A Multi-objective Evolutionary Algorithm for Fuzzy Modeling

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Abstract

In this paper a multi-objective evolutionary algorithm with a single run is proposed in order to consider several objectives dealing with transparency and compactness in obtaining a fuzzy model besides the standard accuracy objective. In this way the use of Pareto-optimal solutions within the evolutionary algorithm let us obtain attractive fuzzy models with respect to compactness, transparency and also accuracy. The results of the combination of Pareto-based multi-objective evolutionary algorithms and fuzzy modeling is compared with other approaches in the literature.

Keywords: Takagi-Sugeno fuzzy model, Pareto optimality, multi-objective evolutionary algorithm.

1. Introduction

In recent years, fuzzy modeling, as a complement to the conventional modeling techniques, has become an active research topic and found successful applications in many areas. However, most fuzzy models are presently built based only on operator's experience and knowledge, but when a process is complex there may not be an expert available [29]. In this kind of situation the use of unsupervised learning techniques is of fundamental importance. The problem can be stated as follows. Given a set of data for which we presume some functional dependency, the question arises whether there is a suitable methodology to derive (fuzzy) rules from the data that characterize the unknown function as precisely as possible. Recently, several approaches have been proposed for automatically generating fuzzy if-then rules from numerical data without domain experts [18].

This paper deals with fuzzy model parameter estimation and structure selection. In fuzzy model identification, we can, in general, take into account three criteria to be optimized: compactness, transparency and accuracy. Different measures for these criteria are proposed here. Compactness is related to the size of the model, i.e. the number of rules, the number of fuzzy sets and the number of inputs for each rule. Transparency is related to linguistic interpretability [3, 25] and locality of the rules. Often one is interested in the local behavior of the global nonlinear model. Such information can be obtained by constraining the model-structure during identification. Transparency and model interpretability for data-based fuzzy models received a lot of interest in recent literature [20, 16, 2, 17].

Evolutionary Algorithms (EA) [1, 6] have been recognized as appropriate techniques for multi-objective optimization because they perform a search for multiple solutions in parallel [5, 11, 26]. EAs have been applied to learn both the antecedent and consequent part of fuzzy rules, and models with both fixed and varying number of rules have been considered [28, 14]. Also, EAs have been combined with other techniques like fuzzy clustering [10, 12, 7] and neural networks [15, 23]. This has resulted in many complex algorithms and, as recognized in [3] and [25], often the transparency and compactness of the resulting rule base is not considered to be of importance. In such cases, the fuzzy model becomes a black-box, and one can question the rationale for applying fuzzy modeling instead of other techniques like, e.g., neural networks. If the fuzzy model or a neural network is handled as a black-box model it will typically store the information in a distributed manner among the neurons or fuzzy sets and their associated connectivity [19].

Most evolutionary approaches to multi-objective fuzzy modeling consist of multiple EAs, usually designed to achieve a single task each, which are applied sequentially to obtain a final solution. In these cases each EA optimizes the problem attending to one criterion separately which is an impediment for the global search.

Simultaneous optimization of all criteria is more appropriate. Other approaches are based on classical multi-objective techniques in which multiple objectives are aggregated into a single function to be optimized [7]. In this way a single EA obtains a single compromise solution. Current evolutionary approaches for multi-objective optimization consist of a single multi-objective EA, based on the Pareto optimality notion, in which all objectives are optimized simultaneously to find multiple non-dominated solutions in a single run of the EA. These approaches can also be considered from the fuzzy modeling perspective [8, 13]. The advantage of the classical approach is that no further interaction with the decision maker is required, however it is often difficult to define a good aggregation function. If the final solution cannot be accepted, new runs of the EA may be required until a satisfying solution is found. The advantages of the pareto approach are that no aggregation function has to be defined, and the decision maker can choose the most appropriate solution according to the current decision environment at the end of the EA run. Moreover, if the decision environment changes, it is not always necessary to run the EA again. Another solution may be chosen out of the family of non-dominated solutions that has already been obtained.

In this paper we propose a single multi-objective EA to find, with a low necessity for human intervention, multiple non-dominated solutions for fuzzy modeling problems. In section 2, fuzzy modeling and the criteria taken into account, are discussed. The main components of the multi-objective EA are described in section 4. Section 5 proposes several optimization models for fuzzy modeling and a decision making strategy. In section 6, experiments with the EA for a test problem are shown and compared with results in literature. Section 6 concludes the paper and indicates lines for future research.

2. Fuzzy Model Identification

2.1. Fuzzy model structure

We consider rule-based models of the Takagi-Sugeno (TS) type [27] which are especially suitable for the approximation of dynamic systems. The rule consequents are often taken to be linear functions of the inputs:

$$R_i : \text{If } x_1 \text{ is } A_{i1} \text{ and } \dots x_n \text{ is } A_{in} \text{ then} \quad (1)$$
$$\hat{y}_i = \zeta_{i1}x_1 + \dots, \zeta_{in}x_n + \zeta_{i(n+1)}, \quad i = 1, \dots, M$$

Here $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ is the input vector, $\hat{y}_i$ is the output of the ith rule, A_{ij} $(j = 1, \dots, n)$ are fuzzy sets defined in the antecedent space by membership functions $\mu_{A_{ij}} : \mathbb{R} \to [0,1]$, $\zeta_{ij} \in \mathbb{R}$ $(j = 1, \dots, n+1)$ are the consequent parameters, and M is the number of

rules. The total output of the model is computed by aggregating the individual contributions of the rules:

$$\hat{y} = \sum_{i=1}^{M} p_i(\mathbf{x})\hat{y}_i \quad (2)$$

where $p_i(\mathbf{x})$ is the normalized firing strength of the ith rule:

$$p_i(\mathbf{x}) = \frac{\prod_{j=1}^{n} \mu_{A_{ij}}(x_j)}{\sum_{i=1}^{M} \prod_{j=1}^{n} \mu_{A_{ij}}(x_j)} \quad (3)$$

We apply the frequently used trapezoidal membership functions to describe the fuzzy sets A_{ij} in the rule antecedents:

$$\mu_{A_{ij}}(x) = \max \left(0, \min \left(\frac{x - a_{ij}}{b_{ij} - a_{ij}}, 1, \frac{c_{ij} - x}{c_{ij} - d_{ij}} \right) \right) \quad (4)$$

2.2. Multi-objective Identification

Identification of fuzzy models from data requires the presence of multiple criteria in the search process. In multi-objective optimization, the set of solutions is composed of all those elements of the search space for which the corresponding objective vector cannot be improved in any dimension without degradation in another dimension. These solutions are called *non-dominated* or *Pareto-optimal*. Given two decision vectors $\mathbf{x}$ and $\mathbf{y}$ in a universe U, $\mathbf{x}$ is said to *dominate* $\mathbf{y}$ if $f_i(\mathbf{x}) \leq f_i(\mathbf{y})$, for all objective functions f_i, and $f_j(\mathbf{x}) < f_j(\mathbf{y})$, for at least one objective function f_j, for minimization. A decision vector $\mathbf{x} \in U$ is said to be *Pareto-optimal* if no other decision vector dominates $\mathbf{x}$.

The Pareto-optimality concept should be integrated within a decision process in order to select a suitable compromise solution from all non-dominated alternatives. In a decision process, the decision maker expresses preferences which should be taken into account to identify preferable non-domination solutions. Approaches based on weights, goals and priorities have been used more often.

2.3. Rule set simplification techniques

Automated approaches to fuzzy modeling often introduce redundancy in terms of several similar fuzzy sets that describe almost the same region in the domain of some variable. According to some similarity measure, two or more similar fuzzy sets can be merged to create a new fuzzy set representative for the merged sets [24]. This new fuzzy set substitutes the ones merged in the rule base. The merging process is repeated until fuzzy sets for each model variable cannot be merged, i.e., they are not similar. This simplification may results in several identical rules, which are removed from the rule set.

We consider the following similarity measure between two fuzzy sets A and B:

$$S(A,B) = \frac{|A \cap B|}{|A \cup B|} \qquad (5)$$

If $S(A,B) > \theta_S$ (we use $\theta_S = 0.6$) then fuzzy sets A and B are merged in a new fuzzy set C as follows:

$$\begin{aligned}
a_C &= min\{a_A, a_B\} \\
b_C &= \alpha b_A + (1-\alpha)b_B \\
c_C &= \alpha c_A + (1-\alpha)c_B \\
d_C &= max\{d_A, d_B\}
\end{aligned} \qquad (6)$$

where $\alpha \in [0,1]$ determine the influence of A and B on the new fuzzy set C.

3. Criteria for Fuzzy Modeling

We consider three main criteria to search for an acceptable fuzzy model: (i) accuracy, (ii) transparency, and (iii) compactness. It is necessary to define quantitative measures for these criteria by means of appropriate objective functions which define the complete fuzzy model identification.

The accuracy of a model can be measured with the *mean squared error*:

$$MSE = \frac{1}{K} \sum_{k=1}^{K} (y_k - \hat{y}_k)^2 \qquad (7)$$

where y_k is the true output and $\hat{y}_k$ is the model output for the kth input vector, respectively, and K is the number of data samples.

Many measures are possible for the second criterion, transparency. Nevertheless, in this paper we only consider one of most significant, *similarity*, as a first starting point. The similarity S among distinct fuzzy sets in each variable of the fuzzy model can be expressed as follows:

$$\begin{aligned}
S &= \max_{\substack{i,j,k \\ A_{ij} \neq B_{ik}}} S(A_{ij}, B_{ik}), \\
i &= 1,\ldots,n, \; j = 1,\ldots,M, \; k = 1,\ldots,M
\end{aligned} \qquad (8)$$

This is an aggregated similarity measure for the fuzzy rule-based model with the objective to minimize the maximum similarity between the fuzzy sets in each input domain.

Finally, measures for the third criterion, the compactness, are the number of rules M and the number of different fuzzy sets L of the fuzzy model. We assume that models with a small number of rules and fuzzy sets are compact.

In summary, we have considered three criteria for fuzzy modeling, and we have defined the following measures for these criteria:

Criteria	Measures
Accuracy	MSE
Transparency	S
Compactness	M, L

4. Multi-Objective Evolutionary Algorithm

The main characteristics of the Multi-Objective Evolutionary Algorithm are the following:

1. The proposed algorithm is a Pareto-based multi-objective EA for fuzzy modeling, i.e., it has been designed to find, in a single run, multiple non-dominated solutions according to the Pareto decision strategy. There is no dependence between the objective functions and the design of the EA, thus, any objective function can easily be incorporated. Without loss of generality, the EA minimizes all objective functions.

2. Constraints with respect to the fuzzy model structure are satisfied by incorporating specific knowledge about the problem. The initialization procedure and variation operators always generate individuals that satisfy these constraints.

3. The EA has a variable-length, real-coded representation. Each individual of a population contains a variable number of rules between 1 and max, where max is defined by a decision maker. Fuzzy numbers in the antecedents and the parameters in the consequent are coded by floating-point numbers.

4. The initial population is generated randomly with a uniform distribution within the boundaries of the search space, defined by the learning data and model constraints.

5. The EA search for among simplified rule sets, i.e, all individuals in the population has been previously simplified (after initialization and variation), which is an added ad hoc technique for transparency and compactness. So, all individuals in the population have a similarity S between 0 and 0.6.

6. Chromosome selection and replacement are achieved by means of a variant of the preselection scheme. This technique is, implicitly, a niche formation technique and an elitist strategy. Moreover, an explicit niche formation technique has been added to maintain diversity respect to the number of rules of the individuals.

In each iteration of the EA, two individuals are picked at random from the population. These individuals are crossed $nChildren$ times and children mutated producing $2 \cdot nChildren$ offspring. Afterwards, a nondominated individual (random if there are several) among the first offspring replaces the first parent, and a nondominated individual (random if there are several) among the second offspring replaces to the second parent only if:

- the offspring dominates the parent, and

- the number of rules of the offspring is equal to the number of rules of the parent, or the niche count of the parent is greater than $minNS$ and the niche count of the offspring is smaller than $maxNS$.

The niche count of an individual I is the number of individuals in the population with the same number of rules as I. The added explicit niche formation technique ensures that the number of individuals with M rules, for all $M \in [1, max]$, is greater or equal to $minNS$ and smaller or equal to $maxNS$.

7. The EAs variation operators affect at the individuals at different levels: (i) the rule set level, (ii) the rule level, and (iii) the parameter level.

Remark: All the procedures in this section can be obtained from the authors on request.

5. Optimization Model and Decision Making

After preliminary experiments in which we have checked different optimization models, the following remarks can be maded:

1. The minimization of the number of rules M of the individuals has negative influence on the evolution of the algorithm. The reason is than this parameter is not an independent variable to optimize, as the amount of information in the population decreases when the average number of rules is low, which is not good for exploration. Then, we do not minimize the number of rules during the optimization, but we will take it into account at the end of the

run, in a posteriori articulation of preferences applied to the last population.

2. It is very important to note that a very transparent model will be not accepted by a decision maker if the model is not accurate. In most fuzzy modeling problems, excessively low values for similarity hamper accuracy, for which these models are normally rejected. Alternative decision strategies, as *goal programming*, enable us to reduce the domain of the objective functions according to the preferences of a decision maker. Then, we can impose a goal g_S for similarity, which stop minimization of the similarity in solutions for which goal g_S has been reached.

3. The measure L (number of different fuzzy sets) is considerably reduced by the rule set simplification technique. So, we do not define an explicit objective function to minimize L.

According to the previous remarks, we finally consider the following optimization model:

$$\begin{aligned} Minimize \quad & f_1 = MSE \\ Minimize \quad & f_2 = max\{g_S, S\} \end{aligned} \qquad (9)$$

At the end of the run, we consider the following a posteriori articulation of preferences applied to the last population to obtain the final compromise solution:

1. Identify the set $X^* = \{x_1^*, \ldots, x_p^*\}$ of non-dominated solutions according to:

$$\begin{aligned} Minimize \quad & f_1 = MSE \\ Minimize \quad & f_2 = S \\ Minimize \quad & f_3 = M \end{aligned} \qquad (10)$$

2. Choose from X^* the most accurate solution x_i^*; remove x_i^* from X^*;

3. If solution x_i^* is not accurate enough or there is no solution in the set X^* then STOP (no solution satisfies);

4. If solution x_i^* is not transparent or compact enough then go to step 2;

5. Show the solution x_i^* as output.

Computer aided inspection shown in Figure 1 can help in decisions for steps 2 and 3.

6. Experiments and results

In this section, the multi-objective EA is applied to the identification of a fuzzy model for the pressure dynamics of a laboratory fed-batch fermentor. With a constant

input air flow-rate, the pressure in the fermentor tank (variable y) is controlled by the opening of the outlet valve (variable u). Based on simplified assumptions, a first-principle physical model can be derived for this process. The setting of the outlet valve results in a certain transient behavior of the pressure, which can be described by a first-order nonlinear differential equation. As shown in [22], a fuzzy linear model provides a simple and elegant solution to the problem. In this way, the process can be represented by means of a TS rule set of the following form:

$$\text{If } y(k) \text{ is } A_i \text{ and } u(k) \text{ is } B_i \text{ then}$$
$$y(k+1) = a_iy(k) + b_iu(k) + c_i, \quad i = 1, \dots, M.$$

This rule base represents a nonlinear first-order regression model $y(k+1) = f(y(k), u(k))$, where $y(k)$ and $u(k)$ are the pressure and the valve position at time k, respectively. The membership functions of the antecedent A_i and B_i, as well as the consequent parameters a_i, b_i and c_i are estimated from the data by the proposed multi-objective EA, by using the optimization model (9) ($g_S = 0.3$, $max = 5$) and the proposed a posteriori decision strategy.

The following values for the parameters of the EA were used in the simulations: population size 100, crossover probability 0.8, mutation probability 0.4, number of children for the preselection scheme 10, minimum number of individuals for each number of rules $minNS = 5$, and maximum number of individuals for each number of rules $maxNS = 20$. All crossover and mutation operators are applied with the same probability. The EA stops when the solutions satisfy the decisor maker.

We compared our results, with those obtained by the two different approaches proposed in [21] and [4]. Solution in [21] is obtained by means of hyperplanar fuzzy clustering (Gustafson-Kessel's algorithm [9]) with projections of the fuzzy clusters in each domain and making the extensional hull of the fuzzy sets obtained to approximate them by trapezoidal fuzzy sets. The obtained solution is transparent and compact, containing 3 rules and 6 fuzzy sets with mean squared error $4.516 \cdot 10^{-4}$. In [4], in order to obtain the coeficient of the linear consequent using the recursive least-squares algorithm (or a stationary Kalman Filter), the grade of membership of the data to the antecedent of the fuzzy rules is considered using directly the grade of membership of the data to the fuzzy clusters found in product space of input-output variables. Moreover as in this kind of model we are searching for local linear models presents in the data, a modification we have adopt is not to use only a Fuzzy C-Means algorithm, but as an inicialization to a Gustafson-Kessel fuzzy clustering algorithm. In this way, this algorithm is adequate to detect the fuzzy partitions that better fulfill the assumption of fuzzy linear

No. rules	No. fuzzy sets	MSE	S
1	1	$9.989 \cdot 10^{-4}$	0.0
2	4	$4.845 \cdot 10^{-5}$	0.235
3	5	$2.778 \cdot 10^{-5}$	0.248
4	6	$2.470 \cdot 10^{-5}$	0.232
5	7	$2.306 \cdot 10^{-5}$	0.232

Table 1. Non-dominated solutions according to (10) obtained with the multi-objective EA.

models. The obtained solution is compact but non transparent, containing 4 rules and 4 n-dimensional fuzzy sets, with mean squared error $6.4 \cdot 10^{-5}$. Solution in this paper is obtained with a single multi-objective EA and it has been chosen among different alternatives, which is an advantage for an appropriate decision process. Non-dominated solutions according to (10) are summarized in Table 1, with an indication of the number of rules, number of different fuzzy sets, obtained MSE for training data and similarity S of the fuzzy sets. The accurate, transparent and compact solution with 2 rules, 4 different fuzzy sets, mean squared error $4.845 \cdot 10^{-5}$ and similarity 0.235 is finally chosen with the proposed a posteriori articulation of preferences. This solution is showed in Figure 1 by means of different graphics for the obtained model. Figure 1(a) shows the local model, the surface generated by the model is shown in Figure 1(b), fuzzy sets for each variable are showed in Figure 1(c), and finally, the prediction error is showed in Figure 1(d).

7. Conclusions and future research

This paper remarks some initial results in the combination of Pareto-based multi-objective evolutionary algorithms and fuzzy modeling. Criteria such as accuracy, transparency and compactness have been taken into account in the optimization process. Some of these criteria have been partially incorporated into the EA by means of ad hoc techniques, such as rule set simplification techniques. An implicit niche formation technique (preselection) in combination with other explicit techniques with low computational costs have been used to maintain diversity. These niche formation techniques are appropriate in fuzzy modeling if excessive amount of data are required. Excessive computational times would result if sharing function were used. Elitism is also implemented by means of the preselection technique. A goal based approach has been proposed to help to obtain more accurate fuzzy models. Results obtained are good in comparison with other more complex techniques reported in literature, with the advantage that the proposed technique identifies a set of alternative solutions. We also proposed an easy decision process with a

posteriori articulation of preferences to choose finally a compromise solution.

One of the main differences between the proposed EA and other approaches for fuzzy modeling is the reduced complexity because we use a single EA for generating, tuning and simplification processes. Moreover, human intervention is only required at the end of the run to choose one of the multiple non-dominated solutions found by the EA.

In our future works we will consider other and more complex fuzzy modeling test problems in order to check the robustness of the EA, other measures to optimize transparency, e.g., similarity in the consequent domain instead or together with of the antecedent domain, scalability of the algorithm, and applications in the real word by means of research projects.

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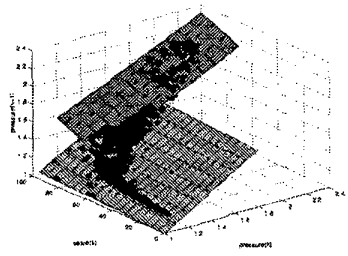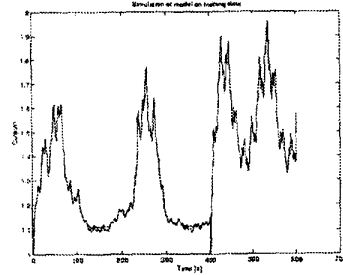
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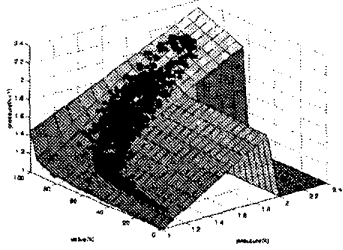
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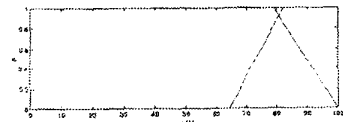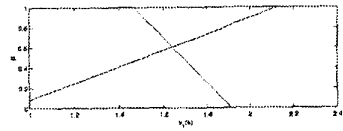
(c) Fuzzy sets

(d) Error prediction

Figure 1. Accurate, transparent and compact fuzzy models for the pressure dynamics of a laboratory fed-batch fermentor.