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Integrating multiplicative preference relations in a multipurpose decision-making model based on fuzzy preference relations

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Abstract

The aim of this paper is to study the integration of multiplicative preference relation as a preference representation structure in fuzzy multipurpose decision-making problems. Assuming fuzzy multipurpose decision-making problems under different preference representation structures (ordering, utilities and fuzzy preference relations) and using the fuzzy preference relations as uniform representation elements, the multiplicative preference relations are incorporated in the decision problem by means of a transformation function between multiplicative and fuzzy preference relations. A consistency study of this transformation function, which demonstrates that it does not change the informative content of multiplicative preference relation, is shown. As a consequence, a selection process based on fuzzy majority for multipurpose decision-making problems under multiplicative preference relations is presented. To design it, an aggregation operator of information, called ordered weighted geometric operator, is introduced, and two choice degrees, the quantifier-guided dominance degree and the quantifier-guided non-dominance degree, are defined for multiplicative preference relations. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Decision making in situations with multiple criteria and/or persons is a prominent area of research in normative decision theory. This topic has been widely studied [2,8,11,21,23]. We do not distinguish between "persons" and "criteria", and interpret the decision process in the fuzzy framework of multipurpose decision-making (MPDM) [4], assuming that the fuzzy property of human decisions can be satisfactorily modeled by fuzzy sets theory as in [8,11,13,14].

In an MPDM problem, we have a set of alternatives to be analyzed according to different purposes in order to select the best one(s). For each purpose a set of evaluations about the alternatives is known. Then, a classical choice scheme for an MPDM problem follows two steps before it achieves a final decision [4,6,19]: "aggregation" and "exploitation". The aggregation phase defines an (outranking) relation which indicates the global preference between every ordered pair of

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alternatives, taking into consideration the different purposes. The exploitation phase transforms the global information about the alternatives into a global ranking of them. This can be done in different ways, the most common one being the use of a ranking method to obtain a score function [18].

In [4], we consider MPDM problems where, for each purpose (expert or criterion), the information about the alternatives could be supplied in different ways. With a view to build a more flexible framework and to give more freedom degree to represent the evaluations, we assumed that they could be provided in any of these three ways: (i) *as a preference ordering of the alternatives*, (ii) *as a fuzzy preference relation* and (iii) *as a utility function.* There we presented a decision process to deal with this decision situation which, before applying the classical choice scheme, made the information uniform, using fuzzy preference relations as the main element of the uniform representation of the evaluations, and then obtained the solution by means of a selection process based on the concept of *fuzzy majority* [10] and on the *Ordered Weighted Averaging* (OWA) operator [30].

In this paper, we increase the flexibility degree of our decision model proposed in [4]. We give a new possibility for representing the evalutions about the alternatives, i.e., to use *multiplicative preference relations.* This representation structure of evaluations has been widely used (see [9,20–22,27,29]). In [20,21] Saaty designed a choice scheme, called *Analytic Hierarchy Process* (AHP), for dealing with decision problems where the evaluations about the alternatives are provided by means of the multiplicative preference relations. We incorporate the multiplicative preference relations in our decision model presenting a transformation mechanism between multiplicative and fuzzy preference relations and analyzing its consistency. Then, as a consequence, we propose an alternative choice scheme to the classical one designed by Saaty. Following our selection process given in [4], we design a new choice scheme using the concept of fuzzy majority and a new aggregation operator, called *ordered weighted geometric* (OWG) operator.

In order to do this, the paper is set out as follows. The MPDM problem under four evaluation structures is presented in Section 2. A transformation mechanism between multiplicative and fuzzy preference relations is proposed in Section 3. Section 4 is devoted to presenting the OWG operator and to design the new scheme choice for dealing with decision problems under multiplicative preference relations. In Section 5 some concluding remarks are pointed out. Finally, the fuzzy majority concept and the OWA operator are presented in Appendix A.

2. The MPDM problem under four evaluation structures of preferences

Let $X=\{x_1,x_2,\dots,x_n,\ (n\geqslant 2)\}$ be a finite set of alternatives. The alternatives will be classified from best to worst (ordinal ranking), using the information known according to a finite set of general purposes (experts or criteria). In the following, without loss of generality, we will use the term experts, i.e., $E=\{e_1,e_2,\dots,e_m,\ (m\geqslant 2)\}$. As each expert, $e_k\in E$, is characterized by his own ideas, attitudes, motivations and personality, it is quite natural to consider that different experts will provide their preferences in a different way. Then, we assume that the experts' preferences over the set of alternatives, X, may be represented in one of the following four ways:

1. *A preference ordering of the alternatives.* In this case, an expert, e_k, provides his preferences on X as an individual preference ordering, $O^k=\{o^k(1),\dots,o^k(n)\}$, where $o^k(\cdot)$ is a permutation function over the index set, $\{1,\dots,n\}$, for the expert, e_k [6,24]. Therefore, according to the viewpoint of each expert, an ordered vector of alternatives, from the best one to the worst one, is given.
2. *A fuzzy preference relation.* With this representation, an expert's preferences on X is described by a fuzzy preference relation, $P^k\subset X\times X$, with membership function, $\mu_{P^k}:X\times X\to[0,1]$, where $\mu_{P^k}(x_i,x_j)=p_{ij}^k$ denotes the preference degree or intensity of the alternative x_i over x_j [10,12,14,25]: $p_{ij}^k=\frac{1}{2}$ indicates indifference between x_i and x_j, $p_{ij}^k=1$ indicates that x_i is absolutely preferred to x_j, and $p_{ij}^k>\frac{1}{2}$ indicates that x_i is preferred to x_j. In this case, the preference matrix, P^k, is assumed additive reciprocal, i.e., by definition [17,25] $p_{ij}^k+p_{ji}^k=1$ and $p_{ii}^k=\frac{1}{2}$.

3. *A multiplicative preference relation.* With this representation, an expert's preferences on X are described by a positive preference relation, $A^k \subset X \times X$, $A^k = [a_{ij}^k]$, where a_{ij}^k indicates a ratio of preference intensity for alternative x_i to that of x_j, i.e., it is interpreted as x_i is a_{ij}^k times as good as x_j. According to Miller's study [16], Saaty suggests measuring a_{ij}^k using a ratio scale, and precisely the 1–9 scale [21]: $a_{ij}^k = 1$ indicates indifference between x_i and x_j, $a_{ij}^k = 9$ indicates that x_i is absolutely preferred to x_j, and $a_{ij}^k \in 2, 3, \ldots, 8$ indicates intermediate evaluations. In order to guarantee that A^k is "self-consistent", only some pairwise comparison statements are collected to construct it. The rest of the values are those which satisfy the following conditions [21]:
   (a) Multiplicative reciprocity property: $a_{ij}^k \cdot a_{ji}^k = 1 \; \forall i, j$.
   (b) Saaty's consistency property: $a_{ij}^k = a_{it}^k \cdot a_{tj}^k \; \forall i, j, t$.

   Therefore, we consider multiplicative preference relations assessed in Saaty's discrete scale, which has only the following set of values: $\{\frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \ldots, \frac{1}{2}, 1, 2, \ldots, 7, 8, 9\}$.
4. *An utility function.* In this case, an expert, e_k, provides his preferences on X as a set of n utility values, $U^k = \{u_i^k, i = 1, \ldots, n\}, u_i^k \in [0, 1]$, where u_i^k represents the utility evaluation given by the expert e_k to the alternative x_i [15,26].

In this context, the resolution process of the MPDM problem consists of obtaining a set of solution alternatives, $X_{\mathrm{sol}} \subset X$, from the preferences given by the experts. Since the experts provide their preferences in different ways, the first step of the resolution process of the MPDM problem must be to obtain a uniform representation of the preferences. As we pointed out in [4], we consider fuzzy preference relations as the main element of the uniform representation of the preferences. Once this uniform representation has been achieved, we can develop from it a selection process to achieve the set of solution alternatives. In this sense, the resolution process of the MPDM problem presents the scheme given in Fig. 1. This resolution process is developed in the following two steps [4]: (i) *making the information uniform* and (ii) *the application of a selection process*. As follows, we summarize the most important results in each step.

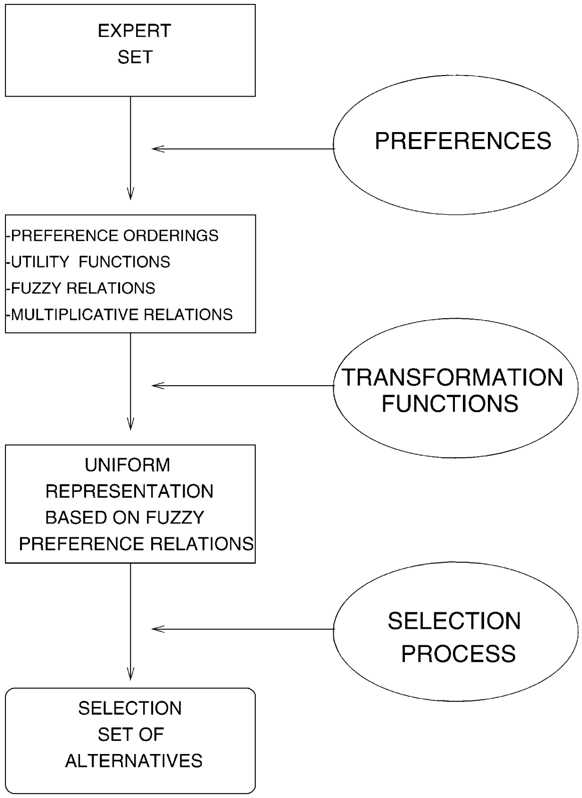

Fig. 1. Resolution process of the MPDM problem.

2.1. Making the information uniform

In the classical resolution processes for an MPDM problem, a set of preferences supplied in the same way is assumed [19]. A common representation of the preferences is necessary in order to facilitate the combination of preferences to achieve a consensus final decision. Due to their apparent merits, many authors use fuzzy preference relations as the base element of the uniform representation [4,7,10,12,14,17,25,28]. The use of fuzzy preference relations in decision-making situations to represent an expert's opinion about a set of alternatives, appears to be a useful tool in modelling decision processes, overcoat when we want to aggregate experts' preferences into group preferences, that is, in the resolution processes of the MPDM problems. Furthermore, preference orderings and utility values are included in the family of fuzzy preference relations [26] and most of the existing results on MPDM are obtained under fuzzy preference relations.

To make the information uniform, it is neccessary to obtain transformation functions relating the different evaluation structures of preferences with fuzzy preference relations. These transformation functions derive an individual fuzzy preference relation from each evaluation structure of preferences. In [3,4] we studied the transformation of preference orderings and utility values into fuzzy preference relations. This study can be summarized in the following proposition.

Proposition 1. *Suppose that we have a set of alternatives, $X = \{x_1, \ldots, x_n\}$, and λ_i^k represents an evaluation of alternative x_i indicating the performance of that alternative according to a point of view (expert or criterion), e_k. Then, the intensity of preference of alternative x_i over alternative x_j, p_{ij}^k for e_k is given by the following transformation function*

$$p_{ij}^k = \varphi(\lambda_i^k, \lambda_j^k) = \tfrac{1}{2} \cdot [1 + \psi(\lambda_i^k, \lambda_j^k) - \psi(\lambda_j^k, \lambda_i^k)],$$

where ψ is a function verifying
1. *$\psi(z, z) = \frac{1}{2}$, $\forall z \in \mathscr{R}$ (set of reals).*
2. *ψ is non-decreasing in the first argument and non-increasing in the second argument.*

Then, in order to make uniform the information represented by multiplicative preference relations, in the following section, we will study the case for multiplicative and fuzzy preference relations.

2.2. Application of a selection process

Once the information is uniformed, we have a set of m individual fuzzy preference relations over the set of alternatives X, and we apply a selection process which has two phases [4,6,19]: (i) *aggregation* and (ii) *exploitation.*

2.2.1. Aggregation phase

This phase defines a collective fuzzy preference relation, $P^c = [p_{ij}^c]$, which indicates the global preference between every ordered pair of alternatives according to the majority of experts' opinions. Using the concept of fuzzy majority applied in the aggregation operations by means of an OWA operator [30], P^c is obtained by means of the aggregation of all individual fuzzy preference relations $\{P^1, \ldots, P^m\}$:

$$p_{ij}^c = \phi_Q(p_{ij}^1, \ldots, p_{ij}^m),$$

where Q is a fuzzy linguistic quantifier that represents the concept of fuzzy majority and it is used to compute the weighting vector of the OWA operator, ϕ_Q (see Appendix A).

2.2.2. Exploitation phase

This phase transforms the global information about the alternatives into a global ranking of them, supplying the set of solution alternatives. Using again the OWA operator and the concept of fuzzy majority, but in another sense, two choice degrees of alternatives are applied over the collective fuzzy preference relation: *the quantifier-guided dominance degree* and *the quantifier-guided non-dominance degree*.

1. *Quantifier-guided dominance degree*. For the alternative, x_i, we compute the quantifier-guided dominance degree, $QGDD_i$, used to quantify the dominance that one alternative has over all the others in a fuzzy majority sense as follows:

   $$QGDD_i = \phi_Q(p_{i1}^c, \ldots, p_{in}^c).$$

2. *Quantifier-guided non-dominance degree*. We also compute the quantifier-guided non-dominance degree, $QGNDD_i$, according to the following expression:

   $$QGNDD_i = \phi_Q(1 - p_{1i}^s, \ldots, 1 - p_{ni}^s),$$

   where

   $$p_{ji}^s = \max\{p_{ji}^c - p_{ij}^c, 0\}$$

   represents the degree to which x_i is strictly dominated by x_j. In our context, $QGNDD_i$ gives the degree to which each alternative is not dominated by a fuzzy majority of the remaining alternatives.

Finally, the solution X_{sol} is obtained by means of the application of both choice degrees of alternatives. This application can be carried out according to different choice policies, e.g., *sequential* or *conjunctive* (see [4,6]).

We should point out that in this phase we can use a fuzzy linguistic quantifier Q different from that used in the aggregation phase because the aggregation context is different.

3. Transformation function between multiplicative and fuzzy preference relations

The relationship between multiplicative and fuzzy preference relations is analyzed assuming that in the considered MPDM problem an expert, e_k, provides his preferences on X by means of a multiplicative preference relation, $A^k = [a_{ij}^k]$.

In general, if

$$A' = \{A^k = [a_{ij}^k] \mid a_{ij}^k \cdot a_{ji}^k = 1,\ a_{ij}^k \in [1/9, 9],\\ k = 1, \ldots, m\}$$

is the set of multiplicative preference relations in Saaty's sense, and

$$P' = \{P^k = [p_{ij}^k] \mid p_{ij}^k + p_{ji}^k = 1,\ p_{ij}^k \in [0, 1],\\ k = 1, \ldots, m\}$$

is the set of additive fuzzy preference relations, then we are looking for a continuous function

$$F : A' \to P', \mid F(A^k) = P^k, \quad \forall k.$$

This class of functions is equivalent to the class of functions verifying

$$f : [1/9, 9] \to [0, 1],$$

$$f(x) + f(1/x) = 1,$$

$$f(9) = 1.$$

Function f can be rewritten in the following way:

$$f(x) = \tfrac{1}{2} + h(x)$$

which implies that

$$h(x) + h(1/x) = 0,$$

$$h(9) = \tfrac{1}{2}.$$

On the other hand, it is well known that the general solution of functional equation [1]

$$l(x \cdot y) = l(x) + l(y)$$

is in $[1, +\infty]$:

$$l(z) = C \cdot \ln z, C \in \mathscr{R}.$$

In our situation, the following relationship holds $y = 1/x$, and making $x = 1$ we have

$$0 = h(1) + h(1) = 2 \cdot h(1) = 2 \cdot h(x \cdot y),$$

and therefore, function h verifies

$$h(x) + h(y) = h(x \cdot y)$$

that is

$$h(z) = C \cdot \ln\ z, \quad C \in \mathscr{R}.$$

Since $h(9) = \frac{1}{2}$, then $C = 1/(2 \cdot \ln\ 9)$, and therefore

$$h(z) = \frac{1}{2} \frac{\ln\ z}{\ln\ 9} = \frac{1}{2} \log_9 z.$$

Summarising, we have the following result:

Proposition 2. *Suppose that we have a set of alternatives, $X = \{x_1, \ldots, x_n\}$, and associated with it a multiplicative preference relation $A^k = [a_{ij}^k]$. Then, the corresponding additive fuzzy preference relation, $P^k = [p_{ij}^k]$, associated with A^k is given as follows*:

$$p_{ij}^k = f(a_{ij}^k) = \frac{1}{2}(1 + \log_9 a_{ij}^k).$$

In the following subsection, we study the consistency of this transformation function.

3.1. Consistency of the transformation function between multiplicative and fuzzy preference relations

In this subsection, we demonstrate that the transformation function acts coherently because it does not change the informative content of the multiplicative preference relations when we make the information uniform in the decision model shown in Section 2. To do so, we analyze the consistency of f showing that the ranking among the alternatives derived from A^k is the same one as from $P^k = f(A^k)$. This study of consistency is done on the basis of the selection models presented in [21] (called *multiplicative selection model*) and in [4] (called *fuzzy additive selection model*). In [21] Saaty proposed the *eigenvector method* to achieve a ranking among the alternatives from a multiplicative matrix A^k. In [4], as was mentioned earlier, we proposed a method based on two quantifier-guided choice degrees (the quantifier-guided dominance degree and the quantifier-guided non-dominance degree) to achieve a ranking among the alternatives from a fuzzy matrix P^k. In [5] we presented a similar consistency study for the transformation function given in Proposition 1.

Some important aspects of the multiplicative selection model that we use to demonstrate the consistency of the transformation function f are given in the following subsection.

3.1.1. Multiplicative selection model

The MPDM problem when the experts express their preferences using multiplicative preference relations have been extensively studied by Saaty [20,21]. From $m+1$ multiplicative preference relations, $\{A^1, A^2, \ldots, A^m\}$ expressing the experts' preferences and $B = [b_{vw}]$, $v, w = 1, \ldots, m$, expressing the relative importance degrees among experts, Saaty designed the decision AHP, which obtains the set of solution alternatives by means of the eigenvector method. This method is applied as follows:

1. Applying the exact eigenvector method to each multiplicative preference relation we obtain the normalized eigenvector for each multiplicative preference relation, i.e., $\alpha^k = (\alpha_1^k, \alpha_2^k, \ldots, \alpha_n^k)$, $\forall A^k$, and $b = (b_1, \ldots, b_m)$ for B. These are the local priority vectors corresponding to the maximum eigenvalues of each matrix $\{\lambda_A^1, \ldots, \lambda_A^m, \lambda_B^{m+1}\}$. The computation of the maximum eigenvalues and the corresponding eigenvectors is described below.
   (a) *The computation of the maximum eigenvalues*: We multiply each matrix of comparisons on the right by an estimated solution vector obtaining a new vector. Then, we divide the first component of this vector by the first component of the estimated solution vector, the second component of the new vector by the second component of the estimated solution vector and so on, obtaining another vector. If we take the sum of the components of this vector and divide by the number of components (n for A^k and m for B) we have an approximation to the maximum eigenvalues. These eigenvalues are used in estimating the consistency as reflected in the proportionality of preferences. For example, the closer λ_A^k is to n (the number of activities in the matrix A^k) the more consistent is the result.
   (b) *The computation of the eigenvectors*: The equations
   $$A^k \cdot \alpha^k = \lambda_A^k \cdot \alpha^k, \; k = 1, \ldots, m, \quad \text{and}$$
   $$B \cdot \alpha^{m+1} = \lambda_B^{m+1} \cdot \alpha^{m+1},$$
   are iterated till column vectors α^t $(t = 1, \ldots, m+1)$ satisfying these equations are obtained. The normalized α^t column vectors correspond to the principal eigenvectors of the multiplicative preference relations. The iterations start with initial unit vectors α^t.
2. The global priority vector $\alpha = (\alpha_1, \ldots, \alpha_n)$ is calculated according to the principle of the hierarchical composition:
   $$\alpha = \sum_{k=1}^{m} b_k \cdot \alpha^k.$$
3. Finally, the solution set of alternatives is obtained from α.

According to Saaty when we have consistent multiplicative preference relations A^k, then the following relation is satisfied:

$$a_{ij}^k = \frac{\alpha_i^k}{\alpha_j^k}.$$

In what follows, we consider the local priority vectors normalized in the unit interval $[0, 1]$.

3.1.2. Consistency

The fuzzy additive selection model, assuming that the experts express their preferences using fuzzy preference relations, is based on the two following choice degrees [4]: the quantifier-guided dominance degree and the quantifier-guided non-dominance degree. Both degrees are calculated for the collective fuzzy preference relation P^c. However, if we want to use them in the consistency demonstration of f they have to be defined for any fuzzy preference relation, P^k. Then, to differentiate these degrees from the former ones, we note them by $QGDD_i^k$ and $QGNDD_i^k$ $\forall i$, $\forall P^k$, respectively. The consistency condition of f is expressed in the following propositions.

Proposition 3. *Let $x_i, x_j \in X$, assuming that for a given consistent multiplicative preference relation, A^k, without loss of generality, the eigenvector method* *provides eigenvalues to verify* $\alpha_i^k \leqslant \alpha_j^k$, *then the quantifier-guided dominance degree obtained from the fuzzy preference relation* $P^k = f(A^k)$ *satisfies the following relationship*:

$$QGDD_j^k \geqslant QGDD_i^k$$

Proof. We have to demonstrate the following statement:

$$\text{if } \alpha_i^k \leqslant \alpha_j^k \Rightarrow QGDD_j^k \geqslant QGDD_i^k.$$

By definition, choose a fuzzy linguistic quantifier Q to calculate the weighting vector $W = [w_1, \ldots, w_n]$, then

$$QGDD_i^k = \phi_Q(p_{i1}^k, \ldots, p_{in}^k) = \sum_{t=1}^{n} w_t \cdot q_{it}^k,$$

where q_{it}^k is the tth largest value in the collection $p_{i1}^k, \ldots, p_{in}^k$. Then, applying the transformation function $f(A^k) = P^k$ and the property of consistency of A^k ($a_{it}^k = \alpha_i^k / \alpha_t^k$),

$$\begin{aligned} QGDD_i^k &= \sum_{t=1}^{n} w_t \cdot \left[\frac{1}{2} \cdot (1 + \log_9 a_{it}^k)\right] \\ &= \frac{1}{2} \cdot \left[\sum_{t=1}^{n} w_t + \sum_{t=1}^{n} w_t \log_9 a_{it}^k\right] \\ &= \frac{1}{2} \cdot \left[1 + \log_9 \Pi_t (a_{it}^k)^{w_t}\right] \\ &= \frac{1}{2} \cdot \left[1 + \log_9 \frac{(\alpha_i^k)^{\sum_t w_t}}{\Pi_t (\alpha_t^k)^{w_t}}\right] \\ &= \frac{1}{2} \cdot \left[1 + \log_9 \frac{\alpha_i^k}{C}\right] \quad \text{where } C = \Pi_t (\alpha_t^k)^{w_t}. \end{aligned}$$

Thus, given that the relationship between α_i^k and $QGDD_i^k$ exists, it is clear that the proposition is satisfied. □

Lemma 1. *Let* $x_i, x_j \in X$, *assuming that for a given consistent multiplicative preference relation*, A^k, *without loss of generality, the eigenvector method provides eigenvalues verifying* $\alpha_i^k \leqslant \alpha_j^k$. *If from the strict preference relation* $P^{k,s}$ *associated to the fuzzy preference relation* $P^k = f(A^k)$ *we obtain the following sets of values*:

$$P_t^{k,s} = \{p_{vt}^{k,s} \mid p_{vt}^{k,s} = p_{vt}^k - p_{tv}^k \geqslant 0,\ \forall v\},\ \forall x_t, t \in \{i, j\},$$

then the following relationship is satisfied:

$$\#(P_i^{k,s}) \geqslant \#(P_j^{k,s}).$$

Proof. We know that $p_{vi}^{k,s} = \max\{p_{vi}^k - p_{iv}^k, 0\}, \forall v$. Then, $p_{vi}^{k,s} = p_{vi}^k - p_{iv}^k$ if it satisfies the condition

$$p_{vi}^k - p_{iv}^k \geqslant 0.$$

Thus, applying f ($P^k = f(A^k)$) and assuming the consistency of A^k, i.e., $a_{tv}^k = \alpha_t^k / \alpha_v^k\ \forall t, v$, we obtain the following condition:

$$\frac{\alpha_v^k}{\alpha_i^k} \geqslant \frac{\alpha_i^k}{\alpha_v^k} \Rightarrow (\alpha_v^k)^2 \geqslant (\alpha_i^k)^2.$$

This means that $\forall v | (\alpha_v^k)^2 \geqslant (\alpha_i^k)^2$ then $p_{vi}^{k,s} \in P_i^{k,s}$. Hence, it is obvious that $\#(P_i^{k,s}) \geqslant \#(P_j^{k,s})$. □

Proposition 4. *Let* $x_i, x_j \in X$, *assuming that for a given consistent multiplicative preference relation*, A^k, *without loss of generality, the eigenvector method provides eigenvalues which verify* $\alpha_i^k \leqslant \alpha_j^k$, *then the quantifier-guided non-dominance degree obtained from the fuzzy preference relation* $P^k = f(A^k)$ *satisfies the following relationship*:

$$QGNDD_j^k \geqslant QGNDD_i^k.$$

Proof. Similarly, we have to demonstrate the following statement:

$$\text{if} \quad \alpha_i^k \leqslant \alpha_j^k \Rightarrow QGNDD_j^k \geqslant QGNDD_i^k.$$

By definition, choose a fuzzy linguistic quantifier Q to calculate the weighting vector $W = [w_1, \ldots, w_n]$, then

$$\begin{aligned} QGNDD_i^k &= \phi_Q(1 - p_{1i}^{k,s}, \ldots, 1 - p_{ni}^{k,s}) \\ &= \sum_{t=1}^{n} w_t \cdot (1 - q_{ti}^{k,s}), \end{aligned}$$

where $p_{vi}^{k,s} = \max\{p_{vi}^k - p_{iv}^k, 0\}$, and $[q_{1i}^{k,s}, \ldots, q_{ni}^{k,s}]$ is the vector associated to the collection $[p_{\sigma(1)i}^{k,s}, \ldots, p_{\sigma(n)i}^{k,s}]$, such that

$$p_{\sigma(a)i}^{k,s} \leqslant p_{\sigma(b)i}^{k,s} \quad \text{if } a \leqslant b,\ a, b \in \{1, \ldots, n\},$$

with σ being a permutation over the set of values $\{p_{vi}^{k,s}, \forall v\}$.

On the other hand, we know that $1 - q_{ti}^{k,s} = 2 \cdot p_{i\sigma(t)}^{k}$, $t = n_i, \ldots, n$, $2 \leqslant n_i \leqslant n$, and $1 - q_{ti}^{k,s} = 1$, $t = 1, \ldots, n_i - 1$. Therefore,

$$QGNDD_i^k = \sum_{t=1}^{n_i-1} w_t + \sum_{t=n_i}^{n} w_t \cdot (2 \cdot p_{i\sigma(t)}^k)$$

$$= 1 + \sum_{t=n_i}^{n} w_t \cdot (2 \cdot p_{i\sigma(t)}^k - 1).$$

Then, using the transformation function f

$$QGNDD_i^k = 1 + \sum_{t=n_i}^{n} w_t \cdot \log_9 a_{i\sigma(t)}^k$$

$$= 1 + \log_9 \prod_{t=n_i}^{n} (a_{i\sigma(t)}^k)^{w_t}$$

$$= 1 + \log_9 \frac{(\alpha_i^k)^{\sum_{t=n_i}^n w_t}}{\prod_{t=n_i}^n (\alpha_{\sigma(t)}^k)^{w_t}}$$

$$= 1 + \log_9 \frac{(\alpha_i^k)^{\sum_{t=n_i}^n w_t}}{\prod_{t=n_i}^n (\alpha_{\sigma(t)}^k)^{w_t}}.$$

Similarly, we have for $QGNDD_j^k$ the following expression:

$$QGNDD_j^k = 1 + \log_9 \frac{(\alpha_j^k)^{\sum_{t=n_j}^n w_t}}{\prod_{t=n_j}^n (\alpha_{\sigma(t)}^k)^{w_t}}.$$

We know by Lemma 1 that $n - n_i + 1 \geqslant n - n_j + 1$, i.e., $n_i \leqslant n_j$. Then,

$$(\alpha_j^k)^{\sum_{t=n_j}^n w_t} \geqslant (\alpha_i^k)^{\sum_{t=n_i}^n w_t}.$$

On the other hand, as $\alpha_v^k \in [0, 1]$ it is clear that

$$\prod_{t=n_j}^{n} (\alpha_{\sigma(t)}^k)^{w_t} \leqslant \prod_{t=n_i}^{n} (\alpha_{\sigma(t)}^k)^{w_t}.$$

Therefore, concluding, we have that $QGNDD_j^k \geqslant QGNDD_i^k$. $\square$

Remark. These propositions establish that the dominance and non-dominance choice degrees for a fuzzy preference relation $P^k = f(A^k)$ and Saaty's priority vectors for A^k give the same ordering among the alternatives. However, this does not establish any comparison criterion between Saaty's selection method and the fuzzy majority-based selection method presented in [4].

In the following section, as a consequence of this consistency study, we propose an alternative choice scheme to the AHP proposed by Saaty. The choice scheme follows the structure of the selection process shown in Section 2, and therefore, it is based on the dominance and non-dominance concepts and on the fuzzy majority one.

4. A multiplicative selection model based on fuzzy majority

In the AHP it is assumed that we have a set of $m + 1$ individual multiplicative preference relations, $\{A^1, A^2, \ldots, A^m, B\}$, where B is the importance matrix. Following the scheme of the selection process given in Section 2, we present a selection process based on fuzzy majority to choose the best alternatives from multiplicative preference relations. With a view to design it, we introduce a new aggregation operator guided by fuzzy majority in Section 4.1. In the following subsections we show how to apply this aggregation operator to solve the MPDM problem under multiplicative preference relations representing the experts' preferences.

4.1. The ordered weighted geometric operator

If we have a set of m multiplicative preference relations, $\{A^1, A^2, \ldots, A^m\}$, to be aggregated, normally, the collective multiplicative preference relation, A^c, which expresses the opinion of the group, is derived by means of the geometric mean, i.e.,

$$A^c = [a_{ij}^c], \quad a_{ij}^c = \prod_{k=1}^{m} (a_{ij}^k)^{1/m}.$$

In this context, we can define the ordered weighted geometric (OWG) operator, which provides a family of aggregators having the "and" operator at one extreme, the "or" operator at the other extreme, and the geometric mean as a particular case. The (OWG) operator is based on the OWA operator [30] and on the geometric mean, therefore, it is a special case of OWA operator. It is applied in our selection process to calculate a collective multiplicative preference relation and the quantifier-guided dominance and non-dominance choice degrees from multiplicative preference relations.

Definition 1. Let $\{a_1, a_2, \ldots, a_m\}$ be a list of values to aggregate, then, an OWG operator of dimension m is a function ϕ^G,

$$\phi^{\mathrm{G}} : R^m \to R$$

that has associated a set of weights W and is defined as

$$\phi^{\mathrm{G}}(a_1, a_2, \ldots, a_m) = \prod_{k=1}^{m} c_k^{w_k}$$

where $W = [w_1, \ldots, w_m]$, is an exponential weighting vector, such that, $w_i \in [0,1]$ and $\sum_k w_k = 1$; and C is the associated ordered value vector. Each element $c_i \in C$ is the ith largest value in the collection $\{a_1, \ldots, a_m\}$.

It is noted that different OWG operators are distinguished by their weighting vector. There are three important special cases of OWG aggregations:

1. $\phi^{\mathrm{G}}_{-}(a_1, a_2, \ldots, a_m) = \min_{k=1}^{m}(a_k)$. In this case $W = [0, 0, \ldots, 1]$.
2. $\phi^{\mathrm{G}}_{+}(a_1, a_2, \ldots, a_m) = \max_{k=1}^{m}(a_k)$. In this case $W = [1, 0, \ldots, 0]$.
3. $\phi^{\mathrm{G}}_{GM}(a_1, a_2, \ldots, a_m) = \prod_{k=1}^{m}(a_k)^{1/m}$. In this case $W = [1/m, 1/m, \ldots, 1/m]$.

We can use a process to obtain W similar to that used in the OWA operator (see Appendix A), i.e., the vector may be obtained using a fuzzy linguistic quantifier, Q, representing the concept of fuzzy majority. When a fuzzy quantifer Q is used to compute the weights of the OWG operator ϕ^{G}, then, it is symbolized by ϕ^{G}_{Q}.

The OWG operator satisfies the following properties:

1. The OWG operator is a max–min operator, i.e,

$$\min_k(a_1, \ldots, a_m) \leqslant \phi^{\mathrm{G}}(a_1, a_2, \ldots, a_m) \leqslant \max_k(a_1 \ldots, a_m).$$

2. The OWG operator is commutative.
3. The OWG operator is idempotent.
4. The OWG operator is increasing monotonous.

To conclude, we present the following result that connects the OWG operator with the OWA operator by means of the function $f(y) = \frac{1}{2}(1 + \log_9 y)$.

Proposition 5. *The OWG operator for the set of multiplicative preferences relations $\{A^1, \ldots, A^m\}$ acts as the OWA operator for the set of fuzzy preferences relations $\{P^1, \ldots, P^m\}$ where $p_{ij}^k = f(a_{ij}^k)$.*

Proof. Aggregating the multiplicative preference relations by means of the OWG operator we have

$$\{A^1, \ldots, A^m\} \to A^{\mathrm{c}} \quad \text{where } a_{ij}^{\mathrm{c}} = \phi^{\mathrm{G}}_{Q}(a_{ij}^1, \ldots, a_{ij}^m).$$

Then,

$$A^{\mathrm{c}} \to P^{\mathrm{c}} \quad \text{where } p_{ij}^{\mathrm{c}} = f(a_{ij}^{\mathrm{c}}) = \frac{1}{2}(1 + \log_9 a_{ij}^{\mathrm{c}}).$$

On the other hand, applying the transformation function f over the set of multiplicative preference relations we have

$$\{A^1, \ldots, A^m\} \to \{P^1, \ldots, P^m\}$$
$$\text{where } p_{ij}^k = f(a_{ij}^k) = \tfrac{1}{2}(1 + \log_9 a_{ij}^k),$$

and thus, aggregating the fuzzy preference relations by means of the OWA operator we have

$$\{P^1, \ldots, P^m\} \to P^d \quad \text{where } p_{ij}^d = \phi_Q(p_{ij}^1, \ldots, p_{ij}^m),$$

with P^d symbolizing the collective fuzzy preference relation.

As a consequence of function f being non-decreasing, if a_{ij}^k is the lth largest value of the collection $\{a_{ij}^1, \ldots, a_{ij}^m\}$ then p_{ij}^k is the lth largest value of the collection $\{p_{ij}^1, \ldots, p_{ij}^m\}$, i.e., we have

$$p_{ij}^{\mathrm{c}} = f(a_{ij}^{\mathrm{c}}) = \frac{1}{2}(1 + \log_9 a_{ij}^{\mathrm{c}})$$
$$= \frac{1}{2}(1 + \log_9 \phi^{\mathrm{G}}_{Q}(a_{ij}^1, \ldots, a_{ij}^m))$$
$$= \frac{1}{2}\left(1 + \log_9\left(\prod_{l}^{m}(c_l)^{w_l}\right)\right)$$

$$= \frac{1}{2}\left(1 + \sum_{l}^{m}(w_l \cdot \log_9 c_l)\right)$$

$$= \frac{1}{2}\left(\sum_{l}^{m} w_l + \sum_{l}^{m} w_l \cdot \log_9 c_l\right)$$

$$= \sum_{l}^{m} w_l \cdot \left[\frac{1}{2} \cdot (1 + \log_9 c_l)\right]$$

$$= \sum_{l}^{m} w_l \cdot b_l = \phi_Q(p_{ij}^1, \ldots, p_{ij}^m) = p_{ij}^d. \qquad \square$$

4.2. Dominance and non-dominance choice degrees for multiplicative preference relations

Using the results on consistency shown in Section 3.1.2 and the OWG operator, we can define the dominance and non-dominance choice degrees in a fuzzy majority sense for multiplicative preference relations. They are obtained via the transformation function f from the equivalent choice degrees defined for fuzzy preference relations as follows:

1. Quantifier-guided dominance degree for an alternative x_i, symbolized $MQGDD_i^k$, from a multiplicative preference relation, A^k, is defined according to the following expression:

$$MQGDD_i^k = \frac{1}{2} \cdot (1 + \log_9 \phi_Q^{\mathrm{G}}(a_{i1}^k, \ldots, a_{in}^k).$$

2. Quantifier-guided non-dominance degree for an alternative x_i, symbolized $MQGNDD_i^k$, from a multiplicative preference relation, A^k, is defined according to the following expression:

$$MQGNDD_i^k = 1 + \log_9 \phi_Q^{\mathrm{G}}(r_{i1}^k, \ldots, r_{in}^k),$$

where r_{ij}^k is a preference obtained as $r_{ij}^k = \min\{a_{ij}^k, 1\}$. This expression is achieved given that

$$QGNDD_i = \phi_Q(1 - p_{1i}^s, \ldots, 1 - p_{ni}^s)$$

$$= \phi_Q(\min\{2 \cdot p_{ij}, 1\}, \ldots, \min\{2 \cdot p_{in}, 1\})$$

$$= \phi_Q(1 + \min\{\log_9 a_{i1}, 0\}, \ldots, 1 + \min\{\log_9 a_{in}, 0\})$$

$$= 1 + \sum_j w_j \cdot \log_9(\min\{a_{ij}, 1\}).$$

In the following subsection, we present a selection process based on fuzzy majority using these choice degrees, which allows us to deal with MPDM problems under multiplicative preference relations.

4.3. Multiplicative selection process based on fuzzy majority

Assume an MPDM problem where the experts express their preferences on X by means of the set of multiplicative preference relations $\{A^1, \ldots, A^m\}$ and where a multiplicative importance matrix B on the experts is known. Then, using the scheme of the selection process presented in Section 2, the multiplicative selection process based on fuzzy majority is structured in the following two phases:

1. *Aggregation phase*. This phase defines a collective multiplicative preference relation, $A^{\mathrm{c}} = [a_{ij}^{\mathrm{c}}]$, which indicates the global preference according to the fuzzy majority of experts' opinions. A^{c} is obtained from $\{A^1, \ldots, A^m\}$ and B by means of the following expression:

$$a_{ij}^{\mathrm{c}} = \phi_Q^{\mathrm{G}}(a_{ij}^1 \cdot b_1, \ldots, a_{ij}^m \cdot b_m),$$

where b_k is the kth expert's importance degree derived from B according to some of the choice degrees defined for multiplicative preference relations, and ϕ_Q^{G} is the OWG operator guided by the concept of fuzzy majority represented by the fuzzy linguistic quantifier Q.

2. *Exploitation phase.* Using the quantifier-guided choice degrees defined for multiplicative preference relations, this phase transforms the global information about the alternatives into a global ranking of them, supplying the set of solution alternatives. According to the exploitation scheme designed in [4,6], the choice degrees can be applied in three steps:

Step 1: Using the OWG operator ϕ_Q^{G} we obtain the two choice degrees of alternatives from A^{c}:

$$[MQGDD_1, \ldots, MQGDD_n] \quad \text{and}$$

$$[MQGNDD_1^k, \ldots, MQGNDD_n^k],$$

with

$MQGDD_i = \frac{1}{2} \cdot (1 + \log_9 \phi_Q^{\mathrm{G}}(a_{i1}^{\mathrm{c}}, \ldots, a_{in}^{\mathrm{c}})$

and

$MQGNDD_i^k = 1 + \log_9 \phi_Q^{\mathrm{G}}(r_{i1}^{\mathrm{c}}, \ldots, r_{in}^{\mathrm{c}})$.

The application of each choice degree of alternatives over X allows us to obtain the following sets of alternatives:

$$X^{MQGDD} = \left\{ x_i \mid x_i \in X,\ MQGDD_i = \sup_{x_j \in X} MQGDD_j \right\},$$

$$X^{MQGNDD} = \left\{ x_i \mid x_i \in X, MQGNDD_i = \sup_{x_j \in X} MQGNDD_j \right\},$$

whose elements are called maximum dominance elements and maximal non-dominated elements, respectively.

Step 2: The application of the conjunction selection policy, obtaining the following set of alternatives:

$X^{QGCP} = X^{MQGDD} \cap X^{MQGNDD}$.

If $X^{QGCP} \neq \emptyset$, then end.
Otherwise continue.

Step 3: The application of one of the two sequential selection policies, according to either a dominance or non-dominance criterion, i.e.,

- *Dominance-based sequential selection process MQG-DD-NDD*. To apply the quantifier-guided dominance degree over X, and obtain X^{MQGDD}. If $\#(X^{MQGDD}) = 1$ then end, and this is the solution set. Otherwise, continue obtaining

$$X^{MQG\text{-}DD\text{-}NDD} = \left\{ x_i \mid x_i \in X^{MQGDD}, MQGNDD_i = \sup_{x_j \in X^{MQGDD}} MQGNDD_j \right\}.$$

This is the selection set of alternatives.

- *Non-dominance-based sequential selection process MQG-NDD-DD*. To apply the quantifier-guided non-dominance degree over X, and obtain X^{MQGNDD}. If $\#(X^{MQGNDD}) = 1$ then end, and this is the solution set. Otherwise, continue obtaining

$$X^{MQG\text{-}NDD\text{-}DD} = \left\{ x_i \mid x_i \in X^{MQGNDD}, MQGDD_i = \sup_{x_j \in X^{MQGNDD}} MQGDD_j \right\}.$$

This is the selection set of alternatives.

4.4. Example

Consider the following illustrative example of the classification method of alternatives studied in this paper. Assume that we have a set of three experts, $E = \{e_1, e_2, e_3\}$, and a set of three alternatives, $X = \{x_1, x_2, x_3\}$. Suppose that experts supply their opinions by means of the following multiplicative preference relations:

$$A^1 = \begin{bmatrix} 1 & 3 & 5 \\ \frac{1}{3} & 1 & 2 \\ \frac{1}{5} & \frac{1}{2} & 1 \end{bmatrix}, \qquad A^2 = \begin{bmatrix} 1 & 2 & 7 \\ \frac{1}{2} & 1 & 5 \\ \frac{1}{7} & \frac{1}{5} & 1 \end{bmatrix},$$

$$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{3} & \frac{1}{2} & 1 \end{bmatrix},$$

the important matrix being:

$$B = \begin{bmatrix} 1 & 5 & 3 \\ \frac{1}{5} & 1 & \frac{3}{5} \\ \frac{1}{3} & \frac{5}{3} & 1 \end{bmatrix}.$$

In the decision process we use the fuzzy majority criterion with the fuzzy linguistic quantifier "*at least half*", with the pair $(0, 0.5)$, and the corresponding OWG operator with the weighting vector, $W = [\frac{2}{3}, \frac{1}{3}, 0]$.

4.4.1. Multiplicative selection process based on fuzzy majority

1. *Aggregation phase*. Using the quantifier-guided dominance degree we obtain from B the

following importance vector:

$(b_1 = 0.82, b_2 = 0.46, b_3 = 0.57)$.

Then the collective multiplicative preference relation is

$$A^c = \begin{bmatrix} 0.72 & 1.9 & 3.78 \\ 0.27 & 0.72 & 2.28 \\ 0.17 & 0.36 & 0.72 \end{bmatrix}.$$

2. *Exploitation phase*. The quantifier-guided choice degrees of alternatives acting over the collective multiplicative preference relation supply the following values:

	x_1	x_2	x_3
$MQGDD_i$	0.75	0.6	0.37
$MQGNDD_i$	1	0.95	0.74

These values represent the dominance that one alternative has over "at least half" of the alternatives according to "at least half" of the experts, and the non-dominance degree to which the alternative is not dominated by "at least half" of the alternatives according to "at least half" of the experts, respectively. Clearly, the maximal sets are

$$X^{MQGDD} = \{x_1\} \quad \text{and} \quad X^{MQGNDD} = \{x_1\},$$

therefore, the selection set of alternatives according to complete selection procedures is the singleton $\{x_1\}$.

5. Concluding remarks

In this paper, we have studied how to integrate the multiplicative preference relations in fuzzy MPDM models under different preference representation structures (orderings, utilities and fuzzy preference relations). We have given a consistent method using the fuzzy preference relations as uniform representation element. This study together with our fuzzy MPDM model presented in [4] provides a more flexible framework to manage different structures of preferences, constituting an approximate decision model to real decision situations with experts of different knowledge areas.

Later, we have provided an alternative choice process to the classical AHP for dealing with MPDM problems under multiplicative preference relations. The aim of the multiplicative selection model is that it is based on fuzzy majority represented by a fuzzy linguistic quantifier. Futhermore, to design it, we have introduced a new aggregation operator based on the OWA operators to aggregate multiplicative preference relations, and have extended quantifier-guided dominance and non-dominance degrees to act with multiplicative preference relations.

A. Appendix: Fuzzy majority and OWA operator

The majority is traditionally defined as a threshold number of individuals. Fuzzy majority is a soft majority concept expressed by a fuzzy quantifier, which is manipulated via a fuzzy logic-based calculus of linguistically quantified propositions.

In this appendix we present the fuzzy quantifiers, used for representing the fuzzy majority, and the OWA operators, used for aggregating information. The OWA operator reflects the fuzzy majority calculating its weights by means of the fuzzy quantifiers.

A.1. Fuzzy majority

Quantifiers can be used to represent the amount of items satisfying a given predicate. Classic logic is restricted to the use of the two quantifiers, *there exists* and *for all*, that are closely related, respectively, to the *or* and *and* connectives. Human discourse is much richer and more diverse in its quantifiers, e.g. *about* 5, *almost all*, *a few*, *many*, *most*, *as many as possible*, *nearly half*, *at least half*. In an attempt to bridge the gap between formal systems and natural discourse and, in turn, to provide a more flexible knowledge representation tool, Zadeh introduced the concept of fuzzy quantifiers [31].

Zadeh suggested that the semantic of a fuzzy quantifier can be captured by using fuzzy subsets for its representation. He distinguished between two types of fuzzy quantifiers, *absolute* and *relative*. Absolute quantifiers are used to represent amounts that are absolute in nature such as *about* 2 or *more than* 5. These absolute linguistic quantifiers are closely related to the concept of the count or number of elements. He

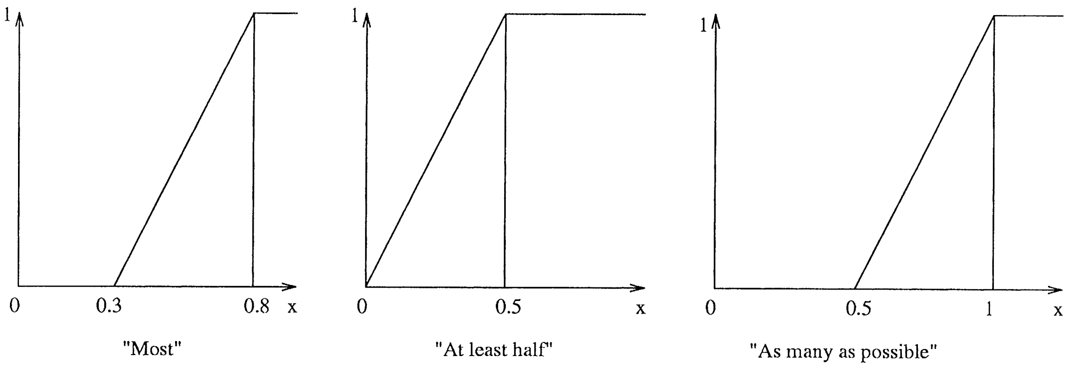

Fig. 2. Relative fuzzy quantifiers.

defined these quantifiers as fuzzy subsets of the non-negative real numbers, $\mathscr{R}^+$. In this approach, an absolute quantifier can be represented by a fuzzy subset Q, such that for any $r \in \mathscr{R}^+$ the membership degree of r in Q, $Q(r)$, indicates the degree to which the amount r is compatible with the quantifier represented by Q. Relative quantifiers, such as *most, at least half*, can be represented by fuzzy subsets of the unit interval, [0,1]. For any $r \in [0,1]$, $Q(r)$ indicates the degree to which the proportion r is compatible with the meaning of the quantifier it represents. Any quantifier of natural language can be represented as a relative quantifier or given the cardinality of the elements considered, as an absolute quantifier. Functionally, fuzzy quantifiers are usually of one of three types, *increasing, decreasing*, and *unimodal*. An increasing-type quantifier is characterized by the relationship $Q(r_1) \geqslant Q(r_2)$ if $r_1 > r_2$. These quantifiers are characterized by values such as *most, at least half*. A decreasing-type quantifier is characterized by the relationship $Q(r_1) \leqslant Q(r_2)$ if $r_1 < r_2$.

An absolute quantifier $Q : \mathscr{R}^+ \to [0,1]$ satisfies

$$Q(0) = 0, \text{ and } \exists k \text{ such that } Q(k) = 1.$$

A relative quantifier, $Q : [0,1] \to [0,1]$, satisfies

$$Q(0) = 0, \text{ and } \exists r \in [0,1] \text{ such that } Q(r) = 1.$$

A non-decreasing quantifier satisfies:

$$\forall a, b \quad \text{if } a > b \text{ then } Q(a) \geqslant Q(b).$$

The membership function of a non-decreasing relative quantifier can be represented as

$$Q(r) = \begin{cases} 0 & \text{if } r < a, \\ \frac{r-a}{b-a} & \text{if } a \leqslant r \leqslant b, \\ 1 & \text{if } r > b \end{cases}$$

with $a, b, r \in [0,1]$.

Some examples of relative quantifiers are shown in Fig. 2, where the parameters, (a, b) are $(0.3, 0.8)$, $(0, 0.5)$ and $(0.5, 1)$, respectively.

A.2. The OWA operator

The OWA operator was proposed by Yager in [30]. It provides a family of aggregation operators which have the "and" operator at one extreme and the "or" operator at the other extreme.

An OWA operator of dimension n is a function ϕ,

$$\phi : [0,1]^n \to [0,1],$$

that is associated with a set of weights. Let $\{a_1, \ldots, a_m\}$ be a list of values to aggregate, then the OWA operator ϕ is defined as

$$\phi(a_1, \ldots, a_m) = W \cdot B^{\mathrm{T}} = \sum_{i=1}^{m} w_i \cdot b_i$$

where $W = [w_1, \ldots, w_m]$, is a weighting vector, such that, $w_i \in [0,1]$ and $\sum_i w_i = 1$; and B is the associated ordered value vector. Each element $b_i \in B$ is the ith largest value in the collection $a_1, \ldots, a_m$.

The OWA operators fill the gap between the operators Min and Max. It can be immediately verified that OWA operators are commutative, increasing monotonous and idempotent, but in general not associative.

A natural question in the definition of the OWA operator is how to obtain the associated weighting vector. In [30], Yager proposed two ways to obtain it. The first approach is to use some kind of learning mechanism using some sample data; and the second approach is to try to give some semantics or meaning to the weights. The final possibility has allowed multiple applications on areas of fuzzy and multi-valued logics, evidence theory, design of fuzzy controllers, and the quantifier-guided aggregations.

We are interested in the area of quantifier-guided aggregations. Our idea is to calculate weights for the aggregation operations (made by means of the OWA operator) using linguistic quantifiers that represent the concept of *fuzzy majority*. In [30], Yager suggested an interesting way to compute the weights of the OWA aggregation operator using fuzzy quantifiers, which, in the case of a non-decreasing relative quantifier Q, is given by the expression

$$w_i = Q(i/n) - Q((i-1)/n), \quad i = 1, \ldots, n.$$

When a fuzzy quantifier Q is used to compute the weights of the OWA operator ϕ, it is symbolized by ϕ_Q.

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