Multiobjective Genetic Fuzzy Systems: Review and Future Research Directions

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In this presentation

1. Evolutionary Multiobjective Optimization

2. Multiobjective Genetic Fuzzy Systems

3. Related Issues and Future Directions
Evolutionary multiobjective optimization (EMO) is a very active research area in evolutionary computation.
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**Major Evolutionary Computation Conferences**

- GECCO 2006 (Seattle, USA, July 8-12)
- CEC 2006 (Vancouver, Canada, July 16-21)
- PPSN 2006 (Reykjavik, Iceland, September 9-13)
- EMO 2007 (Sendai, Japan, March 5-8)
- GECCO 2007 (London, UK, July 7-11)

Many papers are related to multiobjective optimization.
The number of EMO papers is still increasing.
Popularity of EMO Research

Most frequently cited papers published in *IEEE Transactions on Evolutionary Computation* during 1999-2007 (All TEC papers in ISI)


Data from ISI Web of Science, Thomson Scientific (July 21, 2007)
Popularity of EMO Research

Most frequently cited papers published in *IEEE Transactions on Evolutionary Computation* in the recent 5 years (2003-2007)


Data from ISI Web of Science, Thomson Scientific (July 21, 2007)
Multiobjective optimization problem with $k$ objectives:

Maximize $f(x) = (f_1(x), f_2(x), \ldots, f_k(x))$
Comparison between Two Solutions

Maximize \( f(x) = (f_1(x), f_2(x)) \)

- \( A \) dominates \( B \)
- \( B \) is dominated by \( A \)
- \( A \) is better than \( B \)
Maximize \( f(x) = (f_1(x), f_2(x)) \)

A and C are non-dominated with each other.
A Pareto-optimal solution is a solution that is not dominated by any other solutions.
EMO algorithms are designed to efficiently search for Pareto-optimal solutions as many as possible in their single run.
Maximize \( g(x) = w_1 f_1(x) + w_2 f_2(x) \)

Comparison: Weighted Sum Approach

Only a single solution is obtained by the weighted sum approach.

Multiple solutions are obtained by an EMO algorithm.
Difficulties in Weighted Sum Approach

• This approach is sensitive to the specification of the weight vector.
• This approach can not find any Pareto-optimal solutions in a non-convex region of the Pareto front in the objective space.
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Difficulties in Weighted Sum Approach

• This approach is sensitive to the specification of the weight vector.
• This approach can not find any Pareto-optimal solutions in a non-convex region of the Pareto front in the objective space.
Comparison of the Two Approaches

Two-objective maximization problem

EMO Approach

Weighted Sum Approach
Search Direction in Each Approach

Two-objective maximization problem

- **EMO Approach**
- **Weighted Sum Approach**
Difficulties in Fuzzy System Design
Difficulties in Fuzzy System Design

Accuracy-Complexity Tradeoff

Small

Error

Large

Simple

Complexity

Complicated

Interpretable fuzzy system

Ideal fuzzy system

Accurate fuzzy system
Fuzzy System Research in the 1990s

Accuracy maximization: Many studies on
- Universal approximators of nonlinear functions
- Neuro-fuzzy techniques for parameter learning
- Genetic-fuzzy techniques for parameter and structure learning


Research Direction in the 1990s

Interpretable fuzzy system

Accurate fuzzy system

Small Error → Large

Simple ← Complexity → Complicated
A complicated fuzzy system with high accuracy was obtained.
Difficulty in Accuracy Maximization

Error minimization $\rightarrow$ Overfitting to training data
Difficulty in Accuracy Maximization

Error minimization  Overfitting to training data

Error

Test data accuracy

Training data accuracy

Complexity

$S^*$
Search for a good accuracy-complexity tradeoff

Basic Idea
To combine the error minimization and the complexity minimization into a single scalar objective function
Search for a good accuracy-complexity tradeoff

Basic Idea

To combine the error minimization and the complexity minimization into a single scalar objective function

Example: Combination of the average error rate and the number of fuzzy rules

Example of a scalar objective function: Weighted sum

\[ f(S) = w_1 \cdot f_{\text{Error}}(S) + w_2 \cdot f_{\text{Complexity}}(S) \]
Fuzzy System Research in Late 1990s

Search for a good accuracy-complexity tradeoff

Basic Idea
To combine the error minimization and the complexity minimization into a single scalar objective function

Example: Combination of the average error rate and the number of fuzzy rules

Research Direction in Late 1990s

- Interpretable fuzzy system
- Accurate fuzzy system
A fuzzy system with a good accuracy-complexity tradeoff was obtained.
Difficulty in Weighted Sum Approach

Sensitivity to the weight vector:
The obtained fuzzy system strongly depends on the specification of the weight vector.
Minimize $w_1 \cdot Error + w_2 \cdot Complexity$

When the weight for the complexity minimization is large:

A simple fuzzy system is obtained.

Test data accuracy

Training data accuracy
Minimize $w_1 \cdot Error + w_2 \cdot Complexity$

When the weight for the error minimization is large:

A complicated fuzzy system is obtained.

Test data accuracy

Training data accuracy
Minimize $w_1 \cdot \text{Error} + w_2 \cdot \text{Complexity}$

When the two weights are appropriately specified:

A good fuzzy system is obtained. But the best complexity is not always found.
Current Trend in Fuzzy System Research

Multiobjective optimization of accuracy and complexity

Basic Idea
To search for Pareto-optimal solutions with respect to the error minimization and the complexity minimization.
Current Trend in Fuzzy System Research

Multiobjective optimization of accuracy and complexity

Basic Idea
To search for Pareto-optimal solutions with respect to the error minimization and the complexity minimization.

Example: Two-objective problem
- minimize the average error rate
- minimize the number of fuzzy rules

Example of a multiobjective minimization problem

Minimize \( \{ f_{\text{Error}}(S), f_{\text{Complexity}}(S) \} \)
Current Trend in Fuzzy System Research

Multiobjective optimization of accuracy and complexity

Basic Idea
To search for Pareto-optimal solutions with respect to the error minimization and the complexity minimization.

Aggregation Approach

\[ f(S) = w_1 \cdot f_{\text{Error}}(S) + w_2 \cdot f_{\text{Complexity}}(S) \]

Multiobjective Approach

Minimize \( \{ f_{\text{Error}}(S), f_{\text{Complexity}}(S) \} \)
Current Trend in Fuzzy System Research

Multiobjective optimization of accuracy and complexity

Basic Idea
To search for Pareto-optimal solutions with respect to the error minimization and the complexity minimization.

Example: Two-objective problem
- minimize the average error rate
- minimize the number of fuzzy rules

Current Research Direction

- Interpretable fuzzy system
- Accurate fuzzy system

Graph showing the relationship between error, complexity, and accuracy in fuzzy systems.
Current Research Direction

![Diagram showing the trade-off between error and complexity in fuzzy systems. The diagram illustrates that as error decreases, complexity increases. Two regions are highlighted: one for interpretably accurate fuzzy systems and another for more complex and accurate systems.](image-url)
Many Pareto-optimal fuzzy systems can be obtained along the accuracy-complexity tradeoff surface by a single run of an EMO algorithm.
Many Pareto-optimal fuzzy systems can be obtained along the accuracy-complexity tradeoff surface by a single run of an EMO algorithm.
Multiobjective Approach

Many Pareto-optimal fuzzy systems can be obtained along the accuracy-complexity tradeoff surface by a single run of an EMO algorithm.
Two Multiobjective Formulations

Multiobjective Design of Fuzzy Systems
Rule set-level multiobjective optimization

Multiobjective Search for Fuzzy Rules
Rule-level multiobjective optimization
Two Multiobjective Formulations

Multiobjective Design of Fuzzy Systems
Rule set-level multiobjective optimization

Multiobjective Search for Fuzzy Rules
Rule-level multiobjective optimization
Different quality measures of fuzzy rules such as support and confidence in fuzzy data mining are simultaneously optimized.
Maximize \{\text{Confidence, Support}\}

**Confidence maximization:**
\[ c(A_q \Rightarrow \text{Class } h) = \frac{\sum_{p\in \text{Class } h} \mu_{A_q}(x_p)}{m} \]

**Support maximization:**
\[ s(A_q \Rightarrow \text{Class } h) = \frac{\sum_{p\in \text{Class } h} \mu_{A_q}(x_p)}{m} \]

\( \mu(\cdot) \): Membership function
\( m \): Number of patterns
Pareto-Optimal Fuzzy Rules

Wisconsin Breast Cancer Data Set (Breast W)

Class 1

Class 2
Pareto-Optimal Fuzzy Rules

Breast W

Rule A: Very General Fuzzy Rule
(confidence: 0.69, support: 0.65)

Rule B: Very Specific Fuzzy Rule
(confidence: 1.00, support: 0.25)
Breast W

Rule A: Very General Fuzzy Rule
(confidence: 0.89, support: 0.25)

Rule B: Specific Fuzzy Rule
(confidence: 1.00, support: 0.11)
Relation between Pareto-optimal fuzzy rules and Pareto-optimal fuzzy systems

Pareto-Optimal Fuzzy Systems (Breast W)

Error Minimization and Complexity Minimization

Training data accuracy          Test data accuracy

Pareto-optimal fuzzy rules and Pareto-optimal fuzzy systems

Error rate (%) vs. Number of rules

A

B

C

Error rate (%) vs. Number of rules

A

B

C

0 2 4 6 8 10 12

0 2 4 6 8 10 12

Number of rules
Fuzzy rules in a simple fuzzy system A are general rules.

Error rate: 7.8% (training) and 7.4% (test)
Fuzzy rules in A are Pareto-optimal or near Pareto-optimal.
Some fuzzy rules in a complicated fuzzy system B is very specific rules with narrow antecedent fuzzy sets.

Error rate: 0.3% (training) and 3.7% (test)
Many fuzzy rules in B are far from the Pareto-optimal rules.
Fuzzy Rules in Good Fuzzy System C

Some fuzzy rules in a good fuzzy system C are specific but not very specific.

Error rate: 1.2% (training) and 3.4% (test)
A single fuzzy rule in B is far from the Pareto-optimal rules but the other rules are near Pareto-optimal.
Recently EMO algorithms were often used in other areas.
Multiobjective Machine Learning

Multiobjective Design of Decision Trees

Classification Error

Complexity
EMO algorithms can be used for the multiobjective design of various intelligent systems such as

- Fuzzy Rule-Based Systems
- Multilayer Neural Networks
- RBF Networks
- Support Vector Machines
- Decision Trees
- GP Trees
- ...
Multiple objectives are usually involved in the design of any intelligent systems. So you will easily find many future research issues in this research area.

Especially, if you are using an aggregation-based method, you will be able to improve it by the EMO approach.

**Aggregation Approach**

\[ f(S) = w_1 \cdot f_{\text{Error}}(S) + w_2 \cdot f_{\text{Complexity}}(S) \]

**Multiobjective Approach**

Minimize \( \{ f_{\text{Error}}(S), f_{\text{Complexity}}(S) \} \)
Future Research Directions in MGFSs

Formulations of the Interpretability
- The number of fuzzy rules
- The number of antecedent conditions in each rule
- The number of input variables
- The separability of adjacent antecedent fuzzy sets

Handling of Large Data Sets
- Design of efficient EMO algorithms
- Subdivision of data sets
- Parallel implementation

Development of Special-Purpose EMO Algorithms
- Handling of many objectives
- Handling of both discrete and continuous variables
Future Research Directions in MGFSs

Development of New MGFS Methods with
- Multiobjective input selection algorithm
- Multiobjective fuzzy clustering algorithm
- Multiobjective fuzzy partition algorithm
- Multiobjective rule selection algorithm
- . . .

Visualization of Pareto-Optimal Fuzzy Systems
- Visualization of a single fuzzy system
- Visualization of multiple fuzzy systems
- Visualization of accuracy-complexity tradeoff

Ensemble Classifier Design
- Search for multiple fuzzy systems with a large diversity
- Choice of ensemble members and their combination
Future Research Directions in MGFSs

Incorporation of Other Ideas into MGFS

- FUZZ-IEEE 2007 Tutorial by Alexander Gegov on Rule Base Compression in Fuzzy Systems
Welcome to EMOFRBSs

The Evolutionary Multiobjective Optimization of Fuzzy Rule-Based Systems Bibliography Page

Abstract

Since pioneering works by Hisao Ishibuchi in middle nineties, Evolutionary Multiobjective Optimization (EMO) of Fuzzy Rule-Based Systems (FRBSs) is nowadays a well-established research area. This page is intended to collect (possibly all) references to papers dealing with EMO of FRBSs. For a specific bibliography on EMO, please refer to the EMOO bibliography maintained by Dr. Carlos A. Coello Coello. A specific bibliography on FRBSs probably does not exist (if anybody knows any, please let me know). The interested reader in FRBSs literature can, however, contact me for a starting reference list.

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ANY SUGGESTION/CONTRIBUTION IS WELCOME! (on the left, the number of access since May 21, 2007)

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<tr>
<th>Author</th>
<th>Title</th>
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List of 80 MGFSs papers

80 publications until 28-Jun-2007
End of My Presentation

Thank you very much!