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A linguistic decision model for promotion mix management solved with genetic algorithms

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Abstract

Promotion activities are those marketing tools that have a fast and direct impact in the target market. Due to this fact, the selection of such activities is an important decision. This contribution attempts to devise a decision process for this problem in conditions of uncertainty, supplying a linguistic decision model for evaluating the satisfaction of the objectives by the potential solutions. The process uses a genetic algorithm to find a good solution in promotion selection, such that it will both accomplish the communication objectives of the company and minimise the invested amount. © 2002 Published by Elsevier Science B.V.

Keywords: Promotion mix; Linguistic variables; Linguistic decision model; Genetic algorithms

1. Introduction

Promotion may be considered as the process through which an organisation communicates with, and influences, its target market segments with the goal of helping to position its products or services in their desired locations and generating the desired response from the segments.

The promotion mix selection constitutes an important decision, since the customer's behaviour is clearly determined by it. With this decision, the firm must select the marketing tools that accomplished its short-term communication objectives [23].

The aim of this paper is to attempt to devise a decision model for promotion mix management in conditions of uncertainty, supplying a linguistic decision model for evaluating the satisfaction of the objectives by the potential solutions, such that it will both accomplish the communication objectives of the company and minimise the invested amount. Therefore, the way for developing it is different from the traditional approach, since the numerical representation of the information on hold is replaced by a linguistic one, which we think is more suitable for this kind of decisions.

We propose a model that takes into account the possibility of a linguistic information related to the degree of accomplishment of the promotion objectives of the company by the promotion tools available in the market. Therefore, the model uses linguistic variables [36] for obtaining a linguistic valuation of objectives. We shall use the linguistic weighted aggregation (LWA) operator proposed in [13], to get a final linguistic evaluation of the solution combining linguistic satisfaction objective degrees and importance objective linguistic values. The LWA uses the *Min* operator as

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an importance transformation function and the linguistic ordered weighted averaging ($LOWA$) operator as an aggregation operator.

To search the optimum promotion mix, we need a tool able to grasp all the complexity imposed by the maximum level of investment and the linguistic evaluations. For the purpose of this paper we use a genetic algorithm (GA) [21]. GAs are general purpose search algorithms which use principles inspired by natural genetic populations to find solutions to problems. GAs can be efficiently used in high-dimensional, multimodal and complex problems. GAs play a significant role as search techniques for handling complex spaces in many fields, in particular in management problems [1,24,25,18,19]. In this study, the algorithm is characterised by the use of a linguistic fitness function.

In order to do that, the contribution is organised as follows. In Section 2, we introduce the linguistic approach. In Section 3, we will give a short introduction of the *promotion mix management problem*, we introduce the linguistic promotion mix model and show a particular promotion mix application. In Section 4, we will offer a descriptive analysis of the linguistic decision model. In Section 5, the GA designed to achieve a good solution to the selection problem will be presented. In Section 6, we will develop the example-application introduced in Section 3. In Section 7, some concluding remarks are pointed out. Finally, in the Appendix we describe shortly the LWA and $LOWA$ operators.

2. Linguistic approach

Usually, in a quantitative situation, the information required is expressed as numerical values. However, when we are working in qualitative areas, which are characterised by vague or imprecise knowledge, the information cannot be assessed in a precise numerical way. Thus, a more realistic approach would be to use linguistic information instead of numerical one [36]. This linguistic model has been applied to a wide range of problems: information retrieval [3], clinical diagnosis [8], education [22], suppliers selection [17], decision making [31,9,35,15,6], etc.

A linguistic variable differs from a numerical one in that it does not take numbers as a value but words or sentences in a natural or artificial language. Since words, in general, are less precise than numbers, the concept of a linguistic variable serves the purpose of providing a means of approximated characterisation of phenomena, which are too complex or too ill-defined to be amenable to their description in conventional quantitative terms.

Usually, depending on the problem domain, an appropriate linguistic term set is chosen to describe the vague or imprecise knowledge. The elements in the term set will determine the granularity of the uncertainty, that is the level of distinction among different information. Bonissone and Decker studied the use of term sets with an odd cardinal, representing the middle term as “approximately 0.5”, with the remaining terms being placed symmetrically around it and the limit of granularity being 11 or no more than 13 [12].

The semantics of the elements in the term set is given by fuzzy numbers defined on the $[0,1]$ interval and described by membership functions. Since the linguistic assessments given by the individuals are approximate, we can consider that trapezoidal or triangular membership functions are good enough to capture the vagueness of those linguistic assessments, since it may be impossible or unnecessary to obtain more accurate values [11].

The specification of the kind of label set ought to be the first priority in an application. Then, let $S=\{s_i\}$, $i\in H=\{0,\ldots,T\}$ be a finite and totally ordered term set on [0,1] in the usual sense [2,9]. Any label, s_i, represents a possible value for a linguistic variable, that is a vague property or constraint on [0,1]. We consider a term set, S, with its semantics given by linear triangular membership functions. Moreover, it must have the following characteristics:

(1) there is a negation operator: $Neg(s_i)=s_j$ such that $j=T-i$;
(2) the set is ordered: $s_i \geqslant s_j$ if $i \geqslant j$;
(3) there are a maximisation and a minimisation operator:

$$\max(s_i,s_j)=s_i \text{ if } s_i \geqslant s_j;$$

$$\min(s_i,s_j)=s_i \text{ if } s_i \leqslant s_j.$$

A wide study on the choice of a linguistic term set can be found in [14]. In this paper, we have chosen a set of nine linguistic labels as shown in Fig. 1.

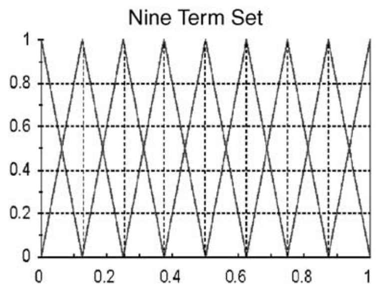

Fig. 1. Linguistic nine term set.

The 3-tuples associated are:

E	Essential (s_8)	(0.875, 1, 1)
VH	Very high (s_7)	(0.75, 0.875, 0.1)
FH	Fairly high (s_6)	(0.625, 0.75, 0.875)
H	High (s_5)	(0.5, 0.625, 0.75)
M	Moderate (s_4)	(0.375, 0.5, 0.625)
L	Low (s_3)	(0.25, 0.375, 0.5)
FL	Fairly low (s_2)	(0.125, 0.25, 0.375)
VL	Very low (s_1)	(0, 0.125, 0.25)
U	Unnecessary (s_0)	(0, 0, 0.125)

Formally speaking, it seems difficult to accept that all individuals should agree on the same membership function associated to linguistic terms, and therefore, there are not any universal distribution concepts. On other hand, it is possible to aggregate the linguistic information by means of fuzzy aggregation operators based on the extension principle (aggregating the associated fuzzy numbers, see [8]) or by means of symbolic aggregation operators (considering ordinal linguistic information, see [9,10]). In this contribution we will consider the latter aggregation approach considering ordinal linguistic information. Therefore, the choice of the membership function distribution is not a crucial task in this paper. A discussion on this topic can be found in [14].

As regards to the information to be aggregated in a linguistic process, clearly there are two types of linguistic information:

1. *Non-weighted linguistic information*. This is the situation in which we have only one set of linguistic values to aggregate, where all the linguistic values have the same importance.
2. *Weighted linguistic information*. This is the situation in which we have a set of linguistic values to aggregate, for example, opinions and each value is characterised by a different importance degree, indicating its weight in the overall set of values.

In both cases, linguistic aggregation operators are needed to appropriately combine the information in such a way that the final aggregation is the "best" representation of the overall opinions, criteria, objectives, etc.

The second case is due to either the introduction of an importance degree for not equally important objectives or the use of objective goals that can be managed in a similar way. To aggregate weighted information, we have to combine linguistic information with the weights, which involves the transformation of the weighted information under the importance degrees [7,32].

Different approaches have been proposed for managing weighted linguistic information 4–6. In this contribution, we use the LWA operator provided in [13]. It is based on the symbolic approach and was defined using the LOWA operator [20,16], the concept of fuzzy majority represented by a fuzzy linguistic quantifiers [37], and two families of linguistic connectives [13].

3. Promotion mix management problem

Promotion is basically communication. As we have mentioned, promotion may be considered as the process through which a company transmit information to its target market with the goal of helping to position its products or services in their desired locations and generating the desired response from the clients.

In the following subsections, we describe the promotion mix components, analyse the linguistic promotion mix model and introduce a particular application that will be solved in Section 6.

3.1. Promotion mix components

Inside of the global promotion concept, three components are included:

Promotion mix objectives. In general, the primary goal of any promotion strategy should be to help the organisation to achieve its marketing objectives. More specifically, it will be the case that the promotion objectives will be to use that blend of promotional devices which will achieve the maximum degree of influence in the target market segments at the minimum cost. The more common generic promotion objectives are [26]:

- Develop potential customers' awareness of existence of a new product or service.
- Refresh existing customers' memories of existence of an organisation, a brand, a product, or service through reminding them of its existence.
- Generate the wished attitude in the different segments of the target market.
- Obtain the desired sales volume.
- Signal to their competitors.

Promotion mix activities and tools. The activities to promote the products or services of the firm are different by their tools to communicate with the customers [29]. The activities are classified into four groups that are:

- **Advertising** is a paid, non-personal presentation of offerings (ideas, goods, or services) by an identified sponsor. Among all of promotion mix components, advertising is the most intuitively relevant one for communicating the product concept. Advertising also reduces barriers between the customers and the firm. Through advertising, customers come to know about the existence of new product.
- **Personal selling** is the communication of persuasive message or series of messages to target customers by an individual paid by the firm. Unlike other components of the promotion mix, it is characterised as a two-way, person-to-person communication between a sales person and a prospective customer.
- **Sales promotion** involves a number of techniques designed to stimulate customer's awareness of interest in the firm's product. Thus, directly or indirectly stimulates purchase or consumption of the product over the short term.
- **Public relation** is all about building and sustaining a good relationship with the public. This covers the customers but also the suppliers, the local community and anyone else the firm deals with as an organisation.

The different promotion activities have their advantages and disadvantages. Each one has its own tools. For example, into the wide concept of advertising, television advertising, radio advertising, newspaper advertising, internet advertising, etc are included. These tools allow to accomplish different promotion objectives to the company. The firm must select those tools included in the activities that are more accurate with their promotion objectives.

Promotion mix budget. The budget allocation for promotion is a fundamental and determinant aspect. To establish the promotion mix budget, many companies use one of the four following methods:

- As much as possible.
- Per cent of sales method.
- Parity with the competitors.
- According with the promotion objectives.

In this paper we follow the method of establishing the promotion mix according to the objectives. This method demands that the firm established its promotion objectives. These objectives determine the tasks to be developed and the expenses. This method is the best because it takes into account the relation between the promotion objectives and the level of investments.

3.2. Linguistic promotion mix model

The first step is establishing the objectives-criteria that the firm is searching with the promotion mix. These objectives are traditionally reduced to maximise the number of buyers, but the company can look for some others like knowledge of the product, preference, good-looking, reliable, loyalty, etc. The reasons are that the objectives are not simple and depend on the life cycle of the product, the available resources, the kind of product, the sort of marketing strategy, etc. The set of objectives selected for a specific promotion program could be:

$$o = (o_1, o_2, \ldots, o_k)$$

The different objectives could be required with different importance. This is the reason for representing the importance values associated to the objectives by means of linguistic weights. So, the linguistic weights required for the above k objectives are

$$\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k), \quad \alpha_i \in W.$$

The labels proposed for the feature weighting are the following:

$$W = \{Essential,\ Very\ high,\ Fairly\ high,\ High,\ Moderate,\ Low,\ Fairly\ low,\ Very\ low,\ Unnecessary\}.$$

The firm must identify all the tools that are available in the market; we denote by n, the number of tools. These tools are used by companies to operate in the target market have an impact whose measure is not a clear impact in it. So, representing their impact by linguistic labels could help the firm to manage the imprecision of this sort of information in a more accurate way.

On the other hand, the satisfaction degree of each objective depends on the number of insertions (number of times applied a tool) that the company makes on the various tools. The result of these accumulative insertions must be an increment of the effect that each tool produces in the target market. The value n_{ij}^h represents the linguistic valuation of the tool h on the objective j making i insertions, with $m_i, \ldots, m_n$ being the respective maximum number of insertions by tool.

$$n^1 = \begin{pmatrix} n_{11}^1, & n_{12}^1, \ldots, n_{1k}^1 \\ \vdots & \vdots \\ n_{m_1 1}^1, & n_{m_1 2}^1, \ldots, n_{m_1 k}^1 \end{pmatrix}, \quad n'_{ij} \in W,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$n^n = \begin{pmatrix} n_{11}^n, & n_{12}^n, \ldots, n_{1k}^n \\ \vdots & \vdots \\ n_{m_n 1}^n, & n_{m_n 2}^n, \ldots, n_{m_n k}^n \end{pmatrix}, \quad n_{ij}^n \in W.$$

After that, the company must know the cost associated with each promotion tool. These costs can vary according with the number of insertions. So the unitary cost will decrease if the firm uses several insertions of the same promotion tool.

The value $\mathscr{C}_i^h$ represents the cost of i insertions of the tool h.

$$\begin{aligned} c^1 &= (c_1^1, c_2^1, \ldots, c_{m_1}^1) \\ &\ \vdots \qquad\qquad\qquad c_i^h \in \Re \\ c^n &= (c_1^n,\ c_2^n, \ldots, c_{m_n}^n). \end{aligned}$$

Finally, the firm usually establishes a maximum level of investment on promotion mix, T, that shows the maximum amount available. Thus, the firm only accepts those promotion mix that have a cost lower or equal than this amount. For example, a possible solution (promotion mix) $S = (S_1, S_2, \ldots, S_n)$ must satisfy $\sum_{h=1}^n S_h \leqslant T$, where S_h represents the investment level on the hth promotion tool, with H_h being the number of insertions made for the tool, with $C_{H_h}^h = S_h$.

3.3. Promotion mix application

Let us suppose that a firm wishes to establish a new promotion campaign. A first step is to determine which objectives need to be reached, and at what degree must the campaign allows the firm to get these goals. Moreover, the different objectives could be required with different importance weights, α_j. Thus, we might have:

Number	Objective name	Importance (α_j)
1	Knowledge	Essential
2	Recognition	Fairly high
3	Acquisition	Fairly high
4	Preference	Very high
5	Loyalty	Fairly high

Let us suppose that there are eight promotion available to the firm in the new promotional campaign.

Number	Tool Name	Promotion activity
1	Television advertising	Advertising
2	Radio advertising	Advertising
3	Newspaper advertising	Advertising
4	Salesman	Personal Selling
5	Discount	Sales promotion
6	Prize	Sales promotion
7	Free sample	Sales promotion
8	Newspaper article	Public relations

Once the objective/weights have been established, the firm must decide how much money to spend in the different promotion tools, T. This step is crucial since the company must select the promotion budget that accomplishes its goals.

For each tool, it is necessary to know the linguistic satisfaction degree in each one of the objectives

Table 1
Television advertising

Television advertising	One insertion	Two insertions
Cost	3000	5000
Knowledge	Very High	Essential
Recognition	High	Very high
Acquisition	Moderate	Moderate
Preference	Very Low	Low
Loyalty	Moderate	High

Table 2
Radio advertising

Radio advertising	One insertion	Two insertions	Three insertions
Cost	2000	4000	6000
Knowledge	Very low	Moderate	Very high
Recognition	Low	Moderate	High
Acquisition	Moderate	Moderate	Moderate
Preference	Very low	Fairly Low	Low
Loyalty	Moderate	Moderate	High

Table 3
Newspaper advertising

Newspaper advertising	One insertion	Two insertions
Cost	4000	7000
Knowledge	High	Very high
Recognition	High	Very high
Acquisition	Fairly high	Fairly high
Preference	Very low	Very low
Loyalty	Moderate	Moderate

Table 4
Salesman

Salesman	One	Two
Cost	2000	4000
Knowledge	Low	Moderate
Recognition	Moderate	Moderate
Acquisition	High	Fairly high
Preference	High	Very high
Loyalty	Fairly high	Fairly high

Table 5
Discount

Discount	One campaign	Two campaigns
Cost	1000	1800
Knowledge	Very low	Moderate
Recognition	Moderate	High
Acquisition	High	High
Preference	Fairly high	Fairly high
Loyalty	High	Fairly high

Table 6
Prize

Prize	One
Cost	10000
Knowledge	High
Recognition	High
Acquisition	Very high
Preference	Low
Loyalty	Moderate

Table 7

Free sample	One campaign	Two campaigns	Three campaigns
Cost	3000	5000	7000
Knowledge	Very low	Fairly low	Fairly low
Recognition	Very low	Very low	Fairly low
Acquisition	Fairly high	Very high	Essential
Preference	High	High	High
Loyalty	Fairly low	Low	Moderate

Table 8
Newspaper article

Newspaper article	One insertion
Cost	8000
Knowledge	Moderate
Recognition	Very High
Acquisition	High
Preference	Essential
Loyalty	Very High

required for the promotion mix. This degree depends on the number of insertions that the firm could make for each tool. Tables 1–8 show the cost of the investment levels and the satisfaction degrees reached on the objectives (n_{ij}^h).

Finally, the maximum level of investment is established as $T = 10,000$.

4. Linguistic promotion mix decision model

In this section, we propose the linguistic decision model for promotion management. As said, the decision model will use the LWA operator presented in [13], providing a method to linguistically evaluate the possible solutions of the problem.

Considering a solution $S=(S_1, S_2, \ldots, S_n)$, we use the vector that represents the number of insertions that the solution suggests to make on each promotion tool, $H=(H_1, \ldots, H_n)$, where S_h represents the investment level on the hth promotion tool, with H_h being the number of insertions made for the tool. Therefore, $n^h_{H_h j}$ shows the tool satisfaction degree of the objective j.

For evaluating the solution, we propose a decision model according to two criteria:

- The first one is used to evaluate the linguistic satisfaction of the promotion mix according to the k objectives, $o=(o_1, o_2, \ldots, o_k)$.

  For each objective, we calculate the linguistic satisfaction objective degree, t_j, as the maximum of the satisfaction degrees made by every tool. In order to do that, we use the max operator because we consider an independent effect of every tool on each objective. Otherwise, if we expect some reinforcement effect among tools, we would use a linguistic t-conorm operator reflecting this effect.

  Then, we use the LWA operator to get the final linguistic evaluation of the solution, solution's suitability, combining linguistic satisfaction objective degrees and importance objective linguistic values. First, we operate on the objective satisfaction degree, t_j, weighted by the required linguistic weights α_j. The min operator is used as an importance transformation function. Then, the $LOWA$ operator is used to aggregate the previous results.

  The LWA is a type of fuzzy majority guided weighted aggregation operator. It satisfies a collection of axioms (see the Appendix A) that provide evidence of rational aggregation of this operator.

  The result seems intuitively reasonable to get the final linguistic objective satisfaction.
- The second one is considered to get the solution cost.

Finally, an evaluation function is considered with the assumption of getting the maximum linguistic objective satisfaction and the minimum cost with the same linguistic objective satisfaction.

In the following, we show the criteria expressions and the evaluation function.

4.1. Criterion 1. Linguistic objective satisfaction of the promotion mix

Step 1. Objetive attainment. For each promotion tool, h, there is an investment amount, S_h, defined in the solution. This investment determines the number of insertions made in the tool, H_h. According to this, we can obtain the tool degree of accomplishing the firm objectives, $n^h_{H_h j}$, $j=1,\ldots,k$. Thus, a link must be established between the degrees that the different promotion tools have on a given objective. To achieve it, the proposal is to use the linguistic operator MAX that indicates the maximum level of objective satisfaction degree.

$$t_j = \mathrm{MAX}(n^1_{H_1 j}, n^2_{H_2 j}, \ldots, n^n_{H_n j}) \quad j = 1, \ldots, k.$$

Step 2. Suitability. To obtain a value of the solution suitability on the objectives established by the firm, we apply an LWA operator using the MIN operator, as an importance transformation between importance weight and objective achievement, and the $LOWA$ operator as an aggregation operator, respectively (see [13]).

Step 2.1. Objective suitability. First, we obtain a linguistic label representing the suitability of the solution in each objective, combining the importance weight with the satisfaction degree, using the classical conjunction MIN. The solution suitability for every objective is obtained in the form of a linguistic label:

$$g_j = \min(\alpha_j, t_j), \quad j = 1, \ldots, k.$$

Step 2.2. Solution suitability. Second, we obtain a label representing the suitability degree of the overall solution, using the LOWA operator with the linguistic quantifier "*most*".

$$Z_S = \phi_Q(\min(\alpha_1, t_1), \ldots, \min(\alpha_k, t_k))$$
$$= \phi_Q(g_1, \ldots, g_k),$$

with ϕ_Q being the $LOWA$ operator whose weigths are obtained by a fuzzy linguistic quantifier Q [37].

Yager proposed an interesting way to compute the weights of the OWA aggregation operator, which, in the case of a non-decreasing proportional fuzzy linguistic quantifier Q, are given by this expression [32]:

$$w_j = Q(j/n) - Q((j-1)/n), \quad j = 1,\ldots,k.$$

With these steps, a linguistic evaluation of the solution in the goals of the promotion mix has been obtained. Nevertheless, the goodness of the solutions will also be determined by the cost incurred to implement that promotion mix.

4.2. Criterion 2. Cost of the promotion mix

To obtain a value of the solution cost, we only have to add all the tool investments. The suitability on these criteria are determined by the lower possible cost. Thus, among those solutions with the same goal level we prefer the one that has a lower cost.

$$\sum_{h=1}^{k} S_h = T_S.$$

As we mentioned, all solutions verifies that $T_S \leqslant T$.

4.3. Evaluation function

Finally, a linguistic label Z_S has been obtained, that is the evaluation for each feasible solution S according to the goal objective, and T_S, that is the cost of the solution.

In order to establish a comparison between two possible solutions we propose the following approach. Let S^1 and S^2 be two candidate solutions, and (Z_{S^1}, T_{S^1}) and (Z_{S^2}, T_{S^2}) be the vectors associated with them. The following expression shows the preference between the solutions:

$$S^1 \text{ is better than } S^2 \quad \Leftrightarrow \quad (Z_{S^1} > Z_{S^2})$$
$$\text{or} \quad (Z_{S^1} = Z_{S^2} \text{ and } T_{S^1} < T_{S^2}).$$

As we said, this expression is justified by the fact of getting the maximum objective satisfaction and the minimum cost with the same objective satisfaction.

5. A genetic algorithm for the linguistic promotion mix selection process

In order to find an optimal solution, we need to apply a meta-heuristic technique. Meta-heuristics are a class of approximate methods, that are designed to attack hard combinatorial optimisation problems where classical heuristics have failed to be effective and efficient. Meta-heuristics provide general frameworks that allow to create new hybrids by combining different concepts derived from: classical heuristics; artificial intelligence; biological evolution; neural systems and statistical mechanics. These families of approaches include GAs, greedy random adaptive search procedure, problem-space search, neural networks, simulated annealing, tabu search, threshold algorithms, ant colony optimisation and their hybrids. For a good and brief tutorial on meta-heuristics, we refer to [28].

Different families of approaches could be used to solve our problem, getting similar results. We have considered the use of GAs, because they constitute a well known search heuristic method that has shown good results in this kind of problems [18,19].

In this section, we first present a brief introduction to GAs and, then the proposal of the biobjective GA is introduced.

5.1. Genetic algorithms

GAs are general purpose search algorithms which use principles inspired by natural genetics to evolve solutions to problems [21,12]. The basic idea is to maintain a population of chromosomes, which represents candidate solutions to the concrete problem being solved, which evolves over time through a process of competition and controlled variation. Each chromosome in the population has an associated *fitness* to determine (*selection*) which chromosomes are used to form new ones in the competition process. The new ones are created using genetic operators such as *crossover* and *mutation*. GAs have got a great measure of success in search and optimisation problems. The reason for a great part of this success is their ability to exploit the information accumulated about an initially unknown search space in order to bias subsequent searches into useful subspaces. This is their key feature, particularly in large, complex, and poorly understood search spaces, where classical search tools are inappropriate, offering a valid approach to problems requiring efficient and effective search techniques.

A GA starts off with a population of randomly generated *chromosomes* (solutions), and advances toward better *chromosomes* by applying genetic operators. The population undergoes evolution in a form of natural selection. During successive iterations, called generations, a new population of chromosomes is formed using a selection mechanism and specific genetic operators such as crossover and mutation. An *evaluation or fitness function* must be devised for each problem to be solved. Given a particular chromosome, a possible solution, the fitness function returns a single numerical fitness, which is supposed to be proportional to the utility or adaptation of the solution represented by that chromosome.

GAs may deal successfully with a wide range of problem areas, particularly in management applications [1]. The main reasons for this success are: (1) GAs can solve hard problems quickly and reliably, (2) GAs are easy to interface to existing simulations and models, (3) GAs are extendible and (4) GAs are easy to hybridise. All these reasons may be summarised in only one: GAs are *robust*. They are not guaranteed to find the global optimum solution to a problem, but they are generally good at finding acceptably good solutions to problems quickly. These reasons have caused that, during the last few years, GA applications have grown enormously in many fields.

It is generally accepted that the application of a GA to solve a problem must take into account the following five components:

1. A genetic representation for the solutions to the problem,
2. a way to create an initial population of solutions,
3. an evaluation function which gives the fitness of each chromosome,
4. genetic operators that alter the genetic composition of offspring during reproduction, and
5. values for the parameters that the GA uses (population size, probabilities of applying genetic operators, etc.).

The basic principles of GAs were first laid down rigorously by Holland [21], and are well described in many books, such as [12,27].

5.2. A genetic algorithm for selecting the best promotion mix

In this paper, the proposed GA will use a discrete codification of the solutions, according to the possible values of the investment of tool insertions. Strings of candidates are generated of the same size as the number of tools available. Each element of the string codifies the investment level of the corresponding promotion tool.

An example of a solution for a case of five promotion tools available and 1000 monetary units available would be

$$S = (200, 400, 100, 100, 200).$$

This solution indicates that the firm expend 200 monetary units in the first promotion tool, 400 monetary units in the second, 100 in the third and the fourth, and finally 200 in the fifth promotion tool, being the total amount invested 1000 monetary units.

In order to operate efficiently with the solutions in the following GA phases, we propose to obtain another vector that represents the number of insertions that the solutions suggest to make on each promotion tool. In this example it could be

$$H = (1, 2, 2, 1, 5).$$

Once the coding has been decided upon, random processes generate a population of these solutions.

5.2.1. Fitness function

To establish the fitness of each solution to the problem, we shall propose to use the linguistic decision model introduced in Section 4. Then, we obtain a label that indicates the goals satisfaction level and a number that shows the cost of each solution.

5.2.2. Parents selection

According to the preference condition established in Section 4, we can obtain an order of the solutions fitness. Then, we assign a selection probability to each one as follows.

- First, we classify the solutions comparing their linguistic goals and cost. With this step, we can order all of them.

- Second, we apply a linear ranking to obtain the selection probabilities:

$$P_i = \frac{1}{N}\left(\eta_{\max} - (\eta_{\max} - \eta_{\min})\frac{i-1}{N-1}\right)$$

with $\eta_{\max} = 2 - \eta_{\min}, \eta_{\min} \in [0,1]$ being two parameters controlling the selection pressure.
- Third, we average the selection probabilities of individuals with equal place in the ordering. So that, all of them are sampled with the same rate.

Moreover, in order not to lose good solutions, an elitism selection [12] has been introduced. This procedure consists of keeping the best individual from a population in a successive generation. In this way, the best solution for a previous population is not lost until outclassed by a more fitted solution.

5.2.3. Crossover

In order to keep the feasibility on the solution population and avoid solutions with cost bigger than the promotion amount available, we propose to use a special crossover operator. The steps are:

1. At the beginning of the crossover process we have two "parents". For example, in a problem of five tools available and a maximum expended amount of 1000 monetary units, their composition could be

$$S_1 = (100, 300, 100, 200, 300),$$

$$S_2 = (500, 100, 50, 100, 150).$$

2. First, we randomly keep some elements in the offspring. Thus, we could obtain

$$S_1' = (100, \cdot, 100, 200, \cdot),$$

$$S_2' = (500, \cdot, 50, 100, \cdot).$$

3. Then, we interchange uniformly the remaining elements to the offspring until the maximum investment level is reached, beginning randomly. The amount of each element of the string must be an element of the cost vectors. So, as we have the number of insertions associated with the investment amount (for S_i, the value H_i, being $C^i_{H_i} = S_i$) we can easily obtain feasible solutions, decreasing the investment amount for getting a feasible solution. Thus, two resulting solutions could be

$$S_1' = (100, 100, 100, 200, 150),$$

$$S_2' = (500, 50, 50, 100, 300).$$

The first place selected was the fifth, and the change between both solutions is possible. The second element of the solution S_2' must be 300, but if we introduce that value into the string, the resulting cost of the solution is bigger than the maximum available. So that solution would not be feasible. According to the aforementioned process we reduce the number of insertion until the total cost is accepted. Due to this, 50 is the cost asociated with some of the investment levels of the second tool.

5.2.4. Mutation

In the same way, to keep the feasibility on the solution population we propose to use a mutation that randomly delete some elements of the solution and then, also randomly, generate their investment. This process must maintain a total investment level lower than the maximum amount available. An example is

1. First, we select a solution. For example, in a problem of five tools available and a maximum expended amount of 1000 monetary units, the solution could be

$$S_1 = (100, 300, 100, 200, 300),$$

2. Then, we randomly keep some elements. So, we could obtain

$$S_1' = (\cdot, 300, 100, \cdot, 300).$$

3. Finally, we randomly generate the remaining elements of the offspring in a random order until the maximum investment level is achieved. As well as the crossover operator, the amounts obtained must belong to the cost vectors. Thus, the resulting solution could be

$$S_1' = (250, 300, 100, 50, 150).$$

6. Example of a practical application

In this section we solve the example introduced in Section 3. The main is to show the decision model evaluation and the GA graphical evolution.

As said, different families of approaches could be used to solve our problem, getting similar results. Therefore, this section must not be considered as an experimental validation of the GA that is far from the objective of the paper.

We divide this section into two subsections according to the following steps: decision model and GA-based selection process.

6.1. Linguistic decision model

Let $S = (3000, 2000, 0, 2000, 0, 0, 3000, 0)$ be a possible solution. We are going to apply the decision model on it for obtaining a linguistic evaluation.

6.1.1. Criterion 1. Linguistic objective satisfaction of the promotion mix

Step 1. Objective attainment

$n^k_{S_i j}$	Investment	Satisfaction objective degree				
Tools		*Knowledge*	*Recognition*	*Acquisition*	*Preference*	*Loyalty*
Television Advertising	3000	Very high	High	Moderate	Very low	Moderate
Radio advertising	2000	Very low	Low	Moderate	Very low	Moderate
Newspaper advertising	0	—	—	—	—	—
Salesman	2000	Low	Moderate	High	High	Fairly high
Discount	0	—	—	—	—	—
Prize	0	—	—	—	—	—
Free sample	3000	Very low	Very low	Fairly high	High	Fairly low
Newspaper article	0	—	—	—	—	—
MAX		Very high	High	Fairly high	High	Fairly high

Step 2. Suitability

Step 2.1. Objective suitability

g_j	Objective suitability				
	Knowledge	*Recognition*	*Acquisition*	*Preference*	*Loyalty*
α_j	Essential	Fairly high	Fairly high	Very high	Fairly high
t_j	Very high	High	Fairly high	High	Fairly high
g_j	Very high	High	Fairly high	High	Fairly high

Step 2.2. Solution suitability

$$Z_S = \phi_Q(VH, FH, FH, H, H) = FH$$

using the quantifier Q as the linguistic quantifier *most*.

We have obtained a linguistic evaluation (*Fairly high*) of the solution tool investments in the promotion goals.

6.1.2. Criterion 2. Cost of the promotion mix

To obtain a value of the solution cost we only have to add all the investment tools.

$$\sum_{h=1}^{8} S_h = T_S = 300 + 2000 + 0 + 2000 + 0 + 0 + 3000 + 0 = 10,000.$$

With the last operation, the cost evaluation of the solution has been obtained. Those solutions that have the minimum cost with the same linguistic suitability degree are preferred to this one.

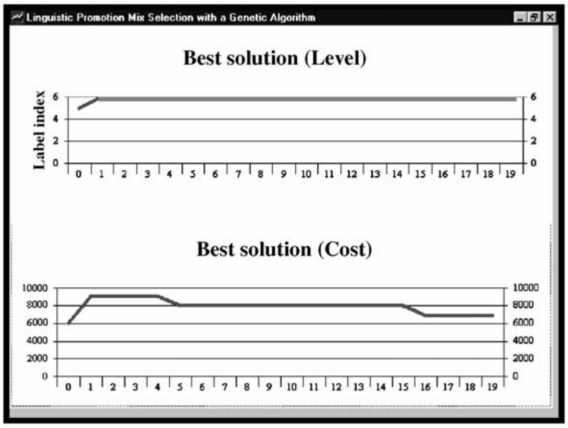

Fig. 2. Genetic evolution of the best population solution.

Therefore, we have obtained a vector for evaluating the solution S:

(*Fairly high*; 10,000).

6.2. GA-based selection process

In this subsection we show the GA-based selection process of this example. So, for the purposes of application of the operational model, the parameters used in finding the solution by means of the proposed model were

- Number of generations: 20.
- Number of individuals: 10.
- Crossover probability: 0.5.
- Mutation probability: 0.35.

We should remark that the use of a high mutation probability was motivated by the need to bring new tools into the chains.

The graphics of the evolution of the best individual in each generation are displayed in Fig. 2. The upper graphic shows the linguistic suitability degree of the best solution, whilst the bottom one indicates the cost of the best solution found till each generation.

In the practical example, the final solution has the following values:

- Linguistic suitability degree: Fairly high,
- Cost: 6.800

Promotion tool	Investment
Television advertising	3000
Radio advertising	0
Newspaper advertising	0
Salesman	2000
Discount	1800
Prize	0
Free sample	0
Newspaper article	0

7. Concluding remarks

The results obtained from this work fall into two clusters. The first one involves the linguistic formulation of a promotion mix selection model that could be adapted to the problem under consideration. The second one is the establishment of a promotion mix procedure based on a linguistic decision model that is used as an evaluation tool for a GA-based selection process.

In this way, an attempt is made to demonstrate the usefulness of the model being proposed in this paper by solving a real problem from the business world.

Finally, to point out that the linguistic formulation for promotion mix management is very general and it can be adopted without doubts to different problems under the same consideration.

Appendix A. LOWA and LWA operators

LOWA and I-LOWA operators

First, we introduce two operators, the LOWA operator presented in [20] and the inverse-linguistic ordered weighted averaging (I-LOWA) operator presented in [13].

Definition (convex combination of m labels (Delgado et al. [10]). Let $A=\{a_1,\ldots,a_m\}$ be a set of labels to be aggregated, $\otimes$ the general product of a label by a positive real number and $\oplus$ the general addition of labels defined in [10], then the convex combination operator of $\boldsymbol{m}$ labels, C^m, is defined as

$$C^m\{w_k,b_k,k=1,\ldots,m\}$$
$$=W/B^T=w_1\otimes b_1\oplus(1-w_1)$$
$$\otimes C^{m-1}\{\beta_h,b_h,h=2,\ldots,m\},$$

where $W = [w_1, \ldots, w_m]$, is a weighting vector, such that: (i) $w_i \in [0,1]$ and, (ii) $\sum w_i = 1$.

$\beta_h = w_h / \sum_2^m w_k, h = 2, \ldots, m$ *and* $B = \{b_1, \ldots, b_m\}$ *is a vector associated to* A, *such that*,

$$B = \sigma(A) = \{a_{\sigma(1)}, \ldots, a_{\sigma(n)}\},$$

where $a_{\sigma(j)} \leqslant a_{\sigma(i)} \forall i \leqslant j$, and σ *being a permutation over the set of labels* A.

Using the above definition, and the ordered weighted averaging (OWA) operator [32], in [20] was defined the $LOWA$ operator.

Definition (LOWA operator). Let A $= \{a_1, \ldots, a_m\}$ be a set of labels to be aggregated, C^m the convex combination operator of $\boldsymbol{m}$ labels, $\otimes$ the general product of a label by a positive real number and $\oplus$ the general addition of labels defined in [10], then the LOWA operator, ϕ, is defined as

$$\phi(a_1, \ldots, a_m) = C^m\{w_k, b_k,\ k = 1, \ldots, m\}$$

If $m = 2$, then C^2 is defined as

$$C^2\{w_i, b_{i,}\ i = 1, 2\} = w_1 \otimes s_j \oplus (1 - w_1) \otimes s_i$$

$$= s_k, s_j, s_i \in S,\ (j \geqslant i)$$

such that $k = \min\{T, i + round(w_1(j - i))\}$, where "round" is the usual round operation, and $b_1 = s_j$, $b_2 = s_i$.

If $w_j = 1$ and $w_i = 0$ with $i \neq j\ \forall i$, then the convex combination is defined as

$$C^m\{w_i, b_i,\ i = 1, \ldots, m\} = b_j.$$

Definition (I − LOWA operator). An I-LOWA operator, ϕ^I, is a type of $LOWA$ operator, in which

$$B = \sigma^I(A) = \{a_{\sigma(1)}, \ldots a_{\sigma(n)}\},$$

where, $a_{\sigma(i)} \leqslant a_{\sigma(j)}\ \forall i \leqslant j$.

If $m = 2$, then it is defined as

$$C^2\{w_i, b_i,\ i = 1, 2\} = w_1 \otimes s_j \oplus (1 - w_1) \otimes s_i$$

$$= s_k, s_j, s_i \in S,\ (j \leqslant i)$$

such that $k = \min\{T, i + round(w_1(j - i))\}$.

Wide studies on these operators can be found in [16,13].

In the OWA operators, the weights measure the importance of a value with independence of the information source. How to calculate the weighting vector of LOWA operator, W, is a basic question to be solved. A possible solution is that the weights represent the concept of fuzzy majority in the aggregation of LOWA operator using fuzzy linguistic quantifiers [37]. Yager proposed an interesting way to compute the weights of the OWA aggregation operator, which, in the case of a non-decreasing proportional fuzzy linguistic quantifier, Q, is given by this expression [32]:

$$w_i = Q(i/n) - Q((i - 1)/n),\ i = 1, \ldots, n$$

being the membership function of Q, as follows:

$$Q(r) = \begin{cases} 0 & \text{if } r < a, \\ \dfrac{r - a}{b - a} & \text{if } a \leqslant r \leqslant b, \\ 1 & \text{if } r > b \end{cases}$$

with $a, b, r \in [0,1]$. Some examples of non-decreasing proportional fuzzy linguistic quantifiers are: "most" $(0.3, 0.8)$, "at least half" $(0, 0.5)$ and "as many as possible" $(0.5, 1)$. When a fuzzy linguistic quantifier, Q, is used to compute the weights of LOWA operator, ϕ, it is symbolised by ϕ_Q. Similarly happens for the I-LOWA operator, i.e., in this case it is symbolised by ϕ_Q^I.

LWA operator

The LWA operator to aggregate linguistic weighted, information is provided in [13], which was defined using the LOWA operator [20], the concept of fuzzy majority represented by a fuzzy linguistic quantifiers [37], and two families of linguistic connectives [13]. In the following we review it.

Before defining the LWA operator, let us present the following two families of linguistic connectives [13]:

(1) Linguistic Conjunction functions $LC^{\rightarrow}$:
  1. The classical MIN operator:

$$LC^{\rightarrow}(c, a) = MIN(c, a).$$

2. The nilpotent MIN operator:

$$LC_2^{\rightarrow}(c,a) = \begin{cases} \mathrm{MIN}(c,a) & \text{if } c > Neg(a), \\ 0 & \text{otherwise.} \end{cases}$$

3. The weakest conjunction:

$$LC_3^{\rightarrow}(c,a) = \begin{cases} \mathrm{MIN}(c,a) & \text{if } \mathrm{MAX}(c,a) = s_T, \\ 0 & \text{otherwise.} \end{cases}$$

(2) Linguistic implication functions $LI^{\rightarrow}$:

1. Kleene–Dienes's implication function:

$$LI_1^{\rightarrow}(c,a) = MAX(Neg(c),a)$$

2. Gödel's implication function:

$$LI_2^{\rightarrow}(c,a) = \begin{cases} s_T & \text{if } c \leqslant a, \\ a & \text{otherwise.} \end{cases}$$

3. Fodor's implication function:

$$LI_3^{\rightarrow}(c,a) = \begin{cases} s_T & \text{if } c \leqslant a, \\ \mathrm{MAX}(Neg(c),a) & \text{otherwise.} \end{cases}$$

Using this families of linguistic connectives as importance transformation functions that integrate the weights and the variables, it is defined the *LWA* operator handling as aggregation operator the *LOWA* or *I-LOWA* operators. It is based on the combination of the *LOWA* and *I-LOWA* operators with several linguistic conjunction functions ($LC^{\rightarrow}$) and several linguistic implication functions ($LI^{\rightarrow}$), respectively.

Definition (LWA operator). The aggregation of a set of weighted individual information $\{(c_1,a_1),\ldots,(c_m,a_m)\}$, being $\mathscr{C}_1$ and a_1 the weights and variable values, respectively, the LWA operator is defined as

$$a_E = LWA[(c_1,a_1),\ldots,(c_m,a_m)]$$
$$= f[g(c_1,a_1),\ldots,g(c_m,a_m)],$$

where $f \in \{\phi_Q, \phi_Q^I\}$ is a linguistic aggregation operator of transformed information and g is an importance transformation function, such that $g \in LC^{\rightarrow}$ if $f = \phi_Q$ and $g \in LI^{\rightarrow}$ if $f = \phi_Q^1$, being $LC^{\rightarrow} = \{LC_1^{\rightarrow}, LC_2^{\rightarrow}, LC_3^{\rightarrow}\}$ and $LI^{\rightarrow} = \{LI_1^{\rightarrow}, LI_2^{\rightarrow}, LI_3^{\rightarrow}\}$.

As it was commented in [13], when the aggregation operator, f, is the *I-LOWA* operator, ϕ_Q^I, and given that ϕ_Q^1 is an aggregation operator with characteristics of a MIN type aggregation operator, then we decide to use the linguistic implication functions, $LI^{\rightarrow}$, as the transformation function type. Something similar happens when f is the *LOWA* operator ϕ_Q.

It can be observed that *LWA* operator tries to reduce the effects of elements with low importance. To do so, when $f = \phi_Q$, the elements with low importance are transformed into small values and when $f = \phi_Q^I$, the elements with low importance are transformed into large values.

As was shown in [13], the *LWA* operator verifies the following axioms: *independence of alternatives*, *commutativity*, *positive sensitivity in its weaker form*, *neutrality with respect to alternatives*, *unrestricted domain*, *and being an* "*orand*" *operator.* The fulfilment of these axioms provides evidence of rational aggregation of these operators in particular frameworks.

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