Some Induced Ordered Weighting Averaging Operators to Solve Decision Problems Based on Fuzzy Preference Relations

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Summary

In [14] Yager and Filev introduced the Induced Ordered Weighted Averaging (IOWA) operator. In this paper, we provide some IOWA operators to aggregate fuzzy preference relations in group decision making problems. In particular, we present the Importance IOWA (I-IOWA) operator, the Consistency IOWA (C-IOWA) operator and the Preference IOWA (P-IOWA) operator. We also provide a procedure to deal with 'ties' with respect to the ordering induced by the application of one of these IOWA operators different to the one proposed by Yager and Filev. Finally, we analyse the reciprocity and consistency properties of the collective fuzzy preference relations obtained using IOWA operators.

Keywords: Aggregation, fuzzy linguistic quantifier, group decision making, IOWA operator.

1 Introduction

In this paper the context of group decision making (GDM) is considered. Then, we suppose that we have a group of experts, $E = \{e_1, \ldots, e_m\}$, which provide preferences about a set of alternatives, $X = \{x_1, \ldots, x_n\}$, by means of the fuzzy preference relations, $\{P^1, \ldots, P^m\}$, $P^k = [p^k_{ij}]$, $p^k_{ij} \in [0, 1]$, which are additive reciprocal, i.e., $p^k_{ij} + p^k_{ji} = 1, \forall i, j, k$. As it is know, such decision situation is solved applying two steps [4, 8]: *aggregation* and *exploitation*. The aggregation step is a necessary and very important task to carry out when we want to obtain a final solution of GDM problems. The aggregation of experts' preferences consists of combining the individual preferences into a collective one in such a way that it summarizes or reflects all the properties contained in all the individual preferences. In the literature, we can find different aggregation operators to aggregate preferences [4].

To aggregate preferences Yager in [11] provided a family of averaging operators, called the Ordered Weighted Averaging (OWA) operators, which are commutative, idempotent, continuous, monotonic, neutral, compensative and stable for positive linear transformations. A fundamental aspect of the OWA operators is the reordering of the arguments to be aggregated, based upon the magnitude of their respective values, which allows us to give importance to values in opposition to the Weighted Average (WA) operators which compute an aggregate value taking into account the reliability of the sources of information. However, it is clear that a set of values can be reordered in a different way to the one used by the OWA operators. To do this, a criterion has to be defined to induce a specific ordering of the arguments to be aggregated before a WA operator can be applied. This is the idea upon which Yager and Filev based the definition of the Induced Ordered Weighted Averaging (IOWA) operator [14].

Thus, it is the reordering step of the arguments to be aggregated where the difference between the OWA operator and the IOWA operator resides. While the OWA operators order the arguments by their value, the IOWA operators induce their ordering by using an additional variable or criterion, called the order inducing variable. In fact, the OWA operator as well as the WA operator are included in the more general class of IOWA operators [15]. This means that the IOWA operators allow us to take control of the aggregation stage of any GDM problem in the sense that importance can be given to the magnitude of the values to be aggregated as the OWA operators do or to the information sources as the WA operators do.

In this paper,in section 3, we introduce three particular cases of IOWA operators to aggregate fuzzy preference relations: the Importance IOWA (I-IOWA) operator, which induces the ordering of the argument values based upon the importance of the information sources; the Consistency IOWA (C-IOWA) operator, which induces the ordering of the argument values based upon the consistency of the information sources; and the Preference IOWA (P-IOWA) operator, which induces the ordering of the argument values based upon the relative preference values associated to each one of them. In section 4, we provide a different procedure to the one proposed by Yager and Filev for dealing with 'ties' in respect of the ordering induced by the application of one of these IOWA operators. This procedure consists of a conjunction application of the above IOWA operators. In section 5, we show that, in general, IOWA operators maintain the reciprocity property of fuzzy preference relations as well as the consistency property. Finally, in section 6, we draw our conclusions.

2 The IOWA Operator

In [14] Yager and Filev introduced a more general type of OWA operator, which they named the Induced Ordered Weighted Averaging (IOWA) operator.

Definition 1 *[14] An IOWA operator of dimension n is a function $\Phi_W : (\Re \times \Re)^n \longrightarrow \Re$, to which a set of weights or weighting vector is associated, $W = (w_1, \ldots, w_n)$, such that $w_i \in [0,1]$ and $\Sigma_i w_i = 1$, and it is defined to aggregate the set of second arguments of a list of n 2-tuples $\{\langle u_1, p_1\rangle, \ldots, \langle u_n, p_n\rangle\}$ according to the following expression,*

$$\Phi_W\left(\langle u_1, p_1\rangle, \ldots, \langle u_n, p_n\rangle\right) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)}$$

being $\sigma : \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$ a permutation such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $\langle u_{\sigma(i)}, p_{\sigma(i)}\rangle$ is the 2-tuple with $u_{\sigma(i)}$ the i-th highest value in the set $\{u_1, \ldots, u_n\}$.

In the above definition the reordering of the set of values to aggregate, $\{p_1, \ldots, p_n\}$, is induced by the reordering of the set of values $\{u_1, \ldots, u_n\}$ associated to them, which is based upon their magnitude. Due to this use of the set of values $\{u_1, \ldots, u_n\}$, Yager and Filev called them the values of an order inducing variable and $\{p_1, \ldots, p_n\}$ the values of the argument variable [14, 15]. As we have mentioned, the main difference between the OWA operator and the IOWA operator resides in the reordering step of the argument variable. In the case of OWA operator this reordering is based upon the magnitude of the values to be aggregated, while in the case of IOWA operator an order inducing variable has to be defined as the criterion to induce that reordering.

An immediate consequence of this definition is that if the order inducing variable is the argument variable then the IOWA operator is reduced to the OWA operator. For a detailed list of properties and uses of the IOWA operators the reader should consult [9, 12, 13, 14, 15].

Note 1. In this paper we will focus on the aggregation of numerical preferences, which is why we assume that the nature of the first argument of the IOWA operators is also numeric, although it could be linguistic ([12, 13, 14, 15]).

Note 2. In the case of using an IOWA operator in the aggregation phase of a GDM problem, the concept of fuzzy majority can be implemented by means of the *fuzzy linguistic quantifiers* [16] which are used to calculate its weights. In the case of a non-decreasing relative quantifier Q, the weights are expressed as follows [11]:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i = 1, \ldots, n.$$

When a fuzzy linguistic quantifier Q is used to compute the weights of the IOWA operator Φ, then it is symbolized by Φ_Q.

Example 1 *If we want to aggregate the following set of 2-tuples $\{\langle 0.65, 0.87\rangle, \langle 0.13, 0.94\rangle, \langle 0.22, 0.75\rangle\}$, using the fuzzy linguistic quantifier "most of", whose corresponding weighting vector is $\left(\frac{1}{15}, \frac{10}{15}, \frac{4}{15}\right)$, then we obtain*

$$\begin{aligned}\Phi_{most}\left(\langle 0.65, 0.87\rangle, \langle 0.13, 0.94\rangle, \langle 0.22, 0.75\rangle\right) &= \\ \tfrac{1}{15} \cdot 0.87 + \tfrac{10}{15} \cdot 0.75 + \tfrac{4}{15} \cdot 0.94 &= \tfrac{12.13}{15}\end{aligned}$$

3 Some IOWA Operators to Aggregate Fuzzy Preference Relations

The result obtained in the aggregation stage using a WA operator summarizes the aggregated values of the information sources (experts) taking into account the reliability of these sources. However, an OWA operator combines the information giving weight to the values in relation to their ordering position, diminishing the importance of extreme values by increasing the importance of central ones. As both the WA and OWA operators are special types of IOWA operator, then by using an IOWA operator the type of aggregation method to be implemented can be chosen.

In this section we present three special cases of IOWA operators for GDM problems with fuzzy preference relations. These IOWA operators allow the introduction of some semantics or meaning in the aggregation, and therefore allow for better control over the aggregation stage. The first two act as the WA operator because the aggregation is based upon the reliability of the information sources, while the third one acts as the OWA operator because the ordering of the argument values is based upon a relative magnitude associated to each one of them.

3.1 The I-IOWA Operator

In many cases, each expert $e_k \in E$ has an *importance degree* associated to them. This *importance degree* can be interpreted as a fuzzy subset, $\mu_I : E \longrightarrow [0,1]$, in such a way that $\mu_I(e_k) \in [0,1]$ denotes the importance degree of the opinion provided by the expert e_k. When this is the case, we call this a heterogeneous GDM problem [5, 7].

Assuming that in our context each value $\mu_I(e_k)$ is a weight indicating the importance of the expert e_k, the general procedure for its inclusion in the aggregation process involves the transformation of the preference values, p_{ij}^k, under the importance degree $\mu_I(e_k)$ to generate a new value, $\bar{p}_{ij}^k$. This activity is carried out by means of a transformation function g :

$$\bar{p}_{ij}^k = g\left(p_{ij}^k, \mu_I(e_k)\right).$$

Examples of functions g used in these cases include the minimum operator [1], the exponential function $g(x,y) = x^y$ [10], or generally any t-norm operator [17].

In GDM problem an alternative proposal to apply this importance degree variable consists in to apply it as the order inducing variable to induce the ordering of the argument values prior to their aggregation. We call this importance degree based IOWA operator as the Importance IOWA (I-IOWA) operator and denote it as Φ_W^I.

Definition 2 *If a set of of experts, $E = \{e_1, \ldots, e_m\}$, provide preferences about a set of alternatives, $X = \{x_1, \ldots, x_n\}$, by means of the fuzzy preference relations, $\{P^1, \ldots, P^m\}$, and each expert e_k has an importance degree, $\mu_I(e_k) \in [0,1]$, associated to them, then an I-IOWA operator of dimension n, Φ_W^I, is an IOWA operator whose set of order inducing values is the set of importance degrees.*

Example 2 *Suppose that we have a set of three experts $E = \{e_1, e_2, e_3\}$ and a set of three alternatives $X = \{x_1, x_2, x_3\}$. Suppose that the importance pairwise comparisons of these three experts are given in the following fuzzy preference relation*

$$I = \begin{pmatrix} 0.5 & 0.87 & 0.75 \\ 0.23 & 0.5 & 0.38 \\ 0.25 & 0.62 & 0.5 \end{pmatrix}$$

As shown in [2], the vector of importance of a consistent fuzzy preference relation induces the same ordering among the set of experts than the vector of quantifier guided dominance degrees, no matter which linguistic quantifier is used. For this reason, we propose to calculate the importance associated to the expert e_i as the total sum of the values of row i, i.e., $\mu_I(e_k) = \sum_j p_{ij}^k$. The normalized vector of importance for this matrix is given by $\mathbf{I} = (0.46, 0.24, 0.30)$.

Suppose that these experts provide the following reciprocal fuzzy preference relations on the set of alternatives

$$P^1 = \begin{pmatrix} 0.5 & 0.75 & 0.87 \\ 0.25 & 0.5 & 0.66 \\ 0.13 & 0.34 & 0.5 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.5 & 0.66 & 0.94 \\ 0.34 & 0.5 & 0.87 \\ 0.06 & 0.13 & 0.5 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.5 & 0.66 & 0.75 \\ 0.34 & 0.5 & 0.66 \\ 0.25 & 0.34 & 0.5 \end{pmatrix}.$$

Using the fuzzy linguistic quantifier "most of" the collective fuzzy preference relation is

$$\begin{aligned} P^c &= \Phi_{most}^I\left(\langle 0.46, P^1\rangle, \langle 0.24, P^2\rangle, \langle 0.30, P^3\rangle\right) \\ &= \begin{pmatrix} 0.5 & 0.67 & 0.81 \\ 0.33 & 0.5 & 0.72 \\ 0.19 & 0.28 & 0.5 \end{pmatrix}, \end{aligned}$$

whose elements can be considered as the preference of one alternative over another for most of the most important experts.

3.2 The C-IOWA Operator

When the experts have equal importance, i.e., in a homogeneous GDM problem, the I-IOWA operator is reduced to the Average Mean (AM) operator. Thus, in this case the application of the I-IOWA operator does not introduce any new meaning and its application is not advisable. However, in a homogeneous situation, each expert can always have a consistency index value associated to them. Usually, for each expert this consistency index value is obtained by analysing their fuzzy preference relation, and then, we can use it as the order inducing variable in the aggregation of preferences by means of IOWA operators.

In decision making problems based on fuzzy preference relations, the study of consistency is associated with the study of the *transitivity property*. In [6], Herrera-Viedma et. al. gave a characterization of the consistency property defined by the additive transitivity property of a fuzzy preference relation $P^k = (p_{ij}^k)$:

$$p_{ij}^k + p_{jl}^k + p_{li}^k = \frac{3}{2}, \forall i, j, l \in \{1, \ldots, n\}.$$

Using this characterization method, a procedure was given to construct a consistent fuzzy preference relation $\widetilde{P}^k$ from a non-consistent fuzzy preference relation P^k [6]. The distance between P^k and $\widetilde{P}^k$ can be interpreted as a measure of the consistency of matrix P^k and hence of the expert who provided it:

$$CI^k = d(P^k, \widetilde{P}^k).$$

The closer CI^k is to 0 the more consistent the information provided by the expert e^k, and thus more importance should be placed on that information. In other words, we could use these values to define the ordering of the argument values to be aggregated, in which case we would be implementing the concept of consistency in the aggregation process of our decision making. This kind of aggregation process defines an IOWA operator that we call the Consistency IOWA (C-IOWA) operator and denote it as Φ_W^C.

Definition 3 *If a set of experts, $E = \{e_1, \ldots, e_m\}$, provides preferences about a set of alternatives, $X = \{x_1, \ldots, x_n\}$, by means of the fuzzy preference relations, $\{P^1, \ldots, P^m\}$, then a C-IOWA operator of dimension n, Φ_W^C, is an IOWA operator whose set of order inducing values is the set of consistency index values, $\{-CI^1, \ldots, -CI^m\}$, associated to the set of experts.*

Example 3 *Suppose that in example 2, the importance values are not provided. The consistency indexes associated to the experts are* $\mathbf{CI} = (0.06, 0.09, 0.01)$, *and the collective fuzzy preference relation obtained by using a C-IOWA operator guided by the same linguistic quantifier "most of" is*

$$\begin{aligned} P^c &= \Phi_{most}^C \left(\langle -0.06, P^1 \rangle, \langle -0.09, P^2 \rangle, \langle -0.1, P^3 \rangle \right) \\ &= \begin{pmatrix} 0.5 & 0.67 & 0.88 \\ 0.33 & 0.5 & 0.8 \\ 0.12 & 0.2 & 0.5 \end{pmatrix}, \end{aligned}$$

whose elements can be considered as the preference of one alternative over another for most of the most consistent experts.

3.3 The P-IOWA Operator

If $P^k = (p_{ij}^k)$ is a fuzzy preference relation on the set of alternatives $\{x_1, \ldots, x_n\}$ then the total sum of the elements of each row i, $\overline{p}_i^k = \sum_j p_{ij}^k$, can be interpreted as the total preference of that alternative x_i. The resulting value obtained by dividing an element of that row, p_{ir}^k, by $\overline{p}_i^k$, $\overline{p}_{ir}^k = \frac{p_{ir}^k}{\sum_j p_{ij}^k}$, can be interpreted as the relative preference contribution of that particular element to the total preference of the alternative x_i.

These relative preference values can be used as the order inducing values of an IOWA operator to aggregate a set of fuzzy preference relations. We call this a Preference IOWA (P-IOWA) operator and denote it as Φ_W^P.

Definition 4 *If a set of experts, $E = \{e_1, \ldots, e_m\}$, provides preferences about a set of alternatives, $X = \{x_1, \ldots, x_n\}$, by means of the fuzzy preference relations, $\{P^1, \ldots, P^m\}$ then a P-IOWA operator of dimension n, Φ_W^P, is an IOWA operator whose set of order inducing values is the set of relative preferences matrices, $\{\overline{P}^k = (\overline{p}_{ij}^k); k = 1, \ldots, m\}$.*

Example 4 *Using the same data as in example 2, the corresponding relative preference matrices, $\overline{P}^k$, are:*

$$\overline{P}^1 = \begin{pmatrix} 0.24 & 0.35 & 0.41 \\ 0.18 & 0.35 & 0.47 \\ 0.13 & 0.35 & 0.52 \end{pmatrix}$$

$$\overline{P}^2 = \begin{pmatrix} 0.24 & 0.31 & 0.45 \\ 0.20 & 0.29 & 0.51 \\ 0.09 & 0.19 & 0.72 \end{pmatrix}$$

$$\overline{P}^3 = \begin{pmatrix} 0.26 & 0.35 & 0.39 \\ 0.23 & 0.33 & 0.44 \\ 0.23 & 0.31 & 0.46 \end{pmatrix}.$$

The collective fuzzy preference relation obtained by using the P-IOWA operator guided by the same linguistic quantifier "most of", is

$$P^c = \Phi_{most}^P(P^1, P^2, P^3) = \begin{pmatrix} 0.5 & 0.67 & 0.84 \\ 0.32 & 0.5 & 0.67 \\ 0.12 & 0.28 & 0.5 \end{pmatrix}.$$

Note 3. *In this example, there is not a tie between $\overline{p}_{12}^1$ and $\overline{p}_{12}^3$ because their actual values are approximately 0.354 and 0.345 respectively.*

4 A Procedure to Deal with Ties Using IOWA operators

When aggregating a set of 2-tuples using IOWA operators, ties could appear and the aggregated values could be different according to the procedure applied. This was not a problem when using OWA operators where ties do not affect the aggregated values. In the case of aggregating fuzzy preference relations, when using IOWA operators, we propose a sequential procedure, different to the one proposed by Yager and Filev in [14]. This procedure is applied in three steps, as follows:

1. If the GDM problem is heterogeneous then the I-IOWA operator is applied; if not the C-IOWA operator is applied.

2. If an I-IOWA operator has been applied in 1 then the ordering of the equally important information is induced based upon their respective consistency index values.

   If a C-IOWA operator has been applied in 1 then the ordering of the equally consistent information is induced based upon their respective relative preference values.

3. Finally, if ties are still present then their ordering is induced based upon their respective magnitude, i.e., the usual OWA operator is applied.

5 Reciprocity and Consistency Properties of the Collective Fuzzy Preference Relation

In GDM models we normally assume that the fuzzy preference relations are reciprocal. However, it is well known that reciprocity is not generally preserved after aggregation is carried out in the resolution process [3]. In example 2 the collective fuzzy preference relation obtained, using the I-IOWA operator, was reciprocal and the one obtained in example 3, using the C-IOWA operator, was also reciprocal. However, the one obtained by applying the P-IOWA was not reciprocal.

In what follows, we will show that IOWA operators acting as WA operators maintain both the reciprocity and the consistency properties. On the other hand, the IOWA operators acting as OWA operators do not generally maintain these properties as shown by example 3 and in [3], where we proved that the subclass of OWA operators do not generally maintain, in the aggregation process, the reciprocity and consistency properties.

5.1 Reciprocity Property

If a group of experts, $E = \{e_1, \ldots, e_m\}$, provides preferences about the alternatives, $X = \{x_1, \ldots, x_n\}$, by means of reciprocal fuzzy preference relations, $\{P^1, \ldots, P^m\}$, $p_{ij}^k + p_{ji}^k = 1, \forall i, j, k$, and if $\{u_1, \ldots, u_m\}$ is a set of order inducing (importance, consistency) values associated to the set of experts, then the collective preference relation, $P^c = (p_{ij}^c)$ obtained by using an IOWA operator Φ_Q guided by a linguistic quantifier Q is also reciprocal.

Indeed,

$$\begin{aligned} p_{ij}^c &= \Phi_Q\left(\langle u_1, p_{ij}^1\rangle, \ldots, \langle u_n, p_{ij}^m\rangle\right) \\ &= \textstyle\sum_{k=1}^m w_k \cdot p_{ij}^{\sigma(k)} \end{aligned}$$

being $\sigma : \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$ a permutation such that $u_{\sigma(k)} \geq u_{\sigma(k+1)},\ \forall k = 1, \ldots, n-1$.

It is clear that

$$\begin{aligned} p_{ji}^c &= \Phi_Q\left(\langle u_1, p_{ji}^1\rangle, \ldots, \langle u_n, p_{ji}^m\rangle\right) \\ &= \textstyle\sum_{k=1}^m w_k \cdot p_{ji}^{\sigma(k)} = \sum_{k=1}^m w_k \cdot \left(1 - p_{ij}^{\sigma(k)}\right) \ , \\ &= 1 - \textstyle\sum_{k=1}^m w_k \cdot p_{ij}^{\sigma(k)} = 1 - p_{ij}^c \end{aligned}$$

that is, P^c verifies the reciprocity property.

5.2 Consistency Property

If the set of fuzzy preference relations are additive consistent [6], i.e.,

$$p_{ij}^k + p_{jl}^k + p_{li}^k = \frac{3}{2}, \forall i, j, l \in \{1, \ldots, n\}, k \in \{1, \ldots, m\},$$

and $P^c = \Phi_Q\left(\langle u_1, P^1\rangle, \ldots, \langle u_n, P^m\rangle\right)$, then

$$p_{ij}^c + p_{jl}^c + p_{li}^c =$$

$$\textstyle\sum_{k=1}^m w_k \cdot p_{ij}^{\sigma(k)} + \sum_{k=1}^m w_k \cdot p_{jl}^{\sigma(k)} + \sum_{k=1}^m w_k \cdot p_{li}^{\sigma(k)} =$$

$$\textstyle\sum_{k=1}^m w_k \cdot \left(p_{ij}^{\sigma(k)} + p_{jl}^{\sigma(k)} + p_{li}^{\sigma(k)}\right) =$$

$$\textstyle\sum_{k=1}^m w_k \cdot \frac{3}{2} = \frac{3}{2},$$

which proves the additive consistency of P^c.

Note 4. The above proof of reciprocity and consistency of the collective fuzzy preference relation is based upon the assumption that the order inducing values are unchanged.

6 Conclusions

In this paper we have studied the use of the IOWA operators in the aggregation of fuzzy preference relations in GDM problems. We have defined three IOWA operators which implement a semantic meaning in the aggregation process: the I-IOWA operator, which induces the ordering of the argument values based upon the importance of the information sources; the C-IOWA operator, which induces the ordering of the argument values based upon the consistency of the information sources; and the P-IOWA operator, which induces the ordering of the arguments based upon the relative preference associated to each one of them. We have also given a sequential procedure to deal with ties in respect of the ordering induced by the application of one of these IOWA operators. This procedure is different to the one proposed by Yager and Filev, and consists of a sequential application of the above IOWA operators. The application of this sequential procedure induces an ordering of the arguments to aggregate without ties, or in the extreme case of their presence these do not affect the aggregated result. Finally, we have shown that the collective fuzzy preference relation verifies the reciprocity and consistency properties under the assumption that the order inducing values are unchanged.

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