A metasemantics to refine fuzzy if-then rules[1]

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Abstract

Fuzzy if-then rules are used to represent fuzzy models. Real data is later used to tune the model. Usually this forces a modification of the initial linguistic terms of the linguistic variables used for the model. Such modifications may lead to a loss in interpretability of the rules. In this paper we suggest using a form of multiresolution to tune the rules, by introducing more linguistic terms in the regions of the universe of discourse where this is needed. Starting with triangular shaped linguistic terms and recalling that symmetric triangles are first degree splines, a form of multiresolution is readily obtained, supporting a better accuracy of the model. It is shown that by using the linguistic modifiers "very" and "more or less" as well as the metasemantic modifiers "between" and "from – to" the interpretability of the original rule may be preserved.

1. Introduction

Fuzzy modelling is a well established area of Soft Computing. An initial model, based on working experience with the target system or advice of experts is built as a rule base consisting of if-then rules using linguistic variables [11]. Test data will be used to tune the model. Most efforts in tuning a model have been dedicated to modify the linguistic terms (see *e.g.* [3], [7], [8]) or modify the operations [6]. Special constrains must be observed to preserve the interpretability of the rules (see *e.g.*.[2] and the references provided there). A third alternative is introduced in the present paper, by using local multiresolution. Since a fuzzy rule base may be seen as a fuzzy partition of the problem space with a conclusion associated to each block, local multiresolution corresponds to refining the fuzzy partition, but only in those blocks where the initial model shows a weak performance. Simple meta-semantic operators provide for an interpretation of the refined rules.

2. Linguistic splines

Linguistic variables represent entities of the real world, but the domain of a linguistic variable is an ordered set of predicates, called linguistic terms, which are specified by fuzzy sets over a same universe. This universe is considered to be finite and represented as an interval [*min, max*], where *min* and *max* represent the lower and upper bounds of the considered universe respectively. (See figure 1)

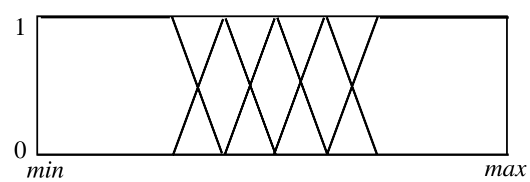

Fig. 1: Structure of a normalized linguistic variable.

A usual first choice when designing a linguistic variable is to use congruent isosceles triangles for the linguistic terms with the possible exception of both linguistic terms at the ends of the universe,

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which may be trapeziums obtained by saturating a triangle. Furthermore the terms are spread in such a way that they constitute a normalized partition of the universe in the sense that for any point of the universe, the sum of the values of the membership functions of all fuzzy sets at that point adds up to 1. (See figure 1. To simplify the figures, in what follows only the support of the fuzzy sets will be shown.)

An isosceles triangle is a first degree spline and has a multiresolution property. (See figure 2).

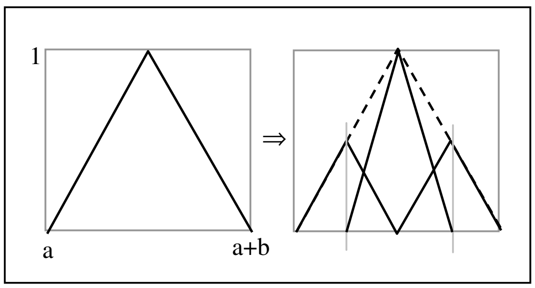

Fig. 2: Multiresolution in the case of an isosceles triangle

Let "LT" denote "linguistic term". Referring to figure 2, the equation of the triangle at the left representing a linguistic term is:

$$LT(x) = \begin{cases} 2(x-a)/b & a \le x \le (a+b)/2 \\ 2(a+b-x)/b & (a+b)/2 \le x \le a+b \\ 0 & \text{otherwise} \end{cases}$$

and with respect to the decomposition at the right,

$$LT(x) = ½LT(2x) + LT(2x-(a+b)/2) + ½LT(2x-(a+b)) \qquad (1)$$

We speak of "multiresolution" in this case, since the information originally represented by 1 triangle becomes represented by 3 triangles *of the same type*, except for scaling of the argument and amplitude.

One can easily see that this situation also may be found at the end trapeziums of a linguistic variable, as shown in figure 3.

Let Trap(x) denote this trapezoidal linguistic term. Its equation is:

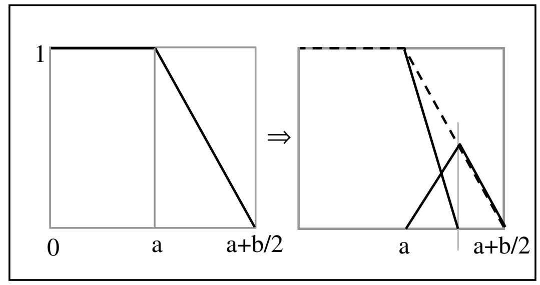

Fig. 3: Multiresolution in the case of a rectangular trapezium

$$Trap(x) = \begin{cases} 1 & 0 \le x \le a \\ (2a + b - 2x)/b & a \le x \le a+(b/2) \\ 0 & \text{otherwise} \end{cases}$$

Define

$$T'(x) = \begin{cases} 1 & 0 \le x \le a \\ (4a + b - 4x)/b & a \le x \le a+(b/4) \\ 0 & \text{otherwise} \end{cases}$$

Then $Trap(x) = T'(x) + ½LT(2x)$. (2)

Consider the case that both an end trapezium and its neighbour triangular linguistic term would be refined as explained above. Then one can easily see that if $a \le x \le a + (b/2)$, two congruent triangles with basis b/2 and height ½ will exactly superpose. As a consequence x will belong *twice* to a refined linguistic term with equation ½LT(2x) or in other words, x will belong (*once*) to an equivalent linguistic term with equation LT(2x).

If this process of refinement is applied to a whole linguistic variable, then the effect to be obtained is as the one illustrated in figure 4.

3. Metasemantics.

In what follows it will be shown how the proposed refining process allows the deduction of a meaning for the resulting new linguistic terms. For this we use the name "metasemantics".

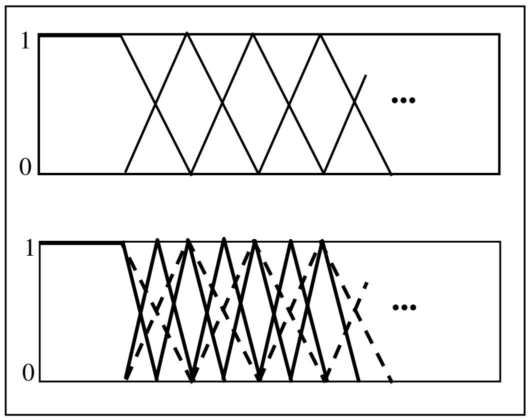

Fig. 4: Increasing total resolution in a linguistic variable.

3.1 The linguistic modifiers “very” and “more or less”

Zadeh [9], [10], [11] introduced the concept of linguistic hedges, which has continued to be a subject of research up to the present days. (See *e.g.* [4], [5]). Consider a fuzzy set representing a vague predicate according to its use in a given natural language and a given context. Linguistic hedges, like “*very*” (or “*strictly*”) and “*more or less*” produce a semantic concentration or dilation of the original predicate, respectively. The name "linguistic modifier" has later been used as a synonym for linguistic hedges. If a fuzzy set has a piecewise linear representation and it is desired to preserve this kind of representation (for instance, to make the interpretation simple, as in the present paper) then the linguistic modifier “*very*” reduces the support by 50% in the case of triangular fuzzy sets and reduces by 50% the interval between support and core in the case of trapezoidal fuzzy sets. This alternative will be used in what follows. Notice that the concentration of a predicate corresponds sometimes to the use of "*very*", as in the case of "near" and "*very* near", meanwhile a concentration of the predicate "lukewarm" rather leads to "*strictly* lukewarm" than to "*very* lukewarm", which does not seem to make sense. A piecewise realization of the linguistic modifier “*more or less*” in the case of a triangle will increase the core to one half of the support. In the case of a (rectangular) trapezium (as shown in figure 3), the linguistic modifier “*more or less*” will increase the core by one half of the difference between support and core.

Let ω be a predicate represented by a triangular fuzzy set as shown in figure 2 and expressed in Equation (1). Figure 2 clearly shows that the narrow triangle in the center of the refinement illustrated at the right hand side corresponds to “*very* ω”. If ω had a trapezoidal fuzzy set representation as shown in figure 3 and expressed in equation (2), then the narrower trapezium at the right hand side of figure 3 would represent “*very* ω”. One can easily see that equation (2) may be extended to cover the case of non rectangular asymmetric trapeziums and, as a limit case, asymmetric triangles.

3.2 The metasemantic operator “between”

Consider the case of two neighbour linguistic terms after applying multiresolution. See figure 5 that illustrates a partial view including *very* LT_j, *very* LT_{j+1} and the new triangular fuzzy set resulting from the superposition of the two half-height terms.

From the interpretation point of view it becomes apparent that the middle triangular fuzzy set may be understood as *between*(*very* LT_j , *very* LT_{j+1}). This metasemantic operator *between* specifies the bounded semantic interval where the meaning of the new fuzzy set has to be defined. (Notice that statements of this kind also appear in natural languages, *e.g*., "Henry comes *between* near ten and noon".)

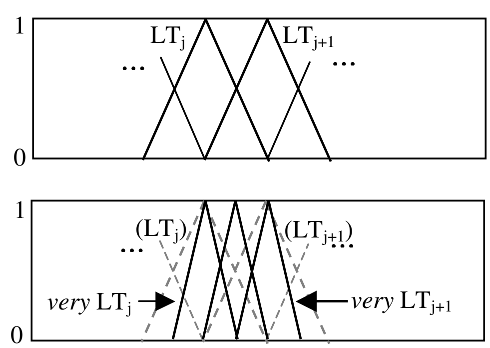

Fig. 5: Partial view before and after mutiresolution

3.3 Partial multiresolution

It is easy to see that the linguistic modifier *very* and the metasemantic operator *between* allow the deduction of a consistent interpretation of the linguistic terms obtained after refinement, if *all* linguistic terms are refined. This however implies almost duplicating the number of linguistic terms of the linguistic variable thus drastically increasing the number of if-then rules of the corresponding rule base using this linguistic variable. It is obvious that refinement of linguistic terms should only be done where a finer granularity is needed to obtain an acceptable model.

In what follows we will show that the refinement of a few linguistic terms –(partial multiresolution)- may produce new asymmetric linguistic terms. Moreover we will discuss a metasemantic method to deduce from the original meaning of the linguistic terms, an interpretation of the linguistic terms of the partially refined linguistic variable.

Consider the case of a linguistic variable with n linguistic terms. Assume that to satisfy the accuracy requirements of a model, where this linguistic variable is being used, a partial refinement has been found to be necessary. Without loss of generality, assume that the linguistic terms LT_1, … LT_j have been refined. Figure 6 illustrates the region (of the universe in which the linguistic variable is defined) where the refined linguistic term LT_j meets its non-refined neighbour LT_{j+1}. Let $LT_j^{-1}(1)$ denote the value of x such that $LT_j(x) = 1$; similarly in the case of $LT_{j+1}^{-1}(1)$. It is easy to see that for all x such that $LT_j^{-1}(1) \leq x \leq LT_{j+1}^{-1}(1)$, x will belong both to the half-height triangle obtained after refining LT_j and to the non-refined linguistic term LT_{j+1}. (Shaded area in figure 6(a). This leads to an equivalent linguistic term with an asymmetric trapezoidal shape as shown in figure 6(b).)

3.4 Analysis

As shown in figure 6(a), the overlap between the last refined linguistic term and the first non-refined one occurs in the shaded interval [r, s], where it is simple to show that

$$r = a + \frac{j-1}{2}b \quad \text{and} \quad s = r + \frac{b}{2} = a + \frac{j}{2}b$$

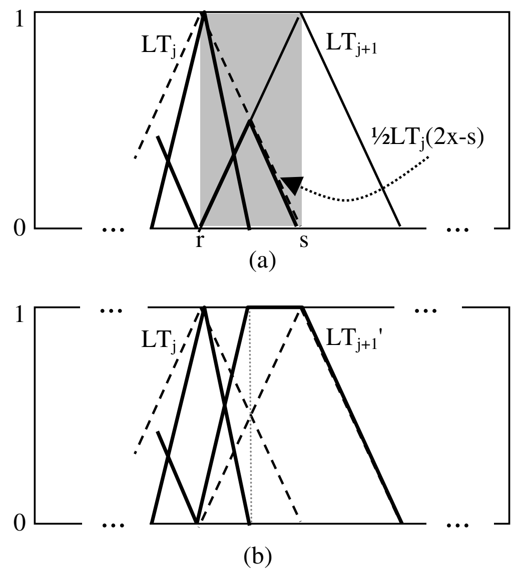

Fig. 6: (a) Overlap of the refined term LT_j and the non-refined term LT_{j+1}. (b) Resulting linguistic terms after resolving the superposition.

Any x in the shaded interval will belong to both the refined triangle $\frac{1}{2}LT_j(2x-s)$) and to LT_{j+1}. The pointwise addition gives as result a modification of LT_{j+1} into LT_{j+1}' as follows:

$$LT_{j+1}'(x) = \begin{cases} 4(x-r)/b & r \leq x \leq r + b/4 \\ 1 & r + b/4 \leq x \leq r + b/2 \\ LT_{j+1}(x) & r + b/2 \leq x \leq r + b \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

This is illustrated in figure 6(b).

Figure 7 shows a decomposition of $LT_{j+1}'(x)$ which allows the following interpretation, based on the t-conorm W* of Łukasiewicz (see *e.g.* [1])

$LT_{j+1}'(x)$ = Maximum { $LT_{j+1}(x)$, W* (*between* [(*very*(LT_j(x)), *very*(LT_{j+1}(x))], *very*(LT_{j+1}(x)))} =
= Maximum {$LT_{j+1}(x)$, Min[1, (*between* [(*very*(LT_j(x)), *very*(LT_{j+1}(x))] + *very*(LT_{j+1}(x)))]}.
. (4)

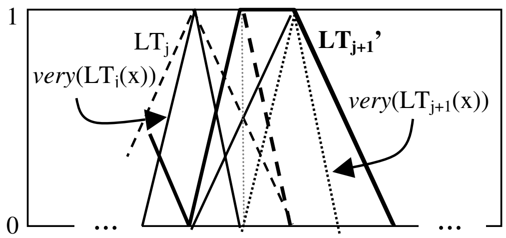

Fig. 7: Decomposition of LT_{j+1}'

Further analysis of figure 7 shows that LT_{j+1}' may be also expressed with a metasemantic operator "*from – to*" defined as follows:

Let A and B be two normal fuzzy sets on a same universe. Then

$$\textit{from } A \textit{ to } B = \begin{cases} A & x \le A^{-1}(1) \\ 1 & A^{-1}(1) \le x \le B^{-1}(1) \\ B & B^{-1}(1) \le x \end{cases} \qquad (5)$$

The analogy with equation (3) is very clear, leading to the following interpretation:

LT_{j+1}'(x) = *from* (*between* [(*very*(LT_j(x)), *very*(LT_{j+1}(x))]) *to* LT_{j+1}(x) (6)

Equation (4) and equation (6) are equivalent, and offer alternative ways to find an interpretation based on the predicates used at the initialization.

Notice that if in equation (5) B were a rectangular trapezium, then $B^{-1}(1)$ would no longer be a point, but an interval. Accordingly, the last condition of equation (5) would become $x \in B^{-1}(1)$. Similarly if A were a rectangular trapezium and B were a triangle.

3.5 Example

A simple example follows to illustrate some of the aspects discussed above. Consider a fuzzy if-then rule using (among others) the linguistic variable "size". Moreover, let the linguistic variable "size" be initialized with three linguistic terms: "small", "average" and "large", as shown in figure 8(a). Let it be assumed that the use of the rule has shown that the corresponding model is not accurate enough and

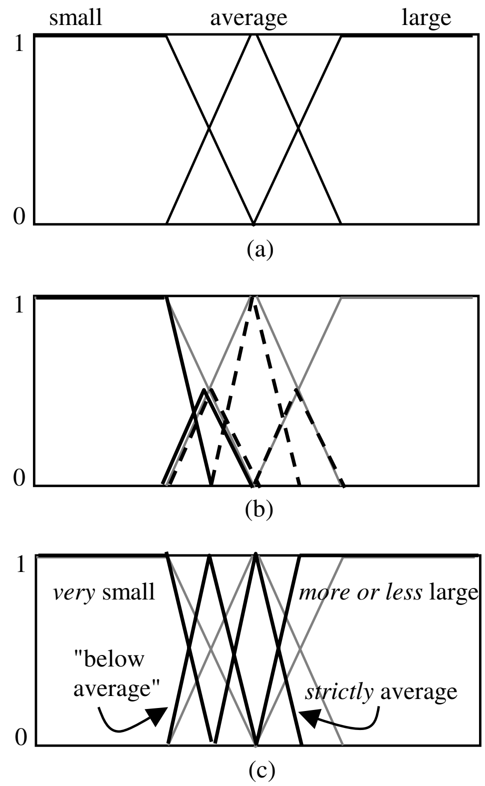

Fig. 8: Partial refinement of the linguistic variable "size". Meaning of the new linguistic terms.

that this is due to a too rough partition of the problem space affected by the variable "size"; particularly by the linguistic terms "small" and "average". It becomes apparent that a partial refinement of this variable is needed. A refinement of the linguistic terms "small" and "average" by means of partial multiresolution leads to the situation illustrated in figure 8(b), where the superposition of new linguistic terms is not yet processed. Figure 8(c) shows the situation after replacing superposed new linguistic terms by an equivalent one.

The analysis of figure 8(c) supports the following preliminary interpretation:

i) The new first linguistic term corresponds to *very* small

ii) The new third linguistic term corresponds to *strictly* average
iii) The new second linguistic term represents *between* [*very* small, *strictly* average]. Notice that in an appropriate context the use of the language might well interpret this as "below average" or possibly "*strictly* below average"
iv) The last linguistic term represents the structure (*from* { *between* [*strictly* average, *very* large] } *to* large). In *this* particular case it may be observed, that the change induced in the last linguistic term corresponds to just increasing its core by one half the length of the interval support \ core meanwhile preserving its support and its piecewise linear character. This is however the effect of the linguistic modifier "*more or less*", as discussed earlier. The last new linguistic term corresponds then to "*more or less* large"

It may be observed that the given example is representative of cases of partial multiresolution comprising a triangular fuzzy set neighbour to one of the border trapeziums. In such a case the process leads smoothly to the linguistic modifier "*more or less*".

4. Conclusions

It was shown that piecewise linear fuzzy sets (triangles, trapeziums) exhibit a multiresolution property. This allows the obtainment of a systematic partial refinement of the linguistic terms of linguistic variables. Some forms of refinement produce a concentration or a dilation of the fuzzy sets representing the terms and may be directly associated to linguistic modifiers thus obtaining readily their meaning. Partial refinements may produce asymmetric linguistic terms. For this case metasemantic operators were proposed which allow tracing the meaning of the refined terms based on the initial ones. This may be used to do fine tuning of models based on sets of fuzzy if-then rules, but keeping consistency with the semantic selected for the initialization.

For the sake of simplicity mainly isosceles triangular fuzzy sets were used, however the proposed method may very simply be extended to cover linguistic variables using asymmetric trapezoidal linguistic terms not only at the borders of the universe of discourse.

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