

## EDITORIAL

### To share the fame in a fair way, $h_m$ modifies $h$ for multi-authored manuscripts

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**Abstract.** The  $h$ -index has been introduced by Hirsch as a useful measure to characterize the scientific output of a researcher. I suggest a simple modification in order to take multiple co-authorship appropriately into account, by counting each paper only fractionally according to (the inverse of) the number of authors. The resulting  $h_m$ -indices for eight famous physicists lead to a different ranking from the original  $h$ -indices.

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#### 1. Introduction

About two years ago the physicist Hirsch [1] proposed an easily computable index  $h$  to characterize the scientific output of a researcher. This  $h$ -index (or Hirsch index) is defined as the number of papers of a scientist that have been cited  $h$  or more times. It has received immediate attention [2, 3] in the scientific community and has been critically evaluated [3, 4]

and applied to various communities [5], as can be seen by the 72 citations that Hirsch's paper [1] has already accumulated within 2 years which is already more than sufficient to contribute to Hirsch's Hirsch-index. Recently, Hirsch has shown for two samples of 27 and 50 physicists that the correlation between the past and future research output in terms of publications and citations is strongest when measured by the  $h$ -index [6].

One of the criticisms with regard to the  $h$ -index is that it does not take into account multiple co-authorship [5, 6, 7, 8]. Hirsch [1] has already suggested that 'it may be useful . . . to normalize  $h$  by a factor that reflects the average number of co-authors'. Using as normalization the mean number of authors of the papers in the  $h$ -core, i.e. in the  $h$ -defining set of papers, the derived index has been labelled  $h_1$ . As far as I know this has been applied in one study [8] only, although the idea has been discussed more often [5, 7]. However, in my opinion this normalization is not fair as it penalizes authors with some papers with a large number of co-authors, because the average is sensitive to extreme values [8]. Likewise, this division by the mean number of authors has an excessively reductive influence on the contribution of single-author publications to one's  $h$ -index.

## 2. The $h_m$ -index and its visualization

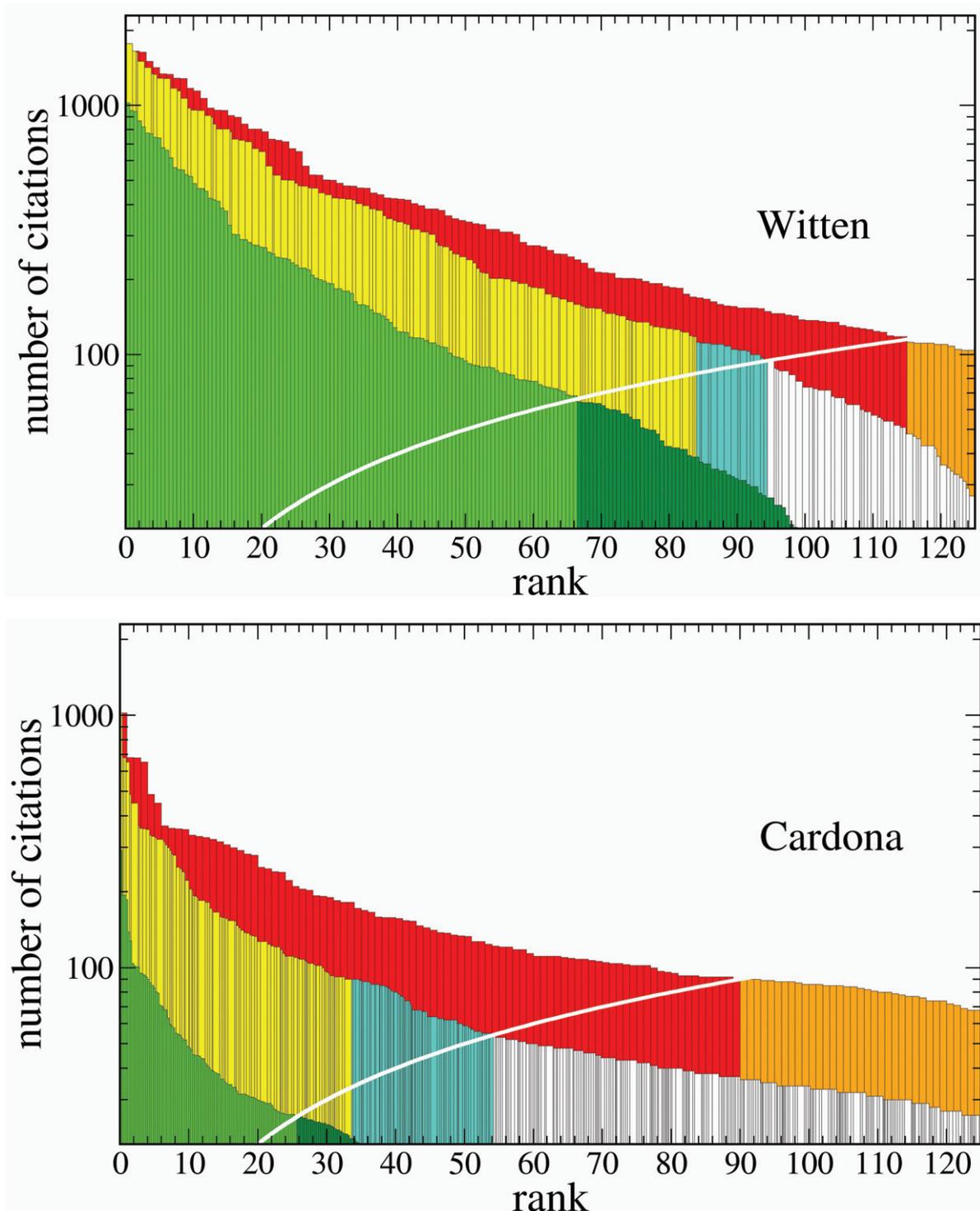
As an alternative, I propose the  $h_m$ -index which is determined in analogy to the  $h$ -index, but counting the papers fractionally according to the number of authors, for example, only as one third for three authors. This yields an effective number which is utilized to define the  $h_m$ -index as that effective number of papers that have been cited  $h_m$  or more times.

The effect of this procedure is visualized in figure 1 in which the citation records of some of the highest-ranked physicists are displayed. For the present investigation, I have selected those scientists who were mentioned by Ball [2]. In addition, I included Hirsch's citation record. All data were obtained in July 2007 via the general search in the Science Citation Index provided by Thomson Scientific in the ISI Web-of-Science (WoS), taking reasonable (and necessary) care that homographs were excluded. Results of my analysis are compiled in table 1.

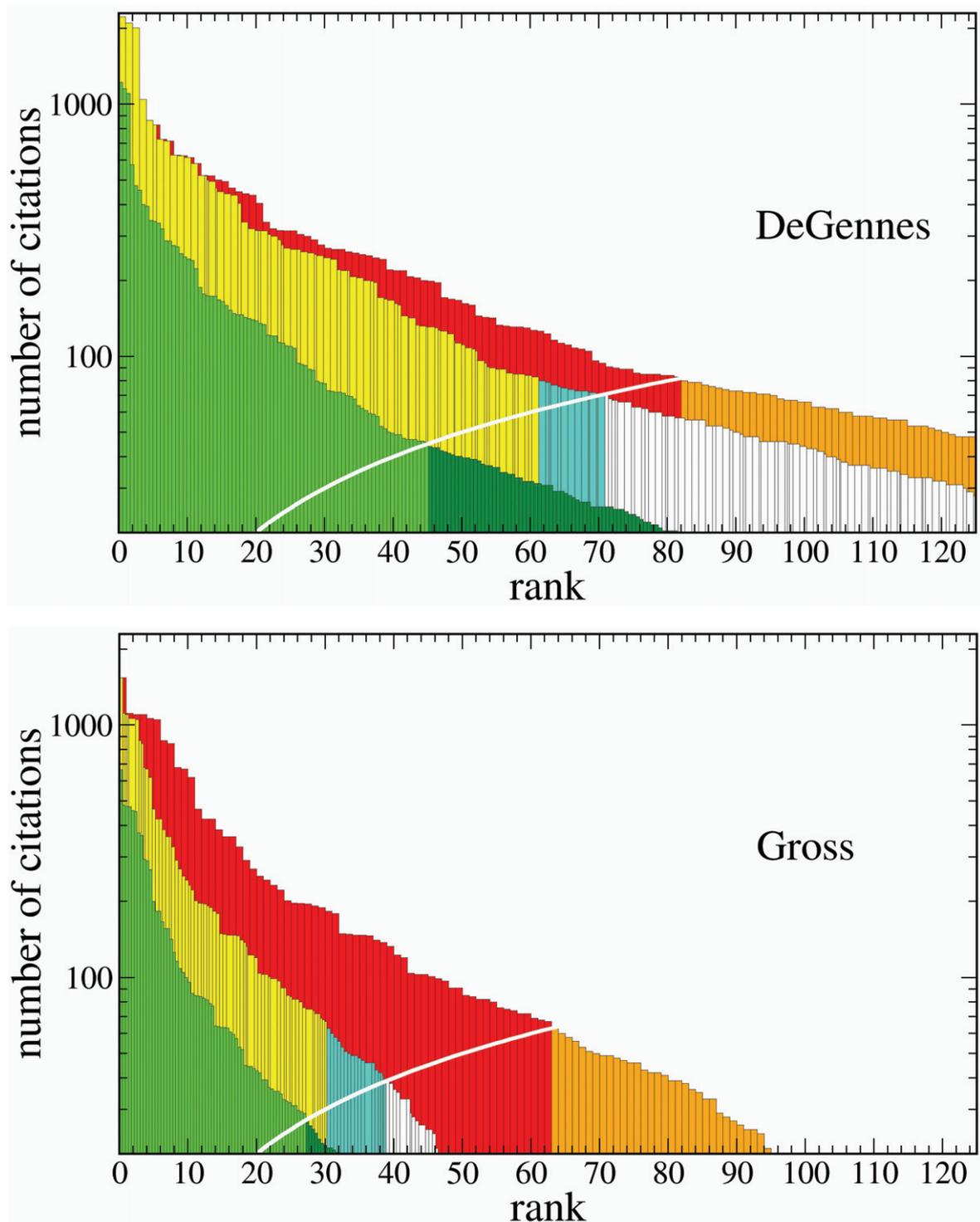
The upper histograms in the plots show the data when all manuscripts are counted in the usual way. One can see distinctive differences like the outstanding high citation counts of deGennes's first three papers. Cardona and Gross have one paper which is cited exceptionally often, as does Wilczek (not shown); Anderson has even seven such papers. The citation count drops relatively fast around and beyond the value of  $h$  for Gross (and for Hirsch, not shown), whereas the plot for Cardona shows a large number of papers with intermediate citation counts around his  $h$ -index. One can expect an increase of his  $h$ -index soon because of the very flat histogram just beyond the present value of his  $h$ -index. A similar behaviour was found for Cohen (not shown).

The middle histograms in the plots show the significant compression of the citation records when the effective number of publications (i.e. counting the papers fractionally) is used to attribute an effective rank instead of the full rank. Now the widths of the histogram bars are no longer equal, but are determined by (the inverse of) the number of authors for each paper.

Obviously, the effective number of papers in the  $h$ -core is much smaller than  $h$ , which means that beyond  $h$  there are papers with more citations than this effective rank. It is straightforward to take these papers into account, as visualized in figure 1. In all the cases, a considerable number of publications with citation counts between  $h$  and  $h_m$  contribute to the  $h_m$ -core, i.e. to the  $h_m$ -defining set. The effect is particularly strong in Cardona's and in



**Figure 1.** The citation counts for four famous physicists. In each plot the upper histogram with wide bars shows the numbers of citations  $c(r)$  versus the rank  $r$  which is attributed to each paper by sorting according to  $c(r)$ , up to the  $h$ -index (red) and beyond (orange). In the middle histograms the effective rank is used so that the original histograms are compressed towards the left (yellow for the first  $h$  papers, turquoise up to the  $h_m$ th paper and white beyond).



**Figure 1.** (Continued.) In the lower histograms the normalization with the mean number of authors of the first  $h$  papers is used, so that the original histograms are compressed to the left as well as downwards (light green up to the  $h_1$ -index and dark green beyond). Note the logarithmic scale for  $c(r)$ . The thick white line displays the function  $c(r) = r$ , so that its intersections with the histograms (from top to bottom) yield the  $h$ -index, the  $h_m$ -index and the  $h_1$ -index, respectively.

**Table 1.** The indices  $h$ ,  $h_m$  and  $h_I$  as well as their ratios  $h/h_I$  and  $h_m/h$  for eight famous physicists. Note that  $h/h_I$  reflects the mean number  $\bar{a}(h)$  of authors of the papers in the  $h$ -core, i.e. in the  $h$ -defining set of papers.  $h_{\text{WoS}}$  denotes the bare index when the WoS data are used without excluding homographs.

Name	$h$	$h_I$	$h/h_I$	$h_m$	$h_m/h$	$h_{\text{WoS}}$
E Witten	115	66.5	1.7	94.4	0.82	115
P W Anderson	99	48.0	2.1	73.5	0.74	100
M L Cohen	97	26.9	3.6	55.9	0.58	107
M Cardona	90	25.6	3.5	54.2	0.60	91
P G deGennes	82	45.1	1.8	70.9	0.86	82
F Wilczek	73	28.5	2.6	45.4	0.62	73
D J Gross	63	27.2	2.3	38.9	0.62	68
J E Hirsch	50	26.0	1.9	39.5	0.79	50

Cohen's case and more or less compensates the relatively strong compression of their histograms which occurs due to the comparatively large number of co-authors. Of course, the  $h_m$ -index remains smaller than the  $h$ -index. Witten and deGennes often have no co-authors, so that their compressed histograms stay close to the original ones.

### 3. The $h_I$ -index and its disadvantages

One can derive the  $h_I$ -index in a similar way as the  $h_m$ -index by counting each paper fractionally but now according to the mean number of authors of the first  $h$  papers and simultaneously scaling the number of citations in the same way. This is visualized by the lower histograms in the plots. Obviously this procedure yields significantly smaller index values, see table 1. I maintain that this reduction in the  $h_I$ -index is excessive. Moreover, it seems unreasonable that clear distinctions that can be seen in the plots are washed out in the values of  $h_I$  which are nearly the same in five of the eight cases considered.

The index  $h_I$  also has the disadvantage that its calculation is restricted to the  $h$ -core, so that further publications with citation counts between  $h_I$  and  $h$  are not taken into account even if they are single-author manuscripts. This appears inappropriate to me, and it can also lead to the strange effect that  $h_I$  might decrease when such a paper attracts further citations and thus advances into the  $h$ -core.

An interesting effect could be observed in Cardona's citation records when the bare WoS data are used without care. About 10% of the papers found are due to homographs, i.e. they have not been written by the famous solid state physicist but by other scientists with the same name. As expected only very few of these 'wrong' papers have high citation counts, but two in the  $h$ -core are conspicuous, because the first has about 900 authors (and the highest overall citation count) and the second has about 700 authors. As a consequence the mean number of authors of the publications in the  $h$ -core appears to be 21, leading to the wrong  $h_I = 4.3$  if these homographs were overlooked. Irrespective of the question whether those publications belong to the data set or not, such a large influence of two papers on the  $h_I$ -index is not appropriate. And it is a strange effect that by including these highly cited papers the  $h_I$ -index is decreased. On the other hand, the influence on the  $h_m$ -index is reasonably negligible, it is increased by 0.07

because these two additional publications (plus a third one with ‘only’ 14 authors which appears wrongly in the  $h_m$ -core) change the effective ranks only a very little.

#### 4. Changing the ranks and sharing the fame

The entries in table 1 have been arranged according to the values of the  $h$ -index for the different authors. It is interesting to note that the differences between the values change significantly when one uses the  $h_m$ -index and that in some cases even the order is changed. Most notably, due to a small number of co-authors deGennes moves forward two positions and his  $h_m$ -index is close to Anderson’s. Hirsch also has few co-authors and thus advances in the  $h_m$ -sorted list and comes close to Wilczek. Witten’s lead is increased. These changes are quantified by the ratio  $h_m/h$  in table 1.

In my opinion these changes are substantial. However, due to the large spread of the original values of the  $h$ -index it is not surprising that the order is not completely mixed up. This can be quantified in terms of Spearman’s rank-order correlation coefficient  $\rho_s(h, h_m) = 0.90$ . The rather high correlation between the  $h$ -index and the  $h_m$ -index is thus not unexpected. Quite on the contrary it shows that the proposed  $h_m$ -index is not completely different from the original  $h$ -index, but just a modification, although (at least in my opinion) an important and necessary modification. In contrast, the excessive changes in the  $h_1$ -index are also reflected in the significantly smaller values of the correlation coefficients  $\rho_s(h, h_1) = 0.55$  and  $\rho_s(h_m, h_1) = 0.69$ .

The highest citation counts of Gross and Wilczek coincide, because they refer to the same publication of which Gross and Wilczek are co-authors. According to the definition of the  $h_m$ -index, this paper is counted half for both authors in the present analysis, so that its impact is equally shared which in my opinion demonstrates the appropriateness of this definition. Short of knowing how much each co-author has contributed to a publication, this seems to me the fairest way of sharing the fame.

The  $h_m$ -index also allows for a straightforward accumulation of the impact when data sets are united in order to quantify the combined impact of the publications of several people like all scientists in an institute or a department. For example, in the case of the Gross–Wilczek paper, the total contribution to their  $h_m$ -indices is two times one half, as it should be, while it is fully taken into account twice for the  $h$ -indices which is unreasonably high. On the other hand, it counts  $1/2.32 + 1/2.56 = 0.82$ -fold for their  $h_1$ -indices, which is too low and thus likewise unjustified.

#### 5. Computing the index

In principle, the graphical presentation of the citation counts in figure 1 is sufficient to determine the values of the indices as indicated by the white line. For clarification, in the following I specify the formal evaluation.

Let  $r$  be the rank that is attributed to a paper when the publication list of an author is sorted by the number  $c(r)$  of citations. This arrangement is offered, e.g. in the WoS data base. Hirsch’s index  $h$  is determined from

$$h = \max_r(r \leq c(r)), \quad (1)$$

where each paper is fully counted for the (trivial) determination of its rank

$$r = \sum_{r'=1}^r 1. \quad (2)$$

Counting a paper with  $a(r)$  authors only fractionally, i.e. by  $1/a(r)$  yields an effective rank

$$r_{\text{eff}}(r) = \sum_{r'=1}^r \frac{1}{a(r')}, \quad (3)$$

which is used to define the  $h_m$ -index as

$$h_m = \max_r (r_{\text{eff}}(r) \leq c(r)) \quad (4)$$

in correspondence with (1).

For the  $h_I$ -index, the average number of authors of the first  $r$  papers is calculated as the mean

$$\bar{a}(r) = \frac{1}{r} \sum_{r'=1}^r a(r'), \quad (5)$$

which is used as a normalization yielding

$$h_I = \frac{h}{\bar{a}(h)}. \quad (6)$$

The same result can be obtained in analogy to the definitions of the  $h$ - and the  $h_m$ -indices, if one employs  $\bar{a}(r)$  for the (trivial) determination of the normalized rank

$$r_I(r) = \frac{1}{\bar{a}(r)} \sum_{r'=1}^r 1 = \frac{r}{\bar{a}(r)}, \quad (7)$$

with which the  $h_I$ -index is then calculated as

$$h_I = \max_r \left( r_I(r) \leq \frac{c(r)}{\bar{a}(r)} \right) \quad (8)$$

similar to equations (4) and (1).

## 6. Concluding remarks

As explained above for the most cited paper of Gross and Wilczek, for the determination of the  $h_m$ -index the credit is equally shared by all co-authors. Lately, Hirsch [6] argued that the  $h$ -index discriminates in papers with multiple authors [9] favouring senior and more able authors, because it allocates a smaller portion of the credit to those authors with a smaller index and thus to those authors that are likely to have contributed less. In his opinion, the reason for this is that only  $h$  citations are allocated to each author although the paper might have attracted many more citations. This reasoning was repeated by Ball [9]. But I think that the argument has to be reversed. Publications with a moderate number of citations contribute already to the  $h$ -index of those co-authors who have a low index, while these papers do not count for those co-authors who have already a higher  $h$ -index. Therefore, the definition of the  $h$ -index

favours junior and less able co-authors. In this sense, the  $h_m$ -index is likewise advantageous for junior and less able co-authors. But as already argued, it seems to be the fairest way to allocate the impact, unless one would be able to determine each individual contribution to a paper quantitatively.

In this respect, it has been suggested [7] to take only those publications into consideration where the evaluated scientist is the first or last author because in some scientific areas these positions can have a special meaning. This is based on the assumption that the first author has contributed most of the work and that the last author is the group leader and has contributed the main ideas. However, the first argument is only convincing, when the author list is not in alphabetical order. Otherwise it is unclear whether the first author is the main contributor or not. On the other hand the inclusion of the group leader in the author list (be it at the end or not) does not necessarily mean that this scientist's contribution was above average. Therefore a restriction of the evaluation to first and last authors does not solve the problem of how to allocate the impact in a fair way, but it certainly favours in an inappropriate way those authors whose names are very high or low in the alphabetical order.

As discussed for Cardona's citation records, the homographs in the bare WoS data can have a significant effect on the computed indices. Therefore care has to be taken to eliminate the homographs from the database. In table 1, I have included the values of the  $h$ -index which one obtains when one does not discriminate the scientists with the same name. Of course, for less common names there is no effect, but for example, in Cohen's case the change of  $h$  is significant. According to the above discussion one might expect that Cardona's  $h$ -index would be enhanced by 2 points due to the mentioned two homographs' publications. However, this is not the case, the difference is only 1 point, because when those two papers were included in the  $h$ -core another publication was pushed out of the  $h$ -core. Nevertheless, the data in table 1 clearly show how necessary it is to take reasonable care to exclude the homographs. Obviously this will in general become even more significant for scientists with smaller indices, because it is more likely to find homographs for smaller citation counts. I have indeed made such an observation when determining the  $h_m$ -indices.

In conclusion, the calculation of the  $h_m$ -index is a little bit more involved than the determination of the  $h$ -index. But in my opinion this is worthwhile, because this modification allows one to take multiple co-authorship appropriately into account. Of course, before the new index is actually utilized for evaluation and comparison purposes, it is necessary to test its validity thoroughly on the basis of empirical data in various research fields. The present investigation has demonstrated that the consideration of multiple co-authorship can substantially modify the Hirsch index. This issue should also be taken as a warning of how problematic and dangerous it is to reduce the complete scientific output of a researcher to a single number.

Other modifications of the  $h$ -index have been proposed, for example, the exclusion of self-citations [10, 11]. This could be applied in a straightforward way to the  $h_m$ -index as well, but it requires a considerably larger effort in establishing the data base. In contrast, the definition of the  $h_m$ -index could be automatically incorporated into the WoS search in the same way as the  $h$ -index was implemented two years ago.

*Note added in proof:* The same fractional counting of papers has been proposed for the Hirsch index by Egghe [12] investigating two fictitious examples and one empirical case.

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