Duopoly Cournot model with piecewise linear marginal costs and stranded costs recovery [1]

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Abstract

We study the influence of long term contracts and stranded costs recoveries in the spot market strategic behavior of Spanish electricity generating companies. A Cournot model with two strategic players and with piecewise linear marginal costs is considered. We prove that for fixed demand, the optimization problem may have one, several or no solution at all. In addition, we show that long term contracts imply a more agressive behavior in the market and that the presence of stranded costs recoveries may result on prices smaller than the competitive equilibrium price. We, finally, illustrate our results with numerical demonstrations.

Key words: Duopoly, piecewise linear marginal costs, competition transition charges, contracts, Spanish electricity market.

1 Introduction

In the Spanish electricity market, to compensate for stranded costs in a previously regulated market, some generating companies are entitled to yearly reimburse the so-called Competition Transition Charges (CTC's) which are paid

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from the public budget. CTC's reimbursements depend on fixed tariff small consumers incomes on the basis of some predetermined CTC's coefficients. When the annual average price in the pool is lower than the forecasted, the amount to be shared in CTC's is greater than the initially expected and, therefore, generators with market shares smaller than their CTC's coefficients could consider profitable to low the price in the spot market.

In the literature, we find different models related to the electricity market. For a survey,we refer the reader to [1]. In connection with our problem, we may recall the following papers:

In [2], the authors study a Cournot model with two symmetric generators with constant marginal costs that compete in contract and spot markets. They observe that contracts lead to a more aggressive behavior in the spot market. The reason is that if a generating company signed a contract, the generating capacity under it is not affected by the price in the spot market. Therefore, the company has less incentives to withdraw capacity in order to raise the spot price. In fact, they prove that if the number of contract rounds tends to infinity, the unique equilibrium is the competitive equilibrium. In that paper, the authors use the restrictive assumption that prices in the spot and contract markets are the same. However, a slightly weaker conclusion regarding the increase of aggressive behavior in the spot market induced by the contract market was obtained without this assumption, see [3,4]. In the works mentioned above, regular convex marginal costs are assumed and therefore the spot market has an unique equilibrium which is obtained by solving the first order condition.

In [5] the authors analyzed competition in the British market using the supply function equilibria approach of [6] that study duopoly under demand uncertainty. In [5], convex marginal costs are assumed and the two strategic generators submit continuously differentiable supply functions to the auctioneer, being the latter assumed to use the same bids to meet demand for different auctions. The equilibria are computed solving a system of ordinal differential equations which lack of an initial condition, and therefore infinite equilibria do exist. These equilibria lie between the price-setting (Bertrand) equilibrium and the quantity setting (Cournot) equilibrium.

More involved models consider, for instance, conjectural variation in the contract market and supply function equilibrium in the spot market, see [7,8].

An additional inconvenient in the regular supply function approach is that, in real markets, schedules are discrete. In [9] the authors modelled the British electricity pool as a first-price, sealed bid, multi unit auction. In this model, generators submit discrete step functions. Although this is a most realistic model, if we assume that the forecasted demand is sharp, the undercutting behavior leads to aggressive competition as is showed in [10].

The paper is organized as follows. In section 2, we present the duopoly model with piecewise linear marginal costs and with contracts. In section 3 we study the existence of solutions to this model. In sections 4 and 5 we compute some numerical simulations, and finally in section 6 we give the conclusions.

2 Mathematical model

On one hand, the iterative nature of the interaction between generators which bid into the pool, the incentives introduced by CTC's and the different strategic behavior in the long term (investing decisions) lead us to consider a non-cooperative model. On the other hand, the aggressive outcome obtained in a multi-unit auction process, the certainty in the forecast demand, and the possibility for generators to change their bids in the different auctions, suggest us to consider a Cournot model.

If we consider a convex setting, the optimization problem may be solved considering the solution of the first order differential optimality condition. However, for a realistic model, no convex marginal costs should be considered. In this article, a stepwise linear marginal costs model is considered resulting, as we will prove, in demands that lead to multiple, unique or non existing equilibria.

We assume that, owning most of the generating capacity, only two companies are strategic players. The other players are price takers. The demand function is price dependent since the two strategic players assume that the price takers generates according to their costs, and for simplicity, company independent minimum and maximum costs for any technology are assumed. Therefore, continuous and piecewise linear marginal cost functions for the two strategic players and price takers are considered. We note that linear price dependence of the demand could be not consistent with constant marginal cost. Assuming that in any auction generators take account of their influence to decide their bidding behavior in any spot auction, we add CTC's to the cost functions. Finally, since hydroelectric generation depends, among other factors, on the amount of water in the dams and on the forecasted uses of water we shall only consider thermal demand.

Within the above framework, the profit function for the strategic players ($i = 1, 2$) is given by

$$\tilde{\pi}_i = p\Big(q_i - x_i\Big) + \alpha_i\Big((Q - x_1 - x_2)(t - p) + x_1(t - f_1) + x_2(t - f_2)\Big) - C_i, \quad (1)$$

where, Q is the thermal demand, $q_i < q_{imax}$ and $x_i \leq q_i$ are the amounts of electricity generated and contracted, respectively, by strategic company i, and $\alpha_i > 0$ and C_i are its corresponding CTC's coefficient and cost function, with $\alpha_1 + \alpha_2 < 1$.

Observe that the amount of electricity generated by price takers is $q_3 = Q - q_1 - q_2$, and then, the effective price in the spot market is given by the aggregated price takers marginal cost function, $p = C_3'(q_3) \in [P_{min}, P_{max}]$. Here, P_{min} and P_{max} stand for the minimum and maximum technology costs. The total amount to be shared in CTC's is given by $(Q - x_1 - x_2)(t - p) + x_1(t - f_1) + x_2(t - f_2)$, being t the tariff (fixed by the regulator) and f_i the price contracted by company i. Dropping the constant terms from (1), the profit functions are rewritten as

$$\pi_i = p_r(q_3)\Big(q_i - x_i - \alpha_i(Q - x_1 - x_2)\Big) - C_i(q_i), \tag{2}$$

where we recall that q_3 is a linear function of q_1, q_2 and we introduced the notation $p_r = C_3'$, for clarity.

As mentioned above, the convexity assumption on the cost functions is unrealistic for the situation we are studying. Instead, we shall just assume that for $i = 1, 2, 3$

$$C_i' \quad \text{is a continuous piecewise linear increasing function, with } C_i'(0) = P_{min}. \tag{3}$$

Using the inverse of the price takers marginal cost function, p_r, we introduce a classification of prices.

Definition 1 *Price p^* is a* regular price *if p_r^{-1} is differentiable at p^*. On the contrary, we say that p^* is a* corner price.

Thus, the optimization functions π_i defined in (2) are, in general, nor convex neither regular in generating quantities corresponding to corner-prices, and therefore the general theory does not help us in deciding whether a unique global equilibrium may exist. Indeed, examples may be given for which global equilibrium does not exist. We therefore study the existence of the following weaker notion of equilibrium.

Definition 2 *We say that there is a* local equilibrium at price *p if there exist quantities q_1, q_2 such that for fixed q_2 (resp. q_1), the function π_1 (resp. π_2) defined in (2) has a local maximum in q_1 (resp. q_2), and the price at these maxima is p.*

3 Local equilibrium at regular prices

Theorem 1 *If $p \in [P_{min}, P_{max}]$ is a regular price then there exists a unique local equilibrium at price p.*

Proof. We split the proof in two parts.

(i) If (q_1, q_2) is a local equilibrium point at a regular price, p, then the first order conditions are well defined and form a coupled system of equations:

$$p_r(q_3) - p_r'(q_3)(1-\alpha_1)q_1 + p_r'(q_3)[(1-\alpha_1)x_1 + \alpha_1 q_2 + \alpha_1 q_3 - \alpha_1 x_2] = C_1'(q_1), \quad (4)$$
$$p_r(q_3) - p_r'(q_3)(1-\alpha_2)q_2 + p_r'(q_3)[(1-\alpha_2)x_2 + \alpha_2 q_1 + \alpha_2 q_3 - \alpha_2 x_1] = C_2'(q_2). \quad (5)$$

Since q_3 is fixed by p, if we consider q_2 as a fixed parameter in (4) then the left hand side of this equation only depends on q_1. We denote it by $g_1(q_1)$. To show the existence of an equilibrium we start checking when function $g_1 - C_1'$ has a root in $(0, q_{1max})$. We have $g_1(0) > C_1'(0)$ since $q_2 \geq x_2$ implies $g_1(0) > 0$, being $C_1'(0) = 0$, by (3). It is clear that $g_1 - C_1'$ is decreasing $[x_1, q_{1max}]$. Finally, we have two possibilities: if

$$q_{1max} > x_1 + \alpha_1(q_{1max} + q_2 + q_3 - x_1 - x_2). \quad (6)$$

then $g_1(q_{1max}) - C_1'(q_{1max}) < 0$, and therefore, by continuity, there exists at least one root. On the contrary, if (6) is not true, then the above inequality is reversed and the optimum for the first player is attained at the boundary $q_1 = q_{1max}$. Finally, monotonicity of $g_1 - C_1'$ implies the uniqueness of the solution, q_1^*, in both cases, solution which we denote by $r_1(q_2)$ and which, actually, is taken as $\max\{q_1^*, x_1\}$. Similar arguments apply to condition (5), allowing us to define $r_2(q_1)$.

(ii) To prove the existence of a local equilibrium, we must check that curves $(r_1(q_2), q_2)$ and $(q_1, r_2(q_1))$ have an intersection point in $Q = [0, q_{1max}] \times [0, q_{2max}]$. Differentiating in (4) we get

$$-p_r'(q_3)(1-\alpha_1)dq_1 + p_r'(q_3)\alpha_1 dq_2 = C_1''(q_1)dq_1, \quad (7)$$

and therefore, for all q_2 (up to a finite number corresponding to corner-prices), we obtain, particularizing in $q_1 = r_1(q_2)$,

$$\frac{dr_1(q_2)}{dq_2} = \frac{\alpha_1 p_r'(q_3)}{(1-\alpha_1)p_r'(q_3) + C_1''(r_1(q_2))} > 0. \quad (8)$$

In a similar way, we obtain from (5)

$$\frac{dr_2(q_1)}{dq_1} = \frac{\alpha_2 p_r'(q_3)}{(1-\alpha_2)p_r'(q_3) + C_2''(r_2(q_1))} > 0. \quad (9)$$

Notice also that $r_1(0)$ and $r_2(0)$ are positive. When $(r_1(q_2), q_2) = (q_1, r_1^{-1}(q_1))$ is an interior point, property (8) allow us to apply the inverse function theorem to get

$$
\begin{aligned}
&\frac{dq_2}{dr_1(q_2)} > \frac{dr_2(q_1)}{dq_1} && (10)\\
\Leftrightarrow\ &[(1-\alpha_1)p_r'(q_3) + C_1''(q_1)][(1-\alpha_2)p_r'(q_3) + C_2''(r_2(q_1))] > \alpha_1\alpha_2 p_r'(q_3)^2, && (11)
\end{aligned}
$$

which is true as a consequence of hypothesis $\alpha_1 + \alpha_2 < 1$. We have then the following two possibilities: (a) There exists an intersection point in the interior of Q, which, due to (10), is unique. (b) If no interior intersection point does exist, then, without loss of generality, we may assume that for some q_2^* we have $r_1(q_2) = q_{1max}$ for $q_2 \geq q_2^*$. If $r_2(q_1)$ does not reach q_{2max}, then the equilibrium is at $(q_{1max}, r_2(q_{1max}))$. However, if for some q_1^* we have $r_2(q_1^*) = q_{2max}$, then the equilibrium is at (q_{1max}, q_{2max}). □

Remark 1 *(i)Functions $r_1(q_2)$ and $r_2(q_1)$ are connected with the notion of* player reaction function *but they are computed for a fixed price. (ii) Generating quantities $q_1(p)$ and $q_2(p)$ are piecewise linear for regular prices.*

The use of functions r_1 and r_2, introduced in the proof of Theorem 1 provide us with an iterative algorithm to approximate the equilibrium.

Corollary 2 *Assume that p is a regular price and let q_1, q_2 be the local equilibrium ensured by Theorem 1. Then, for $i = 1, 2$, we have $q_i^n \to q_i$ as $n \to \infty$, where $q_1^0 = 0$, $q_2^0 = r_2(0)$ and, for $n \in \mathbb{N}$,*

$$q_1^n = r_1(q_2^{n-1}), \quad q_2^n = r_2(q_1^{n-1}).$$

Proof. We may write $q_1^{n+1} = r_1(q_2^n) = r_1(r_2(q_1^{n-1}))$. Since p is a regular price, condition (10) holds, impliying

$$\frac{dr_1(q_2^n)}{dq_2}\frac{dr_2(q_1^{n-1})}{dq_1} < 1, \tag{12}$$

i.e., the mapping $s \to r_1(r_2(s))$ is contractive and therefore, a unique fixed point, q_1 does exist. A similar argument applies to deduce the existence of a unique fixed point, q_2, for the mapping $s \to r_2(r_1(s))$. Let us finally check that (q_1, q_2) is an equilibrium point, i.e., that $(r_1(q_2), q_2) = (q_1, r_2(q_1))$. We have $r_2(r_1(r_2(q_1))) = r_2(q_1)$, i.e., $r_2(q_1)$ is the unique fixed point of $r_2 \circ r_1$, $r_2(q_1) = q_2$. A similar argument shows $r_1(q_2) = q_1$. □

4 Local equilibrium at corner-prices

Definition 3 *Let p^* be a corner-price and assume that the following lateral limits do exist,*

$$q_{i-}(p^*) := \lim_{p \uparrow p^*} q_i(p), \quad q_{i+}(p^*) := \lim_{p \downarrow p^*} q_i(p). \tag{13}$$

If $q_{i-}(p^*) < q_{i+}(p^*)$ (resp., $q_{i-}(p^*) > q_{i+}(p^*)$), then we say that q_i has an increasing (resp., decreasing) jump at p^*. *If both q_1 and q_2 have increasing jumps at p^*, then we may define the non-empty interval I_Q as*

$$I_Q = \Big(q_{1-}(p) + q_{2-}(p) + p_r^{-1}(p^*)\ ,\ q_{1+}(p) + q_{2+}(p) + p_r^{-1}(p^*)\Big). \tag{14}$$

Theorem 1 establishes the existence of equilibrium at regular prices. In the following theorem we analyze the case of corner prices.

Theorem 3 *Let p^* be a corner price. There exists an equilibrium at price p^* if and only if there exist q_1^* and q_2^* satisfying*

$$r_{1-}(q_2^*) \leq q_1^* \leq r_{1+}(q_2^*) \quad \text{and} \quad r_{2-}(q_1^*) \leq q_2^* \leq r_{2+}(q_1^*), \tag{15}$$

with $r_{i\pm}$ defined in (18).

Remark 2 *(i) In the particular case in which no* CTC*'s are considered ($\alpha_1 = \alpha_2 = 0$), condition (15) is equivalent to find q_1^* and q_2^* such that*

$$q_{1-}(p^*) < q_1^* < q_{1+}(p^*), \quad q_{2-}(p^*) < q_2^* < q_{2+}(p^*), \tag{16}$$

which is always possible if $Q \in I_Q$ and if both q_1 and q_2 have increasing jumps at p^*. Hence, (15) is an effective restriction only when CTC*'s are present.*

For (15) to define a non-empty set it is necessary that both q_1 and q_2 have an increasing jump at p^*. In this case, to have equilibrium at price p^* and demand $Q \in I_Q$, (15) must hold. In Section 5, Step 1-3, we show an explicit way to check (15).

(ii) In multiple equilibria do arise following a unique equilibrium situation we expect the equilibrium in the market to continue in the same branch due to the almost continuous variation of demand in consecutive auctions. Consequently, we only expect a jump to other branch if the actual branch doesn't have an equilibrium for the new demand.

Proof. Assume that q_1^* and q_2^* satisfy (15). From the relation $p_r(q_3^*) = p^*$ we determine q_3^* and henceforth $Q^* = q_1^* + q_2^* + q_3^*$. We shall show that for all

$\delta > 0$, we have

$$\frac{\partial \pi_1}{\partial q_1}(q_1^* - \delta, q_2^*) > 0 > \frac{\partial \pi_1}{\partial q_1}(q_1^* + \delta, q_2^*), \tag{17}$$

and similarly for π_2. As in the Proof of Theorem 1, we define function $r_{1+}(q_2^*)$ as being either the unique solution of

$$p^* - p'_{r+}(1-\alpha_1)q_1^* + p'_{r+}[(1-\alpha_1)x_1 + \alpha_1 q_2^* + \alpha_1 q_3^* - \alpha_1 x_2] - C_1'(q_1^*) = 0 \tag{18}$$

where $p'_{r+} = \lim_{q_3 \downarrow q_3^*} p'_r(q_3)$, or if no solution does exist then $r_{1+}(q_2^*) = q_{1max}$. In a similar way, we define r_{1-}, r_{2+}, r_{2-}. Consider $q_1 = q_1^* - \delta$ and $q_3 = q_3^* + \delta$. Since p is piecewise linear, for $\delta > 0$ small enough we have $p_r(q_3) = p^* + \delta p'_{r+}$ and $p'_r(q_3) = p'_{r+}$. Therefore

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_1} &= p_r(q_3) - p'_r(q_3)(1-\alpha_1)q_1 + p'_r(q_3)[(1-\alpha_1)x_1 + \alpha_1 q_2 + \alpha_1 q_3 - \alpha_1 x_2] - C_1'(q_1) \\
&= p^* + \delta p'_{r+} - p'_{r+}(1-\alpha_1)q_1 + p'_{r+}[(1-\alpha_1)x_1 + \alpha_1 q_2^* + \alpha_1 q_3 - \alpha_1 x_2] - C_1'(q_1) \\
&= p^* - p'_{r+}(1-\alpha_1)q_1^* + p'_{r+}[(1-\alpha_1)x_1 + \alpha_1 q_2^* + \alpha_1 q_3^* - \alpha_1 x_2] - C_1'(q_1^*) \qquad (19) \\
&\quad + \delta p'_{r+}(1-\alpha_1) + p'_{r+}(1-\alpha_1)(r_{1+}(q_2^*) - q_1) + C_1'(r_{1+}(q_2^*)) - C_1'(q_1). \qquad (20)
\end{aligned}$$

Expression (19) is non-negative by the definition of r_{1+}, see (18), while (20) is positive since $r_{1+}(q_2^*) > q_1$ and functions C_1', p_r are non-decreasing. Therefore, the left hand side inequality of (17) is proven. In a similar way we prove the right hand side inequality of (17) and the corresponding inequalities for π_2. We omit the proof for brevity, and deduce in this way the existence of an equilibrium.

To prove the necessary condition, we proceed by contradiction. There are four cases, according to how condition (15) is violated. We examine one case, being the others similarly treated. Assume that $r_{1+}(q_2^*) < q_1^*$. Then $C_1'(q_1^*) - C_1'(r_{1+}(q_2^*)) > 0$. For δ small enough we have $C_1'(q_1^* - \delta) - C_1'(r_{1+}(q_2^*)) > (1-\alpha_1)p'_{r+}\delta$ and $q_1^* - \delta > r_{1+}(q_2^*)$. Using expression (19) we deduce that $\frac{\partial \pi_1}{\partial q_1}(q_1, q_2^*) < 0$, and the result follows. $\square$

5 Numerical simulations

In this section we present numerical demonstrations of our algorithms applied to two set of data. With the first, the Spanish electricity market, we intend to study a real situation while with the second, artificially created, we try to understand uncommon situations of the market.

The algorithm for computing the possible equilibria is different according to the type of price we are handling. For regular prices, we use the iterative scheme introduced in Corollary 2 to compute the unique local equilibrium. For corner prices the algorithm is somehow more involved. We use the following properties:

(1) Curves $(q_1, r_{2-}(q_1))$ and $(r_{1+}(q_2), q_2)$ have a unique intersection point which may be computed with an algorithm similar to that of Corollary 2. Analogously for $(q_1, r_{2+}(q_1))$ and $(r_{1-}(q_2), q_2)$. This property is proven reasoning as in (10).

(2) Curves $(q_1, r_{2-}(q_1))$ and $(q_1, r_{2+}(q_1))$ have, at most, one intersection pointin the interior of $[x_1, q_{1max}] \times [x_2, q_{2max}]$. Indeed, it may be shown that the sign of $d(r_{2+} - r_{2-})/dq_1$ in each intersection point does not change. Analogously for $(r_{1-}(q_2), q_2)$ and $(r_{1+}(q_2), q_2)$.

Using these properties we employ the following algorithm to study the existence and localization of demands in I_Q for which we have equilibria at a corner price, p^*.

Step 1. If either q_1 or q_2 do not have an increasing jump at p^* then no equilibrium does exist. On the contrary, we proceed to Step 2.

Step 2. Compute intersection points of the pairs of curves $(q_1, r_{2-}(q_1))$, $(r_{1+}(q_2), q_2)$ and $(q_1, r_{2+}(q_1))$, $(r_{1-}(q_2), q_2)$. If both intersection points belong to the set $[q_{1-}, q_{1+}] \times [q_{2-}, q_{2+}]$, then there exist equilibria for any demand in I_Q. On the contrary, proceed to Step 3.

Step 3. If only one of the intersection points belongs to $[q_{1-}, q_{1+}] \times [q_{2-}, q_{2+}]$, say the intersection point of curves $(r_{1-}(q_2), q_2)$ and $(r_{1+}(q_2), q_2)$, then function $r_{1+} - r_{1-}$ changes sign in the interval $[q_{2-}, q_{2+}]$. We may therefore use the bisection algorithm to compute the intersection point of $(q_1, r_{2-}(q_1))$ and $(q_1, r_{2+}(q_1))$. There exist equilibria for price p^* only for demands belonging to the interval $[q_{1-} + q_{2-} + p_r^{-1}(p^*), q_1^* + q_2^* + p_r^{-1}(p^*)]$.

Step 4. Finally, if none of the intersection points belongs to $[q_{1-}, q_{1+}] \times [q_{2-}, q_{2+}]$ then no equilibrium does exist at price p^*.

Remark 3 *Reasoning as in (2) we deduce that the sign of* $d(r_{2+} - r_{2-})/dq_1$ *at the intersection point of curves* $(q_1, r_{2-}(q_1))$ *and* $(q_1, r_{2+}(q_1))$ *is the opposite of the sign of* $(d(r_{1+} - r_{1-})/dq_2)^{-1}$ *at the intersection point of* $(r_{1-}(q_2), q_2)$ *and* $(r_{1+}(q_2), q_2)$*. This implies that in Step 2, the pairs* (q_{1-}, q_{2-}) *and* (q_{1+}, q_{2+}) *define two equilibria at price* p^**.*

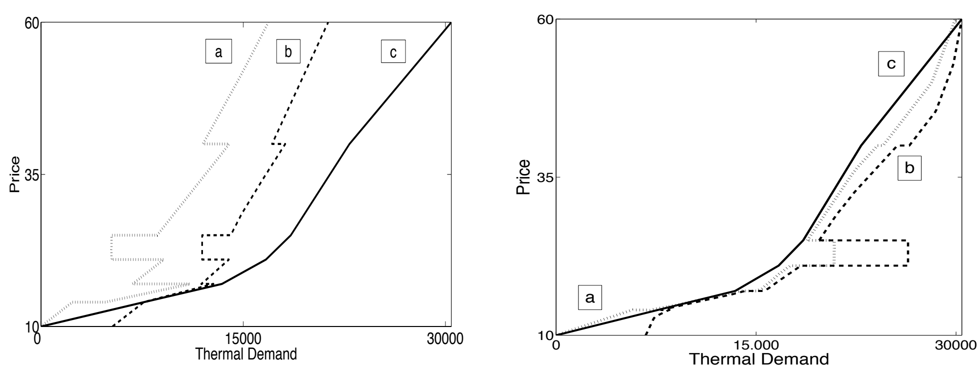
Companies generating capacity in MW

	nuclear	nat. coal	lignite	import. coal	gas	fuel
Endesa	3.481	1.372	2.365	1.631	1.200	2.581
Iberdrola	3.242	1.003	0	214	1.187	3.193
Price takers	905	3.422	895	0	1.962	1.779

5.1 First simulation: the Spanish electricity market

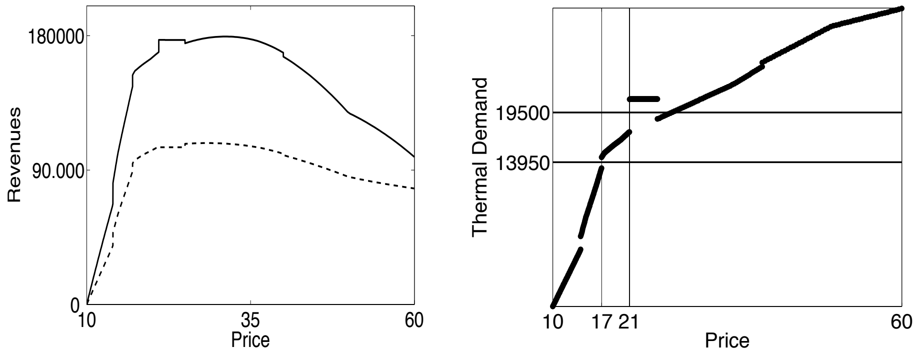
The Spanish market is dominated by two large companies, Endesa and Iberdrola (strategic players). For our purposes, the rest of companies may be aggregated in what we called *price takers.* Notice that strategic behavior may be restricted to thermal generation since other technologies are either long term determined, e.g. hydraulic, or regulated by other means, e.g. eolic. Average generating capacities (see Table above) are of public domain, generating costs are not. However, some information is available in the literature, see [11,12]. For the year 2003 we have the following data: $P_{min} = 10$€, $P_{max} = 60$€, the set of corner prices, $P^* = \{14, 17, 21, 25, 40\}$, the CTC's coefficients $\alpha_1 = 0.48$ (Endesa) and $\alpha_2 = 0.27$ (Iberdrola).

Regular prices. Using the iterative scheme introduced in Corollary 2, we compute the unique local equilibria corresponding to a finite set of regular prices in $(P_{min}, P_{max}) \setminus P^*$. In Fig. 1, we plot demand against price for these equilibria. We observe in Fig. 1 (i) that, if there are no CTC's , the strategic companies have incentives to withhold, resulting in a price higher than in the competitive case. We also observe that, according to the literature [2], the presence of contracts (covering the nuclear capacity with $x_1 = 3481$ and $x_2 = 3242$) leads to lower prices in the spot market. For the same set of regular prices, equilibria for the CTC's model is computed, see Fig. 1(ii). We observe that the presence



(i): No CTC's case (ii): CTC's case

Fig. 1. Thermal demand against price. (a) Without contracts, (b) with contracts at nuclear capacity, (c) competitive case



(i): Price versus revenues (ii): Price vs thermal demand

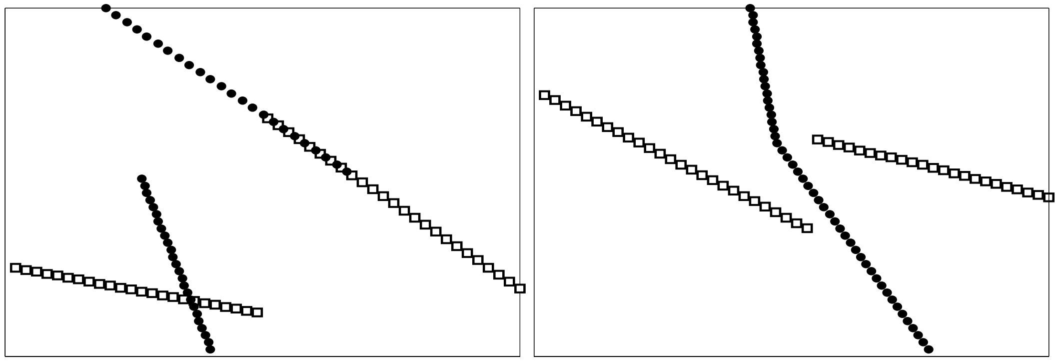
Fig. 2. (i) CTC's case. Continuous line, Endesa, dotted line Iberdrola.

of CTC's may lead to aggressive behavior. Finally, in Fig. 2 (i) we plot price against profits for the strategic players. The presence of CTC's implies that a generator is not always favored in a high prizes scenario.

Corner prices. The set of corner prices, P^*, is split in the subsets $\{14, 21, 40\}$ (q_1, q_2 have increasing jumps), $p = 17$ (q_1 has an increasing jump but q_2 has a decreasing jump), $p = 25$ (both functions have decreasing jumps). Existence of equilibria at corner prices with increasing jumps may be graphically verified by drawing the functions introduced in (15). In Fig. 4(i) we show curves $(q_1, r_{2-}(q_1))$, $(r_{1-}(q_2), q_2)$ (corresponding to $p'_r = p'_{r-}$) and $(q_1, r_{2+}(q_1))$, $(r_{1+}(q_2), q_2)$ (corresponding to $p'_r = p'_{r+}$), for the corner price $p = 21$. Region marked with "c" corresponds to the set of demands (q_1, q_2) verifying condition (15). We observe that there exists equilibrium for any demand in I_Q. Once we got this information, we pass to study the optimization problem for fixed demand.

(i) In Fig. 2(ii), we observe that for demand $Q = 19500$ there exists unique local equilibrium at the regular price $p \approx 27$, while an infinite number of equilibria do exist at the corner price, $p = 21$. In Fig. 3(i) we illustrate the reaction functions behavior for this demand and check the existence of such equilibria.

(ii) In Fig. 3(ii), we show a case without equilibrium. For $p = 17$, we have a decreasing jump in q_2. Taking $Q = 13950 \in [q_-(17), q_+(17)]$, see Fig. 2(ii), we see that the lack of equilibrium is due to the discontinuity in the second player reaction function.



(i): Demand $Q = 19500$ (ii): Demand $Q = 13950$

Fig. 3. Reaction curves for fixed demands

5.2 Second simulation: artificial data

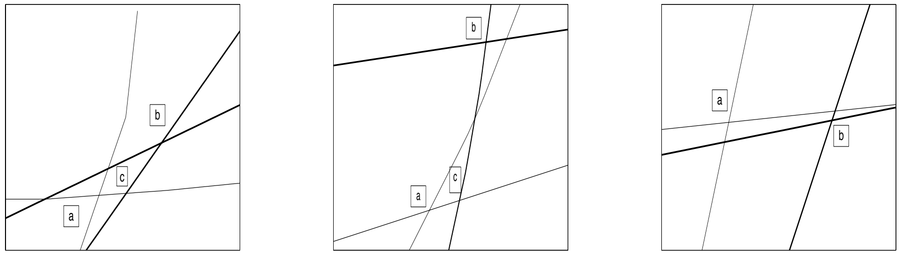
We now turn to illustrate other possible configurations which may arise from the model, and, in particular, the importance of condition (15).

(1) We use the same data than in the previous simulations but for contracts $x_1 = 500$, $x_2 = 0$. In Fig. 4(ii), for the corner price $p = 14$ (q_1 and q_2 have increasing jumps), we have that the curves $(r_{1-}(q_2), q_2)$ and $(r_{1+}(q_2), q_2)$ intersect at the point $(q_1^*, q_2^*) \in [q_{1-}, q_{1+}] \times [q_{2-}, q_{2+}]$. Therefore, while for the generating quantities q_{1-} and q_{2-} there exists an equilibrium, for q_{1+}, and q_{2+} it does not. In fact, there exists equilibrium at $p = 14$ only in the interval $[q_{1-} + q_{2-} + p_r^{-1}(14), q_1^* + q_2^* + p_r^{-1}(14)]$. Observe that the intersection point of $(r_{1-}(q_2), q_2)$ and $(q_1, r_{2+}(q_1))$ is outside the set $[q_{1-}, q_{1+}] \times [q_{2-}, q_{2+}]$.

(2) In this simulation we introduced a change in the set of corner prices by replacing price 14 by 10.5. We have that for the corner price $p = 17$, functions q_1 and q_2 have increasing jumps but, drawing the reaction functions, we check that condition (15) is not satisfied and therefore, even if the jumps are increasing, no equilibrium does exist, see Fig. 4(iii). We observe that the intersections of the two pairs of curves $(r_{1-}(q_2), q_2)$, $(q_1, r_{2+}(q_1))$ and $(q_1, r_{2-}(q_1))$, $(r_{1+}(q_2), q_2)$ lay outside the set $[q_{1-}, q_{1+}] \times [q_{2-}, q_{2+}]$.

6 Conclusions

Consideration of piecewise linear marginal costs and CTC's breaks the usual convex setting of the optimization problem for the profit function. However, some computable conditions in terms of the marginal cost function and the price are given under which local equilibrium does exist. In fact, iterative



(i): Step 2 (ii): Step 3 (iii): Step 4

Fig. 4. Study of the existence of equilibrium at fixed corner price following the algorithm in Section 5. Thin lines: curves $(q_1, r_{2-}(q_1))$ and $(r_{1-}(q_2), q_2)$. Thick lines: curves $(q_1, r_{2+}(q_1))$ and $(r_{1+}(q_2), q_2)$. Points a and b correspond to (q_{1-}, q_{2-}) and (q_{1+}, q_{2+}), respectively, of the algorithm. Region c corresponds to the equilibria.

algorithms are provided for the computation of such equilibria.

In the numerical simulations of the model we observe interesting phenomena related to the introduction of CTC's . In one hand, we see that CTC's may increase competitive behavior when the market share of a company is smaller than its CTC's coefficient. In the other and as a consequence, dependence of profits on demand is not necessary increasing.

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