



A new edited k -nearest neighbor rule in the pattern classification problem

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Abstract

A new edited k -nearest neighbor (k -NN) rule is proposed. For every sample y in the edited reference set, all the k - or $(k + \ell)$ -nearest neighbors of y must be in the class to which y belongs. Here ℓ denotes the number of samples which tie with the k th nearest neighbor of y with respect to the distance from y . The performance of the rule proposed has been investigated using three classification examples. As a result, it is shown that the rule proposed will yield good results in many pattern classification problems. © 2000 Pattern Recognition Society. Published by Elsevier Science Ltd. All rights reserved.

Keywords: Pattern classification problems; k -Nearest neighbor rule; Wilson's edited k -nearest neighbor rule; Edited reference set; Number of samples misclassified

1. Introduction

In pattern classification problems, the k -nearest neighbor (k -NN) rule [1–3] is a simple nonparametric decision rule. In the rule, an input sample is assigned to the class to which the majority among its k -nearest neighboring labeled samples are assigned. The k -NN rule [1–3] has been widely used since it is effective, when probability distributions of the feature variables are not known, and therefore, Bayes decision rule [2] is not applicable.

Wilson [4] proposed an edited k -NN rule to improve the 1-NN rule [1–3]. In his rule, editing the reference set is first performed: Each sample in the reference set is classified using the k -NN rule [1–3] and the set formed by eliminating it from the reference set. All the samples misclassified are then deleted from the reference set. Afterward, any input sample is classified using the 1-NN rule [1–3] and the edited reference set. Wilson's edited k -NN rule [4] has yielded good results in many finite-sample-size problems, although its asymptotic optimality has been disproved [5,6].

Dasarathy [7] has developed a condensing method to edit the reference set: His method provides the minimal consistent subset (MCS) which is used as the edited reference set. All the samples in the reference set can be correctly classified using the 1-NN rule [1–3] and the MCS. Dasarathy [7] has shown that the performance of the 1-NN rule [1–3] suffers little degradation when a given large reference set is replaced by its much smaller MCS. Kuncheva [5] has applied a genetic algorithm [8] to editing the reference set for the k -NN rule [1–3]. Some fitness functions for the genetic algorithm [8] have been proposed, and the fitness function including a penalizing term is found to be effective to edit the reference set.

In the editing method using the MCS [7] and that using the genetic algorithm [5,8], the number of samples in the edited reference set is considerably small, and therefore, the computational burden to classify input samples is much reduced. However, classification accuracy in these two editing methods [5,7] is usually found not to be higher than that in the k -NN rule [1–3]. In Wilson's edited k -NN rule [4], the number of samples in the edited reference set is usually not so small. With respect to classification accuracy, however, his rule is usually better in practical classification problems than

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the editing method using the MCS [7] and that using the genetic algorithm [5,8].

The purpose of this work is to propose a new edited k -NN rule. In the rule proposed, high classification accuracy is more weighted than decrease in the number of samples in the reference set. Three classification examples are presented to test the rule proposed. The performance of the rule proposed has been compared with those of the k -NN rule [1–3] and Wilson's edited k -NN rule [4].

2. A new edited k -NN rule

In this section, a new edited k -NN rule has been proposed. Let $W_0 = \{x_1, x_2, \dots, x_n\}$ be the set of n labeled samples, namely, a reference set. In the rule proposed, the reference set W_0 is edited as follows:

- E-1. For each labeled sample x_i , $1 \leq i \leq n$, find the k -nearest neighbors of x_i from the set $W(x_i)$ which is given by $\{x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n\}$. Let $d_j(x_i)$ be the distance from x_i to its j th nearest neighbor, and assume $d_1(x_i) \leq d_2(x_i) \leq \dots \leq d_k(x_i)$. Let $V(k, x_i)$ be the set of the k -nearest neighbors of x_i . If ℓ different labeled samples in $W(x_i)$ satisfy the condition that the distance from x_i to each of them is equal to $d_k(x_i)$, include these ℓ labeled samples in $V(k, x_i)$, i.e., obtain the $(k + \ell)$ -nearest neighbors of x_i . Check the condition that all the samples in $V(k, x_i)$ should be assigned to the class to which x_i is assigned. If the condition is satisfied, take x_i as a typical sample in the class to which x_i belongs; otherwise, x_i is not any typical sample.
- E-2. Obtain the set of all the typical samples as the edited reference set $W_1(k)$.

Any input sample is classified using the k' -NN rule [1–3] and $W_1(k)$, where k' is not restricted to 1. In the practical use of the rule proposed, the values of k and k' should be optimally determined as follows:

- K-1. Determine the range of k and that of k' , properly. Express any possible pair of k and k' as (k, k') .
- K-2. For each pair (k, k') , obtain $W_1(k)$ and classify each sample x in W_0 using the k' -NN rule [1–3] and the set $W_1(k, x)$. Here $W_1(k, x)$ is the set formed by eliminating x from $W_1(k)$, if x is in $W_1(k)$. If x is not in $W_1(k)$, the set $W_1(k, x)$ is given by $W_1(k)$. Evaluate the number of samples misclassified for each pair (k, k') .
- K-3. Find the pair (k_m, k'_m) which yields the minimum number of samples misclassified, and take it as the optimal pair of k and k' . If more than one pair yield the minimum number of samples misclassified, take all these pairs as the optimal pairs of k and k' .

When more than one pair are obtained as optimal pairs of k and k' , any input sample should be classified

using the k' -NN rule [1–3] and $W_1(k)$ for all these pairs of k and k' . The input sample should be assigned to the majority class which is determined by the classification results obtained for all these pairs of k and k' . If a tie is present among classes determined by the classification results, the input sample cannot be assigned to a class arbitrarily chosen from the classes which tie.

3. Experimental results and discussion

Three classification examples are presented to test the rule proposed. For the sake of comparison, the k -NN rule [1–3] and Wilson's edited k -NN rule [4] are also tested in the examples. The values of k and k' are taken in the range of $1 \leq k \leq 15$ and $1 \leq k' \leq 15$. In Wilson's edited k -NN rule [4], the 1-NN rule [1–3] is usually used with the edited reference set to classify input samples, i.e., k' is restricted to 1. In this work, however, the k' -NN rule [1–3] with $k' \geq 1$ is used in Wilson's edited k -NN rule [4].

With respect to classification accuracy, the rule proposed and Wilson's edited k -NN rule [4] are estimated as follows: Each sample x in the data set of the example is classified using the k' -NN rule [1–3] and the edited reference set, if x is not in the edited reference set. If x is in the edited reference set, the set formed by eliminating x from the edited reference set is obtained as the reference set, and x is classified using the k' -NN rule [1–3] and the reference set thus obtained. The number of samples misclassified is then evaluated.

Classification accuracy of the k -NN rule [1–3] is estimated as follows: Each sample x in the data set of the example is classified by the k -NN rule [1–3] in which the k -nearest neighbors of x are obtained from the set formed by eliminating x from the data set. The number of samples misclassified is then evaluated.

The first example is classification of the Iris data set [9]. The data set consists of 150 flowers which are classified into three classes, namely, Setosa, Versicolor and Virginica. Each class includes 50 flowers. Every flower is characterized by four features, and therefore, it is expressed by a four-dimensional feature vector. Table 1 shows the number of samples taken from each class to form the edited reference set in the rule proposed and that in Wilson's edited k -NN rule [4]. Evaluation of the number of samples misclassified has been systematically performed. The results of the rule proposed and those of Wilson's edited k -NN rule [4] are shown in Tables 2 and 3, respectively. The results of the k -NN rule [1–3] are shown in Table 4.

The second example is classification of the butterfly-type data set which was presented by Pham and Corrochano [10]. The data set consists of 51 two-dimensional points. It is classified into two classes forming the "wings" of the butterfly. Fig. 1 depicts the data set and

Table 1
Number of samples taken from each class in editing the Iris data set

<i>k</i>	Rule proposed				Wilson's edited <i>k</i> -NN rule			
	Setosa	Versicolor	Virginica	Total	Setosa	Versicolor	Virginica	Total
1	50	47	47	144	50	47	47	144
2	50	47	45	142	50	47	47	144
3	50	45	43	138	50	47	47	144
4	50	44	41	135	50	47	47	144
5	50	43	36	129	50	47	48	145
6	50	43	36	129	50	47	47	144
7	50	42	34	126	50	46	49	145
8	50	40	34	124	50	47	48	145
9	50	39	33	122	50	47	48	145
10	50	36	32	118	50	46	48	144
11	50	35	31	116	50	48	48	146
12	50	33	29	112	50	47	48	145
13	50	31	29	110	50	47	48	145
14	50	31	28	109	50	47	49	146
15	50	30	26	106	50	47	49	146

Table 2
Number of samples misclassified by the rule proposed in the Iris data set

<i>k'</i>	<i>k</i>														
	1	2	3	4	5, 6	7	8	9	10	11	12	13	14	15	
1	5	5	4	5	6	6	5	5	7	9	11	10	12	8	
2	5	5	4	5	6	6	5	5	7	9	11	10	12	8	
3	4	4	4	4	8	8	9	9	9	9	9	7	10	9	
4	4	4	4	4	7	7	8	8	9	9	9	8	10	10	
5	4	4	3	4	6	9	9	9	9	9	10	9	10	8	
6	4	4	3	3	8	8	8	8	8	9	9	10	10	9	
7	3	2	4	3	9	9	8	8	7	8	9	9	10	10	
8	3	2	2	3	8	8	8	8	7	9	9	10	10	10	
9	3	3	2	2	8	9	9	9	7	9	9	10	10	10	
10	3	2	2	3	8	9	9	9	9	9	9	9	10	10	
11	3	3	2	4	9	9	9	9	8	8	10	11	11	13	
12	3	3	3	3	9	9	9	9	9	9	10	10	11	11	
13	3	3	3	4	10	9	8	9	8	9	10	10	13	13	
14	3	3	2	5	9	9	9	9	8	10	10	11	11	11	
15	4	3	3	8	11	9	8	8	8	9	13	11	14	10	
Average	3.60	3.33	3.00	4.00	8.13	8.27	8.07	8.13	8.00	8.93	9.87	9.67	10.93	10.00	

shows that there is much overlapping between two classes 1 and 2. The number of samples taken from each class to form the edited reference set is shown in Table 5. The numbers of samples misclassified are shown in Tables 6–8.

The third example is classification of the data set of 162 two-dimensional points which are shown in Fig. 2. The data set was first used by Yan [11]. As shown in Fig. 2,

four classes 1–4 are present in the data set. Table 9 shows the number of samples taken from each class to form the edited reference set. Tables 10–12 show the numbers of samples misclassified.

As is clear from Tables 1, 5 and 9, the number of samples in the edited reference set in the rule proposed is equal to or less than that in Wilson's edited *k*-NN rule [4]. This is due to the fact that for any sample *x* in the

Table 3
Number of samples misclassified by Wilson’s edited k -NN rule in the Iris data set

k'	k									
	1, 2, 3, 4, 6	5	7, 14	8	9	10	11	12	13	15
1	5	5	5	5	5	5	6	6	6	6
2	5	5	5	5	5	5	6	6	6	6
3	4	4	4	4	4	4	5	5	5	5
4	4	4	4	4	4	4	5	5	6	5
5	4	4	4	4	4	4	4	4	4	4
6	4	4	4	4	4	4	4	4	4	4
7	3	5	5	4	4	4	4	4	5	5
8	3	4	4	4	3	3	3	4	3	4
9	3	3	4	3	5	4	5	3	3	4
10	3	4	5	3	4	4	4	3	3	5
11	3	3	3	3	3	3	4	3	3	3
12	3	5	5	4	5	5	5	4	4	5
13	3	4	5	3	5	5	5	3	3	5
14	3	3	4	3	4	4	4	3	3	4
15	4	4	4	4	3	3	3	4	3	4
Average	3.60	4.07	4.33	3.80	4.13	4.07	4.47	4.07	4.07	4.60

Table 4
Number of samples misclassified by the k -NN rule in the Iris data set

$k = k'$															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Average
6	6	6	6	5	6	5	5	5	6	4	5	5	4	4	5.20

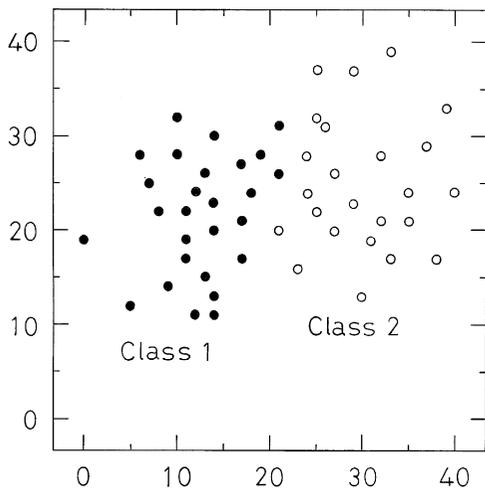


Fig. 1. Butterfly-type data set.

reference set, the condition for x to be included in the edited reference set in the rule proposed is severer than that in Wilson’s edited k -NN rule [4]: In Wilson’s edited k -NN rule [4], the majority among the k -nearest neighbors of x must be in the class to which x belongs for x to be included in the edited reference set. In the rule proposed, however, all the k - or $(k + \ell)$ -nearest neighbors

Table 5
Number of samples taken from each class in editing the butterfly-type data set

k	Rule proposed			Wilson’s edited k -NN rule		
	Class 1	Class 2	Total	Class 1	Class 2	Total
1	27	22	49	27	23	50
2	25	20	45	27	23	50
3	25	18	43	25	24	49
4	24	17	41	26	24	50
5	22	17	39	26	24	50
6	21	15	36	26	24	50
7	21	14	35	27	24	51
8	21	14	35	26	24	50
9	21	14	35	25	23	48
10	19	13	32	26	23	49
11	18	13	31	25	23	48
12	17	11	28	25	23	48
13	16	11	27	25	23	48
14	16	10	26	26	23	49
15	16	10	26	26	23	49

Table 6
Number of samples misclassified by the rule proposed in the butterfly-type data set

k'	k												
	1	2	3, 4	5	6	7, 8, 9	10	11	12	13	14	15	
1	1	1	1	0	0	0	0	0	1	1	1	1	
2	1	1	1	0	0	0	0	0	1	1	1	1	
3	1	0	1	0	1	1	1	1	1	1	1	1	
4	0	0	1	0	1	1	0	0	1	1	1	1	
5	0	1	1	1	1	1	1	1	2	2	2	2	
6	0	1	1	1	1	1	1	1	1	1	1	1	
7	0	1	1	1	1	1	1	1	1	1	2	1	
8	0	1	1	0	1	1	1	1	1	1	1	1	
9	1	1	1	1	1	1	1	1	2	1	2	2	
10	1	1	1	1	1	1	1	1	1	1	2	2	
11	2	1	1	1	1	1	1	1	3	3	3	3	
12	1	1	1	1	1	1	1	1	2	2	2	2	
13	2	1	1	1	1	2	2	1	2	2	2	2	
14	1	1	1	1	1	2	1	1	2	2	2	1	
15	1	1	1	1	2	2	2	2	2	2	3	3	
Average	0.80	0.87	1.00	0.67	0.93	1.07	0.93	0.87	1.53	1.47	1.73	1.60	

of x obtained from the procedure E-1 must be in the class to which x belongs.

In the rule proposed, the condition for a sample to be included in the edited reference set becomes severer with

Table 7
Number of samples misclassified by Wilson's edited k -NN rule in the butterfly-type data set

k'	k					
	1, 2	3	4, 5, 6, 8	7	9, 11, 12, 13	10, 14, 15
1	1	1	1	1	1	1
2	1	1	1	1	1	1
3	2	2	2	2	2	2
4	1	1	1	1	1	1
5	1	1	1	1	1	1
6	1	1	1	1	1	1
7	0	2	1	0	2	1
8	1	2	2	1	1	1
9	2	3	3	3	3	3
10	2	3	3	2	3	3
11	3	3	3	3	3	3
12	2	3	3	3	2	2
13	2	3	3	3	2	2
14	2	3	3	2	2	2
15	2	3	3	2	3	3
Average	1.53	2.13	2.07	1.73	1.87	1.80

Table 8
Number of samples misclassified by the k -NN rule in the butterfly-type data set

$k = k'$															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Average
1	1	2	1	1	1	0	1	3	2	3	3	3	2	2	1.73

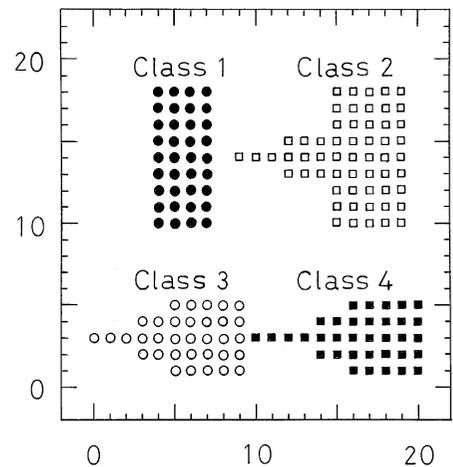


Fig. 2. Data set of 162 points.

Table 9
Number of samples taken from each class in editing the data set of 162 points

k	Rule proposed					Wilson's edited k -NN rule				
	Class 1	Class 2	Class 3	Class 4	Total	Class 1	Class 2	Class 3	Class 4	Total
1	36	57	33	34	160	36	57	33	35	161
2	36	56	33	34	159	36	57	33	35	161
3	36	56	33	33	158	36	57	34	34	161
4	36	56	31	33	156	36	57	34	34	161
5	36	56	31	33	156	36	56	34	34	160
6	35	55	29	33	152	36	56	34	34	160
7	35	55	29	32	151	36	56	34	33	159
8	35	55	29	32	151	36	56	34	33	159
9	33	55	28	32	148	36	56	34	33	159
10	33	55	28	32	148	36	56	34	33	159
11	33	55	24	32	144	36	56	34	33	159
12	33	54	24	32	143	36	56	34	34	160
13	31	54	24	32	141	36	56	34	33	159
14	31	54	24	32	141	36	56	34	33	159
15	31	54	24	31	140	36	56	34	33	159

Table 10
Number of samples misclassified by the rule proposed in the data set of 162 points

k'	k									
	1, 2	3	4, 5	6	7, 8	9, 10	11	12	13, 14	15
1	0	1	0	0	1	1	0	1	0	1
2	0	1	0	0	1	1	0	1	0	1
3	1	1	1	2	2	2	2	2	1	1
4	1	1	1	2	2	1	1	2	1	1
5	3	3	2	2	2	2	2	2	1	1
6	3	3	2	2	2	1	1	2	1	1
7	3	3	2	2	2	1	1	2	2	2
8	3	3	2	2	2	1	1	2	2	2
9	3	3	2	2	3	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2
11	3	3	3	2	3	2	2	2	2	2
12	3	3	3	2	2	2	2	2	2	2
13	3	3	3	2	2	2	2	2	2	2
14	3	3	3	2	2	2	2	2	2	2
15	3	3	3	2	3	3	2	2	2	2
Average	2.27	2.40	1.93	1.73	2.07	1.67	1.47	1.87	1.47	1.60

increasing k . Therefore, the number of samples in the edited reference set becomes smaller with increasing k . This tendency is found in Tables 1, 5 and 9. In the tables, however, the number of samples in the edited reference set in Wilson's edited k -NN rule [4] is not so changed by k . In the special case of $k = 1$ and $\ell = 0$, the edited reference set in the rule proposed is identical with that in Wilson's edited k -NN rule [4]. This special case is found for $k = 1$ in Table 1.

From Tables 2–4, the minimum number of samples misclassified in the Iris data set [9] is estimated to be 2 in the rule proposed, 3 in Wilson's edited k -NN rule [4] and 4 in the k -NN rule [1–3]. Further, it is found from Tables 2–4 that the results of the rule proposed for $k = 2$ and 3 are much better than those of the k -NN rule [1–3] and Wilson's edited k -NN rule [4] for $1 \leq k \leq 15$: The average number of samples misclassified in the range of $1 \leq k' \leq 15$ is very small for $k = 2$ and 3 in the rule

Table 11
Number of samples misclassified by Wilson’s edited k -NN rule in the data set of 162 points

k'	k		
	1, 2	3, 4, 5, 6, 12	7, 8, 9, 10, 11, 13, 14, 15
1	1	0	1
2	1	0	1
3	1	1	1
4	1	1	1
5	2	3	3
6	2	3	3
7	3	3	3
8	2	3	3
9	3	3	3
10	2	3	3
11	2	3	3
12	2	3	3
13	3	3	3
14	3	3	3
15	3	3	3
Average	2.07	2.33	2.47

Table 12
Number of samples misclassified by the k -NN rule in the data set of 162 points

$k = k'$																Average
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		
1	1	1	1	2	2	3	3	3	3	3	2	3	3	3	2.27	

proposed. However, Table 2 shows that the results of the rule proposed for $k \geq 5$ are not good. This will be due to serious decrease of samples in the edited reference set for $k \geq 5$.

From Tables 6–8, the minimum number of samples misclassified in the butterfly-type data set [10] is obtained as zero in the rule proposed, Wilson’s edited k -NN rule [4] and the k -NN rule [1–3]. Tables 6–8 show that in the evaluation of the average number of samples misclassified in the range of $1 \leq k' \leq 15$, the results of the rule proposed for $1 \leq k \leq 11$ are much better than those of the k -NN rule [1–3] and Wilson’s edited k -NN rule [4] for $1 \leq k \leq 15$.

From Tables 10–12, the minimum number of samples misclassified in the data set of 162 points [11] is evaluated as zero in the rule proposed and Wilson’s edited k -NN rule [4], and as 1 in the k -NN rule [1–3]. Tables 10–12 show that in the evaluation of the average number of samples misclassified in the range of $1 \leq k' \leq 15$, the results of the rule proposed for $9 \leq k \leq 15$ are much

better than those of the k -NN rule [1–3] and Wilson’s edited k -NN rule [4] for $1 \leq k \leq 15$.

4. Conclusions

A new edited k -NN rule has been proposed, and its performance is investigated using three classification examples. In the examples, the rule proposed has yielded better results than the k -NN rule [1–3] and Wilson’s edited k -NN rule [4]. This results from the fact that good edited reference sets are obtained in the rule proposed. The condition for a sample to be included in the edited reference set in the rule proposed is severer than that in Wilson’s edited k -NN rule [4]. In the rule proposed, every sample y in the edited reference set for any k must be surrounded by its k - or $(k + \ell)$ -nearest neighbors which are all in the class to which y belongs. Accordingly, every sample y in the edited reference set is very proper as a typical sample in the class to which y belongs.

The number of samples in the edited reference set in the rule proposed is shown to be equal to or less than that in Wilson’s edited k -NN rule [4], and it decreases with increasing k . Therefore, editing the reference set using a too large value of k is not good, and k should be properly determined in some range of k . If the values of k and k' are optimally determined by taking the procedures from $k-1$ to $k-3$, the rule proposed will yield good results in many pattern classification problems.

5. Summary

A new edited k -NN rule has been proposed. In the rule, the condition for a sample x to be included in the edited reference set is that all the k - or $(k + \ell)$ -nearest neighbors of x must be in the class to which x belongs. Here ℓ denotes the number of samples which tie with the k th nearest neighbor of x with respect to the distance from x . Accordingly, this condition is severer than that in Wilson’s edited k -NN rule, and every sample in the reference set in the rule proposed is very proper as a typical sample in the class to which it belongs. The number of samples in the edited reference set in the rule proposed is equal to or less than that in Wilson’s edited k -NN rule, and it decreases with increasing k .

Three classification examples are presented to test the rule proposed. The k -NN rule and Wilson’s edited k -NN rule are also performed in the examples. The rule proposed is found to yield better results than the k -NN rule and Wilson’s edited k -NN rule. Any input sample can be classified using the k' -NN rule and the edited reference set. If the values of k and k' are optimally determined by taking the procedures presented in this work, the rule proposed will yield good results in many pattern classification problems.

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