Anatomy of the Attraction Basins: Breaking with the Intuition

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***Abstract*—This is a summary of the motivations, contributions, experiments and conclusions of the article titled Anatomy of the Attraction Basins: Breaking with the Intuition that has been accepted for publication in Evolutionary Computation journal (doi: 10.1162/EVCO_a_00227) [1].**

I. Introduction

Solving combinatorial optimization problems efficiently requires the development of algorithms that consider the specific properties of the problems. In this sense, local search algorithms are designed over a neighborhood structure that partially accounts for these properties. Considering a neighborhood, the space is usually interpreted as a natural landscape, with valleys and mountains. Under this perception, it is commonly believed that, if maximizing, the solutions located in the slopes of the same mountain belong to the same attraction basin, with the peaks of the mountains being the local optima. Unfortunately, this is a widespread erroneous visualization of a combinatorial landscape. Thus, our aim is to clarify this aspect, providing a detailed analysis of, first, the existence of plateaus where the local optima are involved, and second, the properties that define the topology of the attraction basins, picturing a reliable visualization of the landscapes. Some of the features explored in this paper have never been examined before. Hence, new findings about the structure of the attraction basins are shown. The study is focused on instances of permutation-based combinatorial optimization problems considering the 2-exchange and the insert neighborhoods. Particularly, we work with the permutation flowshop scheduling problem (PFSP), the linear ordering problem (LOP) and the quadratic assignment problem (QAP). As a consequence of this work, we break away from the extended belief about the anatomy of attraction basins.

II. Analysis of the topology of the attraction basins

A. Local optima and local optimal plateaus

A best-improvement local search algorithm returns a local optimum that necessarily match one, and only one, of the three following options: (i) be a strict local optimum, (ii) belong to a local optimal plateau, or (iii) belong to an open plateau. Thus, we show the number of strict local optima, the number of local optimal plateaus and the number of open plateaus that appear in the instances chosen for the analysis. In general, for the three analyzed problems, the presence of plateaus is remarkable. One of the main conclusions derived from this study is that, usually, the algorithm gets trapped inside the plateaus. That is, although we find instances with plateaus composed by just two solutions, this algorithm is not designed to detect and escape from them.

B. Roundness of the attraction basins

An attraction basin is considered to be round if all the solutions at distance 1, 2, ... until a certain distance r from the local optimum or the plateau are within the attraction basin. We record, for the smaller instances, for each local optimum, the proportion of solutions belonging to its attraction basin that are at different distances from it. On average, the local optima are located in the frontier of the attraction basins, as they have a number of neighboring solutions belonging to a different attraction basin. However, on average, we also find solutions at the longest distances from them that do belong to their attraction basins. This structure clearly differs from the concept of roundness.

C. Centrality of the local optima

We aim to study the position of the local optima within the attraction basins. We focus on the centrality of the local optima inside the attraction basins. The local optima (or the plateaus) are considered to be centered if they minimize the average distance to the rest of the solutions in the attraction basin. We observe that, in general, the local optima of the instances have a lower average distance than the rest of the solutions of the attraction basins. We conclude that the local optima are located close to the barycenter of the attraction basins, as they have the minimal (or almost the minimal) average distances to the rest of the solutions: in general, they are almost centered within the attraction basins.

D. Interior and frontier of the attraction basins

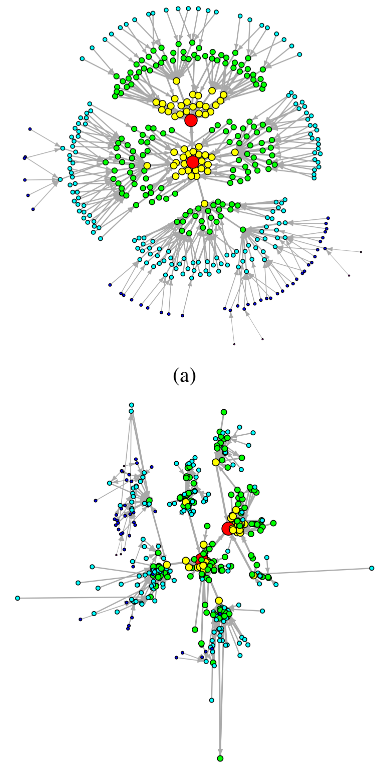
There is a really low number of solutions in the interior of the attraction basins. Surprisingly, the average number of neighboring attraction basins is really high (close to the total number of local optima). Basically, almost all of the attraction basins are neighboring attraction basins. Moreover, those solutions that are close to the local optimum have a large proportion of their neighbors inside the same attraction basin. The solutions that are far from the local optimum have a small number of neighbors in the same attraction basin, while the number of different neighboring attraction basins is large. If we take all the solutions of an attraction basin, the connectivities with other different attraction basins are higher for those solutions at long distances from the local optima.

The attraction basins are intertwined in the search space: the paths drawn by the local search algorithm are interconnected with each other or, at least, they are close to each other.

III. Visualization of the attraction basins

We give a representation of a specific attraction basin by means of a network showing all the paths encountered until the local optimum is reached. Each node of the graph represents one solution belonging to the attraction basin. Edges between nodes indicate that the node at the end of the edge is the best neighbor of the node at the start of the edge. The color of the nodes changes with the distance to the local optimum. Particularly, red, yellow, green, light blue, dark blue and purple are used to represent the solutions at distances 0 (the local optima), 1, 2, 3, 4 and 5, respectively. The size of the nodes and the width of the edges also decrease as the distance to the local optimum increases.

Figure 1 presents two different graphs illustrating the same attraction basin of a local optimal plateau of a PFSP instance when using the 2-exchange neighborhood. Figure 1(a) represents this attraction basin considering the steps that the algorithm takes until it reaches the local optimal plateau (and not the distances between the solutions). In an attempt to visualize this attraction basin in a more realistic way, we force the graph to take into account the distances between all the solutions that belong to the attraction basin (Figure 1(b)). The real structure of the attraction basins is more complex than one could try to imagine. More visualizations can be found at the website: http://www.sc.ehu.es/ccwbayes/members/leticia/AnatomyOfAB/visualization/visualizeOneAB.html.



(a)

(b)

Fig. 1. Visualization of an attraction basin of a local optimal plateau found an instance of the PFSP with the 2-exchange neighborhood, considering only the steps of the algorithm (a) and taking into account, as far as possible, the distances between all the solutions (b).

IV. Conclusion

The attraction basins can be understood as long intertwined rivers, that flow into the different local optima, instead of being mountains in a landscape. In fact, each attraction basin is composed of several of those rivers ending at the same local optimum, while at the same time, each of them could have different tributaries. Moreover, the end of those rivers can be made up of more than one local optimum that have the same objective function value, forming a plateau. The local optima or the plateaus composed by local optima are centered in the attraction basin. Nevertheless, we should be cautious with this perception, because by understanding the combinatorial optimization landscapes as if we were in a 3D natural landscape, we could be misunderstanding the real anatomy. The understanding of the landscapes in combinatorial optimization has been one of the main challenges when developing and improving algorithms. This work not only breaks with an erroneous extended belief about the attraction basin shapes, but also provides valuable information for the design of new algorithms based on local search.

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References

[1] L. Hernando, A. Mendiburu and J.A. Lozano. Anatomy of the Attraction Basins: Breaking with the Intuition. *Evolutionary Computation*, 1-32. 2018. In Press.