OWA OPERATORS IN THE CALCULATION OF THE AVERAGE GREEN-HOUSE GASES EMISSIONS

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Abstract

This study proposes, through weighted averages and ordered weighted averaging operators, a new aggregation system for the analysis of average gases emissions. We introduce the ordered weighted averaging operators gases emissions, the induced ordered weighted averaging operators gases emissions, the weighted ordered weighted averaging operators gases emissions and the induced probabilistic weighted ordered weighted averaging operators gases emissions. These operators represent a new way of analyzing the average gases emissions of different variables like countries or regions. The work presents further generalizations by using generalized and quasi-arithmetic means. The article also presents an illustrative example regarding the calculations of the average gases emissions in the European region.

Keywords: Green-house gases emission, aggregation operators, decision making, ordered weighted average

1. Introduction

In the very recent decades, because of an enormous growth of the population and the necessity to provide food for them from one hand and the other hand an immethodical consumption of fossil fuel, our planet is experiencing an unexampled growth in terms of green-house gases (GHG) emission such as CO_2, CH_4 and N_2O in its atmosphere that cause an ascending amount of global warming year by year and a drastic climate change [6,20].

There are many works that study the ways that can lead the GHG emission toward the minimization. [19], evaluate the potential influence of vehicle electrification on grid infrastructure and road-traffic green-house emission. [5] study the impact of electrical power generation on GHG emission in Europe.

Besides, although these works exist but it seems vital to present a comprehensive forecast about the future of countries based on the experts' opinions to provide a clear plan and make a suitable decision to decrease this emission in any of the studied sectors and under various conditions.

Aggregation operators in the related literature with the aim of decision making are diverse and each of them can be used to collect the information [1,2,9-14, 21,23,25]. These techniques give importance to the variables according to certain available subjective or objective findings [17].

A very popular aggregation operator is the weighted average. This aggregation operator is flexible to use in a wide range of problems. Another popular aggregation operator is the ordered weighted average (OWA) [24]. The OWA operator provides a parametrized family of aggregation operators between the minimum and the maximum, weighting the data according to the attitudinal character of the decision-maker. Based on this operator and with the purpose of expanding it, many authors expand and generalize it [3,7,18,22]. There are several types for the concept of expanding and generalizing and the most important item is the form of integrating OWA operator with some key concepts such as, using the induced variables, the probability and the weighted average. [23] propose some new aggregation operators such as the induced ordered weighted geometric averaging (IOWGA) operator, generalized induced ordered weighted averaging (GIOWA) operator, hybrid weighted averaging (HWA) operator.

The purpose of this work is to concentrate on the analysis of the use of the aggregation operators in the calculation of green-house gases (GHG) emission with the aim of developing better decision-making techniques. To this end, the paper studies several aggregation operators including the WA [1], OWA [24], OWAWA and IOWAWA [11], IOWA [27], POWAWA and IPOWAWA operator [12]. With the use of each operator, a new operator for GHG emission is produced including the OWA GHG emission (OWAGE), induced OWA GHG emission (IOWAGE), ordered weighted averaging-weighted average GHG emission (OWAWAGE), induced OWAWA GHG emission (IOWAWAGE), probabilistic OWAWA GHG emission (POWAWAGE) and induced probabilistic OWAWA GHG emission (IPOWAWAGE).

The work also presents further generalizations by using generalized and quasi-arithmetic means obtaining the generalized OWAGE (GOWAGE). The aim of this approach is to show a more general framework in the analysis of averages by using complex aggregations including with geometric and quadratic averages. The study presents a wide range of particular types of aggregations under this approach.

The work presents an application regarding the calculation of the average gases emissions in Europe. For doing so, the paper considers a multi-expert aggregation problem where four experts analyze the expected average emissions of each European country for the next period. From, the analysis develops several aggregation methods based on the tools developed in the paper including the OWAGE, IOWG and OWAWAGE operators. The main advantage of the OWA operator is the possibility of under or overestimate the information according to the attitudinal character of the decision maker. Thus, depending on the degree of optimism or pessimism of the decision maker, the results may lead to different decisions and interpretations of the information.

2. Preliminaries

2.1. The weighted average (WA)

The WA [1] is one of the most common aggregation operators found in the literature. It has been used in a wide range of applications. It can be defined as follows.

Definition 1. A WA operator of dimension n is a mapping $WA: R^n \to R$ that has an associated weighting vector V, with $v_j \in [0,1]$ and $\sum_{i=1}^{n} v_i = 1$, such that:

$$WA(a_1, \ldots, a_n) = \sum_{i=1}^{n} v_i a_i \qquad (1)$$

where a_i represents the argument variable.

2.2. The OWA operators

The OWA operator [24] is an aggregation operator that provides a parametrized family of aggregation operators that include the maximum, the minimum and the average criteria as especial cases and can be defined as follows.

Definition 2. An OWA operator of dimension n is a mapping $OWA: R^n \to R$ that has an associated weighting vector W of dimension n such that sum of the weight is 1 and $w_j \in [0,1]$, then:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \qquad (2)$$

where b_j is the jth largest of the a_i.

2.3. The Induced OWA operator (IOWA)

The IOWA operator [27] is an extension of the OWA operator. The main difference between OWA and IOWA is that the reordering step is not developed with the values of the arguments a_i. In this case, the reordering step is carried out with order inducing variables. The IOWA operator also includes as particular cases the maximum, the minimum and the average criteria. It can be defined as follows.

Definition 3. An IOWA operator of dimension n is a mapping $IOWA: R^n \times R^n \to R$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, such that:

$$IOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \qquad (3)$$

where b_j is the a_i value of the IOWA pair $\langle u_i, a_i \rangle$ having the jth largest u_i. u_i is the order-ranking variable and a_i is the argument variable.

2.4. The probabilistic ordered weighted averaging-weighted average (POWAWA)

The POWAWA [17] operator uses probabilities, weighted average and OWA in the same formulation. It unifies these three concepts by considering the degree of importance that each concept has in the aggregation, depending on the situation considered. The POWAWA operator is defined as follows.

Definition 4. A POWAWA operator of dimension n is a mapping $POWAWA: R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$POWAWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j \tag{4}$$

where b_j is the jth largest of the a_i, each argument a_i has an associated weight v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, a probability p_i with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0,1]$, $\hat{v}_j = C_1 w_j + C_2 v_j + C_3 p_j$, with C_1, C_2 and $C_3 \in [0,1]$, $C_1 + C_2 + C_3 = 1$ and v_j p_j are the weights v_i and p_i ordered according to b_j, that is to say, according to the jth largest of the a_i.

2.5. The Induced probabilistic OWAWA operator

The IPOWAWA [12] is an aggregation operator that extends POWAWA operator that uses order-inducing variables that represent complex reordering processes of an aggregation. Thus, it is an aggregation operator that uses induced variables, the probability, the weighted average and the OWA operator. Moreover, it can assess complex reordering processes by using order-inducing variables. Its main advantage is that it provides a more robust formulation than the POWAWA operator because it includes a wide range of cases. It can be defined as follows.

Definition 5. The IPOWAWA operator of dimension n is a mapping $IPOWAWA: R^n \times R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$IPOWAWA(\langle u_1, e_1 \rangle, \langle u_2, e_2 \rangle, \ldots, \langle u_n, e_n \rangle) = \sum_{j=1}^{n} \hat{v}_j b_j \tag{5}$$

where b_j is the a_i value of the IPOWAWA pair $\langle u_i, e_i \rangle$ having the jth largest u_i, u_i is the order-inducing variable, each argument a_i has an associated weight v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, a probability p_i with $\sum_{i=1}^{n} p_i$ and $p_i \in [0,1]$, $\hat{v}_j = C_1 w_j + C_2 v_j + C_3 p_j$, with C_1, C_2 and $C_3 \in [0,1]$, $C_1 + C_2 + C_3 = 1$, v_j and p_j are the weights v_i and p_i ordered according to b_j, that is to say according to the jth largest of the e_i.

3. Calculation of the average green-house gases (GHG) emission with OWA operators

The purpose of this paper is to calculate the average GHG emission. The average GHG emission represents a numerical value that reports the information of the GHG emission. To calculate this item, using many aggregation operators is possible likewise normal arithmetic mean. These possible aggregation operators could be WA, OWA, IOWA or a combination of them such as OWAWA, IOWAWA, etc. Through using them we prepare some possibilities for the future of GHG emission in different scenarios in a spectrum from the worst case to the best case based on experts' opinions.

The basic operator for analyzing a set of GHG emission is OWAGE. The OWAGE operator is an aggregation operator that analyses an average GHG emission under uncertainty situation. It can be defined as follows for the set of GHG emission $A = \{e_1, e_2, \ldots, e_n\}$:

$$OWAGE(e_1, e_2, \ldots, e_n) = \sum_{j=1}^{n} w_j f_j \tag{6}$$

where f_j is the jth largest of the e_i.

The other significant aggregation operator is the induced OWA (IOWA) that its reordering step is developed with order including variables. So, by using the IOWA operator we obtain IOWA GHG emission (IOWAGE) that can be defined as follows:

$$IOWAGE(\langle u_1, e_1 \rangle, \langle u_2, e_2 \rangle, \ldots, \langle u_n, e_n \rangle) = \sum_{j=1}^{n} w_j f_j \tag{7}$$

where f_j is the e_i value of the IOWA pair $\langle u_i, e_i \rangle$ having the jth largest u_i. u_i is the order-ranking variable and e_i is the argument variable.

It is important to mention that this operator is based on considering no extra information. One of the very important aspects of the average GHG emission is the importance of each of them and in other words, their weights in comparison with each other. To this end it is better to use some approaches of information aggregation that combine OWA operators and WA. In the literature there are some aggregation operators with this structure like, the WOWA operator [21], the hybrid average [9] and the OWAWA operators [8]. In this work we apply OWAWA to obtain the OWAWA GHG emission (OWAWAGE) and it is defined as follows for a set of GHG emission $A = \{e_1, e_2, \ldots, e_n\}$:

$$OWAWAGE(e_1, e_2, \ldots, e_n) = \sum_{j=1}^{n} \hat{v}_j f_j \tag{8}$$

where f_j is the *j*th largest of the e_i, each argument e_i has an associated weight (WA) v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$ and v_j is the weight (WA) v_i ordered according to b_j, that is, according to the *j*th largest of the e_i.

To focus more deeply on our contributions, we implement IOWAWA which is a combination of IOWA operators and WA in the same formulation. By using the IOWAWA operator we obtain IOWAWA GHG emission (IOWAWAGE) that can be defined as follows:

$$IOWAWAGE(\langle u_1, e_1\rangle, \langle u_2, e_2\rangle, \ldots, \langle u_n, e_n\rangle) = \sum_{j=1}^{n} \hat{v}_j f_j \tag{9}$$

where f_j is the e_i value of the IOWAWA pair $\langle u_i, e_i\rangle$ having the *j*th largest u_i, u_i is the order including variable and e_i is the argument variable, each argument e_i has an associated weight (WA) v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_i$ with $\beta \in [0,1]$ and v_j is the weight (WA) v_i ordered according to f_j, that is, according to the *j*th largest u_i.

Besides, the other aspect that can be considered and leads results to a better form is probabilities in the attitudinal character of the decision-maker. For this reason, we apply POWAWA operator. By applying the Eq. (7) we could obtain the probabilistic OWAWA GHG emission (POWAWAGE). It can be defined as follows:

$$POWAWAGE(e_1, e_2, \ldots, e_n) = \sum_{j=1}^{n} \hat{v}_j f_j \tag{10}$$

where f_j is the *j*th largest of the e_i, each argument e_i has an associated weight v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, a probability p_i with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0,1]$, $\hat{v}_j = C_1 w_j + C_2 v_j + C_3 p_j$, with C_1, C_2 and $C_3 \in [0,1]$, $C_1 + C_2 + C_3 = 1$ and v_j p_j are the weights v_i and p_i ordered according to f_j, that is to say, according to the *j*th largest of the e_i.

Let us analyze the different families of IOWAWAGE and POWAWAGE in the following paragraphs

First, we are considering the two main cases of the IOWAWAGE operator that are found by analyzing the coefficient β. Basically:

- If $\beta = 0$, we get the WA.
- If $\beta = 1$, the IOWA operator.
- If $\beta = 1$ and the ordered position of u_i is the same than the ordered position of f_i such that f_j is the *j*th largest of e_i, the OWA operator.
- Note that when β increases, we are giving more importance to the IOWAGE operator and when β decreases, we give more importance to the WA.

Another group of interesting families are the maximum-WAGE, the minimum-WAGE, the step-IOWAWAGE operator and the usual average.

- The maximum-WAGE is found when $w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = Max\{e_i\}$.
- The minimum-WAGE is formed when $w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = Min\{e_i\}$.

The arithmetic-WAGE is obtained when $w_j = 1/n$ for all *j*, and the weighted average is equal to the OWA when the ordered position of *i* is the same as the ordered position of *j*. The arithmetic-WAGE (A-WAGE) can be formulated as follows:

$$A-WAGE(\langle u_1, e_1\rangle, \langle u_2, e_2\rangle, \ldots, \langle u_n, e_n\rangle) = \frac{1}{n}\beta a_i + (1-\beta)\sum_{i=1}^{n} v_i e_i, \tag{11}$$

Note that if $v_i = 1/n$, for all *i*, then, we get the unification between the arithmetic mean (or simple average) and the IOWAGE operator, that is, the arithmetic-IOWAGE (A-IOWAGE). The A-IOWAGE operator can be formulated as follows:

$$A-IOWAGE(\langle u_1, e_1\rangle, \langle u_2, e_2\rangle, \ldots, \langle u_n, e_n\rangle) = \beta\sum_{j=1}^{n} w_j b_j + (1-\beta)\frac{1}{n} e_i. \tag{12}$$

Following the OWA literature [13, 25, 27], we can develop many other families of IOWAWA operators such as:

- The olympic-IOWAWAGE operator ($w_1 = w_n = 0$, and $w_j = 1/(n-2)$ for all others).

- The general olympic-IOWAWAGE operator ($w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $w_{j*} = 1/(n-2k)$, where $k < n/2$).
- The centered-IOWAWAGE (if it is symmetric, strongly decaying from the center to the maximum and the minimum, and inclusive).

Many other particular cases can be studied by looking at different expressions of the weighting vectors and the coefficients C_1, C_2 and C_3. For example:

- If $C_1 = 1$, we obtain the OWAGE operator.
- If $C_2 = 1$, the weighted GHG emission (WGE).
- If $C_3 = 1$, the probabilistic GHG emission (PGE).
- If $C_1 = 0$, the probabilistic weighted averaging GHG emission (PWAGE).
- If $C_2 = 0$, the probabilistic OWA GHG emission (POWAGE).
- If $C_3 = 0$, the OWAWA GHG emission (OWAWAGE) [14].

4. Generalizations with generalized and quasi-arithmetic means

Generalization of the OWA operators is possible to do by generalized and quasi-arithmetic averaging aggregation operators that as the most common one generalized OWA (GOWA) [26] and then quasi-arithmetic OWA (Quasi-OWA) [4] are formed. These functions apply a general framework including particular cases. The GOWA operator applied to the analysis of gases emissions is called GOWA gases emissions (GOWAGE) and is defined as follows.

Definition 6. A GOWAGE operator of dimension n is a mapping $GOWA: R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, such that:

$$GOWAGE(e_1, e_2, ..., e_n) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda} \quad (13)$$

where b_j is the jth largest of the e_i, and λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$.

Like the section 3, this operator also has the particular cases of the maximum, the minimum and the generalized mean (GM). Besides, there are some special cases that can be obtained by maneuvering on the values of λ, such as:

- If $\lambda = 1$, the usual OWAGE operator.
- If $\lambda \rightarrow 0$, the ordered weighted geometric average gases emissions (OWGAGE).
- If $\lambda = 2$, the ordered weighted quadratic average gases emissions (OWQAGE).

Quasi-arithmetic OWA gases emissions (Quasi-OWAGE) operator is the other generalization that uses the quasi-arithmetic means instead of the generalized means. So, it replaces the parameter λ by a strictly continuous monotonic function g.

Definition 7. A Quasi-OWAGE operator of dimension n is a mapping Quasi-OWAGE: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, then:

$$Quasi-OWAGE(e_1, e_2, ..., e_n) = g^{-1}\left(\sum_{j=1}^{n} w_j g\left(b_{(j)}\right) \right) \quad (14)$$

where b_j is the jth largest of the e_i and g is a strictly continuous monotonic function.

5. Conclusions

The purpose of this study is to concentrate on the analysis of the use of the aggregation operators in the calculation of GHG emission with the aim of developing better decision-making techniques. In this study we reviewed some of the important operators of the family of OWA. This review started with simple WA and continued with OWA operator. Moreover, we also analyzed some operators that form by combination of two or more aggregation operators. So, these operators are, IOWAGE, OWAWAGE, IOWAWAGE, POWAWAGE and IPOWAWAGE.

In addition, through these formulations, we found some particular cases in either IOWAWAGE or POWAWAGE operators such as, olympic-IOWAWAGE, S-IOWAWAGE, centered-IOWAWAGE, maximum, minimum and arithmetic probabilistic weighted average, and arithmetic probabilistic ordered weighted average. Furthermore, some other generalizations are developed by using generalized and quasi-arithmetic means obtaining the GOWAGE and the Quasi-OWAGE operators.

The study provides a simple example to review the function of two simple aggregations operators of average green-house gases emission. During this example we review weighted average gases emission (WAGE) and ordered weighted average gases emission (OWAGE) to represent the difference between the result of the calculation based on these operators.

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