Enhancing the Performance of a Multivariable Fuzzy Controller by Means of Multiobjective Genetic Programming and Statistical Analysis

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Abstract

The paper addresses the issue of performance optimisation of a MIMO fuzzy controller for a gas turbine aero-engine. The proposed method attempts to improve the performance of the controller by looking at the accuracy of the input-output mapping of the control parameters. A multiobjective genetic programming approach is utilized to search for suitable input-output structures, able to satisfy the rigorous performance criteria imposed on military engines and simultaneously to ensure the accuracy of the output surfaces. The effectiveness of the approach is verified by performing statistical tests of significance on the design data. In an effort to reduce the computational burden associated with controller design via optimisation, a response surface method is also considered.

Keywords: fuzzy control, multiobjective genetic programming

1. Introduction

The bottleneck of military gas turbine engine (GTE) optimisation is the satisfaction of strict and conflicting performance and constraints requirements. The task is more problematical when one attempts to enhance the performance of already *a priori* optimised engines. The cross-coupled control parameters and the highly non-linear behaviour of the plant further complicate the optimisation problem. Moreover, the use of nonlinear thermodynamic models to evaluate various performance criteria becomes increasingly expensive as the number of control parameters rises. Consequently, the overall optimisation of a GTE may be a difficult and costly process.

Previous work [1] dealt with the design and implementation of a multivariable fuzzy controller, as a feasible and promising alternative to the existing digital PI controllers. It was shown that the performance of the feedback system could be drastically affected by the choice of fuzzy mapping parameters. Following the conclusions arisen form this earlier work, the proposed technique investigates the relationship between the accuracy of fuzzy emulation of the original mapping, and the system performance.

Bearing in mind that the system behaviour is strongly influenced by the correctness of Sugeno approximation, the main problem reduces at finding the most suitable functions, which models the original relationship between the control variables. Mathematically speaking, the analytical solution of a "best" multivariate function approximation is not straightforward. The alternative presented herein is that of a stochastic search, via evolutionary algorithms. A genetic programming approach appears to be the appropriate tool for search and optimisation, particularly due to its ability to manipulate structures, rather then simply parameters. In order to accommodate multiple objectives, the optimisation is treated in a Pareto-optimal fashion. A special consideration is place upon the development of response surface models for the performance evaluation in the multiobjective genetic programming (MOGP) problem. Rather than evaluating the set of objective functions for every potential solution at each iteration of the optimisation, approximations of these functions – response surfaces – are used to reduce the time complexity. This allows greater exploration of the fuzzy controller space within a given period of time and therefore allows more candidate solutions to be considered. The most satisfactory solutions may then be assessed against a full non-linear model of the engine and their performance determined.

2. Multivariable Fuzzy Control

Recently, the application of multivariable fuzzy control, modelling and identification techniques has received considerable attention in the literature [2,3,4,5]. This interest is justified by the necessity to control more realistic scenarios, where the non-linearity, multivariability and system uncertainty handling is mandatory. Efforts have been made to address and solve different aspects of fuzzy control as the system stability [6], robustness [7] or the complexity of the control algorithm in terms of inference procedures [8]. However, some basic elements of MIMO fuzzy control seem to be overlooked. Little attention has been paid to the effects on the system response of the order, or the

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optimality of the regression coefficients in the consequent part of the Takagi-Sugeno rules.

Typically, the Takagi-Sugeno system expresses the outputs y_i as linear combinations of the inputs x_i, e.g.:

Rule i: IF x_1 is A_{i1} AND x_2 is A_{i2} THEN

$$y_1 = a_{01} + a_{11}x_1 + a_{21}x_2 \qquad (1)$$

$$y_2 = a_{01} + a_{11}x_1 + a_{21}x_2$$

where A_i is the linguistic label associated with a membership function. With this formulation, a non-linear system can be represented as a piece-wise linearised system, around specific operating points. However, for highly dimensional non-linear systems this kind of linear approximation may be inappropriate. Even ensuring a very dense distribution of the membership functions over the input parameter space may not be sufficient to accurately approximate the original system. Furthermore, by partitioning the input parameter space into more regions, the complexity of the fuzzy rule-base and of the inference algorithm will increase. With an augmented rule-base to be handled, and a larger number of partitions of the space, the prospect of successfully optimising the fuzzy controller parameters lessens. Opting for a smaller number of membership functions, and consequently a reduced size of rule-base, can diminish the risk of not finding the optimal control configuration. In return, one can act on the degree of the polynomial describing the outputs of the system. For instance, second-order polynomials may be enough to capture the non-linearity of the system in certain operating regimes and this is demonstrated in earlier work.

The investigation could be carried on with the identification of the relevant terms in an output function expression. This procedure will be presented in section 6. The goal of this identification is to eliminate the terms that could negatively affect the quality of the fitting of the data to the output surface. On the other hand, in this manner the functions are determined so that the parsimony principle is satisfied – a minimal number of terms are used, explicitly, the significant ones.

3. Response Surface Methodology And Tests Of Significance

The response surface methodology (RSM) is the practice of associating regression models with the objective functions in an optimisation problem [9,10]. This approach attempts the reduction of the computational burden required for the evaluation of the responses affected by several variables. The method is also known to filter out the noise existent in the parameter space due to the calculated responses. In RSM, statistical and mathematical techniques are applied to a set of experimental data in order to produce polynomial approximations of the calculated responses of interest.

The model fitting is based upon a collection of design experiments, or data points, equally distributed through the parameter space. The fitting of the polynomial surface to the data yields estimation in unknown coefficients. This estimate is often calculated as the solution of a set of least squares equations for the candidate design points.

For n observations, the model equations are expressed in a matrix notation as:

$$y = X\beta + \varepsilon, \qquad (2)$$

where $y \in R^{n\times 1}$ is the predicted response, $X \in R^{n\times k}$ is the observation matrix for k regressors and $\beta \in R^{k\times 1}$ is the regression vector and $\varepsilon \in R^{n\times 1}$ is the error.

With these considerations, the estimate of the regression coefficients in a least squares sense is:

$$\beta = (X'X)^{-1} X'y \qquad (3)$$

The adequacy of the fit (or, conversely, the lack of fit) of the predicted response model is tested by using an Analysis-of-Variance (ANOVA) table. This table is an indicator of the partitioning of the residual sum of square error and pure error, which helps determine the structure of the polynomial model.

In the proposed model, various hypotheses on the parameters can be tested, on the form:

$$H_0 : C \cdot \beta_p = m \qquad (4)$$

where C includes the coefficients in certain linear combinations of β_p so that the elements of $C \cdot \beta_p$ are linearly independent. For example, if the coefficients β_p are $(\beta_0, \beta_1, \beta_2)$ and the hypotheses to be tested is $\beta_1 = 0$, then C will be a vector and m will be a scalar in the form:

$$C = [0\ 1\ 0],\ m=[0]. \qquad (5)$$

For more details the reader is directed to Searle, [11].

The cost associated with the design optimisation of large, complex control strategies frequently remains high. One aim of the RSM is to alleviate the computational expense and structural requirements of repeatedly calculating objective functions and therefore to speed up the design and optimisation process. The problem of dimensionality also needs to be considered when formulating a response surface problem. A larger number of variables or a higher polynomial order can jeopardise the simplicity concept.

In the following sections it is shown how RSM can be embedded in MOGP formulation to help alleviate the burden of evaluating many objective functions at each iteration of the algorithm. For the gas turbine engine example considered here, a full nonlinear simulation of 10 seconds takes of the order of a few minutes of CPU time on a Sparc Ultra. In contrast, using response surface models, with order 3 or 4, only requires tens of seconds of CPU time.

4. MOGP Optimisation

In general terms, evolutionary algorithms replicate the Darwinian process of natural evolution by progressively improving populations of potential solutions according to the philosophy of "survival of the fittest". Perhaps the most renowned representatives of evolutionary algorithms are the genetic algorithms (GAs), proposed by Holland in 1975.

Genetic programming (GP) [12] represents a branch of evolutionary computation, more specifically a subclass of genetic algorithms. The most prominent feature that differentiates GP from GA is the type of genotype (individuals), which they handle. Whilst a GA uses a binary or real-valued encoding of the individuals, computer programmes represent the GP's genotype. The genetic operators manipulated by GA or GP approaches are virtually the same. However, they are different in the way they act and in the results they produce - dependent on the individual's type.

For the traditional GA, the selection, the crossover and the mutation preserve the individual's structure (length, content). Since the main entity of the GP is a program, the outcome of genetic operators acting on a population of individuals is new programs, structurally different from the parental population. The GP operates with a terminal set, comprising variables, and a function set, consisting in mathematical or logical operators.

The Pareto-optimality scheme is frequently employed to solve multiobjective optimisation problems. The Pareto-optimal philosophy demonstrated to be a very realistic and versatile approach for tackling multiobjective optimisation problems. It clearly indicates to outperform traditional non-linear programming methods, including epsilon-constraint, weighted sum or goal attainment. Additionally, Pareto-optimal approaches are able to handle multimodality and discontinuities in the function space, a deficiency in other non-linear programming techniques.

In most cases there will not be one ideal 'optimal' solution, rather a set of Pareto-optimal solutions for which an improvement in one of the design objectives will lead to a degradation in one or more of the remaining objectives. Such solutions are also known as non-inferior or non-dominated solutions to the multiobjective optimisation problem. The integration of the Pareto-optimal scheme into the GP structure was already achieved and presented in the literature [13]. Similar to the multiobjective genetic algorithms approaches, goals and priorities information may also be embedded into the GP structure, in order to distinguish the solutions corresponding to certain demands. [14].

For this study, the MOGP is deemed the ideal environment for a simultaneous consideration of multiple objectives modelling several objective functions. The resulting trade-off solutions should be able to satisfy the system requirements and simultaneously to generate minimal polynomial structures that minimize the residuals.

5. Multivariable Fuzzy Control

The design example considered here is that of the Rolls-Royce Spey gas turbine engine. The Spey engine is a twin-spool turbofan, used in both military and civil aviation. The control philosophy is mainly concerned with thrust regulation. However, other parameters also play an active role in the control. In general, these secondary control parameters are normally regarded as safeguards, which ensure surge free control and prevention of stall. Additionally, mechanical and thermodynamic constraints tighten the control requirements to guarantee proper functionality of the engine components [15].

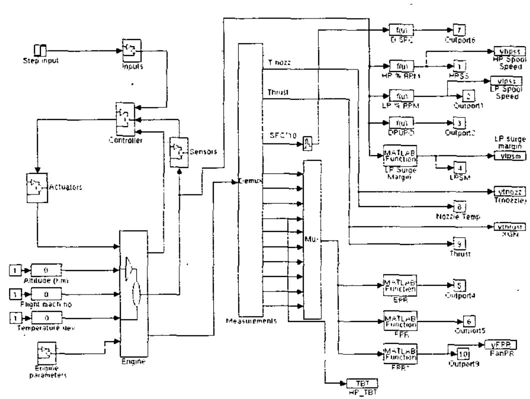

Figure 1: Spey engine SIMULINK model.

The configuration of a typical dry-engine controller comprises a collection of digital PI controllers, which infer thrust and compressor surge margin from measurable parameters, Fig. 1. These control parameters are not directly measurable. The outputs from the controller that regulate the engine thrust and the thermodynamic stability limits are the fuel flow demand, the nozzle area angle and the inlet guide vane angle. The goal of this study is to develop a multivariable fuzzy counterpart of the original controller block for the dry engine.

The fuzzy controller amalgamates all the individual inputs from the original PI controllers. The outputs of this multivariable structure may be linear or non-linear functions of the input variables (1). The MIMO fuzzy controller thus has 4 inputs and 3 outputs, as described in table 1:

Controller inputs	
1. High press. spool speed demand	(NHdem)
2. High press. spool speed measured	(NHmeas)
3. Low press. spool speed measured	(NL)
4. Bypass duct Mach no.	(DPUPO)
Controller outputs	
1. Fuel flow demand	(WFdem)
2. Nozzle area	(NA)
3. Inlet guide vane angle	(IGV)

Table 1: Parameters of the MIMO fuzzy controller.

The fuzzy controller should at least be capable of replicating the behaviour of the original system with PI controllers. To determine the rule-base structure, including the parameters of the consequent part, the signals generated by the original controller are used for parameter identification. Providing that the number of membership functions are predetermined, a pseudo-inverse or least square approach may be used to compute the coefficients of the polynomials of eqn. (2).

However, the previous study illustrated that a linear output function is not sufficient to ensure a good functionality of the system. The principal reasons invoked there was the lack of fit of the predicted response resulted from the fuzzy controller. With strong interactions between input parameters and outputs exhibiting highly non-linear behaviour, a first order polynomial is simply not enough to guarantee reliable closed-loop system response. The study also indicated that increasing the number of partition the input spaces not only does not yield better results, but also has the drawback of higher associated computational costs. The proposed solution to the multivariable control problem consisted of using second-order polynomials to model the output functions. With that implementation, the performance proved to outperform the original Rolls-Royce controller.

The question emerged from these findings is whether or not a better mapping of the input-output fuzzy parameter would lead to increased performance of the closed-loop system. The designer is faced now with one more aspect to be considered. Hence, the accuracy of the mapping can be quantified and included in the evolutionary algorithm as an objective to be minimised. Secondly, the procedure preserves all the facilities of the MOGA approach, which attempts the controller tuning for maximum performance.

In terms of mathematics, the simplest way to control the accuracy of the fitness whilst preserving the low-order of the polynomial is to vary the tolerance level associated with the solution of the system of equations described in (2). However, the approach does not allow a great improvement in the fitness error. Another alternative, which has not been considered in this study, is to employ multivariate function expansions in an attempt to find the best least squares approximations to WFdem, NA and.IGV (independently).

6. Enhancing The Controller Performance With MOGP

The proposed approach seeks to improve the precision of the mapping by employing a MOGP to search for potential solutions to the problem. The choice of MOGP over a simple single-objective GP is justified by the need to achieve at least three objectives: the minimization of the residuals, of the number of terms and of the polynomial degree. These objectives will ensure a good fitting of data to the output surfaces and will also guarantee a parsimonious structure. Moreover, there is the opportunity to perform, simultaneously, the "ordinary" controller tuning routine, realised with MOGA – the maximization of the engine performance. On the other hand, the evaluation of system performance is assessed automatically, for each newly emerged polynomial structure.

As already mentioned in previous sections, a concern associated with optimisation problems is to reduce the simulation time and the computational complexity. One way to achieve this target is to use response surfaces to model the objective functions related to the engine performance. Instead of running an engine simulation to evaluate the objective functions, statistical approximations of these objectives are calculated. Previous work indicated that the approach is highly effective [16,17].

The response models of the selected outputs of the engine are constructed by considering the three outputs
from the MIMO fuzzy controller (assuming constant operating conditions). For example, thrust will be the non-linear function:

$$\text{thrust} = f(\text{WFdem, NA, IGV}) \qquad (6)$$

A set of 250 points equally distributed over the input parameter space was used to build the response models of the outputs to be employed in the MOGA-based optimisation. A further 750 points were then used to validate these models. In this problem, polynomials of order 4 were chosen to model the objective functions. The computation of the average value of residuals for each of the response surface models indicates a good fit of the polynomials to the data, as shown in Table 2.

Output	Mean (residuals)
Thrust	2.77e-11
Surge margin	3.76e-10
Turbine temp.	-4.83e-09

Table 2: Performance values for response surfaces.

A set of non-dominated solutions was evolved from a MOGP with 100 individuals, which ran for 500 generations. In Fig. 4, the MOGA GUI shows one of these solutions and the corresponding objective function goals. Clearly, this solution meets all of the design goals.

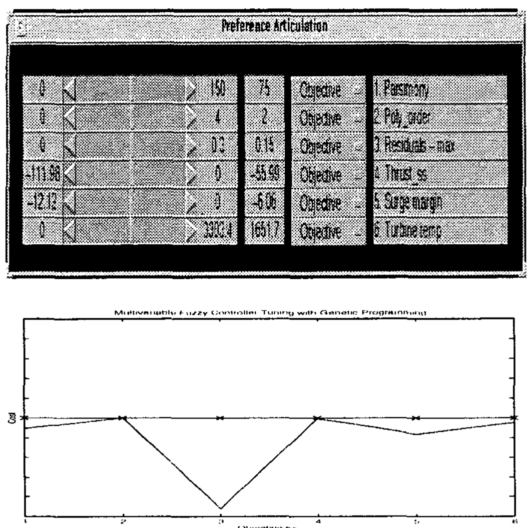

Figure 4: Preference articulation and performance.

For the solution shown in fig.4 the significant terms are indicated by 1, in the associated second-order model:

$$Y = a_0 + a_1 \cdot x_1 + .. + a_4 \cdot x_4 + a_{12} \cdot x_1 \cdot x_2 + .. +$$
$$+ a_{34} \cdot x_3 \cdot x_4 + a_{11} \cdot x_1^2 + .. + a_{44} \cdot x_4^2$$
(7)

Out1: 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0
Out2: 1 1 1 1 1 1 0 1 0 1 0 0 1 1 0
Out3: 1 0 1 0 1 0 1 0 0 0 1 0 0 1 0

For the solution displayed in Fig. 4, the performance of the non-linear system is verified directly on the full thermodynamic SIMULINK model of the engine. The indicators of the GTE behaviour that are the most relevant from a control point of view, and simultaneously exhibit the strongest conflict, thrust, surge margin and turbine blade temperature, are shown in Fig. 5. These outputs are in response to a step change of 55-100% of the high-pressure spool-speed. The dashed line indicates the system with MOGP-MIMO fuzzy control, the dotted one indicates the MOGA-MIMO controller, and the continuous line denotes the original controller. Here, the MOGP-developed fuzzy controller has outperformed both the original and the MOGA-emerged controller, in all of the indicators. Although is improvement of system performance is marginal, the approach is able to challenge and outperform *a priori* drastically optimised structures. A more obvious picture is offered by the numerical values of the control parameters for each of the cases under consideration (table 3). Table 3 compares the best-case performance in each of these measures for all three controllers.

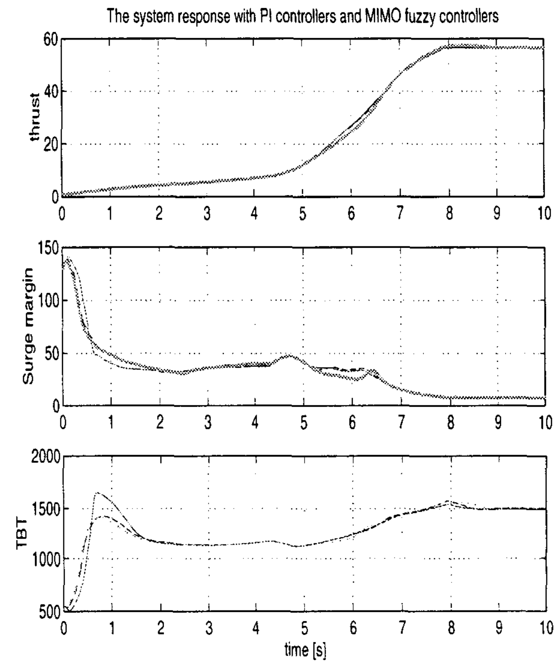

Figure 5: Performance of fine-tuned MIMO controllers versus original PI control.

Performance measure	MOGA-MIMO	MOGP-MIMO	DERA-PI
Thrust@steady state [kN]	56.177	56.380	55.997
Minimum surge margin [%]	6.767	7.059	6.602
Maximum turbine temp. [°C]	1573.152	1570.740	1651.693

Table 3: PI and fuzzy controller performance.

7. Conclusions

This paper has demonstrated an approach to enhance the performance of a multivariable non-linear Takagi-Sugeno fuzzy controller for a gas turbine engine. As the engine controller has to satisfy many competing design objectives, a multiobjective optimiser has been employed to fine-tune the most promising fuzzy controller structures. The core of this work was to look at most feasible spaces of potential solution, by directing the attention towards the models offering best fit. For this purpose, the significant terms in the polynomial structure were found and the solutions verified on the non-linear engines. The solutions yielded by MOGP could also be compared with the results of test of significance. However, the MOGP, and simulation of the engine itself, are computationally demanding and satisfactory solutions could not normally be expected within a reasonable time if applied directly. To address this problem, it has been shown that response surface models offer an attractive mechanism for reducing the time complexity of the initial optimisation. The most suitable structures may then be tested on the actual model of the engine. Furthermore, the system with the significant terms has been shown to attain very good performance for the design objectives considered here, outperforming both the original and the MOGA-optimised controllers.

8. Acknowledgement

The first author gratefully acknowledges the financial support from the CVCP and an Overseas Research Studentship from The University of Sheffield. The non-linear model of the Spey engine and its PI controllers were developed by DERA (Pyestock).

References

[1] Bica, B., Ceconvciuc, C. Chipperfield, A. J. and Fleming, P. J., "Multivariable fuzzy controller design using multiple response surfaces", submitted to, *UKACC, Control 2000* 1999.

[2] Bossley, K. M., Brown, M. and Harris, C. J., "Neurofuzzy identification of an underwater vehicle", *Int. Journal of Sys. Science*, vol.30, no.9, pp 901-913, 1999.

[3] Sousa, J. M., Babuska, R. and Verbruggen, H. B., "Fuzzy predictive control applied to an air-conditioning system", *Control Engineering Practice*, Vol. 5, pp. 1395 – 1406, 1997.

[4] [6] Craven, P. J., Sutton, R., Burns, R. S. and Dai, Y-M, "Multivariable intelligent control strategies for an autonomous underwater vehicle", *Int. Journal of Sys. Science*, vol.30, no.9., pp 965-980, 1999.

[5] Gegov, A. E., "Multilayer fuzzy control of multivariable systems by direct decomposition", *Int. Journal of Sys. Science*, vol.29, no.8., pp 851-862, 1998.

[6] Ordonez, R. and Passino, K. M., " Stable multi-input multi-output direct adaptive fuzzy control", *Proc. of the American. Cont. Conf.*, 1997.

[7] Trebi-Ollennu, A. and White, B. A., "Robust output tracking for MIMO nonlinear systems: an adaptive fuzzy system approach", *IEE Proc. Control Theory Applic*, Vol.144, No.6, 1997.

[8] Kouatli, I. M., " A simplified fuzzy multivariable structure in a manufacturing environment", *Journal of Intelligent Manufacturing* Vol. 5, pp. 365-387, 1994.

[9] Montgomery, D. C., "Design and analysis of experiments", *John Wiley & Sons, Inc.*, 1997.

[10] Khuri, A. I. And Cornell, J. A., "Response surfaces design and analyses", *Marcel Dekker, Inc., ASQC Quality Press*, 1987.

[11] Searle, S.R., "Linear Models", *New York: John Wiley*, pp. 180-188, 1971.

[12] Koza, J. "Genetic Programming: On the Programming of Computers by Means of Natural Selection", *MIT Press*, 1992.

[13] Rodriguez-Vazquez, K. and Fleming, P. J., "Multiobjective Genetic Programming for Gas Turbine Engine Model Identification", *Proc. UKACC*, Swansea, pp. 1385-1390,1998.

[14] Fonseca, C. M. and Fleming, P. J., "An overview of evolutionary algorithms in multiobjective optimization", *Evolutionary Computing*, Vol. 3, No. 1, 1-16,1995.

[15] [Dodd, G.J., "Engine control specification", *SERC Control's research programme, Report No. 60.702 (Amended),* Lucas Aerospace Limited, 1994.

[16] Bica, B., Chipperfield, A. J. and Fleming, P. J., "Towards gain scheduling control of gas turbine engines: a multiobjective approach", submitted to, *IEEE Trans. on Industrial Electronics.*, 1999.

[17] Silva, V. V. R. and Fleming, P. J., "Multiobjective non-linear robust control design for gas turbine engines using evolutionary computation", *Proc. Math. Theory Networks Sys. Conf. –MTNS'98*, pp. 1087 – 1090, 1998.