

Free Double Hierarchy Hesitant Fuzzy Linguistic Term Sets in Group Decision-Making

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Abstract—Hesitant fuzzy linguistic term sets have been an active field of research in recent times. Notwithstanding its usefulness to capture the human way of reasoning using linguistic expressions involving different levels of precision, in some situations they do not depict enough details. In this paper, we present a new kind of linguistic term sets, called free double hierarchy linguistic term sets, and their corresponding free double hierarchy hesitant fuzzy linguistic elements, in order to describe the complexity of linguistic expressions used by the decision makers in a more accurate and precise way. Furthermore, an order and a distance between free double hierarchy hesitant fuzzy linguistic elements are introduced to present an approach based on the TOPSIS method to rank alternatives with free double hierarchy hesitant fuzzy linguistic information by taking into consideration the opinions of a group of decision makers.

Index Terms—Linguistic modeling, Group decision-making, Free double hierarchy linguistic term sets, Free double hierarchy hesitant fuzzy linguistic term sets, TOPSIS

I. INTRODUCTION

This work summarizes the contribution presented by Montserrat-Adell et al. of a new methodology for Group Decision-Making (GDM) under linguistic assessments [4]. In this paper, an extension of the Double Hierarchy Hesitant Fuzzy Linguistic Term Sets (DHHFLTSS) is proposed. Different approaches involving linguistic assessments have been presented in the decision-making literature to deal with the uncertainty connate with human preferences [2], [3]. Furthermore, different levels of precision can be used to give more realistic assessments when uncertainty increases [5]. To model this kind of situations, HFLTSS were introduced in [6].

With the aim of allowing the decision makers (DMs) to express themselves in a more natural way while being more precise on their assessments, Gou et al. introduced the DHSFLTSS [1] by adding a second hierarchy to each linguistic term of the first scale. This allows expressions such as *very good* or *slightly bad* to arise.

This new extension, called the Free Double Hierarchy Hesitant Fuzzy Linguistic Term Sets (FDHFLTSS), overcomes the main issues presented by the DHHFLTSS. The introduction of this new structure, allows each DM involved in a GDM situation to choose the second hierarchy LTS that he or she thinks that suits it better, with as many terms as desired. These

terms can be different for each term of the first hierarchy and for each DM.

Furthermore, an order and a distance between FDHFLTEs are also presented in this paper in order to compare and quantify distances between linguistic assessments provided by the decision makers by means of the aforementioned structure. These order and distance are used to introduce a free double hierarchy approach based on the well-known multi-criteria decision-making TOPSIS ranking method, enabling us to rank alternatives that have been assessed by means of free double hierarchy hesitant fuzzy linguistic information.

The rest of this paper is structured as follows: Section II summarizes the main contributions of the original paper and Section III presents the most remarkable conclusions and lines of futures research of the topic.

II. FREE DOUBLE HIERARCHY

DHLTSS are a useful tool to describe, in a mathematical way, possible linguistic assessments provided by the decision makers in a group decision-making problem. Yet they present some shortcomings such as the fact that the second hierarchy scale is the same for all the linguistic terms of the first hierarchy. However, whilst “*extremely*” applies well for some linguistic terms such as “*good*” (resulting in “*extremely good*”), it does not apply that well for some other linguistic terms, leading to confusing, or even meaningless, linguistic terms such as “*extremely regular*”. Same thing happens with “*almost perfect*”, which is a clear and common linguistic expression, and “*almost regular*”, which is not clear. In order to fix this issue, we propose the following extension of the DHLTSS:

Definition 1: Let $\mathcal{S} = \{s_t \mid t = -\tau, \dots, -1, 0, 1, \dots, \tau\}$ and $\mathcal{O}^t = \{o_k^t \mid k = -\zeta_t, \dots, -1, 0, 1, \dots, \zeta_t\}$ for all $t \in \{-\tau, \dots, -1, 0, 1, \dots, \tau\}$ be the first and second hierarchies of LTSs respectively. Then, a *Free Double Hierarchy Linguistic Term Set (FDHFLTSS)*, $\mathcal{S}_{\mathcal{O}}^{\mathcal{F}}$, can be expressed in a mathematical form as:

$$\mathcal{S}_{\mathcal{O}}^{\mathcal{F}} = \{s_{t < o_k^t} \mid \begin{array}{l} t = -\tau, \dots, -1, 0, 1, \dots, \tau; \\ k = -\zeta_t, \dots, -1, 0, 1, \dots, \zeta_t \end{array} \}.$$



Each $s_{t \langle o_k^t \rangle}$ is called a *Free Double Hierarchy Linguistic Term (FDHLT)*, where o_k^t expresses the second hierarchy linguistic term when the first hierarchy linguistic term is s_t .

Remark 1: For symmetry reasons, as it can be seen in Figure 1, the FDHLTs contained in $\mathcal{O}_{-}^{-\tau} = \{s_{-\tau \langle o_{-\zeta_{-\tau}}^{-\tau} \rangle}, s_{-\tau \langle o_{-\zeta_{-\tau+1}}^{-\tau} \rangle}, \dots, s_{-\tau \langle o_{-1}^{-\tau} \rangle}\}$ as well as in $\mathcal{O}_{+}^{\tau} = \{s_{\tau \langle o_{1}^{\tau} \rangle}, s_{\tau \langle o_{2}^{\tau} \rangle}, \dots, s_{\tau \langle o_{\zeta_{\tau}}^{\tau} \rangle}\}$ should be dismissed. Therefore, from now on, $\mathcal{O}^{-\tau}$, \mathcal{O}^{τ} and $\mathcal{S}_{\mathcal{O}}^{\mathcal{F}}$ are used throughout this paper, without loss of generality, for $(\mathcal{O}^{-\tau} \setminus \mathcal{O}_{-}^{-\tau})$, $(\mathcal{O}^{\tau} \setminus \mathcal{O}_{+}^{\tau})$ and $\mathcal{S}_{\mathcal{O}}^{\mathcal{F}} \setminus (\mathcal{O}_{-}^{-\tau} \cup \mathcal{O}_{+}^{\tau})$ respectively to simplify the notation.

Example 1: Let $\mathcal{S} = \{s_{-2} = \text{"null"}, s_{-1} = \text{"bad"}, s_0 = \text{"regular"}, s_1 = \text{"good"}, s_2 = \text{"perfect"}\}$ be the first hierarchy LTS with $\tau = 2$ and let

$$\begin{aligned} \mathcal{O}^{-2} &= \{o_{0}^{-2} = \text{"completely"}, o_{1}^{-2} = \text{"almost"}, \\ &\quad o_{2}^{-2} = \text{"close to"}\}, \\ \mathcal{O}^{-1} &= \{o_{-3}^{-1} = \text{"extremely"}, o_{-2}^{-1} = \text{"unusually"}, \\ &\quad o_{-1}^{-1} = \text{"very"}, o_{0}^{-1} = \text{"simply"}, o_{1}^{-1} = \text{"pretty"}, \\ &\quad o_{2}^{-1} = \text{"slightly"}, o_{3}^{-1} = \text{"hardly"}\}, \\ \mathcal{O}^0 &= \{o_{-2}^0 = \text{"very low"}, o_{-1}^0 = \text{"low"}, o_{0}^0 = \text{"medium"}, \\ &\quad o_{1}^0 = \text{"high"}, o_{2}^0 = \text{"very high"}\}, \\ \mathcal{O}^1 &= \{o_{-3}^1 = \text{"hardly"}, o_{-2}^1 = \text{"slightly"}, \\ &\quad o_{-1}^1 = \text{"pretty"}, o_{0}^1 = \text{"simply"}, o_{1}^1 = \text{"very"}, \\ &\quad o_{2}^1 = \text{"unusually"}, o_{3}^1 = \text{"extremely"}\}, \\ \mathcal{O}^2 &= \{o_{-2}^2 = \text{"close to"}, o_{-1}^2 = \text{"almost"}, \\ &\quad o_{0}^2 = \text{"completely"}\} \end{aligned}$$

be the respective second hierarchy LTSs for each s_t , for $t = -2, \dots, 2$ (with $\zeta_{-2} = \zeta_0 = \zeta_2 = 2$ and $\zeta_{-1} = \zeta_1 = 3$), defining the associated FDHLTS $\mathcal{S}_{\mathcal{O}}^{\mathcal{F}}$ represented in Figure 1. Thus, the possible linguistic assessments such as “slightly bad” or “almost perfect” can be expressed by means of FDHLTs of $\mathcal{S}_{\mathcal{O}}^{\mathcal{F}}$ as $s_{-1 \langle o_2^{-1} \rangle}$ and $s_{2 \langle o_2^1 \rangle}$.

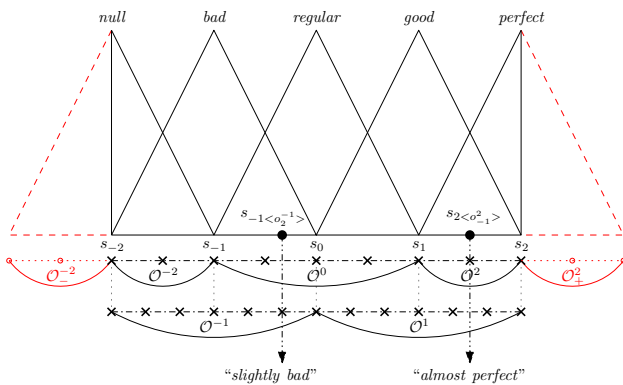


Fig. 1. FDHLTS $\mathcal{S}_{\mathcal{O}}^{\mathcal{F}}$ from Example 1.

III. CONCLUSIONS

Based on the weak points that HFLTSS and DHHFLTSS have in the GDM problems, a new structure is presented in this paper to capture linguistic assessments with more details.

This structure enables the decision makers to be more accurate when evaluating an alternative by means of linguistic terms.

On the one hand, Free Double Hierarchy Linguistic Term Sets are introduced as a double hierarchy LTS in which the second hierarchy LTS can be different for each term of the first hierarchy LTS. Thus, each decision maker can choose the second hierarchy LTS that better suits each linguistic term of the first hierarchy according to his/her criterion.

On the other hand, Free Double Hierarchy Hesitant Fuzzy Linguistic Elements (FDHFFLEs) and Free Double Hierarchy Hesitant Fuzzy Linguistic Term Sets (FDHFFLTSs) are defined as a useful tool to depict the hesitancy inherent in human reasoning.

Lastly, an order and a distance between FDHFFLEs are defined to enable us to present a free double hierarchy approach based on the TOPSIS method, called the FDHFFL-TOPSIS. This method is useful to sort alternatives in a GDM situation when the decision makers provide their assessments by means of free double hierarchy linguistic information in order to be more precise.

Future research is focused on two main directions: on the one hand, other methods to aggregate free double hierarchy hesitant fuzzy linguistic information will be studied as well as new measures within the set of FDHFFLTSs such as distance definitions, similarity measures or preference relations.

On the other hand, the structure of FDHFFLTSs will also be applied on the field of recommender systems among end-users that express their opinions by means of this kind of linguistic information.

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REFERENCES

- [1] X. J. Gou, H. C. Liao, Z. S. Xu, F. Herrera, “Double hierarchy hesitant fuzzy linguistic term set and MULTIMOORA method: A case of study to evaluate the implementation status of haze controlling measures,” *Information Fusion* 38, pp. 22–34, 2017.
- [2] F. Herrera, E. Herrera-Viedma, L. Martínez, “A fuzzy linguistic methodology to deal with unbalanced linguistic term sets,” *IEEE Transactions on Fuzzy Systems* 16 (2), pp. 354–370, 2008.
- [3] E. Herrera-Viedma, F. Herrera, F. Chiclana, “A consensus model for multiperson decision making with different preference structures,” *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans* 32, pp. 394–402, 2002.
- [4] J. Montserrat-Adell, Z. S. Xu, X. J. Gou, N. Agell, “Free Double Hierarchy Hesitant Fuzzy Linguistic Term Sets: An application on ranking alternatives in GDM,” *Information Fusion* 47, pp.45–59, 2019.
- [5] F. Prats, L. Roselló, M. Sánchez, N. Agell, “Using L-fuzzy sets to introduce information theory into qualitative reasoning,” *Fuzzy Sets and Systems* 236 (1), pp. 73–90, 2014.
- [6] R. M. Rodríguez, L. Martínez, and F.Herrera, “Hesitant fuzzy linguistic terms sets for decision making,” *IEEE Transactions on Fuzzy Systems* 20 (1), pp. 109–119, 2012.