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AN INTUITIONISTIC FUZZY SETS APPLICATION TO THE K - NN METHOD

S. T. Hadjitodorov

Department of Biomedical Informatics, Central Laboratory of Biomedical Engineering, Bulgarian Academy of Sciences, Acad. G. Bonchev str. bl.105, 1113 Sofia, Bulgaria
E - mail: STHAD@BGCICT.BITNET

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ABSTRACT: A version for increasing of the classification accuracy of one of the basic statistical nonparametrical methods, the K - NN method, is proposed. Here the idea of including fuzzy information in K-NN method is applied in a new way. The distances are modified by means of the pattern degrees of membership and nonmembership to the classes to which the pattern belongs. Thus for each of the labeled samples its typicalness is taken into consideration.

1. INTRODUCTION

It is known [1,3,5,6,9,10,11,12,] that the K - nearest neighbors (K- NN) method is one of the most often used and most accurate methods in pattern recognition. Its basic assumption is that inside of the small hypersphere around a given pattern the probability density function is approximately constant [5]. That's why the classifiers have to be restricted with a small value of K. In cases of small sample size and overlapping classes this requirement causes accuracy decreasing. Many modifications are published which improve the method with respect to increasing the classification accuracy and minimization of classification time. Fuzzy versions of this method are also developed. In [11,12], for example, the degree of membership of a given pattern (vector) to a given class is determined by weighted averaging of the degree of membership of the K-NNs. The reciprocal values of the distances between the vector and its K-NNs are used as weights. In [10] the degree of membership of a pattern with unknown classification to a class is calculated as a mean of the class membership of the K-NNs of the pattern. Some generalizations of this method are developed in [1,3]. In [14] an attempt to combine the advantages of the fuzzy description of the patterns and classes is made. The idea is to include expert experience (knowledge) in pattern recognition in a new way. In this way the pattern distribution in the space is implicitly taken into consideration. The goal is to cope with cases of small sample size and overlapping classes. In that paper a transformation of the distances using fuzzy description (degree of membership) is proposed and it is proved that the probability of error is less than or equal to the same in case of classical K - NN. Some fuzzy decision rules were combined with these modified distances. The combination of statistical and fuzzy approaches in pattern recognition intuitively leads to an improvement of the recognition results. Here an improvement of this method by means of application of degree of nonmembership besides the degree of membership is described

2. THEORY

Let the feature set Z is given by

$$Z = \{ Z_j \}, j=1,2, \dots , n \tag{1}$$

The set of the values of the feature Z_j is ,

$$Z_j = \{ z_{jk} \}, k = 1,2, \dots, v. \tag{2}$$

The set of patterns (the sample, training set) is

$$X = \{ x_l \}, l = 1,2, \dots ,N;\ x_{l\,i} \text{ is an n-dimensional vector.} \tag{3}$$

The set of the classes is

$$\Omega = \{ \omega_i\},\ i = 1,2, \dots ,M. \tag{4}$$

The fuzzy description can be used not only in cases of insufficient and small training set but it can be used also in case of large training set in order to improve the classification accuracy. Here we propose the using of the degrees of membership $\mu(x)$ and nonmembership $\nu(x)$ for classification purposes, i.e. we propose to use the intuitionistic fuzzy sets (IFSs). It is known that the IFS A* in E is an object having the form [15,16]

$$A^* = \{\langle x, \mu(x), \nu(x)\rangle, x \in E\} \tag{5}$$,

where μ_A: $E \rightarrow [0,1], \nu_A : E \rightarrow [0,1]$ define the degree of membership and nonmembership of the element $x \in E$ to the set A which is a subset of E, respectively, and for every $x \in E$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{6}$$

Here we consider the degree of membership as typicalness and confidence for membership and the degree of nonmembership as nontypicalness and diffidence for membership.

2.1. Membership determination

In order to implement IFS in a K-NN rule we should have at our disposal $\mu(x)$ and $\nu(x)$ for each pattern from the training set. These memberships can be determined from the sample X in several different ways: on the basis of the geometrical properties of the classes[1,7,8,12]; on the basis of the probabilistic measures and expert knowledge[1,4,5,13,14].

For the purposes of the transformation of the distances we propose the following way for evaluating of the degree of membership and nonmembership to the class ω_i to which x (a pattern from the training) belongs:

$$\mu_{\omega_i}(x) = e^{-d_m}, \tag{7}$$

where d_m is the distance between the pattern x and the mean vector for class ω_i, $x_{im} = 1/N_i\ \Sigma x_i$, N_i is the number of the training patterns belonging to class ω_i;

$$\nu_{\omega_i}(x) = e^{-d_{mN}}, \tag{8}$$

where d_{mN} is the distance between the pattern x and the mean vector of the class ω_N that is the nearest mean vector among the other classes (to which x doesn't belong) i.e.

$$d_{mN}= \min \{d_{mj}\} , \; j=1,2,\ldots,M; \; i\neq j \tag{9}$$

In such way each class ω_i, i=1,2,...,M is presented as an IFS. The value of $\mu_{\omega_i}(x)$ determine the subset of level to that the pattern belongs, i.e. the typicalness of x for class ω_i. If (6) is not satisfied then $\nu_{\omega_i}(x)$ is evaluated according to

$$\nu_{\omega_i}(x) = \min \{(1-\mu_{\omega_i}(x),\nu_{\omega_i}(x)\} \tag{10}$$

2.2 Distances ransformation

Let the pattern x_u with unknown classification is given. Our goal is to classify that pattern into one of the classes using the idea of K - NN method. Therefore for x_u the distances d_l between x_u and the patterns from the sample x_l , l = 1,2, ... ,N are determined and the smallest K are chosen. No restrictions are imposed on the kind of the metrics, in that the distances are calculated. The basic idea is to transform these distances and to take into account the degree of membership (that fix the subset of level and the typicalness of x_l for the class) and the degree of nonmembership. We propose the transformations to be performed according to:

$$D_l = d_l/\mu_l; \qquad D'_l= d_l.\nu_l, \tag{11}$$

To take into consideration the fact that D_l is calculated to each x_l which belong to fixed class ω_i (11) gets the form:

$$D_{li}= d_{li}/\mu_{li}; \qquad D'_{li}= d_{li}.\nu_{li}, \tag{12}$$

After the transformation we obtain two sets S_k and S'_k of K distances. These transformed distances are proposed to be used for classification. In [14] the following proposition for application of transformed distances D_l is proved:

PROPOSITION: If the transformed distances are used as distances in K - NN method then the probability of error is less than or equal to the same in case of nontransformed distances.

Similar proposition could be easily generalized for both distances D_{li} and D'_{li}.

2.3. Classification scheme

A two level classification scheme is proposed. On the first level the classical K-NN rule for the two sets of NN S_k and S'_k is applied, i.e. if $k_1,k_2, \ldots,k_M$, are the number of the patterns (among K) which respectively belong to $\omega_1, \omega_2 , \ldots \omega_M$, then :

$$x_u\in \omega_i \;, \text{ if } k_i = \max\{k_1,k_2, \ldots,k_M\} \tag{13}$$

Let with respect to S_k , $x_u\in \omega_i$ and with respect to S'_k , $x_u\in \omega_{i'}$. Then on the second level the decision rule is:

$$\text{if } i = i' \Rightarrow x \in \omega_i$$

$$\text{if } i \neq i' \Rightarrow \text{refusal from classification} \qquad (14),$$

i.e. if the decisions from S_k and S'_k coincide we have simple classification,otherwise we have ambiguous classification and refusal from decision

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