Some remarks on "Preference stability over time: The time cohesiveness measure"

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***Abstract*—This work introduces a non-traditional approach about the problem of measuring the stability of agents' preferences over time under the assumption of considering dichotomous opinions. The general concept of *time cohesiveness measure* is introduced as well as a particular formulation based on the consideration of any two successive moments of time, *the sequential time cohesiveness measure*. Moreover, some properties of the novel measure are also provided. Finally, a case of study is presented. This essay presents the main contributions of the paper entitled "*Preference stability along time: The time cohesiveness measure*" published in the journal *Progress in Artificial Intelligence*.**

***Index Terms*—Time cohesiveness measure; Dichotomous opinions; Preference stability; Patients' preferences**

I. Introduction

Intertemporal decision making is an important research area and it has been obtaining attention from several research fields such as Economics, Health Economics, Social Choice, Psychology, Marketing, Decision Analysis, Neuroscience, and so on. One of the main topics of this area is the study of preference stability over time. Traditionally, intertemporal preferences have ussually been considered permanent by theoretical and empirical studies (see [1], [2] and [3], among other) and the research to date has tend to explore preference stability over time by means of statistical methods.

In order to enhance the preference stability topic, the aim of this contribution is to develop a new tool capable of measuring preference stability from a non-traditional perspective. For this purpose, the notion of preference stability is considered in the same vein that the notion of cohesiveness. This seems natural because the measurement of preference stability resembles the notion of measurement of cohesiveness over time in the sense that the maximum value captures the notion of full stability, that is, unanimity along time, while the minimum value captures the notion of total lack of stability, that is, total disagreement along time.

Taking into account the previous contributions on preference stability and cohesiveness measure, this paper is focused on an intertemporal decision-making problem where a set of agents express their opinions on an alternative along different moments of time. To be precise, agents have to approve or disapprove the alternative under study at diverse points of time. Thus, the paper objective is to determine how much stability or cohesiveness agents opinions conveys to the group on the alternative along time. In order to measure such stability, a new general approach is defined, the *time cohesiveness measure*. Moreover, an specific formulation of the time cohesiveness measure is introduced, the *sequential time cohesiveness measure* as well as a study of its analytic properties. Under this approach, the stability of preferences is understood like the probability that for a randomly chosen moment of time, two randomly chosen agents have the same opinion at such a time and its consecutive.

Furthermore, the measurement proposed is put in practice in a real case of study to emphasize its applicability. In particular, the stability of preferences for life-sustaining treatments in terminally cancer patients' last year of life is analysed.

This contribution is structured as follows. Section 2 introduces the notation and the novel proposals to measure preference stability. Section 3 includes a brief description of the paper application. Finally, some closing comments are provided.

II. The time cohesiveness measure: Notation and definitions

Let $\mathbf{N} = \{1, 2, ..., N\}$ a set of agents or experts. Agents express their opinions on an alternative, x, at different time moments $\mathbf{T} = \{t_1, \ldots, t_T\}$ by means of dichotomous opinions.

A *time preference profile* of a set of agents $\mathbf{N}$ on an alternative x at T different time moments is a matrix $\mathbf{P} = \left(P_{it_j}\right)_{N\times T}$ where P_{it_j} is the opinion of the agent i over alternative x at t_j moment, in the sense

$$P_{it_j} = \begin{cases} 1 & \text{if agent } i \text{ approves } x \text{ at the } t_j \text{ time,} \\ 0 & \text{otherwise.} \end{cases}$$

Let $\mathbb{P}_{N\times T}$ denote the set of all such $N \times T$ matrices.

A time preference profile $\mathbf{P}$ is *unanimous* if alternative x is approved (resp. disapproved) over $\mathbf{T}$ by all agents. In matrix terms, if the time preference profile $\mathbf{P} \in \mathbb{P}_{N\times T}$ is constant, $\mathbf{P} = (1)_{N\times T}$ (resp. $\mathbf{P} =$

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$(0)_{N\times T}$). Any permutation σ of the agents $\{1, 2, ..., N\}$ determines a time preference profile $\mathbf{P}^\sigma$ by permutation of the rows of $\mathbf{P}$, that is, row i of the profile $\mathbf{P}^\sigma$ is row $\sigma(i)$ of the profile $\mathbf{P}$.

Definition 1: A *time cohesiveness measure* for a group of agents $\mathbf{N} = \{1, ..., N\}$ on an alternative x is a mapping $\tau : \mathbb{P}_{N\times T} \to [0, 1]$ that assigns a number $\tau(\mathbf{P}) \in [0, 1]$ to each time preference profile $\mathbf{P}$, with the properties:

i) $\tau(\mathbf{P}) = 1$ if and only if $\mathbf{P}$ is unanimous (full stability).
ii) $\tau(\mathbf{P}^\sigma) = \tau(\mathbf{P})$ for each permutation σ of the agents and $\mathbf{P} \in \mathbb{P}_{N\times T}$ (anonymity).

Definition 2: The *sequential time cohesiveness measure* for a group of agents $\mathbf{N} = \{1, ..., N\}$ on an alternative x is the mapping $\tau_S : \mathbb{P}_{N\times T} \to [0, 1]$ given by

$$\tau_S(\mathbf{P}) = \sum_{b\in\{0,1\}} \frac{1}{T-1} \cdot \frac{\sum_{j=1}^{j=T-1} n_{b,b}^{t_j,t_{j+1}} \cdot (n_{b,b}^{t_j,t_{j+1}} - 1)}{N(N-1)}$$

where $n_{0,0}^{t_j,t_{j+1}}$ denotes the number of agents that disapprove alternative x at t_j and keep their opinion at the following point of time t_{j+1}. Similarly, $n_{1,1}^{t_j,t_{j+1}}$ denotes the number of agents that approve alternative x at t_j and keep their opinion at the following point of time t_{j+1}.

Intuitively, it measures the probability that for a randomly chosen moment of time, two randomly chosen agents of a group have the same opinion upon an alternative at the moment of time selected and its consecutive.

The sequential cohesiveness measure verifies the following meaningful properties: reversal invariance, time-reducibility, replication monotonicity, minimum time stability, leaving minimum time stability, time monotonicity and convergence to full stability.

III. A case study of preference stability in Clinical Decision Making

So as to implement our proposal for measuring the stability of preferences over time of a group of agents, this contribution is inspired and motivated by the study of Tang et al. [4]. In [4], the authors examined the stability of life-sustaining treatment preferences at end of life of cancer patient's last year by means of an statistical approach. Authors collected patients' preferences about life support choices by the *Life Support Preferences Questionnaire* (LSPQ) [5].

Based on this study, a finite set of 257 patients is considered in this contribution. Theses patients expressed their opinions by dichotomous opinions on a finite set of 3 treatments for life-sustaining at end of life being: cardiopulmonary resuscitation (CPR), dying in an intensive care unit (ICU) and mechanical ventilation support (MSV). Patients expressed their preferences about approving o disapproving the aforementioned treatments at four different time moments along their illness. Thus, patients' opinions can be formalized by means of a time preference profile for each treatment $\mathbf{P}^{CPR}$, $\mathbf{P}^{ICU}$ and $\mathbf{P}^{MSV}$. The information provided by the three previous time preference profiles can be group in Table I.

Treatment	$n_{0,0}^{t_1,t_2}$	$n_{1,1}^{t_1,t_2}$	$n_{0,0}^{t_2,t_3}$	$n_{1,1}^{t_2,t_3}$	$n_{0,0}^{t_3,t_4}$	$n_{1,1}^{t_3,t_4}$
CPR	190	34	210	24	228	15
ICU	142	79	156	63	184	26
MSV	170	44	187	38	209	25

Table I: Number of patients that approve and disapprove different treatments at different moments of time

Using Definition 2, the sequential time cohesiveness measure for each profile, that is, for each treatment can be computed. Table II shows such values including all moments of time and all patients.

Treatment	Profile	$\tau_S(\mathbf{P})$
CPR	$\mathbf{P}^{CPR}$	0.676
ICU	$\mathbf{P}^{ICU}$	0.449
MVS	$\mathbf{P}^{MVS}$	0.562

Table II: Values of the sequential time cohesiveness measure for each treatment

Moreover, these results were explored in depth in [6]. The set of patients was partitioned, differentiating between patients with and without metastases.

IV. Closing comments

In this work, a non-traditional perspective on preference stability topic is set out. The problem of measuring the degree of cohesiveness in a setting where agents express their opinions on an alternative at different times by means of an approval or disapproval evaluation is explored. A general concept of *time cohesiveness measure* is introduced and a particular formulation based on the consideration of any two successive times is proposed, namely *the sequential time cohesiveness measure*. Some properties which make our proposal appealing are also provided. The applicability of our proposal to real situations is emphasized by means of adapting a factual problem in Clinical Decision Making. Concretely, the case of terminally cancer patients' last year of life is studied using the new sequential time cohesiveness measure.

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