



Definitions of time series in citation analysis with special attention to the h -index

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ABSTRACT

The structure of different types of time series in citation analysis is revealed, using an adapted form of the Frandsen–Rousseau notation. Special cases where this approach can be used include time series of impact factors and time series of h -indices, or h -type indices. This leads to a tool describing dynamic aspects of citation analysis. Time series of h -indices are calculated in some specific models.

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1. Introduction

We began this investigation when we realized that the study of the evolution of the h -index, h -type indicators and more generally citation indicators is a topic not yet fully addressed. Hirsch (2005) claimed that the lifetime achievement h -index of a scientist grows linearly in time and provided some evidence. By and large this evidence was corroborated by Kelly and Jennions (2006). Yet, as one needs more and more citations to attain a one-point higher h -index it seems intuitively clear that the growth of a scientist's h -index should follow a concavely increasing curve as predicted by the Egghe–Rousseau Power Law Model (2006). Is Hirsch nevertheless correct and if so, why? Or could it be that the growth curve of the h -index is generally S-shaped as it is the case for some of the examples given in (Anderson, Hankin, & Killworth, 2008)? This problem will not be addressed in this article, but, as a first step, we intend to provide precise definitions of time series for h -indices. Such definitions are necessary to avoid possible confusion. We provide a general scheme and notation for indicating exactly which time series is studied.

Time series are used to better understand the underlying mechanism that produces them. They can also be used in forecasting. This aspect is interesting in the framework of research evaluation: how will a scientist or research group most likely perform in the future? Of course, the first question is: is this type of time series capable of predicting features that lie in the future? This question has been studied recently by Hirsch (2007) who found that the h -index series (our type 5, see further) is indeed a good predictor for future scientific achievements.

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When we started writing this contribution it became soon clear that in a similar way as for h -indices time series for journal impact factors can be defined. As there already exists a precise notation for all types of impact factors (Frandsen & Rousseau, 2005) we adapt it to the topic studied in this contribution.

The article is organized as follows. The next section explains the adaptation of the notation introduced by Frandsen and Rousseau (2005). Then time series of citation data, based on a publication–citation matrix (in short: p - c matrix) are defined and discussed in Sections 3 and 4. These two sections contain the essential ideas of this article. Sections 5 and 6 focus on the h -index, presenting different time series of h -indices for two very simple models. We conclude in Section 7.

2. The adapted Frandsen–Rousseau notation for publication and citation indicator calculations

We assume that the focus is on one set of articles. This set can be a scientist's research record, a journal, as in most examples, but it can also be the set of all journals in one particular field, or even all journals in a database. For this journal or scientist we intend to calculate an impact factor or an h -index (or a similar indicator). It might seem somewhat odd (and in practice not recommended) to calculate a person's impact factor, but, as long as this person publishes at least one article a year this is formally possible, and if the scientist does not publish that year the corresponding impact is naturally zero.

Consider a p - c matrix (Ingwersen, Larsen, Rousseau, & Russell, 2001) consisting of N publication years, from year Y to year $Y + N - 1$ (the columns) and M citation years, from year Y to year $Y + M - 1$ (the rows). Hence the p - c matrix is an $M \times N$ -matrix. In theory a p - c matrix can be infinitely long in the two dimensions but we will stick to the realistic case of a matrix with a finite number of rows and columns, even when this makes some formulations somewhat complicated. Moreover, we will always assume that the number of columns is at most equal to the number of rows: $N \leq M$. In this article the words *series* and *sequence* will be used as synonyms. We only consider empirical data sets and do not consider probabilistic p - c models as studied in, e.g. Glänzel (2004).

In (Frandsen & Rousseau, 2005) a framework for describing general impact factors has been introduced, yet in its original form this framework cannot be used and an adaptation is necessary (we thank Leo Egghe for pointing this out to us). Consequently, the following quadruple is used instead: $(Y_p, n_p, Y_{c,q}, n_{c,q})$, where

Y_p is the first year of the publication period;

n_p denotes the length of the publication period;

$Y_{c,q}$ is the first year (=oldest year) of the citation period for the q th publication year; $q = 1, \dots, n_p$, where $q = 1$ refers to the oldest year and $q = n_p$ to the most recent year;

$n_{c,q}$ denotes the length of the citation period for the q th publication year.

This quadruple will be called the F–R notation. Given a p - c matrix, it contains the elements necessary for the calculation of one h -index or one impact factor. If some publication data fall outside the limits of the p - c matrix the whole calculation is not performed. We assume here that time is considered in steps of one year, but the notation also applies for other time steps.

We show how this notation can be used to describe series of impact factors and h -indices alike. Recall that citations are always drawn from a pool, such as the Web of Science, Scopus, a local database such as the Chinese CSCD, or subsets thereof. We will further assume that this pool is known and will not consider this aspect anymore.

3. Types of time series of citation indicators

We keep a publication set fixed and study series of citation indicators derived from this set. We define now general time series of indicators and characterize what they say about the set of publications.

Time series are of the form $(s_k^{\{\text{number}\}})_{k=1, \dots, \text{end}}$, where k is an index ranging from time (year) 1 to some end time. As we consider several time series they are numbered by a superscript between square brackets. Specifics for each case are shown in Table 1. For a general element of the time series (index k) Table 1 gives the following elements, in that order, necessary for the calculation of the index: the first year of the publication period (Y_p), the length of the publication period (n_p), the first year of the citation period for the q th publication year ($Y_{c,q}$) and the length of the citation period for the q th publication year ($n_{c,q}$). Hence, for a given p - c matrix a time series is, in the F–R notation, completely determined by the quadruple $(Y_p, n_p, Y_{c,q}, n_{c,q})$. Note that the first, and in general the q th publication year may differ according to k , the element of the time series considered. If q is not mentioned in $Y_{c,q}$ or $n_{c,q}$ this just means that this year or this period does not depend on q , and hence is the same for all q . As we focus on the p - c matrix we do not include in this table the simple time series that uses only one publication year (or one publication) and one citation year for each element in the series, or the corresponding cumulative case (see, e.g. Franses, 2003). For completeness sake we just mention that such time series are of the form $s_k = (Y_p, 1, Y_c + k - 1, 1)$, $k = 1, \dots, M - Y_c + Y_p$ for the case of one citation year; or $s_k = (Y_p, 1, Y_c, k)$, $k = 1, \dots, M - Y_c + Y_p$, for the cumulative case, where usually $Y_p = Y_c$.

Table 1 shows some types of time series characterized in this manner. Clearly many more time series are possible, but we think these are the most interesting ones. We would like to point out that in (Frandsen & Rousseau, 2005) only fixed impact factors are proposed, while here we consider time series, leading to a tool to study dynamical aspects of citation analysis.

Table 1
Characterizations of time series of citation indicators

Type	Range of index (k)	Data elements needed for the calculation of the k th element of the sequence (F-R notation); $q = 1, \dots, n_p$
1	1 to N	$(Y+k-1, 1, Y+k-1, M-k+1)$
2	1 to N	$(Y, k, Y+q-1, M-q+1)$
3	1 to $\min(N-1, M-2)$	$(Y+k-1, 2, Y+k+1, 1)$
4	1 to M	$(Y, \min(k, N), Y+k-1, 1)$
5	1 to M	$(Y, \min(k, N), Y+q-1, k+1-q)$
6	1 to M	$(Y, \min(N, M-k+1), Y+k+q-2, 1)$
7	1 to M	$(Y, N, Y+q-1, \min(k, M-q+1))$
8	1 to $\min(N, M-w+1)$; $w > 0$ being a given citation window	$(Y+k-1, 1, Y+k-1, w)^a$
9	1 to $N-w+1$; $w > 0$ being a given publication window	$(Y+k-1, w, Y+k+q-2, w-q+1)$
10	1 to N	$(Y+N-k, k, Y+N-k+q-1, k-q+1)$, where $M=N$

^a If data for the complete window of length w are not available the calculation is not performed.

Each of these times series can be visualized using a p - c matrix. The one for the first time series is shown here (Fig. 1), while the other ones are shown in the Appendix A. In these visualizations the shaded areas contain the data used in the calculation of the elements of the time series.

4. Comments on these definitions and some examples

Time series are used to study the dynamics of citation analysis, revealing trends and fluctuations. They may lead to (careful) predictions. The type 1 series leads to a series of diachronous indicators, making use of all data available in the p - c

Publication year / Citation year	Y	Y+1	Y+2	Y+3
Y				
Y+1				
Y+2				
Y+3				
Y+4				
	Y	Y+1	Y+2	Y+3
Y				
Y+1				
Y+2				
Y+3				
Y+4				
	Y	Y+1	Y+2	Y+3
Y				
Y+1				
Y+2				
Y+3				
Y+4				
	Y	Y+1	Y+2	Y+3
Y				
Y+1				
Y+2				
Y+3				
Y+4				

Fig. 1. An illustration of the calculation of a type 1 time series (case $M=5, N=4$).

matrix. If year $Y+M-1$ is the latest year for which data are available this is a natural approach, although publication years are treated unevenly. The type 2 series is similar to type 1 but uses cumulative data. In the application for journal impacts the type 3 case leads to the calculation of a series of standard Garfield-Sher impact factors (a type of synchronous data). This approach is already contested in the impact factor case (Ingwersen et al., 2001; Epstein, 2004) and does not seem useful for h -indices. Type 4 is a general synchronous approach, making use of all data available in the p - c matrix, again, with the disadvantage that the first citation years are treated differently from the other ones. Yet, if $N \ll M$ then most indicators are based on the same number of data points, namely N . Type 5 is the cumulative case of type 4. If one considers a scientist's set of publications then the h -index as proposed by Hirsch (the lifetime achievement h -index) grows as a type 5 indicator (at least if $M=N$). Type 6 are isochronous indicators as used, e.g. in Liang's theory on rhythm indicators (Liang, 2005; Egghe, Liang, & Rousseau, 2008). Type 7 is the cumulative version of type 6. Type 8 is a diachron case (as is type 1) but now with a fixed window, so that publication years are treated in the same way. If $w = 1$ one obtains a series of immediacy indices (in the impact case). Type 9 is a cumulative, truncated case of type 8. Finally, type 10 follows Liang's backward looking approach, the first ever study of time series of h -indices (Liang, 2006). In this case it seems natural to take $N=M$. Note that here the first publication year ($q=1$) differs according to the index k .

4.1. Examples

Most of the time series proposed here are rarely or never used in practice. We assume that this is largely due to the fact that researchers are not aware of the different possibilities for time series studies. Another point is that the Web of Science does not make it easy to collect the necessary data. An example of a type 1 series for the Hirsch index of the *Journal of the American Society for Information Science* has been provided by Rousseau (2006). It was suggested that a normalisation with respect to the number of published articles was preferable. Type 1 and type 2 series were used in (Liu, Rao, & Rousseau, 2008) for the study of the field of horticulture (h -indices of journals). Type 3 series for impact factors of journals are given in Thomson Scientific's JCR, and are studied in many particular cases, see, e.g. (López-Abente & Muñoz-Tinoco, 2005). The second author's type 5 time series is presented in (Rousseau & Jin, 2008). An example of a type 8 time series is presented in (Gupta, 1997) using a window of 5 years and steps of 10 year between publication years (and using percentages with respect to all citations). Type 9 is used in Thomson Scientific's Essential Science Indicators (ESI) with $w = 5$; it has also been used in (Jin & Rousseau, 2007) with $w = 10$. Type 10 is the case studied by Liang (2006) for the physicists studied in Hirsch' original article (Hirsch, 2005).

In the following sections we concentrate on the h -index leaving the better known citation impact case to the reader. We investigate how the h -index series look like under some simple, admittedly unrealistic, conditions.

5. Hirsch' elementary baseline model

We consider Hirsch' baseline model in which each year a fixed number of articles, p , is published, and each article receives *each* year a number of citations equal to $c > 0$. We consider M citation years and N publication years, $N \leq M$. Of course this model is not realistic at all, but it provides a kind of baseline. Moreover, it can be considered an 'average' model if we use for p and c the average number of publications and citations received during the period covered by the p - c matrix. A series, generally denoted as $(s_k)_k$ is now denoted $(h_k)_k$, as we deal with h -indices.

Type 1: For the type 1 series $(h_k^{[1]})_{k=1, \dots, N}$ the number of articles involved in its calculation is always p , and the element $h_k^{[1]} = \min(c(M-k+1), p)$. This series is non-increasing, strictly decreasing if $Mc \leq p$ and constant if $p \leq (M-N+1)c$.

Type 2: For the type 2 series $(h_k^{[2]})_{k=1, \dots, N}$ the number of publications involved in the calculation of the k th element is equal to kp . The first element $h_1^{[2]} = \min(p, Mc)$; for $h_k^{[2]}$, $k=2, \dots, N$ the following holds:

$$\text{if } h_{k-1}^{[2]} < (k-1)p \text{ then } h_k^{[2]} = h_{k-1}^{[2]}$$

$$\text{if } h_{k-1}^{[2]} = (k-1)p \text{ and } (k-1)p > (M-k+1)c \text{ then } h_k^{[2]} = (k-1)p$$

$$\text{if } h_{k-1}^{[2]} = (k-1)p \text{ and } (k-1)p = (M-k+1)c \text{ then } h_k^{[2]} = (k-1)p$$

$$\text{if } h_{k-1}^{[2]} = (k-1)p \text{ and } (k-1)p < (M-k+1)c \text{ then } h_k^{[2]} = \min(kp, (M-k+1)c)$$

Type 3: For the type 3 series $(h_k^{[3]})_{k=1, \dots, \min(N-1, M-2)}$ the number of publications involved in the calculation of the k th element is equal to $2p$. All $h_k^{[3]}$ are the same in this model and equal to $\min(2p, c)$

Type 4: For the type 4 series $(h_k^{[4]})_{k=1, \dots, M}$ the number of articles involved in its calculation is kp , if $1 \leq k \leq N$ and is Np if $N \leq k \leq M$. It is easy to see that in this model $h_1^{[4]} = \min(p, c)$, and generally $h_k^{[4]} = \min(c, kp)$ for $1 \leq k \leq N$, and $h_k^{[4]} = \min(c, Np)$, $N \leq k \leq M$.

Type 5: This is the case considered by Hirsch (for scientists). The number of articles involved in its calculation is kp , if $1 \leq k \leq N$ and is Np if $N \leq k \leq M$. Again $h_1^{[5]} = \min(p, c)$ but the calculation of the other type 5 h -indices is more complicated. Calculating $h_2^{[5]}$ is already rather difficult. Indeed, if $1 \leq 2c \leq p$ then $h_2^{[5]} = \min(p, 2c)$; if $(p < 2c$ and

$p+1 > c$) then $h_2^{[2]} = p$ and when $p+1 \leq c$ then $h_2^{[5]} = \min(c, 2p)$. Clearly this calculation becomes more and more difficult for increasing values of k . For this reason we apply a simplification. Assuming that all publications up to year t_k ($\leq k$) contribute to the $h_k^{[5]}$ -index (the idea for this simplification is due to Hirsch (2005)) $h_k^{[5]}$ is p times t_k , where t_k is the solution of $t_k p = (k - t_k + 1)c$ (with the extra requirement that $t_k \leq \min(k, N)$), i.e. $h_k^{[5]} = p(k+1)c/(p+c)$ for $1 \leq k \leq M$ (note that this number is usually not a natural number). If this equation has no acceptable solution, i.e. when the solution $t_k > \min(k, N)$, then $h_k^{[5]} = \min(kp, Np)$. In this model h usually increases linearly in k (this is: in time), an exception being a situation where $c \geq Np$.

- Type 6: The number of citations involved in the calculation of type 6 h -indices is always c and the number of publications is Np for $1 \leq k \leq M - N + 1$, and $(M - k + 1)p$ for $M - N + 2 \leq k \leq M$. We note that if $c \leq p$ then $h_k^{[6]} = c$, leading to a constant sequence. In general $h_k^{[6]} = \min(c, Np)$ for $1 \leq k \leq M - N + 1$, and $h_k^{[6]} = \min(c, (M - k + 1)p)$ for $M - N + 2 \leq k \leq M$.
- Type 7: The number of articles involved in the calculation of type 7 h -indices is Np . For $1 \leq k \leq M - N + 1$, $h_k^{[7]} = \min(kc, Np)$. For $M - N + 2 \leq k \leq M$ one must first consider $\min((M - k + 1)p, ck)$. If this minimum is ck , then $h_k^{[7]} = ck$. Otherwise one uses Hirsch' simplification and looks for that value x such that $(M - k + x)p = (k - x + 1)c$. Solving this for x yields: $x = [(k + 1)c - (M - k)p]/(p + c)$. Hence $h_k^{[7]} = (M + 1)cp/(p + c)$, unless there is no acceptable solution, in which case $h_k^{[7]} = Np$.
- Type 8: The number of publications involved in the calculation of a type 8 h -index is always p . All h -indices are the same and equal to $\min(p, wc)$.
- Type 9: The number of publications involved in a type 9 h -index is $p \cdot w$. Using again Hirsch' simplification we see that $h_k^{[9]}$ is p times t_k where t_k is the solution of $t_k p = (w - t_k + 1)c$ (with $t_k \leq N - w + 1$). In this case all h -indices are the same and equal to $[c(w + 1)/(p + c)]p$. If this equation has no acceptable solution, i.e. when the solution $t_k > N - w + 1$ then $h_k^{[9]} = wp$ for all k .
- Type 10: Because of the special assumptions, namely a fixed number of publications and citations, this series of h -indices is exactly the same as the type 5 case for $M = N$.

6. Time series in the power law model

In the power law model the h -index is equal to $T^{1/\alpha}$ where T is the total number of publications under consideration and where the number of articles with t citations is given by a power law of the form C/t^α . As in the first model we assume that the number of publications is the same each year and equal to p . No assumptions are made about the number of citations, except that they follow a power law. The exponent of the variable t is in general different in each case and for each element in a series of h -indices. For simplicity we will use the symbol α in a generic sense. Moreover, we do not investigate if the exponent α can be determined for one particular series if it is known for another one. Though certainly interesting, this problem is not studied in this contribution.

Assuming that for all ten types of series citations are indeed distributed according to a power law, we obtain the following series.

- Type 1: For all k $h_k^{[1]} = (p)^{1/\alpha}$. If the exponents α do not differ too much from year to year this is a constant sequence in the power law model. Because of the time factor we expect that in reality this sequence decreases anyway (with increasing α for shorter time periods).
- Type 2: Here we have: $h_k^{[2]} = (kp)^{1/\alpha}$. This sequence is expected to increase.
- Type 3: Now $h_k^{[3]} = (2p)^{1/\alpha}$ which, again, is approximately constant.
- Type 4: For the fourth sequence: $h_k^{[4]} = (kp)^{1/\alpha}$ for $1 \leq k \leq N$, and $h_k^{[4]} = (Np)^{1/\alpha}$ for $N \leq k \leq M$. This sequence is expected to increase first, and to stay constant at the end.
- Type 5: This series looks the same as the type 4 series: $h_k^{[5]} = (kp)^{1/\alpha}$ for $1 \leq k \leq N$, and $h_k^{[5]} = (Np)^{1/\alpha}$ for $N \leq k \leq M$. Consequently, its general behaviour is expected to be the same as for type 4. Yet, because of the different citation periods, the exponents are different (but remember that we use the symbol α in a generic sense). Terms in the $(h_k^{[4]})_k$ series are expected to be smaller than those for the $(h_k^{[5]})_k$ series.
- Type 6: If $m = \min(N, M - k + 1)$ then, for type 6 sequences $h_k^{[6]} = (mp)^{1/\alpha}$. These series are constant (if alpha stays constant) in the beginning, but decrease at the end.
- Type 7: In the power law model type 7 sequences are of the form: $h_k^{[7]} = (Np)^{1/\alpha}$ for $1 \leq k \leq M$, which is probably not constant as it is likely that the exponent alpha depends on k .
- Type 8: This series has the form $h_k^{[8]} = (p)^{1/\alpha}$, which is formally the same as type 1. If exponents were constant the series itself would be constant. Yet, as for type 1 we expect that this series decreases over time (because exponents α will probably increase with k). Moreover, because of the different citation period we expect that the h -indices of type 1 are larger than those of type 8 (corresponding to smaller exponents α for the type 1 case than for the type 8 case).

Type 9: This series has the form $h_k^{[9]} = (wp)^{1/\alpha}$, which is again assumed to be constant (under unchanged citation conditions).

Type 10: Finally, also under power law conditions the backward series looks like the type 5 one. This is in line with the analysis performed in (Egghe, 2008) at least for the special case of a constant number of publications.

In this model no h -index increases linearly in time. Besides the power law model also other models have been used to describe the behaviour of the h -index. According to the elementary model studied by Hirsch (2005) and Burrell's stochastic model (Burrell, 2007), the type 5 series is approximately linearly increasing in time. This is not true in the power law model.

7. Conclusion

The empirical p - c matrix combined with the adapted F-R notation clearly draws attention to fundamentally different approaches to time series in citation analysis. Time series are used to study the dynamics of citation analysis, expounding trends and fluctuations. They may lead to predictions. We hope that the series we designed will be useful in addressing problems related to the development and characterisation of scientific research, as represented by citation indicators such as impact factors and h -type indices. We would like to point out that the time series shown in Table 1 are just examples: the same approach and notation can be used for other time series. The specific models for the h -index time series considered in this article are just the simplest possible. Also here there is much room for improvement.

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Appendix A. Visualizations of data used in the calculation of different types of time series

Figures below illustrate calculations for types 2 and 3 (Fig. 2), types 4 and 5 (Fig. 3), types 6 and 7 (Fig. 4), types 8 and 9 (Fig. 5), and type 10 (Fig. 6).

Type 2 and type 3

Publication year / Citation year	Y	Y+1	Y+2	Y+3	Publication year / Citation year	Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				

Fig. 2. An illustration of the calculation of the types 2 and 3 time series (case $M=5, N=4$).

Type 4 and type 5

Publication year / Citation year	Y	Y+1	Y+2	Y+3	Publication year / Citation year	Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				

Fig. 3. An illustration of the calculation of the types 4 and 5 time series (case $M=5, N=4$).

Type 6 and type 7

Publication year / Citation year	Y	Y+1	Y+2	Y+3	Publication year / Citation year	Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3
Y					Y				
Y+1					Y+1				
Y+2					Y+2				
Y+3					Y+3				
Y+4					Y+4				

Fig. 4. An illustration of the calculation of the types 6 and 7 time series (case $M=5, N=4$).

Type 8 and type 9

Publication year / Citation year	Y	Y+1	Y+2	Y+3	Publication year / Citation year	Y	Y+1	Y+2	Y+3	Y+4
Y					Y					
Y+1					Y+1					
Y+2					Y+2					
Y+3					Y+3					
Y+4					Y+4					
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3	Y+4
Y					Y					
Y+1					Y+1					
Y+2					Y+2					
Y+3					Y+3					
Y+4					Y+4					
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3	Y+4
Y					Y					
Y+1					Y+1					
Y+2					Y+2					
Y+3					Y+3					
Y+4					Y+4					
	Y	Y+1	Y+2	Y+3		Y	Y+1	Y+2	Y+3	Y+4
Y					Y					
Y+1					Y+1					
Y+2					Y+2					
Y+3					Y+3					
Y+4					Y+4					

Fig. 5. An illustration of the calculation of the type 8 (with $w = 3$; case $M=5, N=4$) and type 9 time series (with $w = 3$; case $M=5, N=5$).

