Multiobjective Genetic Fuzzy Systems
- Accurate and Interpretable Fuzzy Rule-Based Classifier Design -

Hisao Ishibuchi
Osaka Prefecture University, Japan
1. Introduction to Fuzzy Rule-Based Classification
   - Is Fuzzy Rule-Based Classification a Popular Research Area?

2. Fuzzy Rule-Based Classifier Design
   - Accuracy Improvement
   - Scalability to High-Dimensional Problems
   - Complexity Minimization

3. Multiobjective Fuzzy Rule-Based Classifier Design
   - Formulation of Multi-objective Problems
   - Accuracy-Complexity Tradeoff Analysis
   - Maximization of Generalization Ability

4. Current Hot Issues and Future Research Directions
   - Search Ability of EMO for Fuzzy System Design
   - Definition of Interpretability of Fuzzy Systems
   - Explanation Ability of Fuzzy Rule-Based Systems
   - Various Classification Problems: Imbalanced, Online, ...
Application Areas of Fuzzy Systems

- Fuzzy Control
- Fuzzy Clustering
- Fuzzy Classification

Control and clustering are well-known application areas!

Question: Is “fuzzy classification” popular?
Application Areas of Fuzzy Systems

- Fuzzy Control
- Fuzzy Clustering
- Fuzzy Classification

Control and clustering are well-known application areas!

Question: Is “fuzzy classification” popular?
Citation Report: Topic=(Fuzzy) AND Topic=(Control) Timespan=All Years, Databases=SCI-EXPANDED.

This report reflects citations to source items indexed within Web of Science. Perform a Cited Reference Search to include citations to items not indexed within Web of Science.

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Citations in Each Year

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Results: 9,421

9421 Fuzzy Control Papers

Use the checkboxes to remove individual items from this Citation Report or restrict to items processed between 1984 and 2010.

1. Title: FUZZY IDENTIFICATION OF SYSTEMS AND ITS APPLICATIONS TO MODELING AND CONTROL
   Author(s): TAKAGI T, SUGENO M
   Source: IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS Volume: 15 Issue: 1 Pages: 116-132 Published: 1985

Web of Science
## Application Areas of Fuzzy Systems

### Fuzzy Control: Well-Known Application Area

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<th>Title</th>
<th>Author(s)</th>
<th>Source</th>
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<th>Issue</th>
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<td>7.</td>
<td>Fuzzy Basis Functions, Universal Approximation, and Orthogonal Least-Squares Learning</td>
<td>Wang LX, Mendel JM</td>
<td>IEEE Transactions on Neural Networks</td>
<td>3</td>
<td>5</td>
<td>807-814</td>
<td>SEP 1992</td>
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Application Areas of Fuzzy Systems

Fuzzy Clustering: Well-Known Application Area

Citation Report

Topic=(Fuzzy) AND Topic=(Clustering)
Timespan=All Years. Databases=SCI-EXPANDED.

This report reflects citations to source items indexed within Web of Science. Perform a Cited Reference Search to include citations to items not indexed within Web of Science.

Results found: 2,968

2,968 Fuzzy Clustering Papers

Use the checkboxes to remove individual items from this Citation Report or restrict to items processed between 1984 and 2010.

1. Title: A REVIEW ON IMAGE SEGMENTATION TECHNIQUES
   Author(s): PAL NR, PAL SK
   Source: PATTERN RECOGNITION Volume: 26 Issue: 9 Pages: 1277-1294 Published: SEP 1993

Web of Science
Application Areas of Fuzzy Systems

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1. Title: A REVIEW ON IMAGE SEGMENTATION TECHNIQUES  
Author(s): PAL NR, PAL SK  
Source: PATTERN RECOGNITION  
Volume: 26  Issue: 9  Pages: 1277-1294  Published: SEP 1993  

2. Title: UNSUPERVISED OPTIMAL FUZZY CLUSTERING  
Author(s): GATH I, GEVA AB  
Source: IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE  
Volume: 11  Issue: 7  Pages: 773-781  Published: JUL 1989  

3. Title: A VALIDITY MEASURE FOR FUZZY CLUSTERING  
Author(s): XIE XLL, BENI G  
Source: IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE  
Volume: 13  Issue: 8  Pages: 841-847  Published: AUG 1991  

4. Title: FCM - THE FUZZY C-MEANS CLUSTERING-ALGORITHM  
Author(s): BEZDEK JC, EHRlich R, FULL W  
Source: COMPUTERS & GEOSCIENCES  
Volume: 10  Issue: 2-3  Pages: 191-203  Published: 1984  

5. Title: ON CLUSTER VALIDITY FOR THE FUZZY C-MEANS MODEL  
Author(s): PAL NR, BEZDEK JC  
Source: IEEE TRANSACTIONS ON FUZZY SYSTEMS  
Volume: 3  Issue: 3  Pages: 370-379  Published: AUG 1995  

6. Title: An on-line self-construction neural fuzzy inference network and its applications  
Author(s): Juang CF, Lin CT  
Source: IEEE TRANSACTIONS ON FUZZY SYSTEMS  
Volume: 6  Issue: 1  Pages: 12-32  Published: FEB 1998  

7. Title: Color image segmentation: advances and prospects  
Author(s): Cheng HD, Jiang XH, Sun Y, et al.  
Source: PATTERN RECOGNITION  
Volume: 34  Issue: 12  Pages: 2259-2281  Published: DEC 2001  

Web of Science
Application Areas of Fuzzy Systems

Fuzzy Classification: Well-Known?

Citation Report

[Image with graphs showing published items and citations by year]

Results: 4,144 4144 Fuzzy Classification Papers

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<th>Year</th>
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<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>Total</th>
<th>Average Citations per Year</th>
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<td>7949</td>
<td>7635</td>
<td>0</td>
<td>43,485</td>
<td>1672.50</td>
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</tbody>
</table>

1. Title: Status of land cover classification accuracy assessment
   Author(s): Foody GM
   Source: REMOTE SENSING OF ENVIRONMENT Volume: 80 Issue: 1 Pages: 185-201
   Published: APR 2002
### Application Areas of Fuzzy Systems

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1. **Title:** Status of land cover classification accuracy assessment  
   **Author(s):** Foody GM  
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   **Issue:** 1  
   **Pages:** 185-201  
   **Published:** APR 2002

2. **Title:** ROUGH FUZZY-SETS AND FUZZY ROUGH SETS  
   **Author(s):** Dubois D, Prade H  
   **Source:** INTERNATIONAL JOURNAL OF GENERAL SYSTEMS  
   **Volume:** 17  
   **Issue:** 2-3  
   **Pages:** 191-209  
   **Published:** 1990

3. **Title:** FUZZY-SETS IN APPROXIMATE REASONING .1. INFERENCE WITH POSSIBILITY DISTRIBUTIONS  
   **Author(s):** Dubois D, Prade H  
   **Source:** FUZZY SETS AND SYSTEMS  
   **Volume:** 40  
   **Issue:** 1  
   **Pages:** 143-202  
   **Published:** MAR 5 1991

4. **Title:** FUZZY MIN MAX NEURAL NETWORKS .1. CLASSIFICATION  
   **Author(s):** Simpson PK  
   **Source:** IEEE TRANSACTIONS ON NEURAL NETWORKS  
   **Volume:** 3  
   **Issue:** 5  
   **Pages:** 776-786  
   **Published:** SEP 1992

5. **Title:** Color image segmentation: advances and prospects  
   **Author(s):** Cheng HD, Jiang XH, Sun Y, et al.  
   **Source:** PATTERN RECOGNITION  
   **Volume:** 34  
   **Issue:** 12  
   **Pages:** 2259-2281  
   **Published:** DEC 2001

6. **Title:** SELECTING FUZZY IF-THEN RULES FOR CLASSIFICATION PROBLEMS USING GENETIC ALGORITHMS  
   **Author(s):** Ishibuchi H, Nozaki K, Yamamoto N, et al.  
   **Source:** IEEE TRANSACTIONS ON FUZZY SYSTEMS  
   **Volume:** 3  
   **Issue:** 3  
   **Pages:** 260-270  
   **Published:** AUG 1995

7. **Title:** Decision templates for multiple classifier fusion: an experimental comparison  
   **Author(s):** Kancheva IJ, Bezdek JC, Duin RPW  
   **Source:** PATTERN RECOGNITION  
   **Volume:** 34  
   **Issue:** 2  
   **Pages:** 299-314  
   **Published:** FEB 2001

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**Dubois D, Prade H**  
**Fuzzy If-Then Rules**  
**Fuzzy Min-Max NN**  
**Web of Science**
Application Areas of Fuzzy Systems
Control, Clustering, and Classification

Results found: 9,421
Sum of the Times Cited [?] : 90,485
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h-index [?] : 94

9421 (Fuzzy Control)

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h-index [?] : 71

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Average Citations per Item [?] : 10.49
h-index [?] : 80

4144 (Fuzzy Classification)
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Application Areas of Fuzzy Systems
Control, Clustering, and Classification

Published Items in Each Year
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   Source: PATTERN RECOGNITION Volume: 34 Issue: 2 Pages: 299-314 Published: FEB 2001
Fitness Function

\[ w_1 \text{Accuracy}(S) - w_2 \text{Complexity}(S) \]

Accuracy Maximization and Complexity Minimization
The number of selected fuzzy rules

\[ w_1 \text{Accuracy}(S) - w_2 \text{Complexity}(S) \]

The number of correctly classified training patterns
Fuzzy Rules for Classification
Accurate and Interpretable Fuzzy Rule-Based Classifier Design

Basic Form
If $x_1$ is small and $x_2$ is small then Class 2
Fuzzy Rules for Classification
Accurate and Interpretable Fuzzy Rule-Based Classifier Design

Basic Form
If $x_1$ is small and $x_2$ is small then Class 2
If $x_1$ is small and $x_2$ is medium then Class 2
Fuzzy Rules for Classification
Accurate and Interpretable Fuzzy Rule-Based Classifier Design

Basic Form
If \( x_1 \) is small and \( x_2 \) is small then Class 2
If \( x_1 \) is small and \( x_2 \) is medium then Class 2
If \( x_1 \) is small and \( x_2 \) is large then Class 1
**Fuzzy Rules for Classification**

Accurate and Interpretable Fuzzy Rule-Based Classifier Design

---

### Basic Form

If \( x_1 \) is *small* and \( x_2 \) is *small* then Class 2

If \( x_1 \) is *small* and \( x_2 \) is *medium* then Class 2

If \( x_1 \) is *small* and \( x_2 \) is *large* then Class 1

\[ \ldots \]

If \( x_1 \) is *large* and \( x_2 \) is *large* then Class 3

---

**High Interpretability**

Easy to Understand!
Classification Boundary
Accurate and Interpretable Fuzzy Rule-Based Classifier Design

Basic Form
If $x_1$ is small and $x_2$ is small
then Class 2

If $x_1$ is small and $x_2$ is medium
then Class 2

If $x_1$ is small and $x_2$ is large
then Class 1

... 

If $x_1$ is large and $x_2$ is large
then Class 3

High Interpretability
Easy to Understand!
Fuzzy Rules for Classification
Basic form does not always have high accuracy

Basic Form
If $x_1$ is *small* and $x_2$ is *small* then Class 2
If $x_1$ is *small* and $x_2$ is *medium* then Class 2
If $x_1$ is *small* and $x_2$ is *large* then Class 1
... 
If $x_1$ is *large* and $x_2$ is *large* then Class 3

High Interpretability
Low Accuracy
Fuzzy Rules for Classification
Another form has a rule weight (certainty)

Basic Form
If $x_1$ is *small* and $x_2$ is *medium* then Class 2

Rule Weight Version
If $x_1$ is *small* and $x_2$ is *medium* then Class 2 with 0.158
Classification Boundary
Accurate and Interpretable Fuzzy Rule-Based Classifier Design

Basic Form
If $x_1$ is small and $x_2$ is medium then Class 2

Rule Weight Version
If $x_1$ is small and $x_2$ is medium then Class 2 with 0.158
**Fuzzy Rules with Rule Weights**

Accurate and Interpretable Fuzzy Rule-Based Classifier Design

---

**Basic Form**

If $x_1$ is small and $x_2$ is medium then Class 2

**Rule Weight Version**

If $x_1$ is small and $x_2$ is medium then Class 2 with 0.158

---

Title: DISTRIBUTED REPRESENTATION OF FUZZY RULES AND ITS APPLICATION TO PATTERN-CLASSIFICATION

Author(s): ISHIBUCHI H, NOZAKI K, TANAKA H

Source: FUZZY SETS AND SYSTEMS Volume: 52 Issue: 1 Pages: 21-32 Published: NOV 25 1992

Fuzzy Rules in This Presentation

Fuzzy Rules with Rule Weights

1.0

Class 1  Class 2  Class 3

Basic Form
If $x_1$ is *small* and $x_2$ is *medium* then Class 2

Rule Weight Version
If $x_1$ is *small* and $x_2$ is *medium* then Class 2 with 0.158

Use of Rule Weights: Controversial Issue

(1) Rule weight adjustment can be replaced with membership learning.

Neuro-fuzzy systems have recently gained a lot of interest in research and application. These are approaches that learn fuzzy systems from data. Many of them use rule weights for this task. In this paper we discuss the influence ...

Fuzzy Rules in This Presentation

Fuzzy Rules with Rule Weights

1.0

Class 1  Class 2  Class 3

Basic Form
If \( x_1 \) is small and \( x_2 \) is medium then Class 2

Rule Weight Version
If \( x_1 \) is small and \( x_2 \) is medium then Class 2 with 0.158

Use of Rule Weights: Controversial Issue

(2) Membership learning can be partially replaced with weight adjustment.

[PDF] Effect of rule weights in fuzzy rule-based classification systems
H Ishibuchi, T Nakashima - algorithms - Citeseer
... Hisao Ishibuchi, Member, IEEE, and Tomoharu Nakashima, Member, IEEE ... Hisao Ishibuchi, Member, IEEE, and Tomoharu Nakashima, Member, IEEE ... Google Scholar
Cited by 170

Fuzzy Rules for Classification

Another Form: Multiple Consequents

Basic Form
If $x_1$ is small and $x_2$ is medium then Class 2

Rule Weight Version
If $x_1$ is small and $x_2$ is medium then Class 2 with 0.158

Multiple Consequents
If $x_1$ is small and $x_2$ is medium then Class 1 with 0.579, Class 2 with 0.421, Class 3 with 0.000
Fuzzy Rules for Classification

Another Form: Multiple Consequents

Basic Form
If \( x_1 \) is small and \( x_2 \) is medium then Class 2

Rule Weight Version
If \( x_1 \) is small and \( x_2 \) is medium then Class 2 with 0.158

Multiple Consequents


Fuzzy Rule-Based Systems have been successfully applied to pattern classification problems. In this type of classification systems, the classical Fuzzy Reasoning Method (FRM) classifies a new example with the consequent of the rule with ...

Cited by 127 - Related articles - BL Direct O. Cordon et al., IJAR (2001)

Google Scholar
Other Forms of Fuzzy Rules
Handling of Classification as Function Approximation

Integer Consequent
If $x_1$ is small and $x_1$ is large then $y = 1$
If $x_1$ is large and $x_2$ is large then $y = 3$

Binary Consequent
If $x_1$ is small and $x_1$ is large then $(y_1, y_2, y_3) = (1, 0, 0)$
If $x_1$ is large and $x_2$ is large then $(y_1, y_2, y_3) = (0, 0, 1)$
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Accuracy Improvement
Use of Fine Fuzzy Partition

Class 1  Class 2  Class 3

Classifications:
- SML
- LM
- MS
- SN
- IN
- TIN
- T
Accuracy Improvement
How to choose an appropriate partition?

Too Fine Fuzzy Partition

==> Over-Fitting
(Poor Generalization Ability)
Accuracy Improvement
How to choose an appropriate partition?

One Idea: Use of All Partitions (Multiple Fuzzy Grid Approach)

Title: DISTRIBUTED REPRESENTATION OF FUZZY RULES AND ITS APPLICATION TO PATTERN-CLASSIFICATION
Author(s): ISHIBUCHI H, NOZAKI K, TANAKA H
Source: FUZZY SETS AND SYSTEMS Volume: 52 Issue: 1 Pages: 21-32 Published: NOV 25 1992

Web of Science
Ishibuchi et al., Fuzzy Sets and Systems (1992)
Accuracy Improvement
Learning of Membership Functions

Class 1  
Class 2  
Class 3

Accuracy Improvement Diagram showing data points for Class 1, Class 2, and Class 3.
Various learning methods such as neuro-fuzzy and genetic-fuzzy methods are available.

Title: A neuro-fuzzy method to learn fuzzy classification rules from data
Author(s): Nauck D, Kruse R
Source: FUZZY SETS AND SYSTEMS Volume: 89
Issue: 3 Pages: 277-288 Published: AUG 1 1997

Various learning methods such as neuro-fuzzy and genetic-fuzzy methods are available.

Interpretability is degraded.

Title: A neuro-fuzzy method to learn fuzzy classification rules from data
Author(s): Nauck D, Kruse R
Source: FUZZY SETS AND SYSTEMS Volume: 89 Issue: 3 Pages: 277-288 Published: AUG 1 1997

Each fuzzy rule can be generated and adjusted independently from other rules. ==> High Accuracy

Membership functions can be heavily overlapping.

==> Poor Interpretability
Each fuzzy rule can be generated and adjusted independently from other rules. ==> High Accuracy

Membership functions can be heavily overlapping.

==> Poor Interpretability

S. Abe and M. S. Lan, IEEE Tras. on FS (1995)
Accuracy Improvement

Use of Multi-Dimensional Membership Functions

If $x$ is $\mathbf{A}$ then Class 2

$\mathbf{A}$: Multi-dimensional Fuzzy Set
(Membership Function)
If \( x \) is \( A \) then Class 2

**A**: Multi-dimensional Fuzzy Set
(Membership Function)

Fuzzy rules are flexibility.

\[ \Rightarrow \text{High Accuracy} \]

Each membership function is multi-dimensional.

\[ \Rightarrow \text{Poor Interpretability} \]
Accuracy Improvement
Use of Multi-Dimensional Membership Functions

If \( x \) is \( A \) then Class 2

\( A \): Multi-dimensional Fuzzy Set (Membership Function)

Title: Feature selection by analyzing class regions approximated by ellipsoids
Author(s): Abe S, Thawonmas R, Kobayashi Y
Source: IEEE Transactions on Systems Man and Cybernetics, Part C: Applications and Reviews
   Volume: 28 Issue: 2 Pages: 282-287 Published: MAY 1998

Title: A fuzzy classifier with ellipsoidal regions for diagnosis problems
Author(s): Abe S, Thawonmas R, Kayama M
Source: IEEE Transactions on Systems Man and Cybernetics, Part C: Applications and Reviews
   Volume: 29 Issue: 1 Pages: 140-149 Published: FEB 1999

S. Abe et al., IEEE TSMC-C (1998)

S. Abe et al., IEEE TSMC-C (1999)

Web of Science
Accuracy Improvement
Use of Tree-Type Fuzzy Partitions

If $x_1$ is small then Class 2.
If $x_1$ is large and $x_2$ is small then Class 3.
If $x_1$ is large and $x_2$ is large then Class 1.
Accuracy Improvement
Use of Tree-Type Fuzzy Partitions

Title: **INDUCTION OF FUZZY DECISION TREES**
Author(s): YUAN YF, SHAW MJ
Source: **FUZZY SETS AND SYSTEMS** Volume: 69
Issue: 2 Pages: 125-139 Published: JAN 27 1995

Title: **Fuzzy decision trees: Issues and methods**
Author(s): Janikow CZ
Source: **IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS PART B-CYBERNETICS** Volume: 28
Issue: 1 Pages: 1-14 Published: FEB 1998

![Diagram]

\[ x_1 \text{ is small} \]
\[ x_1 \text{ is large} \]
\[ x_2 \text{ is small} \]
\[ x_2 \text{ is large} \]
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Difficulty of High-Dimensional Problems
Exponential Increase of Fuzzy Rules

Basic Form
If $x_1$ is \textit{small} and $x_2$ is \textit{small} then Class 2
If $x_1$ is \textit{small} and $x_2$ is \textit{medium} then Class 2
\ldots
If $x_1$ is \textit{large} and $x_2$ is \textit{large} then Class 3

Number of Fuzzy Rules:
2-D Problem: $3 \times 3$
3-D Problem: $3 \times 3 \times 3$
4-D Problem: $3 \times 3 \times 3 \times 3$
5-D Problem: $3 \times 3 \times 3 \times 3 \times 3$
Scalability Improvement
Use of Independent Membership Functions

Fuzzy rules are generated in the multi-dimensional space. => No Exponential Increase
If x is A then Class 2

A: Multi-dimensional Fuzzy Set
(Member of Function)

Fuzzy rules are generated in the multi-dimensional space.
=> No Exponential Increase
An appropriate stopping condition prevents the exponential increase in the number of fuzzy rules.
Scalability Improvement
Hierarchical Fuzzy Systems

Diagram:
- Top level: subsystem
- Second level:
  - Left: subsystem
  - Center: subsystem
  - Right: subsystem
- Bottom level:
  - Left: $x_1$, $x_2$
  - Center: $x_3$, $x_4$
  - Right: $x_5$
Scalability Improvement
Hierarchical Fuzzy Systems

Title: SELF-TUNING FUZZY MODELING WITH ADAPTIVE MEMBERSHIP FUNCTION, RULES, AND HIERARCHICAL STRUCTURE-BASED ON GENETIC ALGORITHM
Author(s): SHIMOJIMA K, FUKUDA T, HASEGAWA Y
Source: FUZZY SETS AND SYSTEMS Volume: 71 Issue: 3 Pages: 295-309 Published: MAY 12 1995

Accuracy & Scalability Improvement

==> Poor Interpretability

- Use of Fine Fuzzy Partition (Accuracy)
- Use of Rule Weights (Accuracy)
- Membership Function Learning (Accuracy)
- Fuzzy Rules with Independent Membership Functions (Accuracy and Scalability)
- Multi-Dimensional Fuzzy Systems (Accuracy and Scalability)
- Tree-Type Fuzzy Partitions (Accuracy and Scalability)
- Hierarchical Fuzzy Systems (Accuracy and Scalability)
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Complexity Minimization
Fuzzy Rule Selection

Rule Selection (Nine Rules ==> Four Rules)

Nine Rules

Four Rules
Rule Selection (Nine Rules ==> Four Rules)
The same classification boundaries are generated.
Rule Selection (Nine Rules ==> Four Rules)
The same classification boundaries are generated.

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Author(s): ISHIBUCHI H, NOZAKI K, YAMAMOTO N, et al.
Source: IEEE TRANSACTIONS ON FUZZY SYSTEMS
Published: AUG 1995

Use of “Don’t Care” Conditions

Nine Rules ==> Seven Rules (If $x_1$ is large then Class 3)
Complexity Minimization
Use of “Don’t Care” Conditions

Nine Rules ==> Seven Rules (If $x_1$ is large then Class 3)
Slightly different classification boundaries are obtained.

Nine Rules

Seven Rules
Nine Rules ==> Seven Rules (If $x_1$ is large then Class 3)

Slightly different classification boundaries are obtained.

Complexity Minimization
Use of "Don’t Care" Conditions

The use of “Don’t Care” conditions can be viewed as an scalability improvement method. If $x_1$ is small and $x_{10}$ is ...

Complexity Minimization
Use of “Don’t Care” Conditions

The use of “Don’t Care” conditions can be also viewed as input selection for each rule (rule-wise input selection).

[PDF] Performance evaluation of fuzzy classifier systems for multidimensional pattern classification problems with continuous attributes. In our method, each fuzzy if–then rule is handled... Google Scholar

Complexity Minimization
Merging Similar Membership Functions
Complexity Minimization
Merging Similar Membership Functions

Similar Membership Functions
Complexity Minimization
Merging Similar Membership Functions

Similar Membership Functions
=> One Membership Function
Complexity Minimization
Merging Similar Membership Functions

Class 1  Class 2  Class 3

Similar Membership Functions
==> One Membership Function

[PDF] ▶ Similarity measures in fuzzy rule base simplification
M Setnes, R Babuska, U Kaymak, HR van ... - IEEE Transactions on Systems, Man, repository.tudelft.nl

Abstract—In fuzzy rule-based models acquired from numerical data, redundancy may be present in the form of similar fuzzy sets that represent compatible concepts. This results in an unnecessarily complex and less transparent ...

Cited by 259 - Related articles - View as HTML - BL Direct - All 8 versions

Complexity Minimization
Merging Similar Membership Functions

Class 1  Class 2  Class 3

Similar Membership Functions
==> One Membership Function

- Interpretability is improved.
- Accuracy is degraded.
Complexity Minimization
Merging Similar Membership Functions

Similar Membership Functions
=> One Membership Function

- Interpretability is improved.
- Accuracy is degraded.

Interpretability-Accuracy Tradeoff
Complexity Minimization
Projection of Multi-Dimensional Fuzzy Sets

Change of Fuzzy Rule Form

If \( x \) is \( A_q \) then ...

If \( x_1 \) is \( A_{q1} \) and \( x_2 \) is \( A_{q2} \) then

- Interpretability is improved.
- Accuracy is degraded.

Interpretability-Accuracy Tradeoff
Interpretability-Accuracy Tradeoff
(Accuracy-Simplicity Tradeoff)

- Interpretable fuzzy system
- Accurate fuzzy system

Error: Large vs. Small
Complexity: Simple vs. Complicated

Graph showing the tradeoff between interpretability and accuracy/simplicity.
Accuracy Maximization
Main Research Direction Since the Early 1990s

- Fuzzy Neuro Learning
- Genetic Fuzzy Optimization
Possible Difficulties

Accuracy Maximization

- Poor Interpretability
- Overfitting to Training Data
Possible Difficulties

Accuracy Maximization

- Poor Interpretability
- Overfitting to Training Data
Difficulty in Accuracy Maximization

Accuracy maximization → Overfitting

Test data accuracy

Training data accuracy

Error

Complexity

$S^*$
Accuracy-Complexity Tradeoff

Curve Fitting

Error

Feasible Area

Complexity
Accuracy-Complexity Tradeoff

Curve Fitting

![Graph showing the tradeoff between accuracy and complexity](image-url)
Accuracy-Complexity Tradeoff

Curve Fitting

Error vs. Complexity

Graph showing a tradeoff curve with error on the y-axis and complexity on the x-axis.
Accuracy-Complexity Tradeoff

Curve Fitting

Error vs. Complexity

Error vs. $x$

$y$ vs. $x$
Accuracy-Complexity Tradeoff

Curve Fitting

Error

Complexity

\[ x \]

\[ y \]
Accuracy-Complexity Tradeoff

Curve Fitting

Error

Test Data Error

Training Data Error

Complexity

0 0.2 0.4 0.6 0.8 1

0 0.2 0.4 0.6 0.8 1

0 0.2 0.4 0.6 0.8 1
Accuracy-Complexity Tradeoff

Curve Fitting

Test Data Error

Training Data Error
Possible Difficulties

Accuracy Maximization

- Poor Interpretability
- Overfitting to Training Data

In the design of fuzzy systems, emphasis should be placed on their linguistic interpretability.

Interpretability maintenance while maximizing the accuracy.
Accuracy and Interpretability Maximization
Active Research Direction Since the Late 1990s

Compromise between Accuracy and Complexity
(Search for a good accuracy-complexity tradeoff)
Accuracy and Interpretability Maximization
Active Research Direction Since the Late 1990s

Compromise between Accuracy and Complexity
(Search for a good accuracy-complexity tradeoff)

Some Ideas
- Aggregated Objective Function: To combine the error minimization and the complexity minimization into a single scalar fitness function
Compromise between Accuracy and Complexity
(Search for a good accuracy-complexity tradeoff)

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- Aggregated Objective Function: To combine the error minimization and the complexity minimization into a single scalar fitness function
- Constraint Condition: To use constraint conditions on the position and the shape of membership functions
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Accuracy and Interpretability Maximization
Active Research Direction Since the Late 1990s

Accuracy Maximization and Complexity Minimization

[PDF] GA-fuzzy modeling and classification: complexity and performance
Manuscript received (...); revised (...). This work was supported in part by the
Research Council of Norway. The authors are with the Delft University of
Technology, Faculty of Information Technology and Systems, Control ...
Cited by 253 - Related articles - View as HTML - BL Direct - All 10 versions

[PDF] Compact and transparent fuzzy models and classifiers
Abstract—In our previous work we showed that genetic algo-
rithms (GAs) provide a powerful tool to increase the accuracy of fuzzy models for both
systems modeling and classification. In addi-
tion to these results, we ...
Cited by 146 - Related articles - View as HTML - BL Direct - All 3 versions

[BOOK] Interpretability issues in fuzzy modeling
J Casillas, O Cordón, F Herrera, L Magdalena, 2003 - books.google.com
Dr. Jorge Casillas Dr. Luis Magdalena E-mail: casillas@decsai. ugr. es E-mail:
Ilayos@ mat. upm. es Dr. Oscar Cord< 5n Dpto. Matematicas Aplicadas E-mail:
ocordon@ decsai. ugr. es a las Tecnologias de la Information Dr. Francisco ...
Cited by 167 - Related articles - All 2 versions
Compromise between Accuracy and Complexity
(Search for a good accuracy-complexity tradeoff)

Basic Idea

- **Aggregated Objective Function:** To combine the error minimization and the complexity minimization into a single scalar fitness function
- **Constraint Condition:** To use constraint conditions on the position and the shape of membership functions
- **Two-Step Fuzzy System Design:** 1st Step: Search for accurate and complicated fuzzy rule-based systems. 2nd Step: Simplification of obtained fuzzy rule-based systems.
Compromise between Accuracy and Complexity
(Search for a good accuracy-complexity tradeoff)

Aggregated Objective Function
- To combine the error minimization and the complexity minimization into a single scalar objective function
Compromise between Accuracy and Complexity
(Search for a good accuracy-complexity tradeoff)

Aggregated Objective Function
- To combine the error minimization and the complexity minimization into a single scalar objective function

Example: Combination of the average error rate and the number of fuzzy rules

Example of a scalar objective function: Weighted sum

\[ f(S) = w_1 \cdot f_{\text{Error}}(S) + w_2 \cdot f_{\text{Complexity}}(S) \]
Fuzzy systems were automatically generated, trained, and simplified.
Sensitivity to the weight vector:

The obtained system strongly depends on the specification of the weight vector.
Minimize \( w_1 \cdot \text{Error} + w_2 \cdot \text{Complexity} \)

When the weight for the complexity minimization is large:

- A simple system is obtained.

The graph shows the trade-off between error and complexity, with the objective of minimizing the weighted sum. The point \( S^* \) indicates a balance where both error and complexity are minimized, leading to an optimal system.
Minimize \( w_1 \cdot \text{Error} + w_2 \cdot \text{Complexity} \)

When the weight for the error minimization is large:

- A complicated system is obtained.

![Graph showing error and complexity with a minimum point labeled \( S^* \).]
Difficulty in Weighted Sum Approach

Minimize \[ w_1 \cdot \text{Error} + w_2 \cdot \text{Complexity} \]

When the two weights are appropriately specified:

A good system is obtained. But the best complexity is not always found.

Test data accuracy

Training data accuracy
Multiobjective Fuzzy System Design
Currently An Active Research Issue

Basic Idea
To search for a number of non-dominated fuzzy systems with respect to the accuracy maximization and the interpretability maximization (instead of searching for a single fuzzy system).

Aggregation Approach

\[ f(S) = w_1 \cdot f_{Error}(S) + w_2 \cdot f_{Complexity}(S) \]

Multiobjective Approach

Minimize \{f_{Error}(S), f_{Complexity}(S)\}
Multiobjective Fuzzy System Design
Currently An Active Research Issue

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Aggregation Approach

\[ f(S) = w_1 \cdot f_{Error}(S) + w_2 \cdot f_{Complexity}(S) \]

Multiobjective Approach

Minimize \( \{ f_{Error}(S), f_{Complexity}(S) \} \)

Search for Pareto Optimal Fuzzy Rule-Based Systems
Multiobjective Fuzzy System Design
Currently An Active Research Issue

Interpretable fuzzy system
Accurate fuzzy system
1. Introduction to Fuzzy Rule-Based Classification
   - Is Fuzzy Rule-Based Classification a Popular Research Area?

2. Fuzzy Rule-Based Classifier Design
   - Accuracy Improvement
   - Scalability to High-Dimensional Problems
   - Complexity Minimization

3. Multiobjective Fuzzy Rule-Based Classifier Design
   - Formulation of Multi-objective Problems
   - Accuracy-Complexity Tradeoff Analysis
   - Maximization of Generalization Ability

4. Current Hot Issues and Future Research Directions
   - Search Ability of EMO for Fuzzy System Design
   - Definition of Interpretability of Fuzzy Systems
   - Explanation Ability of Fuzzy Rule-Based Systems
   - Various Classification Problems: Imbalanced, Online, ...
The Evolutionary Multiobjective Optimization of Fuzzy Rule-Based Systems Bibliography Page

Abstract

Since pioneering works by Prof. Hisao Ishibuchi in middle nineties, Pareto-based Evolutionary Multiobjective Optimization (EMO) of Fuzzy Rule-Based Systems (FRBSs) is nowadays a well-established research area. It is a branch of the more general Evolutionary/Genetic Fuzzy Systems (see F. Herrera, "Genetic Fuzzy systems: Taxonomy, current research trends and
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Multi-Objective Fuzzy Rule-Based Systems

(Fuzzy Rule* OR Fuzzy Rule-Based System*) AND (Multi-Objective OR Multiobjective OR Two-Objective OR Three-Objective OR Multiple Criteria)

Multi-Objective Fuzzy Rule Selection for Fuzzy Rule-Based Classifier Design

Two Objectives:
- The number of correctly classified training patterns
- The number of selected fuzzy rules
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<th>Author(s)</th>
<th>Source</th>
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<td>Three-objective genetics-based machine learning for linguistic rule extraction</td>
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Multi-Objective Fuzzy Rule Selection for Fuzzy Rule-Based Classifier Design

Two Objectives:
- The number of correctly classified training patterns
- The number of selected fuzzy rules

Three Objectives:
- The number of correctly classified training patterns
- The number of selected fuzzy rules
- Total number of antecedent conditions (Total rule length)
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Author(s): Gonzalez J, Rojas I, Ortega J, et al.

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Multi-Objective Neural Network Design and Learning
Multiobjective Neural Networks

Error

0

Complexity
Multiobjective Decision Trees (GP)
Multi-Objective Fuzzy Rule-Based Systems

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We will compare these applications in a cost/complexity framework, and examine the driving factors that led to the use of FLC's in each application. We will emphasize the role of fuzzy logic in developing supervisory controllers and in maintaining explicit tradeoff criteria used to manage multiple control strategies.
Paper Title: Industrial Applications of Fuzzy-Logic at General-Electric

Author(s): BONISSONE PP, BADAMI V, CHIANG KH, KHEDKAR PS, MARCELLE KW, SCHUTTEN MJ


Times Cited: 39  References: 28  Citation Map

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Web of Science
Recent Publications on Multi-Objective Fuzzy System Design

1. Title: A Multiobjective Evolutionary Approach to Concurrently Learn Rule and Data Bases of Linguistic Fuzzy-Rule-Based Systems  
   Source: IEEE TRANSACTIONS ON FUZZY SYSTEMS  
   Volume: 17  Issue: 5  Pages: 1106-1122  
   Published: OCT 2009

2. Title: Optimum energy management of a photovoltaic water pumping system  
   Author(s): Sallem S, Chaabene M, Kamoun MBA  
   Source: ENERGY CONVERSION AND MANAGEMENT  
   Volume: 50  Issue: 11  Pages: 2728-2731  Published: NOV 2009

3. Title: Energy management algorithm for an optimum control of a photovoltaic water pumping system  
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   Volume: 86  Issue: 12  Pages: 2671-2680  Published: DEC 2009

4. Title: Reactive power dispatch considering voltage stability with seeker optimization algorithm  
   Author(s): Dai CH, Chen WR, Zhu YF, et al.  
   Source: ELECTRIC POWER SYSTEMS RESEARCH  
   Volume: 79  Issue: 10  Pages: 1462-1471  Published: OCT 2009

5. Title: Learning concurrently partition granularities and rule bases of Mamdani fuzzy systems in a multi-objective evolutionary framework  
   Author(s): Antonelli M, Ducange P, Lazzerini B, et al.  
   Source: INTERNATIONAL JOURNAL OF APPROXIMATE REASONING  
   Volume: 50  Issue: 7  Pages: 1066-1080  Published: JUL 2009
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Author(s): Alcala R (Alcala, Rafael)¹, Ducange P (Ducange, Pietro)², Herrera F (Herrera, Francisco)¹, Lazzerini B (Lazzerini, Beatrice)², Marcelloni F (Marcelloni, Francesco)²

Source: IEEE TRANSACTIONS ON FUZZY SYSTEMS Volume: 17 Issue: 5 Pages: 1106-1122 Published: OCT 2009

Times Cited: 0 References: 52

Abstract: In this paper, we propose the use of a multiobjective evolutionary approach to generate a set of linguistic fuzzy-rule-based systems with different tradeoffs between accuracy and interpretability in regression problems. Accuracy and interpretability are measured in terms of approximation error and rule base (RB) complexity, respectively. The proposed approach is based on concurrently learning RBs and parameters of the membership functions of the associated linguistic labels. To manage the size of the search space, we have integrated the linguistic two-tuple representation model, which allows the symbolic translation of a label by only considering one parameter, with an efficient
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Author(s): Alcala R (Alcala, Rafael)\(^1\), Ducange P (Ducange, Pietro)\(^2\), Herrera F (Herrera, Francisco)\(^1\), Lazzerini B (Lazzerini, Beatrice)\(^2\), Marcelloni F (Marcelloni, Francesco)\(^2\)

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Multi-Objective Fuzzy System Design Research
Active Geographical Regions
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   - Formulation of Multi-objective Problems
   - Accuracy-Complexity Tradeoff Analysis
   - Maximization of Generalization Ability

4. Current Hot Issues and Future Research Directions
   - Search Ability of EMO for Fuzzy System Design
   - Definition of Interpretability of Fuzzy Systems
   - Explanation Ability of Fuzzy Rule-Based Systems
   - Various Classification Problems: Imbalanced, Online, ...
Many non-dominated fuzzy systems can be obtained along the tradeoff surface by a single run of an EMO algorithm.

EMO: Evolutionary Multi-Objective Optimization
The obtained non-dominated fuzzy systems show the tradeoff between the complexity and the training data accuracy (not the tradeoff between the complexity and the test data accuracy).
The obtained non-dominated fuzzy systems show the tradeoff between the complexity and the training data accuracy (not the tradeoff between the complexity and the test data accuracy).

- Tradeoff for test data accuracy should be examined.
- This can be done since we have many fuzzy systems.
Example: Obtained Rule Sets (Heart C)

Obtained rule sets help us to find the optimal complexity of fuzzy systems. (Rule sets with six, seven and eight rules may be good)
A rule set with High-Generalization Ability

A rule set with eight fuzzy rules

<table>
<thead>
<tr>
<th>Rule</th>
<th>$x_1$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_6$</th>
<th>$x_7$</th>
<th>$x_8$</th>
<th>$x_{10}$</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>Consequent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>Class 1 (0.46)</td>
</tr>
<tr>
<td>$R_2$</td>
<td>DC</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>Class 1 (0.23)</td>
<td></td>
</tr>
<tr>
<td>$R_3$</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>Class 1 (0.81)</td>
</tr>
<tr>
<td>$R_4$</td>
<td>△</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>Class 2 (0.63)</td>
</tr>
<tr>
<td>$R_5$</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>Class 2 (0.20)</td>
</tr>
<tr>
<td>$R_6$</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>Class 2 (1.00)</td>
</tr>
<tr>
<td>$R_7$</td>
<td>△</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>DC</td>
<td>△</td>
<td>Class 3 (0.35)</td>
</tr>
<tr>
<td>$R_8$</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>DC</td>
<td>DC</td>
<td>△</td>
<td>△</td>
<td>Class 3 (0.24)</td>
</tr>
</tbody>
</table>

Some human users may prefer simpler rule sets.
A very simple rule set with only two fuzzy rules

<table>
<thead>
<tr>
<th>Number of rules</th>
<th>Error rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
</tr>
</tbody>
</table>

**Consequent**

<table>
<thead>
<tr>
<th>Class 1</th>
<th>Class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.26)</td>
<td>(1.00)</td>
</tr>
</tbody>
</table>

**Rules**

- $R_1$: $x_{10}$ \(\square\) $x_{11}$ \(\triangle\) Class 1
- $R_2$: $x_{10}$ \(\square\) $x_{11}$ \(\square\) Class 2
Contents of This Presentation

1. Introduction to Fuzzy Rule-Based Classification
   - Is Fuzzy Rule-Based Classification a Popular Research Area?

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   - Scalability to High-Dimensional Problems
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Why is the fuzzy system design difficult?

1. Large Search Space: Difficulty in Search

The search space exponentially increases with the number of attributes (i.e., with the dimensionality of the pattern space).
**Basic Form**

If $x_1$ is *small* and $x_2$ is *small* then Class 2

If $x_1$ is *small* and $x_2$ is *medium* then Class 2

... 

If $x_1$ is *large* and $x_2$ is *large* then Class 3

**Number of Fuzzy Rules:**

- 2-D Problem: $3 \times 3$
- 3-D Problem: $3 \times 3 \times 3$
- 4-D Problem: $3 \times 3 \times 3 \times 3$
- 5-D Problem: $3 \times 3 \times 3 \times 3 \times 3$
Use of Don’t Care

If $x_1$ is small and $x_2$ is small
then Class 2

If $x_1$ is small and $x_2$ is medium
then Class 2

... 

If $x_1$ is large and $x_2$ is don’t care
then Class 3

Number of Fuzzy Rules:
2-D Problem: $(3+1) \times (3+1)$
3-D Problem: $(3+1)^3$
4-D Problem: $(3+1)^4$
5-D Problem: $(3+1)^5$
Number of Fuzzy Rules and Number of Rule Sets

Search Space Size: Large

Example: Classification problem with 50 attributes and 3 linguistic values for each attribute

The total number of fuzzy rules (i.e., antecedent condition combinations):

\[(3+1) \times \ldots \times (3+1) = 4^{50} = 2^{100}\]

The total number of fuzzy rule sets with 20 rules (i.e., combinations of 20 fuzzy rules):

\[N^{C_{20}} \approx 2^{2000} \text{ where } N = 2^{100}\]
Number of Fuzzy Rules and Number of Rule Sets

Search Space Size: Large

Example: Classification problem with 50 attributes and 1-7 fuzzy partition for each attribute

The total number of fuzzy rules (i.e., antecedent condition combinations):

\[(1+2+ \ldots 7) \times \ldots = 28^{50} > 2^{400}\]

The total number of fuzzy rule sets with 20 rules (i.e., combinations of 20 fuzzy rules):

\[\binom{N}{20} \sim 2^{8000} \text{ where } N > 2^{400}\]
1. Large Search Space: Difficulty in Search

The search space exponentially increases with the number of attributes (i.e., with the dimensionality of the pattern space). It is likely that the entire tradeoff curve cannot be covered well by the obtained non-dominated fuzzy rule-based systems.
1. Large Search Space: Difficulty in Search
   The search space exponentially increases with the number of attributes (i.e., with the dimensionality of the pattern space). It is likely that the entire tradeoff curve cannot be covered well by the obtained non-dominated fuzzy rule-based systems.

2. Possibility of Over-Fitting: Difficulty in Learning
   The improvement in the training data accuracy does not always mean the improvement in the test data accuracy.
Why is the fuzzy system design difficult?

1. Large Search Space: Difficulty in Search
The search space exponentially increases with the number of attributes (i.e., with the dimensionality of the pattern space). It is likely that the entire tradeoff curve cannot be covered well by the obtained non-dominated fuzzy rule-based systems.

2. Possibility of Over-Fitting: Difficulty in Learning
The improvement in the training data accuracy does not always mean the improvement in the test data accuracy. This means that the fitness function improvement does not always lead to better fuzzy rule-based classifiers (when the training data accuracy is used in the fitness function).
Why is the fuzzy system design difficult?

2. Possibility of Over-Fitting: Difficulty in Learning

The improvement in the training data accuracy does not always mean the improvement in the test data accuracy.
Why is the fuzzy system design difficult?

2. Possibility of Over-Fitting: Difficulty in Learning

The improvement in the training data accuracy does not always mean the improvement in the test data accuracy.

![Diagram showing the relationship between error, complexity, training data accuracy, and test data accuracy. The diagram illustrates poor diversity and diversity improvement.](image-url)
Why is the fuzzy system design difficult?

2. Possibility of Over-Fitting: Difficulty in Learning

The improvement in the training data accuracy does not always mean the improvement in the test data accuracy.
Why is the fuzzy system design difficult?

2. Possibility of Over-Fitting: Difficulty in Learning

The improvement in the training data accuracy does not always mean the improvement in the test data accuracy.
Recent Studies: Improvement in training data accuracy leads to Improvement in test data accuracy.


Our Experimental Results

MoFGBML Algorithm (Framework: NSGA-II)

Multi-Objective Fuzzy Genetics-Based Machine Learning

Analysis of interpretability-accuracy tradeoff of fuzzy systems by multiobjective fuzzy genetics-based learning 
H Ishibuchi, Y Nojima - International Journal of Approximate Reasoning, 2007 - Elsevier
This paper examines the interpretability-accuracy tradeoff in fuzzy rule-based classifiers using a multiobjective fuzzy genetics-based machine learning (GBML) algorithm. Our GBML algorithm is a hybrid version of Michigan and ...  
Ishibuchi & Nojima, IJAR 2007

NSGA-II Basic Setting
- Population size: 200 individuals
- Termination Condition: 2000 generations
- Multiple Fuzzy Partitions: Granularities 1-5

Three Variants of MoFGBML Setting
- Diversity Improvement Method (Mating, EJOR 2008)
- Termination Condition: 20000 generations
- Multiple Fuzzy Partitions: Granularities 7
Experimental Results (Glass)

<table>
<thead>
<tr>
<th>Number of fuzzy rules</th>
<th>Error rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Training Data</td>
</tr>
<tr>
<td></td>
<td>Test Data</td>
</tr>
</tbody>
</table>

**NSGA-II Basic Setting**

Training Data
Accuracy Improvement
Experimental Results (Glass)

<table>
<thead>
<tr>
<th>Number of fuzzy rules</th>
<th>Error rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test Data</td>
</tr>
<tr>
<td></td>
<td>Training Data</td>
</tr>
</tbody>
</table>

NSGA-II Basic Setting

NSGA-II with Mating
Experimental Results (Glass)

- **NSGA-II Basic Setting**
- **NSGA-II with Mating**
Experimental Results (Glass)

NSGA-II 20,000 Generations

NSGA-II with Mating
Experimental Results (Glass)

![Graphs showing error rate against number of fuzzy rules for different granularities.](#)

- **NSGA-II Granularities 1-7**
- **NSGA-II with Mating**
Our Experimental Results
Simple Changes of Objectives (GEFS 2010, Spain)

Original Formulation
f1(S): Error Rate (%)
f2(S): Number of Fuzzy Rules

Simple Modification
\[ g1(S) = f1(S) - \alpha f2(S) \]
\[ g2(S) = f2(S) + \alpha f1(S) \]
Our Experimental Results
Simple Changes of Objectives (GEFS 2010, Spain)

(a) Glass data.
(b) Diabetes data.
Our Experimental Results
Simple Changes of Objectives (GEFS 2010, Spain)

Original Formulation

\[
f_1(S): \text{Error Rate (\%)}
\]

\[
f_2(S): \text{Number of Fuzzy Rules}
\]

Simple Modification

\[
g_1(S) = f_1(S) - \alpha f_2(S)
\]

\[
g_2(S) = f_2(S) + \alpha f_1(S)
\]

Four-Objective

\[
g_1(S) = f_1(S) - \alpha f_2(S)
\]

\[
g_2(S) = f_1(S) + \alpha f_2(S)
\]

\[
g_3(S) = f_2(S) - \alpha f_1(S)
\]

\[
g_4(S) = f_2(S) + \alpha f_1(S)
\]
Our Experimental Results
Four-Objective Formulation (GEFS 2010, Spain)

(a) Glass data. (b) Diabetes data.
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Handling of Interpretability in Our Former Studies


Interpretability Maximization = Complexity Minimization
- Minimization of the number of fuzzy rules
- Minimization of the number of antecedent conditions
Interpretability of Fuzzy Systems


Interpretability Maximization = Complexity Minimization
- Minimization of the number of fuzzy rules
- Minimization of the number of antecedent conditions
Interpretability of Fuzzy Systems


Interpretability Maximization = Complexity Minimization
- Minimization of the number of fuzzy rules
- Minimization of the number of antecedent conditions

Many other factors are related to the interpretability
Interpretability of Fuzzy Systems


Interpretability Maximization = Complexity Minimization
- Minimization of the number of fuzzy rules
- Minimization of the number of antecedent conditions

Many other factors are related to the interpretability

Special Sessions and Many Related Papers
- IFSA 2009 Conference
- ISDA 2009 Conference (4 Papers with Interpretability in Their Titles)
- FUZZ-IEEE 2010 Conference
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Another Issue in Interpretability
Explanation of Classification Results

Explanation Ability

The ability of fuzzy rule-based systems to explain why a new pattern is classified as a particular class.

Example: Classification of a pattern $\diamond: x_A = (0.05, 0.05)$
Comparison between Rule Sets 1 and 2

Classification of $\diamond : x_A = (0.05, 0.05)$

(1) Rule Set 1: Nine Rules

(2) Rule Set 2: Four Rules
Comparison in Explanation Capability
Responsible Rules for Classification

R₁: If \( x₁ \) is small and \( x₂ \) is small then Class 2

R₅: If \( x₁ \) is medium and \( x₂ \) is medium then Class 2
R₁ seems to be a better explanation for the classification of $x_A$.

R₁: If $x_1$ is small and $x_2$ is small then Class 2

R₅: If $x_1$ is medium and $x_2$ is medium then Class 2
Comparison between Rule Sets 1 and 2

Rule Set 1 seems to have higher explanation ability while Rule Set 2 is simpler than Rule Set 1.

(1) Rule Set 1: Nine Rules

(2) Rule Set 2: Four Rules
Comparison between Rule Sets 1 and 4

Classification of $\diamond: \quad x_A = (0.05, 0.05)$

(1) Rule Set 1: Nine Rules

(4) Rule Set 4: Three Rules
Comparison in Explanation Capability
Responsible Rules for Classification

R_1: If x_1 is small and x_2 is small then Class 2

R_{1234}: If x_1 is small or medium and x_2 is small or medium then Class 2
Comparison in Explanation Capability

Responsible Rules for Classification

$R_1$ seems to be a better explanation for the classification of $x_A$.

$R_1$: If $x_1$ is small and $x_2$ is small then Class 2

$R_{1234}$: If $x_1$ is small or medium and $x_2$ is small or medium then Class 2
Comparison between Rule Sets (1) and (4)

Rule Set 1 seems to have higher explanation ability while Rule Set 4 is simpler than Rule Set 1.
Classification Capability
Another Example $x_B = (0.95, 0.50)$

Classification of $\diamondsuit$: $x_B = (0.95, 0.50)$
Comparison between Rule Sets 1 and 3

Classification of $\diamond : x_B = (0.95, 0.50)$

(1) Rule Set 1: Nine Rules

(3) Rule Set 3: Seven Rules
**Comparison in Explanation Capability**

**Responsible Rules for Classification**

\[ R_8: \text{If } x_1 \text{ is large and } x_2 \text{ is medium then Class 3} \]

\[ R_{789}: \text{If } x_1 \text{ is large then Class 3} \]
Comparison in Explanation Capability
Responsible Rules for Classification

Which is a better explanation for the classification of $x_B$ between $R_8$ and $R_{789}$?

$R_8$: If $x_1$ is large and $x_2$ is medium then Class 3

$R_{789}$: If $x_1$ is large then Class 3
Comparison in Explanation Capability
Responsible Rules for Classification

Which is a better explanation for the classification of \( x_B \) between R_8 and R_789? It is a very difficult question for me to answer.

R_8: If \( x_1 \) is large and \( x_2 \) is medium then Class 3

R_789: If \( x_1 \) is large then Class 3
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We have a lot of different types of classification problems where fuzzy rule-based classifiers have not been well-utilized and have a large potential usefulness:

1. Imbalanced Data
2. Semi-Supervised Learning
3. Active Learning
4. On-Line Learning
5. . . .
6. . . .
Eyke HÜLLERMEIER is with the Department of Mathematics and Computer Science at Marburg University (Germany), where he holds an appointment as a Full Professor and heads the Knowledge Engineering & Bioinformatics Lab. He holds M.Sc. degrees in mathematics and business computing, a Ph.D. in computer science, and a Habilitation degree, all from the University of Paderborn (Germany). His research interests are focused on machine learning and data mining, fuzzy set theory, uncertainty and approximate reasoning, and applications in bioinformatics. He has published numerous research papers on these topics in leading journals and major international conferences. He is on the editorial board of several journals, including Fuzzy Sets and Systems, Soft Computing, and the International Journal of Data Mining, Modeling and Management. Moreover, he is a board member of the European Society for Fuzzy Logic and Technology (EUSFLAT), a coordinator of the EUSFLAT working group on Learning and Data Mining, and head of the IEEE CIS Task Force on Machine Learning.
The purpose of this talk is twofold. First, it is intended to convey an idea of the state-of-the-art in fuzzy logic-based machine learning, to be understood as the application of formal concepts, methods, and techniques from fuzzy set theory and fuzzy logic in the field of machine learning and related research areas, such as data mining and knowledge discovery. In this regard, potential contributions that fuzzy logic can make to machine learning will be especially highlighted, though some deficiencies of this line of research will also be pointed out. Second, some promising directions of future research in this field shall be sketched and promoted, including problems of ranking and preference learning, the representation of uncertainty in model induction and prediction, and the use of fuzzy modeling techniques for feature generation.
Fuzzy Classifiers on Various Problems

ISDA 2009 Invited Talk by Hisao Ishibuchi

Machine Learning

Fuzzy

MoML

MoFuzzy

Basic

Advanced
Conclusions

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   - Definition of Interpretability of Fuzzy Systems It is not always high.
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   We have still a lot of interesting research issues.
Appendix: Comparison of the Two Approaches

Two-objective maximization problem

Experimental results of a single run of each approach

**EMO Approach**

**Weighted Sum Approach**
Appendix: Two-Dimensional Antecedent Fuzzy Sets

(a) A two-dimensional fuzzy vector.        (b) An ellipsoidal antecedent fuzzy set.
Appendix: Interval Rules vs Fuzzy Rules

(a) Interval Rules

(b) Fuzzy Rules
Appendix: Interval Rules vs Fuzzy Rules

(a) Interval Rules

(b) Fuzzy Rules