so transcendental (but are still important), where it is urgent to obtain a solution to the problem, and thus, a minimum consensus value as close as possible to 0.5 could be required.

In any case, when the consensus measure cr reaches γ the CSS will stop and the selection process will be applied to obtain the solution. However, as we said before, the global consensus measure may not converge to this minimal consensus threshold.

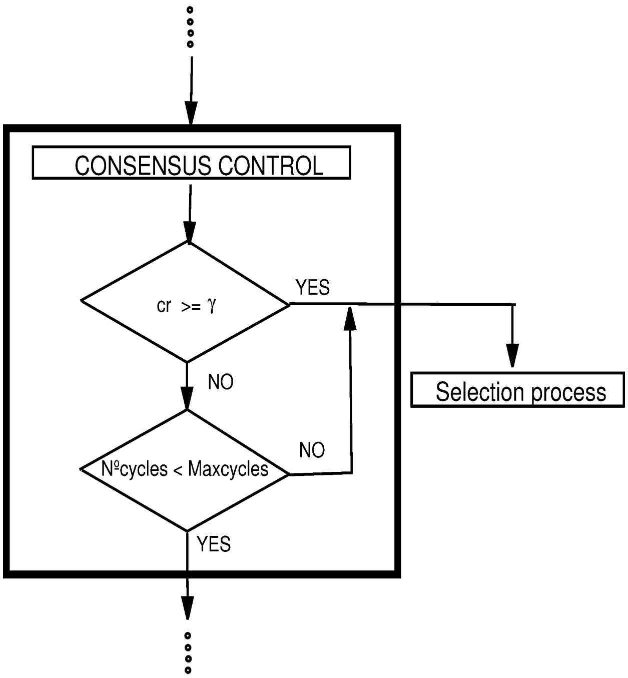

Fig. 5. Consensus control.

In order to avoid this, two parameters, N^0 cycles, to control the number of executed discussion rounds, and *Maxcycles*, to control the maximum number of rounds already executed, are incorporated into the CSS model. This is shown in Fig. 5.

D. Production of Advice

When the consensus level is lower than the minimum threshold value, the experts' opinions must be modified. This is done in a group discussion session in which the CSS model uses proximity measures to identify those experts furthest away from the collective opinion, and a guidance advice system to generate recommendations to support the experts in changing their opinions. Both, the proximity measures and the guidance advice system, are explained in detail in the following subsections.

1) Computation of Proximity Measures: Proximity measures evaluate the agreement between the individual experts' opinions and the group opinion. Thus, to calculate them, a collective preference relation, $\mathbf{P}_{\mathbf{e_c}} = (p_c^{lk})$, has to be obtained by means of the aggregation of the set of (uniformed) individual preference relations $\{\mathbf{P}_{\mathbf{e_1}}, \ldots, \mathbf{P}_{\mathbf{e_m}}\}$

$$p_c^{lk} = \psi\left(p_1^{lk}, \ldots, p_m^{lk}\right)$$

with ψ an "aggregation operator."

Because $p_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$ then $p_c^{lk} = (\alpha_{c0}^{lk}, \ldots, \alpha_{cg}^{lk})$ with $\alpha_{cj}^{lk} = \psi(\alpha_{1j}^{lk}, \ldots, \alpha_{mj}^{lk})$, which means that p_c^{lk} is also a fuzzy set defined on S_T. Clearly, the similarity functions defined in expression (2) can be used to evaluate the agreement between each individual expert's preferences, $\mathbf{P}_{\mathbf{e_i}}$, and the collective preferences, $\mathbf{P}_{\mathbf{e_c}}$. Therefore, the measurement of proximity is carried out in two steps.

1) A *proximity matrix*, $\mathrm{PM}_i = (\mathrm{pm}_i^{lk})$, for each expert e_i, is obtained where $\mathrm{pm}_i^{lk} = s(p_i^{lk}, p_c^{lk})$.
2) Computation of proximity measures. Again, we calculate proximity measures at three different levels.

Level 1) *Proximity on pairs of alternatives*. Given an expert e_i, his/her proximity measure on a pair of alternatives, (x_l, x_k), called pp_i^{lk}, is defined to measure the proximity between his/her preference value on that pair of alternatives and the group's one. In our case, this is expressed by the element (l,k) of the proximity matrix PM_i, i.e.,

$$\mathrm{pp}_i^{lk} = \mathrm{pm}_i^{lk} \qquad \forall l,k = 1,\ldots,n \ \wedge \ l \neq k.$$

Level 2) *Proximity on alternatives.* Given an expert e_i, his/her proximity measure on an alternative, x_l, called pa^l, is defined to measure the proximity between his/her preference values on that alternative and the group's ones. For this, we take the average of the proximities on pairs of alternatives of x_l.

$$\mathrm{pa}_i^l = \frac{\sum_{k=1}^n \mathrm{pp}_i^{lk}}{n}. \tag{7}$$

Level 3) *Proximity on the relation.* Given an expert e_i, his/her proximity measure on the relation, P_{e_i}, called pr_i, is defined to measure the global proximity between his/her preference values on all alternatives and the group's ones. It is computed as the average of all proximity on alternative values, i.e.,

$$\mathrm{pr}_i = \frac{\sum_{l=1}^n \mathrm{pa}_i^l}{n}. \tag{8}$$

If the aforementioned proximity values are close to 1 then they have a positive contribution for the consensus to be high, while if they are close to 0 then they have a negative contribution to consensus. As a consequence, these proximity measures can be used to build a guidance advice system that acts as a feedback mechanism for the experts to change their opinions and to find out which direction that change has to follow in order to obtain the highest degree of consensus possible.

2) Guidance Advice System: As aforementioned, the goal of the guidance advice system is to generate recommendations or advice to the experts in order to achieve a solution set of alternatives with the highest degree of consensus possible. Therefore, the guidance advice system will be applied until a satisfactory consensus level is reached or when a stop condition is satisfied (N^0 cycles reaches *Maxcycle*), as explained in Subsection III-C. There are two reasons why we call this a guidance system.

i) It is able to identify, in a precise way, the experts, alternatives and pairs of alternatives with a negative contribution to consensus, which will allow the CSS to provide appropriate advice on changes of the assessments associated to only those negative contributors.
ii) It is able to advise on the direction of the required changes, by increasing or decreasing the value of the assessments.

To achieve i) and ii), the guidance advice system consists of a set of two types of advice rules, A) identification rules and B) direction rules.

A) **Identification rules (IR)** to identify the experts, the alternatives and the pairs of alternatives that should participate in the change process. Therefore, we define three identification rules.

1) *An identification rule of experts*, to identify those experts that should receive advice on how to change some of their preferences values. Obviously, the first experts to change their opinions are those with the lowest proximity values pe_i. At this point, the number or % of experts *(ne)* that should modify their opinions has to be decided. The choice of the value of *ne* may depend on the type of problem dealt with and/or the amount of time available for carrying out the discussion sessions amongst the experts. If a quick achievement of consensus is desired, then the value of *ne* might be high (for example $ne = 75\%$) while if *ne* is low (for example $ne = 25\%$) then the CSS would need to be executed more time and, therefore, more time will be needed, to reach consensus. This set of experts is denoted as EXPCH. Therefore, the identification rule of experts is the following:

IR.1) $\forall e_i \in E \cap \mathrm{EXPCH}$, then e_i should receive advice on how to change his/her opinions, being

$$\mathrm{EXPCH} = \{e_{\sigma(1)}, \ldots, e_{\sigma(ne)}\}$$

where σ is a permutation over the set of proximities on the relation defined as $\mathrm{pr}_{\sigma(j)} \leq \mathrm{pr}_{\sigma(i)}\ \forall\, j \leq i$.

2) *An identification rule of alternatives*, to identify those alternatives whose associated assessments should be taken into account by the above experts in the change process of their preferences. This set of alternatives is denoted as ALT. To do this, we use the consensus degrees on alternatives $\{\mathrm{ca}^l, l = 1, \ldots, n\}$, being the identification rule of alternatives the following:

IR.2) $\forall x_l \in X \cap \mathrm{ALT}$ then $\forall e_i \in \mathrm{EXPCH}, e_i$ should consider to change some of his/her assessments associated to the set of pairs of alternatives $\{(x_l, x_k), k = 1, \ldots, n\}$, i.e., the following set of preference values

$$\mathbf{P}_{\mathbf{e_i}}[x_l] = \{p_i^{lk}, k = 1, \ldots, n\}$$

being ALT the set of alternatives with associated consensus degrees ca^l lower than the specified consensus threshold γ, i.e.,

$$\mathrm{ALT} = \{x_l \in X | ca^l < \gamma\}.$$

3) *An identification rule of pairs of alternatives*, to identify those particular pairs of alternatives (x_l, x_k) whose respective associated assessments $p_i^{lk} \in \mathbf{P}_{\mathbf{e_i}}[x_l]$ the expert e_i should change. This set of pairs of alternatives is denoted as PALT_i. To do this, we use the proximity measures on pairs of alternatives, being the identification rule of pairs of alternatives the following:

IR.3) $\forall (x_l \in \mathrm{ALT}\ \wedge\ e_i \in \mathrm{EXPCH})$, if $(x_l, x_k) \in \mathrm{PALT}_i$ then e_i should change p_i^{lk}, being PALT_i the set of pairs of alternatives (x_l, x_k) whose proximity values pp_i^{lk} are below a minimum proximity threshold, β, i.e.,

$$\mathrm{PALT}_i = \{(x_l, x_k) | x_l \in \mathrm{ALT}\ \wedge\ e_i \in \mathrm{EXPCH}\ \wedge\ \mathrm{pp}_i^{lk} < \beta\}.$$

Clearly, the greater β the greater the number of changes needed.

B) **Direction rules** to find out the direction of the change to be recommended in each case, i.e., the direction of the change to be applied to the preference assessment p_i^{lk}, with $(x_l, x_k) \in \mathrm{PALT}_i$. To do this, two pairs of direction parameters are obtained, one from p_i^{lk}, and the other from the collective preference assessment p_c^{lk}. These pairs of direction parameters will contain both the position and membership degree associated to a main-label ($ml \in \mathrm{BLTS}$) and a secondary-label ($sl \in \mathrm{BLTS}$), respectively. The main-label will correspond to that with maximum membership degree while the secondary-label will correspond to that with second greatest membership degree. Therefore, for each preference assessment p_i^{lk} to be changed, $(p_i^{lk}(ml_{\mathrm{pos}}), p_i^{lk}(ml_{\mathrm{val}}), p_i^{lk}(sl_{\mathrm{pos}}), p_i^{lk}(sl_{\mathrm{val}}))$ and $(p_c^{lk}(ml_{\mathrm{pos}}), p_c^{lk}(ml_{\mathrm{val}}), p_c^{lk}(sl_{\mathrm{pos}}), p_c^{lk}(sl_{\mathrm{val}}))$ are compared to define the following four direction rules.

DR.1) If $p_i^{lk}(ml_{\mathrm{pos}}) > p_c^{lk}(ml_{\mathrm{pos}})$ then the expert e_i should decrease the assessment associated to the pair of alternatives (x_l, x_k), i.e., p_i^{lk}.

DR.2) If $p_i^{lk}(ml_{\mathrm{pos}}) < p_c^{lk}(ml_{\mathrm{pos}})$ then the expert e_i should increase the assessment associated to the pair of alternatives (x_l, x_k), i.e., p_i^{lk}.

DR.3) If $p_i^{lk}(ml_{\mathrm{pos}}) = p_c^{lk}(ml_{\mathrm{pos}})$ then rules DR.1, DR.2, and DR.3 are applied using the membership values of the main-labels, $p_i^{lk}(ml_{\mathrm{val}})$ and $p_c^{lk}(ml_{\mathrm{val}})$.

DR.4) If $(p_i^{lk}(ml_{\mathrm{pos}}) = p_c^{lk}(ml_{\mathrm{pos}}), p_i^{lk}(ml_{\mathrm{val}}) = p_c^{lk}(ml_{\mathrm{val}}))$, then rules DR.1, DR.2, and DR.3 are applied using the position and membership values of the secondary-labels *sl*.

These direction rules will not be produced when a decrease or increase are suggested to an assessment represented by the first or last label of a linguistic term set, respectively.

The structure of the algorithm that implements the operation of the guidance advice system based on the above advice rules is shown in Table I.

Obviously, the consensus reaching process will depend on the size of the group of experts as well as on the size of the set of alternatives, so that when these sizes are small and when opinions are homogeneous, the consensus level required is easier to obtain [17], [41]. On the other hand, we note that changes in the experts' opinions will produce a change in the collective opinion, especially when the experts opinions are quite different, i.e., in the early stages of the consensus process. In fact, when experts opinions are close, i.e., when the consensus measure approaches the consensus level required, changes in experts' opinions will not produce a great difference in the collective opinion. This will be illustrated with an example in the next section.

TABLE I
OPERATION ALGORITHM OF THE GUIDANCE ADVICE SYSTEM

```
INPUTS:
    γ, ne, β
BEGIN
Compute EXPCH = {e_σ(1), ..., e_σ(ne)}.
Compute ALT = {x_l ∈ X | ca^l < γ}.
FOR i = 1 TO m DO
IF e_i ∈ EXPCH
THEN
    FOR l = 1 TO n DO
    IF x_l ∈ ALT
    THEN
        FOR k = 1 TO n DO
        IF pp_i^lk < β
        THEN
          Compute the values: p_i^lk(ml_pos), p_i^lk(ml_val), p_i^lk(sl_pos), p_i^lk(sl_val)
          Compute the collective preference relation: P_c
          Compute the values: p_c^lk(ml_pos), p_c^lk(ml_val), p_c^lk(sl_pos), p_c^lk(sl_val)
          IF p_i^lk(ml_pos) > p_c^lk(ml_pos)
          THEN
            IF p_i^lk is the first label of S_i
            THEN
              Do not change p_i^lk
            ELSE
            Decrease p_i^lk
            END-IF
          END-IF
          IF p_i^lk(ml_pos) < p_c^lk(ml_pos)
          THEN
            IF p_i^lk is the last label of S_i
            THEN
              Do not change p_i^lk
            ELSE
            Increase p_i^lk
            END-IF
          END-IF
          ELSE
            IF p_i^lk(ml_val) > p_c^lk(ml_val)
            THEN
            IF p_i^lk is the first label of S_i
            THEN
              Do not change p_i^lk
            ELSE
              Decrease p_i^lk
            END-IF
            IF p_i^lk(ml_val) < p_c^lk(ml_val)
            THEN
            IF p_i^lk is the last label of S_i
            THEN
              Do not change p_i^lk
            ELSE
              Increase p_i^lk
            ELSE
              Apply the same rules for the secondary-labels sl
            END-IF
          END-IF
        END-IF
    END-IF
END-IF
END
```

IV. Example of Application of the CSS Model

An investment company wants to invest a sum of money in the best industrial sector, from the set of four possible alternatives:

- Car industry: x_1;
- Food company: x_2;
- Computer company: x_3;
- Arms industry: x_4.

To do this, four consultancy departments within the company are requested to provide information:

τ_{AS_T} :	τ_{BS_T} :	τ_{CS_T} :
$a_0 \mapsto (1,0,0,0,0,0,0,0,0)$	$b_0 \mapsto (1,0.57,0.14,0,0,0,0,0,0)$	$c_0 \mapsto (1,0.68,0.34,0,0,0,0,0,0)$
$a_1 \mapsto (0,1,0,0,0,0,0,0,0)$	$b_1 \mapsto (0.43,0.86,0.7,0.28,0,0,0,0,0)$	$c_1 \mapsto (0.32,0.66,1,0.68,0.34,0,0,0,0)$
$a_2 \mapsto (0,0,1,0,0,0,0,0,0)$	$b_2 \mapsto (0,0.3,0.72,0.86,0.43,0,0,0,0)$	$c_2 \mapsto (0,0,0.32,0.66,1,0.68,0.34,0,0)$
$a_3 \mapsto (0,0,0,1,0,0,0,0,0)$	$b_3 \mapsto (0,0,0.14,0.57,1,0.57,0.14,0,0)$	$c_3 \mapsto (0,0,0,0,0.32,0.66,1,0.68,0.34)$
$a_4 \mapsto (0,0,0,0,1,0,0,0,0)$	$b_4 \mapsto (0,0,0,0,0.43,0.86,0.7,0.28,0)$	$c_4 \mapsto (0,0,0,0,0,0,0.32,0.66,1)$
$a_5 \mapsto (0,0,0,0,0,1,0,0,0)$	$b_5 \mapsto (0,0,0,0,0,0.3,0.72,0.86,0.43)$	
$a_6 \mapsto (0,0,0,0,0,0,1,0,0)$	$b_6 \mapsto (0,0,0,0,0,0,0.14,0.57,1)$	
$a_7 \mapsto (0,0,0,0,0,0,0,1,0)$		
$a_8 \mapsto (0,0,0,0,0,0,0,0,1)$		

- Risk analysis department: e_1;
- Growth analysis department: e_2;
- Social-political analysis department: e_3;
- Environmental impact analysis department: e_4.

Each department is directed by an expert who provides his/her preferences about the alternatives using the following linguistic term sets:

- e_1 and e_2 provide their preferences by using a linguistic term set of granularity 5, **C**;
- e_3 provides preferences using a linguistic term set of granularity 9, **A**;
- e_4 provides preferences using a linguistic term set of granularity 7, **B**.

Label set A	**Label set B**	**Label set C**
$a_0 = (0,0,0.12)$	$b_0 = (0,0,0.16)$	$c_0 = (0,0,0.25)$
$a_1 = (0,0.12,0.25)$	$b_1 = (0,0.16,0.33)$	$c_1 = (0,0.25,0.5)$
$a_2 = (0.12,0.25,0.37)$	$b_2 = (0.16,0.33,0.5)$	$c_2 = (0.25,0.5,0.75)$
$a_3 = (0.25,0.37,0.5)$	$b_3 = (0.33,0.5,0.66)$	$c_3 = (0.5,0.75,1)$
$a_4 = (0.37,0.5,0.62)$	$b_4 = (0.5,0.66,0.83)$	$c_4 = (0.75,1,1)$
$a_5 = (0.5,0.62,0.75)$	$b_5 = (0.66,0.83,1)$	
$a_6 = (0.62,0.75,0.87)$	$b_6 = (0.83,1,1)$	
$a_7 = (0.75,0.87,1)$		
$a_8 = (0.87,1,1)$		

The linguistic preference relations provided by each one of the experts are

$$\mathbf{P_{e_1}} = \begin{pmatrix} - & c_0 & c_0 & c_2 \\ c_4 & - & c_3 & c_4 \\ c_3 & c_0 & - & c_1 \\ c_2 & c_1 & c_3 & - \end{pmatrix}$$

$$\mathbf{P_{e_2}} = \begin{pmatrix} - & c_2 & c_0 & c_4 \\ c_1 & - & c_1 & c_1 \\ c_3 & c_3 & - & c_1 \\ c_0 & c_4 & c_3 & - \end{pmatrix}$$

$$\mathbf{P_{e_3}} = \begin{pmatrix} - & a_1 & a_4 & a_3 \\ a_5 & - & a_8 & a_4 \\ a_4 & a_1 & - & a_2 \\ a_5 & a_5 & a_7 & - \end{pmatrix}$$

$$\mathbf{P_{e_4}} = \begin{pmatrix} - & b_0 & b_4 & b_5 \\ b_6 & - & b_1 & b_6 \\ b_3 & b_4 & - & b_2 \\ b_0 & b_1 & b_4 & - \end{pmatrix}$$

We shall use the proposed CSS model to carry out the consensus process of this GDM problem.

FIRST ROUND

1) **Application of the multigranular transformation function**

Once the experts provide their linguistic preference relations, the CSS will choose an appropriate BLTS, $S_T = \{c_0, \ldots, c_g\}$. In this case, because there is only one linguistic term set **A**, from the set of different domains to be unified, with maximum granularity, then $S_T = \mathbf{A}$. Next, multigranular transformation functions $\{\tau_{\mathrm{AS}_T}, \tau_{\mathrm{BS}_T}, \tau_{\mathrm{CS}_T}\}$ are applied, to make the information uniform; see the table at the top of page **XXX**, showing the application of the multigranular transformation fucntions.

2) **Computation of consensus degrees**

1) *Central values:*

A	**B**	**C**
$cv(a_0) = 0$	$cv(b_0) = 0.5$	$cv(c_0) = 0.67$
$cv(a_1) = 1$	$cv(b_1) = 1.37$	$cv(c_1) = 2.02$
$cv(a_2) = 2$	$cv(b_2) = 2.61$	$cv(c_2) = 4.02$
$cv(a_3) = 3$	$cv(b_3) = 4$	$cv(c_3) = 6.02$
$cv(a_4) = 4$	$cv(b_4) = 4.37$	$cv(c_4) = 7.34$
$cv(a_5) = 5$	$cv(b_5) = 6.61$	
$cv(a_6) = 6$	$cv(b_6) = 7.5$	
$cv(a_7) = 7$		
$cv(a_8) = 8$		

2) *Similarity matrices:*

$$\mathrm{SM}_{12} = \begin{pmatrix} - & 0.58 & 1 & 0.59 \\ 0.34 & - & 0.5 & 0.34 \\ 1 & 0.33 & - & 1 \\ 0.58 & 0.34 & 1 & - \end{pmatrix}$$

$$\mathrm{SM}_{13} = \begin{pmatrix} - & 0.96 & 0.58 & 0.87 \\ 0.71 & - & 0.75 & 0.58 \\ 0.75 & 0.96 & - & 1 \\ 0.88 & 0.63 & 0.88 & - \end{pmatrix}$$

$$\mathrm{SM}_{14} = \begin{pmatrix} - & 0.98 & 0.54 & 0.68 \\ 0.98 & - & 0.42 & 0.98 \\ 0.75 & 0.54 & - & 0.93 \\ 0.56 & 0.92 & 0.79 & - \end{pmatrix}$$

$$\mathrm{SM}_{23} = \begin{pmatrix} - & 0.62 & 0.58 & 0.46 \\ 0.63 & - & 0.25 & 0.75 \\ 0.75 & 0.37 & - & 1 \\ 0.46 & 0.71 & 0.88 & - \end{pmatrix}$$

$$\mathrm{SM}_{24} = \begin{pmatrix} - & 0.56 & 0.54 & 0.91 \\ 0.32 & - & 0.92 & 0.32 \\ 0.75 & 0.79 & - & 0.93 \\ 0.98 & 0.25 & 0.79 & - \end{pmatrix}$$

$$\mathrm{SM}_{34} = \begin{pmatrix} - & 0.94 & 0.95 & 0.55 \\ 0.69 & - & 0.17 & 0.56 \\ 1 & 0.58 & - & 0.92 \\ 0.44 & 0.55 & 0.67 & - \end{pmatrix}.$$

3) *Consensus matrix.*

$$\mathrm{CM} = \begin{pmatrix} - & 0.77 & 0.7 & 0.68 \\ 0.61 & - & 0.5 & 0.59 \\ 0.83 & 0.6 & - & 0.96 \\ 0.65 & 0.57 & 0.84 & - \end{pmatrix}.$$

4) *Consensus degrees.*

Level 1) *Consensus on pairs of alternatives.* The element (l, k) of CM represents the consensus degree on the pair of alternatives (x_l, x_k).

Level 2) *Consensus on alternatives.*

$$\mathrm{ca}^1 = 0.72 \quad \mathrm{ca}^2 = 0.57 \quad \mathrm{ca}^3 = 0.8 \quad \mathrm{ca}^4 = 0.69.$$

Level 3) *Consensus on the relation or global consensus.*

$$\mathrm{cr} = 0.7.$$

3) **Consensus control**

In this step of the CSS model, the global consensus value cr is compared with the consensus threshold γ. In this example, we have decided to use the value, $\gamma = 0.75$. Because $\mathrm{cr} < \gamma$, then it is concluded that there is no consensus amongst the experts, and consequently the CSS computes the proximity measures to support the experts on the necessary changes in their preferences in order to increase cr.

4) **Production of advice**

To compute the proximity measures we first obtain the collective preference relation by aggregating all individual preference relations, $\mathbf{P_{e_i}}$. In our case, we do this by using the average as the aggregation operator ψ

$$p_c^{12} = (0.5, 0.56, 0.2, 0.17, 0.25, 0.17, 0.09, 0, 0)$$
$$p_c^{13} = (0.5, 0.34, 0.17, 0, 0.36, 0.22, 0.18, 0.17, 0)$$
$$p_c^{14} = (0, 0, 0.08, 0.42, 0.25, 0.25, 0.35, 0.38, 0.36)$$
$$p_c^{21} = (0.08, 0.17, 0.25, 0.17, 0.09 \\ 0.25, 0.12, 0.31, 0.5)$$
$$p_c^{23} = (0.19, 0.38, 0.43, 0.24, 0.17 \\ 0.17, 0.25, 0.17, 0.34)$$
$$p_c^{24} = (0.08, 0.17, 0.25, 0.17 \\ 0.34, 0, 0.12, 0.31, 0.5)$$
$$p_c^{31} = (0, 0, 0.04, 0.14, 0.66, 0.47 \\ 0.54, 0.34, 0.17)$$
$$p_c^{32} = (0.25, 0.42, 0.09, 0, 0.19 \\ 0.38, 0.43, 0.24, 0.09)$$
$$p_c^{34} = (0.16, 0.41, 0.93, 0.56, 0.28, 0, 0, 0, 0)$$
$$p_c^{41} = (0.5, 0.31, 0.2, 0.17, 0.25, 0.42, 0.09, 0, 0,)$$
$$p_c^{42} = (0.19, 0.38, 0.43, 0.24, 0.09 \\ 0.25, 0.08, 0.17, 0.25)$$
$$p_c^{43} = (0, 0, 0, 0, 0.27, 0.55, 0.68, 0.66, 0.17).$$

4.1 **Computation of Proximity Measures**

1) *Proximity matrices:*

$$\mathrm{PM}_1 = \begin{pmatrix} - & 0.84 & 0.76 & 0.83 \\ 0.71 & - & 0.72 & 0.69 \\ 0.91 & 0.59 & - & 0.98 \\ 0.81 & 0.82 & 0.99 & - \end{pmatrix}$$

$$\mathrm{PM}_2 = \begin{pmatrix} - & 0.74 & 0.76 & 0.76 \\ 0.63 & - & 0.78 & 0.64 \\ 0.91 & 0.74 & - & 0.98 \\ 0.77 & 0.52 & 0.99 & - \end{pmatrix}$$

$$\mathrm{PM}_3 = \begin{pmatrix} - & 0.88 & 0.82 & 0.7 \\ 1 & - & 0.47 & 0.89 \\ 0.84 & 0.63 & - & 0.98 \\ 0.69 & 0.81 & 0.87 & - \end{pmatrix}$$

$$\mathrm{PM}_4 = \begin{pmatrix} - & 0.82 & 0.65 & 0.85 \\ 0.69 & - & 0.70 & 0.67 \\ 0.84 & 0.82 & - & 0.94 \\ 0.74 & 0.75 & 0.93 & - \end{pmatrix}.$$

2) *Proximity measures.*

Level 1) *Proximity on pairs of alternatives* for expert e_i are given in PM_i.

Level 2) *Proximity on alternatives.*

x_1	x_2	x_3	x_4
$pa_1^1 = 0.81$	$pa_1^2 = 0.71$	$pa_1^3 = 0.83$	$pa_1^4 = 0.87$
$pa_2^1 = 0.75$	$pa_2^2 = 0.68$	$pa_2^3 = 0.88$	$pa_2^4 = 0.76$
$pa_3^1 = 0.8$	$pa_3^2 = 0.79$	$pa_3^3 = 0.82$	$pa_3^4 = 0.79$
$pa_4^1 = 0.77$	$pa_4^2 = 0.68$	$pa_4^3 = 0.87$	$pa_4^4 = 0.8$

Level 3) *Proximity on the relation.*

$$\mathrm{pr}_1 = 0.8 \quad \mathrm{pr}_2 = 0.77 \quad \mathrm{pr}_3 = 0.8 \quad pr_4 = 0.78.$$

4.2. **Guidance Advice System**

A) **Identification rules.**

1) *Set of experts to change their preferences,* EXPCH. The ranking of the experts according to their proximity of the collective preferences is e_1, e_3, e_4, e_2. In this step, we need to set the number of experts that should change their opinions, ne. In our example, we have decided that half of the experts will change their assessments, i.e., $ne = 50\%$, which implies

$$\mathrm{EXPCH} = \{e_4, e_2\}.$$

2) *Set of alternatives whose assessments should be considered in the change process* ALT. In our case, as we fixed a γ value of 0.75, we have:

$$\mathrm{ALT} = \{x_l \in X \,|\, \mathrm{ca}^l < 0.75\} = \{x_1, x_2, x_4\}.$$

3) *Set of pairs of alternatives whose associated assessments should change*, PALT_i. At this point, we need to identify the preference values, p_i^{lk}, that have to be changed. To do this, a proximity threshold $\beta = 0.75$

	$\left(p_i^{lk}(ml_{pos}), p_i^{lk}(ml_{val}), p_i^{lk}(sl_{pos}), p_i^{lk}(sl_{val})\right)$	$\left(p_c^{lk}(ml_{pos}), p_c^{lk}(ml_{val}), p_c^{lk}(sl_{pos}), p_c^{lk}(sl_{val})\right)$
p_2^{12}	$(5, 1, 6, 0.68)$	$(2, 0.56, 1, 0.5)$
p_2^{21}	$(3, 1, 4, 0.68)$	$(9, 0.5, 8, 0.31)$
p_2^{24}	$(3, 1, 4, 0.68)$	$(9, 0.5, 5, 0.34)$
p_2^{42}	$(9, 1, 8, 0.66)$	$(3, 0.43, 2, 0.38)$
p_4^{13}	$(6, 0.83, 7, 0.7)$	$(1, 0.5, 5, 0.36)$
p_4^{21}	$(9, 1, 8, 0.57)$	$(9, 0.5, 8, 0.31)$
p_4^{23}	$(2, 0.86, 3, 0.7)$	$(3, 0.43, 2, 0.38)$
p_4^{24}	$(9, 1, 8, 0.57)$	$(9, 0.5, 5, 0.34)$
p_4^{41}	$(1, 1, 2, 0.57)$	$(1, 0.5, 6, 0.42)$

is fixed, which gives the following two sets of pairs of alternatives:

$$\text{PALT}_2 = \{(x_1, x_2), (x_2, x_1), (x_2, x_4), (x_4, x_2)\}$$

and

$$\text{PALT}_4 = \{(x_1, x_3), (x_2, x_1), (x_2, x_3), (x_2, x_4), (x_4, x_1)\}$$

which gives the following list of preference values:

$$p_2^{12} \quad p_2^{21} \quad p_2^{24} \quad p_2^{42} \quad p_4^{13} \quad p_4^{21} \quad p_4^{23} \quad p_4^{24} \quad p_4^{41}.$$

B) **Direction rules.**

1) *Direction parameters; see the table at the top of the page.*

2) *Application of the direction rules.*

- Because $p_2^{12}(ml_{\text{pos}}) > p_c^{12}(ml_{\text{pos}}), p_2^{42}(ml_{\text{pos}}) > p_c^{42}(ml_{\text{pos}}), p_2^{21}(ml_{\text{pos}}) < p_c^{12}(ml_{\text{pos}})$, and $p_2^{24}(ml_{\text{pos}}) < p_c^{24}(ml_{\text{pos}})$, expert e_2 is advised to decrease the assessment of the first two preference values (DR1) and increase those of the second pair of preference values (DR2).
- Expert e_4 is advised to decrease the value of p_4^{13} (DR1), increase the value of p_4^{23} (DR2) and decrease the value of p_4^{21}, p_4^{24}, and p_4^{41} (DR3): $p_4^{41}(ml_{\text{pos}}) = p_c^{41}(ml_{\text{pos}})$ and $p_4^{41}(ml_{\text{val}}) > p_c^{41}(ml_{\text{val}})$). However, because $p_4^{41} = b_o$ its associated direction rule is not provided by the CSS.

SECOND ROUND

1) **Providing new preferences**

Following the previous advice, the experts e_2 and e_4 have to change their preferences on some pairs of alternatives. Their new preferences are as follows:

$$\mathbf{P_{e_2}} = \begin{pmatrix} - & c_2 & c_0 & c_4 \\ \mathbf{c_2} & - & c_1 & \mathbf{c_2} \\ c_3 & c_3 & - & c_1 \\ c_0 & \mathbf{c_3} & c_3 & - \end{pmatrix}$$

$$\mathbf{P_{e_4}} = \begin{pmatrix} - & b_0 & \mathbf{b_3} & b_5 \\ \mathbf{b_5} & - & \mathbf{b_2} & \mathbf{b_5} \\ b_3 & b_4 & - & b_2 \\ b_0 & b_1 & b_4 & - \end{pmatrix}.$$

2) **Computation of consensus degrees**

1) *Similarity matrices:*

$$\text{SM}_{12} = \begin{pmatrix} - & 0.58 & 1 & 0.58 \\ 0.58 & - & 0.50 & 0.58 \\ 1 & 0.33 & - & 1 \\ 0.58 & 0.50 & 1 & - \end{pmatrix}$$

$$\text{SM}_{13} = \begin{pmatrix} - & 0.96 & 0.58 & 0.87 \\ 0.71 & - & 0,75 & 0.58 \\ 0.75 & 0.96 & - & 1 \\ 0.88 & 0.63 & 0.88 & - \end{pmatrix}$$

$$\text{SM}_{14} = \begin{pmatrix} - & 0.98 & 0.58 & 0.68 \\ 0.91 & - & 0.57 & 0.91 \\ 0.75 & 0.41 & - & 0.93 \\ 0.56 & 0.92 & 0.92 & - \end{pmatrix}$$

$$\text{SM}_{23} = \begin{pmatrix} - & 0.62 & 0.58 & 0.46 \\ 0.88 & - & 0.25 & 1 \\ 0.75 & 0.37 & - & 1 \\ 0.46 & 0.87 & 0.88 & - \end{pmatrix}$$

$$\text{SM}_{24} = \begin{pmatrix} - & 0.56 & 0.58 & 0.91 \\ 0.68 & - & 0.93 & 0.68 \\ 0.75 & 0.92 & - & 0.93 \\ 0.98 & 0.42 & 0.92 & - \end{pmatrix}$$

$$\text{SM}_{34} = \begin{pmatrix} - & 0.94 & 1 & 0.55 \\ 0.8 & - & 0.33 & 0.67 \\ 1 & 0.45 & - & 0.92 \\ 0.44 & 0.55 & 0.80 & - \end{pmatrix}.$$

2) *Consensus matrix.*

$$\text{CM} = \begin{pmatrix} - & 0.77 & 0.72 & 0.67 \\ 0.76 & - & 0.56 & 0.74 \\ 0.83 & 0.57 & - & 0.96 \\ 0.65 & 0.65 & 0.90 & - \end{pmatrix}.$$

3) *Consensus degrees.*

Level 1) *Consensus on pairs of alternatives.* The element (l, k) of CM represents the consensus degree on the pair of alternatives (x_l, x_k).

Level 2) *Consensus on alternatives.*

$$\text{ca}^1 = 0.72 \quad \text{ca}^2 = 0.68 \quad \text{ca}^3 = 0.79 \quad \text{ca}^4 = 0.73.$$

Level 3) *Consensus on the relation or global consensus.*

$$\text{cr} = 0.73.$$

3) **Consensus control**

As we can observe, the changes in the preference values introduced result in an increasing of the global consensus from 0.7 to 0.73, although it is still lower than the minimum consensus threshold value $\gamma = 0.75$. If it were decided that this consensus value is still insufficient, then further preference changes would be necessary, and the CSS would consequently compute the proximity measures.

4) **Production of advice**

The collective preferences values affected by the changes of the individual preferences are

$$p_c^{13} = (0.5, 0.34, 0.21, 0.14, 0.50, 0.14, 0.04, 0, 0)$$
$$p_c^{21} = (0, 0, 0.08, 0.17, 0.25, 0.5, 0.35, 0.38, 0.36)$$
$$p_c^{23} = (0.08, 0.24, 0.43, 0.39, 0.27\ 0.17, 0.25, 0.17, 0.34)$$
$$p_c^{24} = (0, 0, 0.08, 0.17, 0.5, 0.25, 0.35, 0.38, 0.36)$$
$$p_c^{42} = (0.19, 0.38, 0.43, 0.24, 0.17, 0.42, 0.25, 0.17, 0.09).$$

4.1 **Computation of Proximity Measures**

1) *Proximity matrices:*

$$\mathrm{PM}_1 = \begin{pmatrix} - & 0.84 & 0.81 & 0.83 \\ 0.79 & - & 0.76 & 0.77 \\ 0.91 & 0.59 & - & 0.98 \\ 0.81 & 0.82 & 0.99 & - \end{pmatrix}$$
$$\mathrm{PM}_2 = \begin{pmatrix} - & 0.74 & 0.81 & 0.76 \\ 0.80 & - & 0.74 & 0.81 \\ 0.91 & 0.74 & - & 0.98 \\ 0.77 & 0.68 & 0.99 & - \end{pmatrix}$$
$$\mathrm{PM}_3 = \begin{pmatrix} - & 0.88 & 0.77 & 0.7 \\ 0.92 & - & 0.51 & 0.81 \\ 0.84 & 0.63 & - & 0.98 \\ 0.69 & 0.81 & 0.87 & - \end{pmatrix}$$
$$\mathrm{PM}_4 = \begin{pmatrix} - & 0.82 & 0.77 & 0.85 \\ 0.88 & - & 0.81 & 0.86 \\ 0.84 & 0.82 & - & 0.94 \\ 0.75 & 0.74 & 0.93 & - \end{pmatrix}.$$

2) *Proximity measures.*

Level 1) *Proximity on pairs of alternatives* for expert e_i are given in PM_i.

Level 2) *Proximity on alternatives.*

x_1	x_2	x_3	x_4
$pa_1^1 = 0.82$	$pa_1^2 = 0.77$	$pa_1^3 = 0.83$	$pa_1^4 = 0.87$
$pa_2^1 = 0.77$	$pa_2^2 = 0.78$	$pa_2^3 = 0.88$	$pa_2^4 = 0.82$
$pa_3^1 = 0.78$	$pa_3^2 = 0.75$	$pa_3^3 = 0.82$	$pa_3^4 = 0.79$
$pa_4^1 = 0.81$	$pa_4^2 = 0.85$	$pa_4^3 = 0.87$	$pa_4^4 = 0.8$

Level 3) *Proximity on the relation.*

$$\mathrm{pr}_1 = 0.82 \quad \mathrm{pr}_2 = 0.81 \quad \mathrm{pr}_3 = 0.78 \quad \mathrm{pr}_4 = 0.83.$$

It is worth noting the diverse effects the changes in the individual preferences had on the proximity values. In general the new proximity values in the second round are greater than in the first one, although there are a few cases where the effect is the opposite. If we look at the proximity values at the third level, we observe that there is one expert whose proximity value has decreased. However, the average proximity value of the group in this round of the CSS is greater than in the first one.

4.2. **Guidance Advice System**

A) **Identification rules.**

1) *Set of experts to change their preferences, EXPCH.* The ranking of the experts according to their proximity of the collective preferences is e_4, e_1, e_2, e_3. With the same ne value of 0.5

$$\mathrm{EXPCH} = \{e_2, e_3\}.$$

2) *Set of alternatives whose assessments should be considered in the change process,* ALT. In our case, as we fixed a γ value of 0.75, we have

$$\mathrm{ALT} = \{x_l \in X; \mathrm{ca}^l < 0.75\} = \{x_1, x_2, x_4\}.$$

3) *Set of pairs of alternatives whose assessments should change*. There are six preference values, p_i^{lk}, on which the CSS will produce advice rules in this second round which are considerably lower than in the previous round (with the same proximity threshold $\beta = 0.75$). These are

$$p_2^{12} \quad p_2^{23} \quad p_2^{42} \quad p_3^{14} \quad p_3^{23} \quad p_3^{41}.$$

We observe that two of the preference values for the expert e_2 were already obtained in this step in the first round of the CSS. For the first one, p_2^{12}, a direction rule of change was produced but it was not implemented, this being the reason why it appeared again in the second round. The reason for the appearance of p_2^{42} in the second round of the CSS could reside in its associated proximity value. In the first round, this proximity value was very low (0.52) compared to the proximity threshold (0.75), and although it experienced a considerable increase from 0.52 to 0.68, this has proven to be insufficient. This could indicate that the intensity of the proposed change for a preference value should be linked to the magnitude of the difference between the proximity threshold and its proximity value. The appearance, however, of p_2^{23} is mainly due to a side effect of the changes implemented in the first round. In fact, this is one of the few preference assessments whose proximity value has been affected "negatively," decreasing from 0.78 to 0.74, a value, on the other hand, quite close to the proximity threshold (which may be taken into account not to change it).

B) **Direction rules**

1) *Direction parameters.* In the table at the top of the page, $*$ means that there are more than one possible secondary label candidates (eight, actually). However, they do not play any role in the production of direction rules. We also note that both individual and collective main and secondary labels of the preference value p_2^{23}

	$\left(p_i^{lk}(ml_{pos}),p_i^{lk}(ml_{val}),p_i^{lk}(sl_{pos}),p_i^{lk}(sl_{val})\right)$	$\left(p_c^{lk}(ml_{pos}),p_c^{lk}(ml_{val}),p_c^{lk}(sl_{pos}),p_c^{lk}(sl_{val})\right)$
p_2^{12}	$(5,1,6,0.68)$	$(2,0.56,1,0.5)$
p_2^{23}	$(3,1,4,0.68)$	$(3,0.43,4,0.39)$
p_2^{42}	$(7,1,8,0.66)$	$(3,0.43,6,0.42)$
p_3^{14}	$(4,1,*,0)$	$(4,0.42,8,0.38)$
p_3^{23}	$(9,1,*,0)$	$(3,0.43,4,0.39)$
p_4^{41}	$(6,1,*,0)$	$(1,0.5,6,0.42)$

have the same position, which may be another reason for not changing it.

2) *Application of the direction rules.* In this second round, both experts are advised to decrease the assessment of their three preference values.

THIRD ROUND

Taking into account all the suggested rules, the assessments of the preference values changed to the nearest lower label (except for p_2^{23}), the new consensus measures that we obtain are as follows.

Level 1) *Consensus on pairs of alternatives.*

$$\mathrm{CM} = \begin{pmatrix} - & 0.90 & 0.72 & 0.61 \\ 0.76 & - & 0.62 & 0.74 \\ 0.83 & 0.57 & - & 0.96 \\ 0.71 & 0.73 & 0.90 & - \end{pmatrix}.$$

Level 2) *Consensus on alternatives.*

$$\mathrm{ca}^1 = 0.74 \quad \mathrm{ca}^2 = 0.70 \quad \mathrm{ca}^3 = 0.79 \quad \mathrm{ca}^4 = 0.78.$$

Level 3) *Consensus on the relation or global consensus.*

$$\mathrm{cr} = 0.75.$$

The minimum consensus threshold is reached and, therefore, the CSS would stop and the selection process would be applied to obtain the final solution of consensus.

V. Conclusion

A CSS model to automatically model the whole consensus process of multigranular linguistic GDM problems has been presented. There are two main features of this CSS model: (i) it is able to manage consensus processes in problems where experts may have different levels of background or knowledge to solve the problem, and ii) it is able to generate advice on the necessary changes in the experts' opinions in order to reach consensus, which makes the figure of the moderator, traditionally present in the consensus reaching process, unnecessary.

The main purpose of this CSS model is to provide support to the experts throughout the consensus process, however they are responsible for the final decision, not the CSS. In fact, it is the decision makers who decide whether or not to follow the advice generated by the CSS. In any case, this CSS model considerably reduces the time associated with making the decision, and thus it extends the decision makers ability to analyze the information involved in the decision-making process.

This CSS model makes use of a multigranular linguistic methodology based on transformation functions to unify the multigranular linguistic information, and a similarity function based on the central values of the preference degrees has been proposed to calculate consensus degrees and proximity values. This calculation was carried out at the three different levels of representation of information: pairs of alternatives, alternatives and relation. Based on these consensus criteria, a guidance advice system has been designed, to identify, in a precise way, the experts, alternatives and pairs of alternatives with a negative contribution to consensus, which allowed the CSS to provide appropriate advice on changes of the assessments associated to only those negative contributors. Finally, how the CSS recommendation rules are obtained and the improvement they have on the level of consensus in the group have been illustrated using a practical example.

We point out that the multigranular linguistic based CSS model presented in this paper does not incorporate any specific criteria in the advice rule system to provide recommendations to the experts on the appropriate level of degree of change of their assessments in order to obtain the highest degree of consensus possible. In future research, this problem will be addressed to improve the performance of the CSS model.

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