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Rule base reduction for knowledge-based fuzzy controllers with application to a vacuum cleaner☆

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Abstract

A rule base reduction and tuning algorithm is proposed as a design tool for the knowledge-based fuzzy control of a vacuum cleaner. Given a set of expert-based control rules in a fuzzy rule base structure, proposed algorithm computes the *inconsistencies* and *redundancies* in the overall rule set based on a newly proposed measure of equality of the individual fuzzy sets. An inconsistency and redundancy measure is proposed and computed for each rule in the rule base. Then the rules with high inconsistency and redundancy levels are removed from the fuzzy rule base without affecting the overall performance of the controller. The algorithm is successfully tested experimentally for the control of a commercial household vacuum cleaner. Experimental results demonstrate the effective use of the proposed algorithm.

Keywords: Knowledge-based systems; Fuzzy control; Industrial applications; Rule base reduction

1. Introduction

Many new technological products are being developed and updated responding to the needs of the society. This fact is especially visible in the home appliances industry, since the range and features of its products are inevitably related with the households and the life styles of their occupants. With the decreasing cost of microprocessor-based control systems, implementation of more complicated control algorithms becomes more cost effective. Household appliances that are equipped with sensors and more sophisticated control algorithms started dominating the market in the past decade. The motivation for this study stems from this need for human friendly control systems for household appliances.

Most house appliance products are subjected to very different environmental conditions during operation and thus the control methodology in use should cope with these uncertainties. Additionally the control algorithm should be robust to the usually highly nonlinear low cost sensors. Such sensors also directly present uncertainty in the overall control problem.

Apart from these problems, modeling of household appliances constitutes a challenging task. Variations in the parameters of the underlying systems during operation and nonlinear behavior of the components are major obstacles facing such modeling efforts. For example, highly complicated and random disturbances such as the distribution of cloths in the drum of a washing machine or variations of the humidity and temperature of ambient air for a refrigerator, should be considered during the modeling stage.

Conventionally, manual control and rule-based on/off algorithms are employed for controlling the household appliances. In the case of manual control, the whole controlling task is left to the user. Rule-based control is used in the controllers which operate continuously such as refrigerators and recently in other household appliances to reduce the control burden on the user. Conventional rule-based controllers, however, fail to effectively implement the decision mechanism, in other words the performance requirement of the user. Here, the problem to address is the mathematical expression of a set of vague rules which cover ranges rather than exact values of the sensor signal. Hence Fuzzy Logic theory (Zadeh, 1965) lends itself as

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a powerful tool in the development of highly efficient control algorithms.

The applications of fuzzy logic in control engineering had its origins going back to mid 1970s with the control of a pilot scale boiler-steam engine system (Mamdani, 1977). A more systematic treatment of rule-based (expert-based) control is given byin Arzen (1989) and Astrom, Anton, and Arzen (1986). Here control signal is based on rules from expert knowledge of the process to be controlled. During the last two decades, fuzzy logic had found many industrial applications in the control engineering area, especially in Japan. The control system of Sendai subway system, autofocus video camera control, fuzzy car control are some examples of fuzzy logic control applications. A fuzzy rule-based control of a mobile robot is also reported by the author in (Ciliz & Işik, 1989). A systematic methodology for the synthesis and analysis of fuzzy-logic controllers for multi-input multi-output nonlinear dynamic systems with application to a robotic manipulator is proposed by Emami, Goldenberg, and Turksen (2000). A complete treatment of fuzzy modeling and control with a large range of applications can be found in Passino and Yurkovich (1998), Pedrycz and Zadeh (1995) and Yager and Filev (1994). The success of fuzzy logic control in industrial processes paved the way for its applications in household appliances. The problems encountered in the control of household appliances are mainly the difficulties encountered in specifying the performance criteria, availability of only low quality sensor signals and most importantly cost factors which would directly affect the size of the chosen rule base. In this study, we focus on the efficient tuning and reduction of rule base size with application on a household vacuum cleaner where we extend some of the results reported in our work (Ciliz, 2003).

The construction of a fuzzy controller is a relatively simple task compared to its analysis and tuning (Passino & Yurkovich, 1998; Pedrycz & Zadeh, 1995). There has been efforts in the past for obtaining the interactivity levels of the rules, inconsistencies and completeness of the rule set (Pedrycz & Zadeh, 1995). These completeness and inconsistency notions are used in the analysis of the controllers. In the current study a new methodology is proposed to better analyze the consistency of the rule base. The proposed approach computes the inconsistencies of the rules based on the ‘equality of two fuzzy sets’ as an alternative to the ‘possibility approach’ used in (Pedrycz & Zadeh, 1995). As a further refinement for the rule base, a new measure, ‘redundancy level of rules’ is proposed for the analysis of the controller. The motivation behind this analysis is the fact that redundant rules do not explicitly contribute to the performance of the fuzzy controller. Hence it gives the control designer additional incentive to eliminate rules without affecting the overall performance of the controller.

The proposed tuning methodology is applied for the control of a household vacuum cleaner. The experimental tests were performed on a commercial vacuum cleaner at the Research Center of Arçelik Co. which is the largest manufacturer of household appliances in Turkey.

2. Mathematical preliminaries

Let U denote the universal set. A fuzzy subset of U is characterized by a function

$$A(x) : U \rightarrow [0,1] \quad \forall\, x \in A \subset U \tag{1}$$

which associates each member of universal set with a real number between 0 and 1. In this notation, U can be taken as a discrete or continuous set which contains all the elements that the group of fuzzy sets contain. Each number represents the extent to which the particular element of universal set U belongs to A. This helps in achieving a continuous grade of membership. Thus, each element of a fuzzy set is characterized by a membership value. A fuzzy set A which is defined on a discrete universe of discourse can be shown as,

$$A = \mu_1/y_1 + \mu_2/y_2 + \cdots + \mu_n/y_n \tag{2}$$

where μ_i denotes the membership value of y_i and $+$ sign denotes the union operation of the set theory. Replacing $+$ with Σ as a short hand notation, universal set U may be represented as,

$$U = \sum_{i=1}^{n} 1/y_i \tag{3}$$

where 1 denotes the membership value of the elements of the universal set U.

Connectives like ‘and’, ‘or’, and ‘negation’ may be represented as operations on fuzzy sets. B is a *subset* of A if the membership values of set A is greater than or equal to the membership values of the corresponding members of the set B. Formally,

$$\forall y \in U, \quad B \subset A \Leftrightarrow \mu_B(y) \leq \mu_A(y) \tag{4}$$

The *union* C of two fuzzy sets A and B is defined as,

$$C = A + B = \max(\mu_A(y), \mu_B(y))/y \tag{5}$$

where $+$ denotes the union operation. According to this definition, $A+B$ is the smallest fuzzy set which includes both A and B. The *intersection* of A and B is defined as,

$$C = A \cap B = \min(\mu_A(y), \mu_B(y))/y \tag{6}$$

According to this definition, C is the largest fuzzy set which is contained in both A and B.

To describe the relations between fuzzy concepts, *fuzzy relations* are used. A fuzzy relation R from X to Y is a subset of the Cartesian product $X \times Y$. R is defined by a membership function defined on the elements of X and Y as (Pedrycz & Zadeh, 1995),

$$R = \int_{X \times Y} \mu_R(x,y)/(x,y) \tag{7}$$

where the membership value $\mu_R(x, y)$ associated with the ordered pair (x, y) gives the strength of the relation between x and y elements.

A fuzzy rule or relation can be put into the general form,

IF A THEN B

where A and B are fuzzy quantities. In general *fuzzy implication* functions are used to define the membership value of each ordered pair of the relation connecting A and B sets. Here we define the fuzzy implication operation as,

$$R_c = (A \times B) \tag{8}$$

$$R_c = \int_{U \times V} \min(\mu_A(u), \mu_B(v))/(u, v) \tag{9}$$

where U, V are the universal sets for A and B, and $\mu_A(u)$, $\mu_B(v)$ are the membership functions for A and B, respectively. As an example, let us illustrate the calculation of a fuzzy implication for fuzzy sets A and B. Let, $A = \{1/1 + 0.5/2\}$ and $B = \{0.55/1 + 0.8/2\}$. Then

$$R_c = A \times B = A \begin{array}{c} B \\ \begin{vmatrix} 0.55 & 0.8 \\ 0.5 & 0.5 \end{vmatrix} \end{array} \tag{10}$$

where each entry is computed using (9). For instance, the first element of the first row is obtained by taking the minimum of the membership values {1, 0.55} which correspond to the first elements of A and B, respectively (i.e. $0.55 = \min(1, 0.55)$).

2.1. Fuzzy inference

Fuzzy relations basically describe a mapping from input space of the fuzzy rule to its output space. Thus, similar to ordinary functions one can obtain the implications of a fuzzy relation for all of its input space. Using fuzzy inference, it is possible to obtain the output of a fuzzy rule even if the input *does not completely* match the conditions defined in the input section of the rule. Fuzzy inference can be achieved through the use of *compositional operators*. Although there are different kinds of compositional operators described in the literature, Sup–Min inference mechanism is one of the most commonly used operators in fuzzy logic control applications. For the fuzzy rule of the form IF X THEN Y where X and Y are two fuzzy sets, for X_1 which is a fuzzy set defined on the same universal set with X. Then, given X_1, output of the fuzzy rule can be computed as,

$$X_1 \oplus R = \int_{X \times Y} \max_y(\min(\mu_R(x, y), \mu_{X_1}(x))) \tag{11}$$

where $\oplus$ defines the Sup–Min inference operation (Pedrycz & Zadeh, 1995). Next, let us give an example for the inference mechanism just outlined above.

Example. Let A and B be two fuzzy sets given as,

$$A = 1/1 + 0.8/2 + 0.3/3 + 0.5/4 + 0.7/5$$

and

$$B = 0.3/1 + 0.9/2 + 0.1/3$$

Then for a rule IF A THEN B, first the fuzzy relation $R_c = A \times B$ is obtained using the fuzzy implication defined in (9) as,

$$R_c = A \times B = \begin{vmatrix} 0.3 & 0.9 & 0.1 \\ 0.3 & 0.8 & 0.1 \\ 0.3 & 0.3 & 0.1 \\ 0.3 & 0.5 & 0.1 \\ 0.3 & 0.7 & 0.1 \end{vmatrix} \tag{12}$$

Then for an arbitrary fuzzy predicate A_1 which is defined on the same universe with A,

$$A_1 = 0.8/1 + 0.9/2 + 0.3/3 + 0.7/4 + 0.9/5$$

the *inferred* fuzzy set B_1 (output of the above defined rule) can be obtained using the inference mechanism defined in (11) as,

$$B_1 = A_1 \oplus R = 0.3/1 + 0.9/2 + 0.1/3 \tag{13}$$

3. Design tools for fuzzy control

A dynamic process which is defined in terms of a set of 'IF X THEN Y' type fuzzy rules can be controlled through a *fuzzy inference mechanism* which is briefly outlined above. Fuzzy quantities X and Y are related to each other using the fuzzy implication function defined in (9) in the previous section. The choice of primary fuzzy sets and the fuzzy implication function which is responsible for fuzzy inference mechanism and the defuzzification method, which transforms the fuzzy quantities to crisp values that are suitable to apply to the plant, are important features of a fuzzy control algorithm. Another problem that must be addressed is the determination of universal sets of input and control (output) variables (Astrom et al., 1986; Emami et al., 2000). Then, primary fuzzy sets are defined for each variable covering the universal set of the signal. If the universes are assumed to be discrete and finite, each universe can be treated as a set of elements. Primary fuzzy sets are defined as vectors which contain the membership values of corresponding elements of the universal set.

The next step is the construction of the fuzzy relations, R_i, $i = ,\ldots, N$ corresponding to each fuzzy rule by using the implication function defined in (9), with N being the total number of rules in the rule set. As the final step, the overall relation R is computed by taking the union of the individual fuzzy relations by using the union operator defined in (5),

$$R = R_1 + R_2 + \cdots + R_n$$

During the execution of the controller, the input signals are obtained and quantized at each sampling instant, then the corresponding fuzzy set is obtained based on the previously defined membership functions over the universe of discourse.

The final control action is obtained through the use of the inference mechanism which is defined in (11) on the previously obtained rule base. A fuzzy logic controller is a *deterministic system* in the sense that it always yields the same control action as long as the input state of the controller is the same. Here it is assumed that the reader is familiar with the basic principles of fuzzy logic control. For a detailed treatment of fuzzy rule-based control, the interested reader is referred to Passino and Yurkovich (1998) and Pedrycz and Zadeh (1995). Next, we discuss the design considerations and the proposed methodology for the fuzzy logic controller.

One of major handicaps of fuzzy logic control has been the difficulties in the stability analysis and tuning. Since the controller is not based on an explicit model of the process, the behavior of the closed loop system cannot be easily examined as in the conventional type controllers. There have been many efforts in the literature for the analysis of fuzzy control systems in Astrom et al. (1986), Ciliz and Işik (1988), Emami et al. (2000), and Lee (1990), and in references compiled in Passino and Yurkovich (1998), Pedrycz and Zadeh (1995), and Yager and Filev (1994). Despite these efforts, establishing a formal method of analysis for fuzzy logic systems is still an active research topic. As noted in Pedrycz and Zadeh (1995), there are some important issues that have to be carefully analyzed during the design phase of a fuzzy logic controller. These can be summarized as (a) completeness (b) interaction and (c) consistency of the control rules.

By completeness, it is meant that the controller can generate a *control output* for any input fuzzy state X. In other words, at least one rule is fired at all times. Another important problem in fuzzy rule-based control is the *consistency* of the control rules. If the outcome of the two or more control rules with similar predicate conditions fire contradictory control actions, this would lead to unsatisfactory performance of the overall control scheme. Inconsistency is evident if for a given input of the controller, the resulting fuzzy set is multimodal, i.e. the set clusters exist at more than one point of the universe of discourse. The final outcome of the fuzzy controller may suggest one action through the use of the defuzzification process (for example after the use of an α *cut* for final action determination) (Passino & Yurkovich, 1998). Some tools for locating contradictory rules and determining the level of inconsistency of each rule are available in the literature. The 'i'th and 'k'th rules are said to be consistent when a slight difference between *predicates* X_i and X_k of the rules produces a slight difference between their *corresponding actions* Y_i and Y_k. Formally, an *index of inconsistency* of a pair of 'i'th and 'k'th rules, c_{ik}, is defined by,

$$c_{ik} = |\Pi(X_i, X_k) - \Pi(Y_i, Y_k)| \tag{14}$$

$\Pi(X_i, X_k)$ and $\Pi(Y_i, Y_k)$ are called the possibility measures of fuzzy sets (X_i, X_k) and $\{Y_i, Y_k\}$ respectively (Pedrycz & Zadeh, 1995). The *possibility measure* of X_i with respect to X_k is defined as,

$$\Pi(X_i, X_k) = \sup_{x \in X}[\min(X_i(x), X_k(x))] \tag{15}$$

If two sets are identical, possibility measure yields a c_{ik} value of 1. If they are disjoint the corresponding c_{ik} value is equal to 0. Then, if two rules are identical c_{ik} is equal to 0, meaning complete accordance. If X_i and X_k are identical but Y_i and Y_k are disjoint, then c_{ik} becomes 1, meaning complete inconsistency. As an example, the calculation of the possibility measure of two fuzzy sets A and B are illustrated in Fig. 1. In order to compute the inconsistency of the 'i'th rule and the remaining N ones, one can sum the individual inconsistencies over the second index,

$$c_i = \sum_{k=1}^{N} c_{ik} \tag{16}$$

Having obtained the level of inconsistencies between each rule, it is possible to omit the rules with high levels of inconsistencies. Inconsistency measure introduced here can be used effectively for the elimination of inconsistent rules in a large fuzzy rule base. It should be expected that by removing inconsistent rules, the overall performance of the controller would be improved or not changed much, while a reduction in the size of the rule base is achieved.

4. Tuning and rule base reduction

There have been some efforts in the literature for the reduction of rule base size in fuzzy control. An interpolation algorithm is proposed by Koczy and Hirota (1997) where dense rule bases are reduced based on an *interpolation algorithm* so that only minimal number of necessary rules

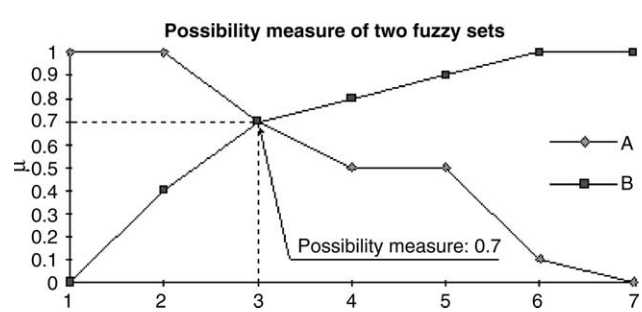

Fig. 1. Computation of the *possibility measure* of two fuzzy sets.

remain in the rule base. A rule base reduction technique with application to a robot manipulator control is given by Bezine, Derbel, and Alimi (2002) where a *boolean approach* is used for measuring equivalence hence inconsistency in the rules. A premise learning methodology combined with a genetic algorithm is proposed by Xiong and Litz (2002) to effectively reduce a large rule base for the effective control of an inverted pendulum.

In this study, the inconsistency concept introduced in Section 3 is further investigated and a new methodology for refining the *inconsistency measure* is introduced, termed as the 'fuzzy matching set approach' (Ciliz, 2003). In addition to this new inconsistency approach, a new metric 'redunredundancy measure' is introduced to eliminate highly redundant rules in the rule base, as well.

Two rules in the rule set are said to be an inconsistent pair if they suggest diverse actions while their inputs are close to each other. Given the rule format IF X_i THEN Y_i, the inconsistency measure of each rule pair can be obtained as in (14). The possibility measure used in (14) gives the extent with which two fuzzy sets overlap. In the possibility approach the measure of equality of two fuzzy sets A and B was taken to be the maximum membership value of $A \times B$ (Pedrycz & Zadeh, 1995). Based on the definition given in (14), rather high equality measures are obtained even if the two sets overlap in a small portion of the universe of discourse. This is demonstrated with the example given in Section 3 and shown in Fig. 1. Obviously this measure which relies on only a single point of universe of discourse is *not dependable*. This led us to investigate a better methodology to refine the inconsistency measure for the control rule base (Ciliz, 2003).

4.1. Fuzzy matching set approach

As an alternative to the possibility measure as a means to represent the equality of two fuzzy sets, the *equality of two fuzzy sets* may be described as a fuzzy set defined on the same universe of discourse. The second method relies on the comparison of fuzzy sets in a certain logical setting. In set theory two sets are said to be equal if simultaneously one is contained in the second and vice versa. This fact is expressed in logical notation as,

$$\{A = B\} \Leftrightarrow \{A \subset B\} \cap \{B \subset A\} \tag{17}$$

Modelling inclusion operator $\subset$ as an operation in fuzzy set theory as 'ϕ' and applying an 'AND' definition as a connective, we can identify the level of equality of fuzzy sets A and B at a point in their universal sets, $x \in X$,

$$\mathrm{fuzeq}(A|B) = [A(x)\phi B(x)] \cap [B(x)\phi A(x)] \tag{18}$$

where

$$A(x)\phi B(x) = \sup(c \in [0,1] | A(x) \cap c \le B(x)) \tag{19}$$

Here we introduce a new index fuzeq($A|B$) to denote the fuzzy equality measure of two sets A and B. Calculating the level of equality of two fuzzy sets for all of their elements, a new fuzzy set that represents the equality of two fuzzy sets can be obtained. As an example, let us illustrate the equality measure between two fuzzy sets.

Example. Let A and B be two fuzzy sets defined on the same universe of discourse U. These sets are defined below

$$A = 1/1 + 0.8/2 + 0.7/3 + 0.6/4 + 0.5/5 + 0.5/6$$
$$B = 0.7/1 + 0.8/2 + 1/3 + 1/4 + 0.8/5 + 0.5/6$$
$$C = 1/1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6$$

Applying the implication operator defined in (19), the grade of inclusions $\|A \subset B\|(x)$ and $\|B \subset A\|(x)$ are calculated at each point of the universal set U. The grade of inclusions is expressed as fuzzy sets given below. The grade of inclusion at a certain point is defined as the maximum $\{c\}$ satisfying the inequality $A(x) \cap c \le B(x)$ when calculating $\|A \subset B\|$. Assuming that intersection is modeled as taking the minimum of two membership values (as in (6)), let us calculate the membership values corresponding to element '1' of the universal set U. Since the membership value of A is greater than B for '1', $\{c\}$ must be equal to the membership value of B to satisfy the equation defined in (18). The inclusion of B in A at '1' is equal to '1' since the membership value of B is less than A at this point of the universe. Similarly, calculating for other elements of U, we obtain,

$$\|A \subset B\|_U = 0.7/1 + 1/2 + 1/3 + 0.6/4 + 1/5 + 1/6$$
$$\|B \subset A\|_U = 1/1 + 1/2 + 0.7/3 + 0.6/4 + 0.5/5 + 1/6$$

Finally, calculating the intersection of these fuzzy sets, the fuzzy set representing the equality of A and B over universal set U is obtained as,

$$\mathrm{fuzeq}(A|B) = 0.7/1 + 1/2 + 0.7/3 + 0.6/4 + 0.5/5 + 1/6 \tag{20}$$

This proposed method of computing the equality of two fuzzy sets will be termed as the fuzzy matching set approach. An illustration of the pointwise definition of equality of two fuzzy sets A and B given above is given in

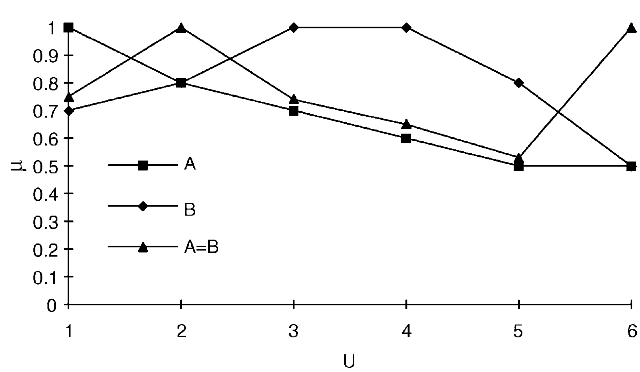

Fig. 2. Illustration of the set fuzeq($A|B$) for '$A=B$'.

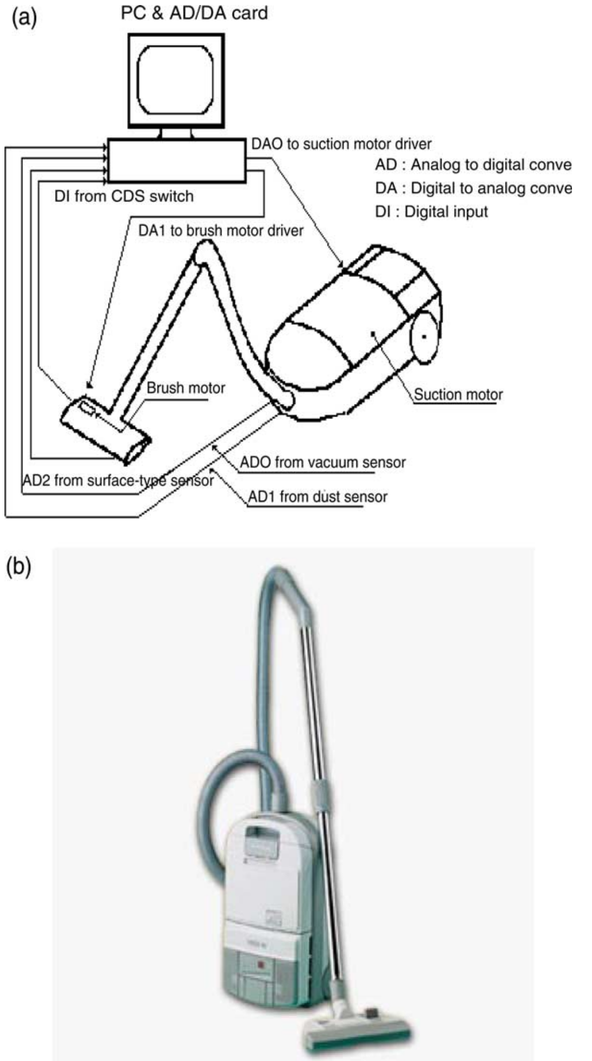

Fig. 3. (a) Experimental set-up for the tests. (b) Experimental test bed: Arcelik household vacuum cleaner.

Fig. 2. The figure illustrates that the membership values of the fuzzy set $\{A=B\}$ becomes higher as the membership values of the fuzzy sets A and B take values closer to each other. Furthermore if we take the average of the membership values of the fuzzy set $\{A=B\}$ we get a better representation of the matching between the fuzzy sets A and B.

4.2. Redundancy of the fuzzy rule base

In addition to the inconsistency level of rule pairs, it is also meaningful to search for the excess or redundant rules in the rule base. The motivation behind this search is obvious, since the redundant rules do not contribute to the performance of the controller. Therefore, it makes sense to minimize the rule set, especially when performing fuzzy inference calculations in real time. In this study, a redundancy measure which is based on the degree with which input and output fuzzy sets of different rules overlap is introduced. This proposed redundancy measure is defined as,

$$r_{ik} = (\mathrm{fuzeq}(X_i|X_k) + \mathrm{fuzeq}(Y_i|Y_k))/2 \tag{21}$$

where fuzeq($\cdot$)is defined as in (18) and i, $k=1,\ldots,N$ where N denotes the number of rules in the rule base. X_i and Y_i denote the predicates and consequences of the 'i' rule, respectively and 'fuzeq' denotes the equality degree of two fuzzy sets computed using the fuzzy matching set approach as defined in (18). According to the basic notion underlying this measure, a fuzzy rule pair is said to be highly redundant if both their input spaces and output spaces closely resemble each other. Consequently, identical rules yield a maximum redundancy value of 1.00 while a pair of totally disjoint rules yield a redundancy value of 0.00. Removing the rules with high redundancy levels would reduce the size of the rule base while not affecting the overall performance of the controller.

Table 1
Fuzzy controller with four sensors design rule base

Rule no.	Antecedents				Consequents	
	Vacuum	Dust	CDS	Surface	Suction	Brush
1		Low		Parkey	Low	Slow
2		Medium		Parkey	Low	Slow
3		High		Parkey	Medium	Slow
4		Low		Carpet	Low	Slow
5	Medium	Medium		Carpet	Medium	Medium
6		High		Carpet	High	Fast
7	Medium	Low		Deep carpet	Low	Medium
8		Medium		Deep carpet	High	Fast
9		High		Deep carpet	High	Fast
10		Low	High		High	Fast
11		Medium	High		High	Fast
12	Very high		Low		Very low	Slow
13	Low	Medium		Carpet	Medium	Medium
14	Low	Low		Deep carpet	Low	Medium

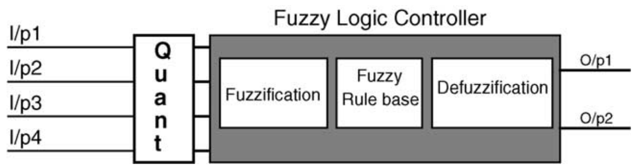

Fig. 4. Fuzzy controller with four sensor inputs and two outputs.

5. Application to the control of a vacuum cleaner

The methods developed in Section 4 are tested for the control of a vacuum cleaner set which is a product of Arçelik Co. of Turkey. A standard vacuum cleaner set is modified with the necessary sensors for generating information to be used in the fuzzy rule base. Four signals from the sensors namely, *dust sensor*, *vacuum sensor*, *cleaning direction sensor* and *surface type sensor*, are used as inputs to the fuzzy inference mechanism. *Suction power* and *cleaning brush speed* are the two outputs of the fuzzy controller. The experimental set-up used for the tests are shown in Fig. 3.

The rule base is formed based on the expert views and user data. Fourteen different action rules are generated based on the experiences of the product development engineers at Arçelik Research Center (Altasli & Ciliz, 1993). Appropriately selected membership functions over the whole range of each signal's universal set are used to determine the fuzzy predicates to be used in the sup–min inference mechanism which is defined in (11). Membership function derivation is discussed in detail in Altasli and Ciliz (1993). The set of control rules is shown in Table 1.

In order to test the developed controller and the tuning algorithms, *six different scenarios* are assumed with varying input conditions based on different carpet types and dirtiness levels. These scenarios are the standard for the product development phase of this appliance (Altasli & Ciliz, 1993). In all the experiments, inputs for these six scenarios are used in the fuzzy controller. For the experimental tests, a commercial development tool, Omron Co.'s FS10AT Development Package is utilized. This is basically a fuzzy control development software package along with a hardware interface card which can be used on a standard PC architecture for interfacing with analog signals (Altasli & Ciliz, 1993). The fuzzy controller block diagram with four sensor inputs and two outputs is shown in Fig. 4.

The inconsistency measures based on the possibility approach given in Section 3 and the newly proposed inconsistency and redundancy measures developed in Section 4 are used in the implementation of the fuzzy controller. For the implementation of the fuzzy controller, expert driven rules tabulated in Table 1 are used. For these 14 rules, the inconsistency levels of each rule pair which were computed based on the possibility measure is presented in Fig. 5. As it can be observed from this table a quite coarse map of inconsistencies were obtained. The indexes show either total inconsistency (1.00) or complete accordance (0.00) for most of the rule pairs. The last line in the index table in Fig. 5 shows the 'total inconsistency' index for each rule.

The inconsistency values for each *pairwise rule* combination were recomputed using the proposed fuzzy matching set approach. The results are presented in Fig. 6. The results demonstrate the fact that we have a considerably finer map of inconsistencies as discussed in Section 4. Therefore, inconsistent rule pairs can be located with much better accuracy.

Rule No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.00	0.52	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.48	1.00	0.00	0.00
2		0.00	0.45	1.00	0.00	0.00	0.00	0.00	0.00	0.48	1.00	1.00	0.00	0.00
3			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45	1.00	0.00	0.00
4				0.00	0.48	0.00	0.69	0.48	0.00	1.00	0.48	1.00	0.48	0.69
5					0.00	0.45	0.48	0.69	0.45	0.48	1.00	0.30	0.65	0.35
6						0.00	0.00	0.55	0.31	1.00	0.55	1.00	0.45	0.00
7							0.00	0.48	0.00	1.00	0.48	0.30	0.35	0.65
8								0.00	0.55	0.52	0.00	1.00	0.69	0.48
9									0.00	1.00	0.55	1.00	0.45	0.00
10										0.00	0.52	0.00	0.48	1.00
11											0.00	0.00	1.00	0.48
12												0.00	0.00	0.00
13													0.00	0.48
14														0.00
Total	4.00	4.45	1.90	7.30	5.33	4.31	4.43	5.44	4.31	8.48	6.99	7.60	5.03	4.13

Fig. 5. Inconsistency matrix computed using the possibility approach.

Rule No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.00	0.18	0.20	0.37	0.22	0.33	0.19	0.27	0.30	0.17	0.24	0.18	0.19	0.16
2		0.00	0.14	0.37	0.22	0.33	0.19	0.27	0.27	0.24	0.17	0.18	0.19	0.16
3			0.00	0.33	0.22	0.33	0.19	0.27	0.27	0.27	0.24	0.20	0.19	0.16
4				0.00	0.27	0.44	0.20	0.33	0.38	0.22	0.34	0.28	0.24	0.20
5					0.00	0.27	0.17	0.18	0.25	0.26	0.20	0.20	0.19	0.17
6						0.00	0.31	0.37	0.24	0.43	0.38	0.28	0.25	0.28
7							0.00	0.20	0.25	0.18	0.23	0.17	0.16	0.15
8								0.00	0.28	0.33	0.23	0.23	0.17	0.18
9									0.00	0.37	0.33	0.23	0.22	0.22
10										0.00	0.28	0.27	0.22	0.16
11											0.00	0.27	0.18	0.20
12												0.00	0.19	0.16
13													0.00	0.12
14														0.00
Total	2.97	2.91	2.99	3.97	2.82	4.24	2.59	3.31	3.58	3.40	3.29	2.82	2.51	2.32

Fig. 6. Inconsistency matrix computed using the fuzzy matching set approach.

Next, the redundancy concept developed in Section 4 is tested for the proposed controller. The redundancy measure (defined in Section 4) of all the combinations of the rule set are also computed (using (21)) and presented in Fig. 7. The last line in the redundancy index table shows the total redundancy level for each rule with all the other rules in the rule base.

In the implementation of the fuzzy controller, the rules having the highest inconsistency and highest redundancy levels are discarded from the rule base. In doing so, the performance of the controller was expected to improve or remain unaffected in the worst case, since other rules could compensate the missing redundant and inconsistent rule.

As a simple test for comparison, the controller outputs for the ‘pipe blockage’ scenario is shown in Figs. 8 and 9. Fig. 8 shows the suction power and brush speed outputs for the controller utilizing all 14 rules of the rule base. In Fig. 9, controller outputs are given with the most inconsistent and the most redundant rule eliminated from the rule base. These results show that the output of the controller has been affected minimally when the most redundant rule is eliminated.

Rule No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1.00	0.91	0.86	0.82	0.85	0.80	0.87	0.83	0.83	0.87	0.84	0.87	0.87	0.88
2		1.00	0.89	0.82	0.85	0.80	0.87	0.83	0.83	0.84	0.87	0.87	0.87	0.88
3			1.00	0.80	0.85	0.80	0.87	0.83	0.83	0.83	0.84	0.87	0.87	0.88
4				1.00	0.83	0.73	0.86	0.79	0.77	0.85	0.79	0.82	0.84	0.86
5					1.00	0.83	0.88	0.87	0.83	0.83	0.86	0.86	0.91	0.87
6						1.00	0.80	0.81	0.88	0.79	0.81	0.82	0.83	0.82
7							1.00	0.86	0.84	0.87	0.85	0.87	0.88	0.93
8								1.00	0.86	0.84	0.88	0.84	0.88	0.87
9									1.00	0.82	0.84	0.84	0.85	0.84
10										1.00	0.86	0.83	0.85	0.88
11											1.00	0.83	0.87	0.86
12												1.00	0.87	0.88
13													1.00	0.90
14														1.00
Total	12.0	12.1	11.9	11.5	12.1	11.5	12.2	11.9	11.8	11.9	12.0	12.0	12.2	12.3

Fig. 7. Redundancy matrix of the rule base computed using the fuzzy matching set approach.

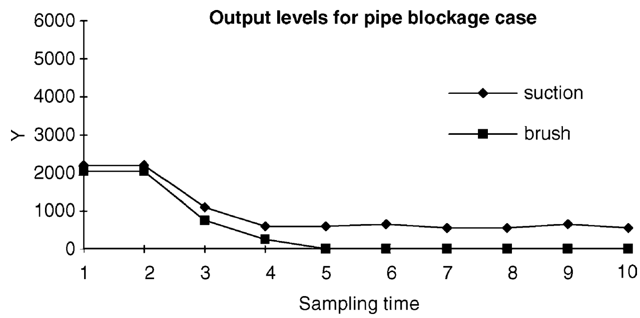

Fig. 8. Output levels of the fuzzy controller with the original rule base for a specific scenario.

As another test, experiments were performed for the other five scenarios (Altasli & Ciliz, 1993) in order to explicitly test the success of removing the *most redundant* rule only. An *error metric* is introduced to compare the difference between the output levels of the controllers utilizing the original and the modified rule base. The error metric is defined as,

$$\text{error} = \frac{\sum_{i=1}^{N} |y_0 - y_m|}{N} \tag{22}$$

where y_0 denotes the original outputs and y_m denotes the outputs of the modified controller and N is the number of rules in the rule base. The error is computed for all sampling instants. The total error values were normalized by dividing with the largest possible error value. Hence an identical controller output would yield 0.0 as the error metric and the metric approaches to 1.0 as the difference between the original output and modified output increases. The error metric values obtained for six different scenarios are listed in Fig. 10. Note that in all six cases, the error values are nearly zero. These results clearly show that the outputs of the controller do not depend on the most redundant rule, as expected. Therefore, it may be concluded that the performance of the controller was not affected when the original rule base is analyzed and tuned based on the redundancy measure. Hence, the results of the experimental tests suggest that the redundancy measure can be successfully used to locate the unnecessary rules in the rule base. This tool is

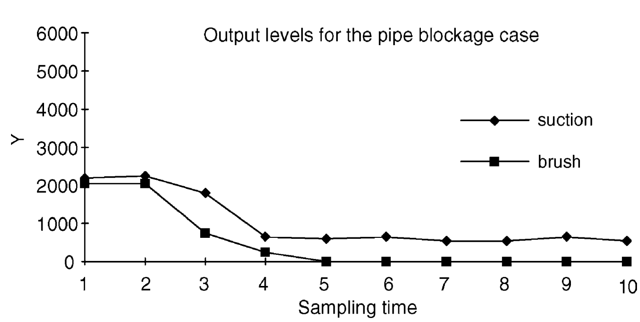

Fig. 9. Output levels of the fuzzy controller without the most inconsistent and the most redundant rule for the same scenario.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Most redundant	0.0125	0.0040		0	0.001	0

Fig. 10. Evaluation of the redundancy measure. The error metric (defined in (22)) values for the six different scenarios.

especially valuable when the fuzzy logic calculations are performed on-line, that is, when the speed and/or memory of the available hardware directly affect the performance of the controller algorithm. Then minimization of the rule base is a requirement.

6. Conclusions

A novel tuning and rule reduction algorithm is proposed as a design tool for the development of knowledge-based fuzzy controllers. The developed algorithm is applied to the fuzzy rule-based control of a household vacuum cleaner.

First a *new equality measure* is defined to compare the equality of two fuzzy sets. This measure is first used in the computation of *inconsistency levels* of the fuzzy rules so that the inconsistent rules can be better distinguished.

Then a new measure, which is termed as the *redundancy level*, is proposed so that the rules with higher redundancy levels can be removed from the fuzzy rule base without affecting the overall performance level of the fuzzy controller. Experimental results are given for the control of a household vacuum cleaner demonstrating the efficient use of the proposed algorithm.

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