## **REVIEW ARTICLE**

# Genetic fuzzy systems: taxonomy, current research trends and prospects

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Abstract The use of genetic algorithms for designing fuzzy systems provides them with the learning and adaptation capabilities and is called genetic fuzzy systems (GFSs). This topic has attracted considerable attention in the Computation Intelligence community in the last few years. This paper gives an overview of the field of GFSs, being organized in the following four parts: (a) a taxonomy proposal focused on the fuzzy system components involved in the genetic learning process; (b) a quick snapshot of the GFSs status paying attention to the pioneer GFSs contributions, showing the GFSs visibility at ISI Web of Science including the most cited papers and pointing out the milestones covered by the books and the special issues in the topic; (c) the current research lines together with a discussion on critical considerations of the recent developments; and (d) some potential future research directions.

Keywords Genetic fuzzy systems ·

Fuzzy rule based systems · Genetic algorithms ·

Evolutionary algorithms · Machine learning · Data mining · Computational Intelligence

# 1 Introduction

Computational Intelligence techniques such as artificial neural networks [89], fuzzy logic [108], and genetic algorithms (GAs) [45, 57] are popular research subjects, since

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they can deal with complex engineering problems which are difficult to solve by classical methods [73].

Hybrid approaches have attracted considerable attention in the Computational Intelligence community. One of the most popular approaches is the hybridization between fuzzy logic and GAs leading to genetic fuzzy systems (GFSs) [27]. A GFS is basically a fuzzy system augmented by a learning process based on evolutionary computation, which includes genetic algorithms, genetic programming, and evolutionary strategies, among other evolutionary algorithms (EAs) [40].

Fuzzy systems are one of the most important areas for the application of the Fuzzy Set Theory. Usually it is considered a model structure in the form of fuzzy rule based systems (FRBSs). FRBSs constitute an extension to classical rule-based systems, because they deal with "IF-THEN" rules, whose antecedents and consequents are composed of fuzzy logic statements, instead of classical ones. They have demonstrated their ability for control problems [85], modelling [87], classification or data mining [64, 75] in a huge number of applications.

The automatic definition of an FRBS can be seen as an optimization or search problem, and GAs are a well known and widely used global search technique with the ability to explore a large search space for suitable solutions only requiring a performance measure. In addition to their ability to find near optimal solutions in complex search spaces, the generic code structure and independent performance features of GAs make them suitable candidates to incorporate a priori knowledge. In the case of FRBSs, this a priori knowledge may be in the form of linguistic variables, fuzzy membership function parameters, fuzzy rules, number of rules, etc. These capabilities extended the use of GAs in the development of a wide range of approaches for designing FRBSs over the last few years. Figure 1

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illustrates this idea, where the genetic process learns or tunes different components of an FRBS.

In the last few years we observe the increase of published papers in the topic due to the high potential of GFSs. Contrary to neural networks, clustering, rule induction and many other machine learning approaches, GAs provide a means to encode and evolve rule antecedent aggregation operators, different rule semantics, rule base aggregation operators and defuzzification methods. Therefore, GAs remain today as one of the fewest knowledge acquisition schemes available to design and, in some sense, optimize FRBSs with respect to the design decisions, allowing decision makers to decide what components are fixed and which ones evolve according to the performance measures.

The predominant type of GFS is that focused on FRBSs. However other kinds of GFSs have been developed, with successful results. They include genetic fuzzy neural networks and genetic fuzzy clustering algorithms. We will not analyze them in this paper. Readers can find an extended introduction to them in [27, Chap. 10].

In this paper we shortly introduce GFSs, propose a taxonomy focused on the FRBS components and sketch our vision of some hot current trends and prospects of GFSs.

The paper starts by briefly presenting FRBSs in Sect. 2. Section 3 introduces a taxonomy of GFSs according to the FRBS components involved in the genetic learning process and taking into account which of them are encoded. Section 4 presents an introduction to GFSs, paying attention to the pioneer GFS contributions, the GFSs visibility at *ISI Web of Science* including the most cited papers and pointing out the milestones covered by the existing books and special issues. Section 5 discusses in depth some current trends and critical considerations on the recent developments. Section 6 presents some suggestions of potential future research directions. Finally, some concluding remarks are pointed out in Sect. 7.



Fig. 1 Genetic fuzzy systems

#### 2 Preliminaries: fuzzy rule based systems

The essential part of FRBSs is a set of IF-THEN linguistic rules, whose antecedents and consequents are composed of fuzzy statements, related by the dual concepts of fuzzy implication and the compositional rule of inference.

An FRBS is composed of a knowledge base (KB), that includes the information in the form of IF-THEN fuzzy rules;

# *IF* a set of conditions are satisfied

THEN a set of consequents can be inferred

and an inference engine module that includes a fuzzification interface, which has the effect of transforming crisp data into fuzzy sets; an inference system, that uses them together with the KB to make inference by means of a reasoning method; and a defuzzification interface, that translates the fuzzy rule action thus obtained to a real action using a defuzzification method.

FRBSs can be broadly categorized into different families. The first includes linguistic models based on collections of IF-THEN rules, whose antecedents are linguistic values, and the system behaviour can be described in natural terms. The consequent is an output action or class to be applied. For example, we can denote them as:

 $R_i$ : If  $X_{i1}$  is  $A_{i1}$  and ... and  $X_{in}$  is  $A_{in}$  then Y is  $B_i$  or

 $R_i$ : If  $X_{i1}$  is  $A_{i1}$  and ... and  $X_{in}$  is  $A_{in}$  then  $C_k$  with  $w_{ik}$ 

with i = 1 to M, and with  $X_{i1}$  to  $X_{in}$  and Y being the input and output variables for regression respectively, and  $C_k$  the output class associated to the rule for classification, with  $A_{i1}$  to  $A_{in}$  and  $B_i$  being the involved antecedents and consequent labels, respectively, and  $w_{ik}$  the certain factor associated to the class. They are usually called linguistic FRBSs or Mamdani FRBSs [79].

The second category based on a rule structure that has fuzzy antecedent and functional consequent parts. This can be viewed as the expansion of piece-wise linear partition represented as

$$R_i$$
: If  $X_{i1}$  is  $A_{i1}$  and ... and  $X_{in}$  is  $A_{in}$  then  $Y = p(X_{i1}, ..., X_{in})$ .

with  $p(\cdot)$  being a polynomial function, usually a linear expression,  $Y = p_0 + p_1 \cdot X_{i1} + \cdots + p_n \cdot X_{in}$ . The approach approximates a nonlinear system with a combination of several linear systems. They are called TS-type fuzzy systems [97].

Other kind of fuzzy models are the approximate or scatter partition FRBSs, which differ from the linguistic ones in the direct use of fuzzy variables [2]. Each fuzzy rule thus presents its own semantic, i.e., the variables take different fuzzy sets as values (and not linguistic terms from a global term set). The fuzzy rule structure is then as follow:

$$R_i$$
: If  $X_{i1}$  is  $\hat{A}_{i1}$  and ... and  $X_{in}$  is  $\hat{A}_{in}$  then Y is  $\hat{G}_i$ .

with  $\hat{A_{ij}}$  to  $\hat{A_{in}}$  and  $\hat{G_i}$  being fuzzy sets. The major difference with respect to the rule structure considered in linguistic FRBSs is that rules of approximate nature are semantics free whereas descriptive rules operate in the context formulated by means of the linguistic semantics.

In linguistic FRBSs, the KB is comprised by two components, a database (DB) and a rule base (RB).

• A DB, containing the linguistic term sets considered in the linguistic rules and the membership functions defining the semantics of the linguistic labels.

Each linguistic variable involved in the problem will have associated a fuzzy partition of its domain representing the fuzzy set associated with each of its linguistic terms. Figure 2 shows an example of fuzzy partition with five labels.

This can be considered as a discretization approach for continuous domains where we establish a membership degree to the items (labels), we have an overlapping between them, and the inference engine manages the matching between the patterns and the rules providing an output according to the rule consequents with a positive matching. The determination of the fuzzy partitions is crucial in fuzzy modelling [8], and the granularity of the fuzzy partition plays an important role for the FRBS behaviour [28].

If we manage approximate FRBSs, then we do not have a DB due to the fact that rules have associated the fuzzy values.

• An RB, comprised of a collection of linguistic rules that are joined by a rule connective ("also" operator). In other words, multiple rules can fire simultaneously for the same input.

The inference engine of FRBSs acts in a different way depending of the kind of problem (classification or regression) and the kind of fuzzy rules (linguistic ones,



Fig. 2 Example of a fuzzy partition

TS-ones...). It always includes a fuzzfication interface that serves as the input to the fuzzy reasoning process, an inference system that infers from the input to several resulting output (fuzzy set, class,...) and the defuzzification interface or output interface that converts the fuzzy sets obtained from the inference process into a crisp action that constitutes the global output of the FRBS, in the case of regression problems, or provide the final class associated to the input pattern according to the inference model.

The generic structure of an FRBS is shown in Fig. 3.

For more information about fuzzy systems the following books may be consulted [27, 64, 75, 108]. For different issues associated to the trade-off between interpretability and accuracy of FRBSs, the two following edited books present a collection of contributions in the topic [18, 19].

Finally, we must point out that we can find a lot of applications of FRBSs in all areas of engineering, sciences, medicine,.... At the present it is very easy to search for these applications using the publisher web search tools focusing the search in journals of different application areas.

# 3 Taxonomy of genetic fuzzy systems

The central aspect on the use of GAs for automatic learning of FRBSs is that the design process can be analyzed as a search problem in the space of models, such as the space of rule sets, by means of the coding of the model in a chromosome.

From the optimization point of view, to find an appropriate fuzzy model is equivalent to code it as a parameter structure and then to find the parameter values that give us the optimum for a concrete fitness function. Therefore, the first step in designing a GFS is to decide which parts of the fuzzy system are subjected to optimization by the GA coding them into chromosomes.

In this section, we first present a taxonomy of GFSs according to the different parts of the fuzzy systems coded by the genetic model. Then we will pay attention to the different genetic learning coding approaches that we can find in the literature, according to the way of coding an RB and the cooperation versus competition among chromosomes, connecting them with the mentioned taxonomy.

#### 3.1 Taxonomy

We divide the GFS approaches into two processes, tuning and learning. It is difficult to make a clear distinction between tuning and learning processes, since establishing a precise borderline becomes as difficult as defining the concept of learning itself. The first fact that we have to take into consideration is the existence or not of a previous KB,



Fig. 3 Structure of an FRBS

including DB and RB. In the framework of GFSs we can shortly introduce the following distintion.

- Genetic tuning. If there exists a KB, we apply a genetic tuning process for improving the FRBS performance but without changing the existing RB. That is, to adjust FRBS parameters for improving its performance, mantaining the same RB.
- Genetic learning. The second possibility is to learn KB components (where we can even include an adaptive inference engine). That is, to involve the learning of KB components among other FRBS components.

We classify the proposals according to these two processes and according to the FRBS components involved in the genetic learning process. In this way, I propose the taxonomy shown in Fig. 4.

There are three main areas in the taxonomy that we can observe in the first tree: genetic tuning, genetic KB learning, and genetic learning of KB components and inference engine parameters.

In the following, we shortly analyze the three areas. We will provide some references as examples for every approach, but we do not present an exhaustive list of papers for every approach, this is far from the paper's objective.

## 3.1.1 Genetic tuning

With the aim of making the FRBS perform better, some approaches try to improve the preliminary DB definition or the inference engine parameters once the RB has been derived. A graphical representation of this kind of tuning is shown in Fig. 5.

The following three tuning possibilities can be considered (see the sub-tree under "genetic tuning").

- 1. Genetic tuning of KB parameters. In order to do so, a tuning process considering the whole KB obtained (the preliminary DB and the derived RB) is used a posteriori to adjust the membership function parameters. Nevertheless, the tuning process only adjusts the shapes of the membership functions and not the number of linguistic terms in each fuzzy partition, which remains fixed from the beginning of the design process. In [68] we can find a first and classic proposal on tuning. We can also find recent proposals that introduce linguistic modifiers for tuning the membership functions, see [17]. This latter approach is close to the inference engine adaptation.
- 2. Genetic adaptive inference systems. The main aim of this approach is the use of parameterized expressions in the Inference System, sometimes called Adaptive Inference Systems, for getting higher cooperation among the fuzzy rules and therefore more accurate fuzzy models without loosing the linguistic rule interpretability. In [6, 31, 32] we can find proposals in this area focused in regression and classification.
- 3. Genetic adaptive defuzzification methods. The most used technique in practice, due to its good performance, efficiency and easier implementation, is to apply the defuzzification function to every inferred rule fuzzy set (getting a characteristic value) and to



Fig. 4 GFSs taxonomy



Fig. 5 Genetic tuning process

compute them by a weighted average operator. This way to work introduces the possibility of using parameter based average functions, and the use of GAs can allow us to adapt the defuzzification methods. In [71] we can find a proposal in this area.

#### 3.1.2 Genetic KB learning

As a second big area we find the learning of KB components. Following, we describe the four approaches that can be found within the genetic learning of a KB (see the second tree under "genetic KB learning").

1. *Genetic rule learning*. Most of the approaches proposed to automatically learn the KB from numerical information have focused on the RB learning, using a

predefined DB. The usual way to define this DB involves choosing a number of linguistic terms for each linguistic variable (an odd number between 3 and 9, which is usually the same for all the variables) and setting the values of the system parameters by an uniform distribution of the linguistic terms into the variable universe of discourse. Figure 6 shows graphically this type of RB learning. The pioneer proposal for this approach can be found in [99].

On the other hand, we also find approaches that are focused on the extraction of some descriptive rules for data mining problems (association rules, subgroup discovery,...) [36, 69].

2. Genetic rule selection. Sometimes we have a big number of rules extracted via a data mining method that only provide us a big number of rules associated to our problem. A big RB and an excessive number of rules makes difficult to understand the FRBS behaviour. Thus we can find different kinds of rules in a fuzzy rule set: irrelevant rules, redundant rules, erroneous rules and conflictive rules, which perturb the FRBS performance when they coexist with others. To face this problem we can use a genetic rule selection process for obtaining an optimized subset of rules from a previous fuzzy rule set by selecting some of them. Figure 7 graphically shows this idea. In [65] we can find the most classic and first contribution in this area and in [62] we can find the first journal paper on multiobjective genetic rule selection.



Fig. 6 Genetic rule learning process

We must point out that rule selection can be combined with tuning approches, trying to get a good rule set together with a tuned set of parameters. In [3, 17] we can find two recent proposal that combines genetic tuning with rule selection. Figure 8 presents the scheme of the hybrid model proposed in [3].

3. Genetic DB learning. There is another way to generate the whole KB that considers two different processes to derive both components, DB and RB. A DB generation process allows us to learn the shape or the membership functions and other DB components such as the scaling functions, the granularity of the fuzzy partitions, .... This DB generation process can use a measure for evaluating the quality of the DB, we can call them as "A priori genetic DB learning". The second possibility is to consider and embedded genetic learning process where the DB generation process wraps an RB learning one working as follows: each time a DB has been obtained by the DB definition process, the RB generation method is used to derive the rules, and some type of error measure is used to validate the whole KB obtained. We should note this operation mode involves a partitioning of the KB learning problem. These two kinds of learning models are



Fig. 7 Genetic rule selection process



Fig. 8 Example of genetic lateral tuning and rule selection

represented in Fig. 9. In [30] we can find a proposal following the embedded genetic DB learning.

4. Simultaneous genetic learning of KB components. Other approaches try to learn the two components of the KB simultaneously. This kind of learning is depicted in Fig. 10. Working in this way, they have the possibility of generating better definitions but there is a need to deal with a larger search space that makes the learning process more difficult and slow. In [55] we can find a contribution that is a reference in the simultaneous genetic KB learning process.



Fig. 9 Genetic DB learning (embedded and a priori)



Fig. 10 Genetic KB learning process

# 3.1.3 Genetic learning of KB components and inference engine parameters

This is the last area of GFSs taxonomy, belonging to a hybrid model between adaptive inference engine and KB components learning. We can find novel approaches that try to find high cooperation between the inference engine via parameters adaptation and the learning of KB components, including both in a simultaneous learning process. In [80] we can find a recent proposal to learn a linguistic RB and the parametric aggregation connectors of the inference and defuzzification in a single step. Figure 11 presents the coding scheme of the model proposed in this paper.

3.2 Genetic learning: rule coding and cooperation/ competition evolutionary process

Although GAs were not specifically designed for learning, but rather as global search algorithms, they offer a set of advantages for machine learning. Many methodologies for machine learning are based on the search of a good model inside the space of possible models. In this sense, they are very flexible because the same GA can be used with different representations. Genetic learning processes cover different levels of complexity according to the structural changes produced by the algorithm, from the simplest case of parameter optimization to the highest level of complexity for learning the rule set of a rule-based system, via the coding approach and the cooperation or competition between chromosomes.

When considering the task of learning rules in a rule based system, a wider range of possibilities is open. When considering a rule based system and focusing on learning rules, the different genetic learning methods follow two approaches in order to encode rules within a population of individuals:

- The "Chromosome = Set of rules", also called the *Pittsburgh* approach, in which each individual represents a rule set [96]. In this case, a chromosome evolves a complete RB and they compete among them along the evolutionary process. GABIL is a proposal that follows this approach [35].
- The "Chromosome = Rule" approach, in which each individual codifies a single rule, and the whole rule set is provided by combining several individuals in a population (rule cooperation) or via different evolutionary runs (rule competition).

In turn, within the "Chromosome = Rule" approach, there are three generic proposals:

- The Michigan approach, in which each individual encodes a single rule. These kinds of systems are usually called learning classifier systems [58]. They are rule-based, message-passing systems that employ reinforcement learning and a GA to learn rules that guide their performance in a given environment. The GA is used for detecting new rules that replace the bad ones via the competition between the chromosomes in the evolutionary process. An interesting study on the topic can be found in [74].
- The *IRL* (*Iterative Rule Learning*) approach, in which each chromosome represents a rule. Chromosomes compete in every GA run, choosing the best rule per run. The global solution is formed by the best rules



**Fig. 11** Example of the coding scheme for learning an RB and the inference connective parameters

obtained when the algorithm is run multiple times. SIA [104] is a proposal that follows this approach.

- The GCCL (genetic cooperative-competitive learning) approach, in which the complete population or a subset of it encodes the RB. In this model the chromosomes compete and cooperate simultaneously. COGIN [49], REGAL [44] and LOGENPRO [106] are examples with this kind of representation.

These four genetic learning approaches (Pittsburgh, Michigan, IRL and GCCL) have been considered for learning KB components, and we can find different examples of them in the literature. Two of the pioneer GFS proposals were focused on the Pittsburgh [99] and Michigan [101] approaches. MOGUL [24, 26, 54] and SLAVE [46] are two proposals that follow the IRL approach in the framework of GFSs. In [63, 67] we find two proposals following the GCCL approach.

#### 4 Genetic fuzzy systems outlook

This section tries to present a quick snapshot of the GFS status stressing the following points:

- pioneer GFSs contributions, four contributions that mark the birth of GFSs in 1991,
- the GFSs visibility at the ISI Web of Science,
- the milestones that cover the books and journal special issues in the topic, and
- the most cited papers, that can also mark milestones as important contributions in the topic.

#### 4.1 Pioneer papers: the birth of GFSs in 1991

Following, are shortly described the four pioneer papers, that introduced the first genetic tuning and genetic RB learning proposals following the Michigan and the Pittsburgh approaches:

*Karr's AI Expert paper (Genetic tuning of the DB)* [68]. The pioneer work in genetic tuning considers linguistic FRBSs. The DB definition is encoded in the chromosome, which contains the concatenated parameters of the input and output fuzzy sets.

Valenzuela-Rendon's ICGA91 paper (Linguistic RB Learning, Michigan approach) [101]. This proposal presents the first GFS based on the Michigan approach for learning RBs with DNF fuzzy rules. It employs a reward distribution scheme that requires knowledge of the correct action, and thus, must be considered as a supervised learning algorithm. The author later extended the original proposal, in order to enable true reinforcement learning [102].

Thrift's ICGA91 paper (Linguistic RB Learning, Pittsburgh approach) [99]. This is the pioneer work on the Pittsburgh approach for learning RBs. This method works by using a complete decision table that represents a special case of crisp relation defined over the collections of fuzzy sets corresponding to the input and output variables. A chromosome is obtained from the decision table by going row-wise and coding each output fuzzy set as an integer including a "null" label as a 0. Therefore, the GA employs an integer coding.

Pham and Karaboga's Journal of Systems Engineering paper (Relational matrix-based FRBS learning) [88]. This is a quite different approach that uses a fuzzy relation R instead of the classical crisp relation (decision table). The GA is used to modify the fuzzy relational matrix of an oneinput, one-output fuzzy model. The chromosome is obtained by concatenating the M·N elements of R, where Mand N are the number of linguistic terms associated with the input and output variables. The elements of R are real numbers in the interval [0,1].

After the publication of these four pioneer proposals we can find an increasing number of contributions in the specialized literature with proposals that cover all the different areas of the taxonomy, with a rich body of literature on this topic and with high visibility. This is shown in the next subsection, we shortly show the visibility of GFSs at the *ISI Web of Science*.

#### 4.2 GFSs visibility at the ISI Web of Science

The *ISI Web of Science* provides seamless access to current and retrospective multidisciplinary information from approximately 8,700 of the most prestigious, high impact research journals in the world. *Web of Science* also provides a unique search method, cited reference searching. With it, users can navigate forward, backward, and through the literature, searching all disciplines and time spans to uncover all the information relevant to their research. Users can also navigate to electronic full-text journal articles (http://scientific.thomson.com/products/wos/).

In the link of "Advanced Search", we consider the query:

TS = [("GA-" OR "GA based" OR evolutionary OR "genetic algorithm\*" OR "genetic programming" OR "evolution strate\*" OR "genetic learning") AND ("fuzzy rule\*" OR "fuzzy system\*" OR "fuzzy neural" OR "neuro-fuzzy" OR "fuzzy control\*" OR "fuzzy logic cont\*" OR "fuzzy class\*" OR "fuzzy if" OR "fuzzy model\*")] TS field is a search based on the "Topic". The numerical results of the query are

Date of analysis: 12 October 2007 Number of papers: 1,241 Sum of the time cited: 6,053 Average citations per item: 4.88

Figures 12 and 13 show the number of publications and citations per year.

We observe an increasing number of publications per year with more than 100 papers per year in the last five ones. The number of citations shows a similar increasing trend in recent years. All this data can allow us to say the field of GFSs has now reached a stage of maturity after the earliest papers published at 1991, and there are also many basic issues yet to be resolved and there is an active and vibrant worldwide community of researchers working on these issues.

4.3 Some GFS milestones: books and special issues

For beginners, in the following we present the GFS milestones associated to the books and special issues published in the specialized literature.

We can find two authored books and three edited ones:

• A. Geyer-Schulz. Fuzzy rule-based expert systems and genetic machine learning. Physica-Verlag, 1995 [43].

This is the first GFS book. It is a very specific book focused on fuzzy classifier systems (Michigan approach) and RB learning with genetic programming.

• O. Cordón, F. Herrera, F. Hoffmann and L. Magdalena. Genetic Fuzzy Systems. Evolutionary Tuning and



Fig. 12 Publications in GFS per year (Web of Science)



Fig. 13 Number of citations per year (Web of Science)

Learning of Fuzzy Knowledge Bases, World Scientific, 2001.

This is the first general GFS book. It covers the overall state of the art of GFSs by that time.

These three following books compile an important number of contributions that gave maturity to the topic.

- F. Herrera and J.L. Verdegay (eds.). Genetic Algorithms and Soft Computing. Physica-Verlag, 1996.
- E. Sanchez, Shibata and L. Zadeh (eds.). Genetic Algorithms and Fuzzy Logic Systems. Soft Computing Perspectives. World Scientific, 1997.
- W. Pedrycz (ed.). Fuzzy Evolutionary Computation. Kluwer Academic Publishers, 1997.

In the following we provide a list of the journal special issues devoted to GFSs, including important contributions to all topics of GFSs.

- F. Herrera. Special Issue on Genetic Fuzzy Systems for Control and Robotics. International Journal of Approximate Reasoning, Vol 17, No 4, November 1997.
- F. Herrera and L. Magdalena. Special Issue on Genetic Fuzzy Systems. International Journal of Intelligent Systems, Vol 13, No 10–11, October–November 1998.
- O. Cordón, F. Herrera, F. Hoffmann and L. Magdalena. Special Issue on Recent Advances in Genetic Fuzzy System. Information Sciences, Vol 136, No 1–4, August 2001.
- O. Cordón, F. Gomide, F. Herrera, F. Hoffmann, L. Magdalena. Special Issue on Genetic Fuzzy Systems. Fuzzy Sets and Systems, Vol 141, No 1, January 2004.
- J. Casillas, M.J. del Jesus, F. Herrera, R. Pérez, P. Villar. Special Issue on Genetic Fuzzy Systems and the Interpretability-Accuracy Trade-off. International Journal of Approximate Reasoning. Vol 44, No 1, February 2007.

- O. Cordón, R. Alcalá, J. Alcalá-Fdez, I. Rojas. Genetic Fuzzy Systems. Special Section on Genetic Fuzzy Systems: What's Next?. IEEE Transactions on Fuzzy Systems. Vol 15, No 4, August 2007.
- B. Carse, A.G. Pipe. Special Issue on Genetic Fuzzy Systems. International Journal of Intelligent Systems. Vol 22, No 9, September 2007.

The collection of papers that we could find on these special issues give us a historical tour on the different stages we can find in the evolution of GFSs research:

- The two first special issues (1997, 1998) contain contributions devoted to learning KB components using the different learning approaches (Michigan, IRL, Pittsburgh) together with some applications. We can find representative approaches of different areas of the taxonomy.
- In the next two special issues (2001, 2004) we can find contributions that exploit the mentioned genetic learning approaches together with contributions that stress new branches such as genetic rule selection, multiobjective genetic algorithms for rule selection, the use of genetic programming for learning fuzzy systems, hierarchical genetic fuzzy systems, coevolutionary genetic fuzzy systems, the combination of boosting and evolutionary fuzzy systems learning, embedded genetic DB learning, and first studies for dealing with high dimensional problems, among others.

I would like to point out the review paper that was published in the last issue [25] that was the first review in the topic, shortly introducing GFS models and applications, trends and open questions. Another short review was presented in [52]. The present paper can be considered as a continuation of those, with the novelty of the taxonomy, the GFSs outlook based on the pioneer papers, the *ISI Web of Science* based visibility and the milestones along the GFSs history and new trends and prospects.

• The last three special issues, published in 2007, emphasize three different directions. Carse and Pipe's special issue collect papers focused in the mentioned areas (multiobjective evolutionary learning, boosting and evolutionary learning,...) and stress some new ones such as evolutionary adaptive inference systems. Casillas et al.'s special issue is focused on the trade-off between interpretability and accuracy, collecting four papers that proposed different GFSs for tackling this problem. Cordón et al.'s special issue focuses its attention on novel GFS proposals under the title "What's Next?", collecting highly innovative GFS proposals that can mark new research trends. The four collected papers are focused on: a new Michigan approach for learning RBs based on XCS [15], GFSs

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for imprecisely observed data (low quality data) [91], incremental evolutionary learning of TS-fuzzy systems [56], and evolutionary fuzzy rule induction for subgroup discovery [36].

#### 4.4 The ten most cited papers at the ISI Web of Science

The search on the *ISI Web of Science* allows us to get the ten most cited papers that can provide a picture on ten important contributions on the topic that are representative approaches of different taxonomy areas. Figure 14 shows the list of ten papers (we should note that we have eliminated a paper devoted to a survey on neuro-fuzzy rule generation that is not devoted to GFSs). Following, we shortly describe them, paying attention to the associated area of the taxonomy and the used learning approach.

Homaifar and Mccormick's paper (IEEE TFS, 166 cites) [55]. Authors proposed the use of GAs to learn a complete KB for control problems, determining both membership functions and RB together in order to address their codependency (KB learning). They considered the simple GA for a Pittsburgh approach, with integer coding for rule consequents (similar to Thrift's proposal) and integer coding for membership function support amplitude (five different amplitude values) in the same chromosome. This contribution is a reference in the topic as a classic Pittsburgh approach for genetic KB learning.

Ishibuchi, Nozaki, Yamamoto et al.'s paper (IEEE TFS, 156 cites) [65]. GAs are used for selecting a small number of fuzzy IF-THEN rules with high classification performance. The proposed algorithm was based on a simple GA with binary coding representing whether a rule should be selected or not from an initial set of candidate rules (obtained from a predefined DB by applying a simple data-driven method). The problem was formulated as a combinatorial optimization problem with two objectives considered by a weighted fitness function: to maximize the number of correctly classified patterns and to minimize the number of rules. This contribution is the most classic contribution for genetic rule selection and one of the departure points for studies in the trade-off between interpretability and accuracy.

Setnes and Roubos' paper (IEEE TFS, 92 cites) [93]. A two-step approach was proposed for function approximation, dynamic systems modeling and data classification problems by learning approximate TS-rules. First, fuzzy clustering was applied to obtain a compact initial KB. Then this model is optimized by a real-coded GA subjected to constraints in order to maintain the semantic properties of the rules. Each chromosome represents the parameters defining each fuzzy model (membership functions of the



Fig. 14 GFS ten most cited papers

antecedents and coefficients of the consequents), thus performing a tuning of the initial model. This approach was also combined with an iterative similarity-driven rule base simplification algorithm as an intermediate stage between KB generation and parameter optimization. This is an important contribution that uses GAs for tuning inside a hybrid method, trying to get a more interpretable approximate TS model.

Ishibuchi, Nakashima and Murata's paper (IEEE TSMC-B, 84 cites) [63]. Authors presented a geneticsbased machine learning method that automatically learns a linguistic RB for pattern classification problems from numerical data. In this method, each linguistic IF-THEN rule is handled as a chromosome. Integer coding was considered to represent the rule antecedents (including the don't care symbol) and the heuristic method proposed in [65] was used to automatically generate the consequent class and certainty factor for each antecedent combination. A fitness value was assigned to each rule. The evolution is not based on the performance of an entire rule set, the solution is not the final population but the best population. It follows a GCCL approach being an important contribution for learning RBs.

Park, Kandel and Langholz's paper (IEEE TSMC, 83 cites) [86]. A new fuzzy reasoning method was used to enhance the performance of fuzzy controllers obtained from prior knowledge provided by an expert. To avoid initial subjective selection of fuzzy reasoning models, the

authors proposed the use of GAs to find simultaneously the optimal fuzzy relation matrix (used in the new fuzzy reasoning method, extending Pham and Karaboga's proposal) and the fuzzy membership functions. In this way, each chromosome is divided into two parts, one for the fuzzy relation matrix and another for the fuzzy membership functions of the DB, following a Pittsburgh approach. It is a classic paper using fuzzy relations for evolving a KB that can be considered as a tuning approach since it considers the prior knowledge provided by the experts.

*Herrera*, *Lozano and Verdegay's paper (IJAR, 66 cites)* [53]. Authors proposed a tuning method for obtaining highperformance fuzzy control rules by means of GAs. The tuning method locally fits the membership functions of the fuzzy rules dealing with the parameters of the membership functions. A chromosome represents the parameters of the membership functions used by each rule in the initial KB, the chromosome represents the concatenated rule parameters. This is the first proposal for getting an approximate FRBS via tuning associated to the rules.

Shi, Eberhart, Chen's paper (IEEE TFS, 59 cites) [95]. A new GFS was proposed for classification, using a GA for evolving the membership function parameters and, the type and the RB including the number of rules inside it. In addition, a fuzzy expert system was designed from the experience and knowledge and was used to adapt the genetic parameters of the GA. The chromosome was a mixture considering all the parts of the linguistic FRBS by using integer coding and following a Pittsburgh approach. This is an interesting approach for evolving KB components for classification problems.

Carse, Fogarty and Muro's paper (FSS, 59 cites) [14]. Carse et al. proposed a novel approach to genetics-based machine learning of fuzzy controllers, called a Pittsburgh Fuzzy Classifier System. This algorithm was based on the reinforcement learning in fuzzy control and on the Pittsburgh model of learning classifier systems. It employs variable length rule-sets simultaneously evolving fuzzy set membership functions and fuzzy rules, that is, the KB for approximate models. In the approach presented, genetic operations (selection, recombination and replacement) and credit assignment were carried out at the level of the complete fuzzy rule-set. The chromosome representing a complete rule-set was a variable length concatenated string of such fuzzy rules. In addition, fuzzy set membership functions are encoded together with each rule, as opposed to using a global collection of fuzzy sets used by all rules (approximate fuzzy rules). Real coding was considered to encode both parts. This is a well-known paper considered as a classic one for evolving an approximate FRBS with a Pittsburgh approach.

Cordón and Herrera's paper (IJAR, 1997, 53 cites). A three-stage GGS based on soft constrained learning was presented, to learn local semantics-based fuzzy rules (approximate fuzzy rules) and linguistic RBs. Both possibilities are presented in the paper. The first stage was composed of an evolutionary generation process following the IRL approach to identify a set of candidate local semantics-based Mamdani rules. The method takes as a base some initial linguistic fuzzy partitions. The second stage was composed of a genetic niching-based selection process based on that presented in [65]. The third stage performs the same genetic tuning proposed in [53] for approximate FRBSs and a genetic tuning based on the global partitions for mantaining a certain interpretability level. It is a classic proposal following the IRL approach for evolving an approximate FRBS and a descriptive FRBS.

Juang, Ling and Ling's paper (IEEE TSMC-B, 2000, 52 cites) [67]. A new genetic reinforcement learning algorithm was proposed in this contribution, the Symbiotic Evolution [82] based fuzzy controller. Each chromosome represents a single TS-type rule, and an n-rule fuzzy system is constructed by selecting chromosomes from the population. In this way, a real coding was used to represent each rule by encoding the parameters of local semantics-based Gaussian-type membership functions and the associated coefficients of the consequent part. It is an interesting GCCL approach for evolving an approximate FRBS.

#### 5 Current research trends In GFSs

This section, are introduced some current trends that have focused the attention of researchers in the last few years and I discuss some critical considerations on the publications in the topic at the present.

#### 5.1 Discussing some current trends

In this subsection, from the abundant GFSs literature published, we focus our attention into six current trends that are of high interest at the present and show considerable potential in the near future.

- 1. Multiobjective genetic learning of FRBSs: interpretability-precision trade-off.
- 2. GA-based techniques for mining fuzzy association rules and novel data mining approaches.
- 3. Learning genetic models based on low quality data (noise data and vague data).
- 4. Genetic learning of fuzzy partitions and context adaptation.
- 5. Genetic adaptation of inference engine components.
- 6. Revisiting the Michigan-style GFSs.
- (1) Evolutionary Multiobjective learning of FRBSs: interpretability-precision trade-off. Multiobjective evolutionary algorithms (MOEAs) are one of the most active research areas in the field of evolutionary computation, due to population-based algorithms being capable of capturing a set of non-dominated solutions in a single run of the algorithm. A large number of algorithms have been proposed in the literature [23, 33]. Among them, NSGA-II [34] and SPEA2 [110] are well known and frequently used MOEAs.

Obtaining high degrees of interpretability and accuracy is a contradictory aim, and, in practice, one of the two properties prevails over the other. Nevertheless, a new tendency in the fuzzy modeling scientific community that looks for a good balance between interpretability and accuracy is increasing in importance. The improvement of the interpretability of rule based systems is a central issue in recent research, where not only the accuracy is receiving attention but also the compacting and the interpretability of the obtained rules [76, 81].

In multiobjective GFSs is desirable to design genetic learning algorithms in which the learning mechanism itself finds an appropriate balance between interpretability and accuracy. We consider objectives based on accuracy and objectives that include different complexity/interpretability measures. Figure 15 from [61] illustrates this idea where each ellipsoid denotes a fuzzy system. There exists a large



Fig. 15 Non-dominated fuzzy systems

number of non-dominated fuzzy systems along the accuracy-complexity trade-off curve.

There exists an important number of contributions focused on this topic:

- (a) Multiobjective genetic rule selection [62, 66].
- (b) Multiobjective genetic RB learning [20, 21, 94, 103].
- (c) Multiobjective genetic tuning [5].
- (d) Multiobjective genetic data mining (including fuzzy association rules, subgroup discovery, ...) [9, 69].

Whereas the definition of accuracy in a certain application is straightforward, the definition of interpretability is rather problematic. Most researchers would agree in interpretability involving aspects as: the number of rules is enough to be comprehensible, rule premises should be easy in structure and contain only a few input variables, linguistic terms should be intuitively comprehensible, etc.

There is a need to propose new interpretability metrics that consider not only the number of rules but other aspects as the number of labels of a rule, the shape of the membership functions, etc. with a better understanding and formalization of the notions of "interpretability", "comprehensibility" or "simplicity". More research in evaluation metrics is needed for giving an interpretability measure associated to the FRBSs, allowing us to compare different FRBSs for a problem from the interpretability point of view, and including them as objectives into MOEAs.

Another interesting issue for the future research that was pointed out by Ishibuchi in [61] is the "theoretical analysis for maximizing the generalization ability of fuzzy systems. Multiobjective GFSs can be used for empirical analysis. Theoretical analysis such as statistical learning theory [22] seems to be required."

(2) GA-based techniques for mining fuzzy association rules and novel data mining approaches. Fayyad et al. defined knowledge discovery (KD) as the nontrivial process of identifying valid, novel, potentially useful, and ultimately understandable patterns in data [41]. KD may not be viewed as a synonymous of DM, but they are intimately related. KD is a wide ranging process which covers distinct stages: the comprehension of the problem, the comprehension of the data, pre-processing (or preparation) of the data, DM and post-processing (assessment and interpretation of the models). The DM stage is responsible for automatic KD of a high level and from information obtained from real data. Some of the important problems that DM and KD deal with are: rule extraction, identification of associations, feature analysis, linguistic summarization, clustering, classifier design and novelty/anomaly detection.

The interpretability is crucial in the field of DM/KD where knowledge should be extracted from databases and represented in a comprehensible form, or for decision support systems where the reasoning process should be transparent to the user. In fact, the use of linguistic variables and linguistic terms in a discovered process has been explored by different authors.

Frequent pattern mining has been a focused theme in DM research over a decade. Association analysis is a methodology that is useful for the discovery of interesting relationships hidden in large data sets. The uncovered relationships can be represented in the form of association rules or sets of frequent items. Abundant literature can be found presenting a tremendous progress in the topic [51, 98].

As it was claimed in [39], the use of fuzzy sets to describe association between data extends the types of relationships that may be represented, facilitates the interpretation of rules in linguistic terms, and avoids unnatural boundaries in the partitioning of the attribute domains.

Linguistic variables with linguistic terms can contribute in a substantial way to advance in the design of association rules and the analysis of data to establish relationships and identify patterns, in general [60]. On the other hand, GAs in particular, and EAs in general, are widely used for evolving rule extraction and patterns association in DM/KD [42]. The conjunction in the GFS field provides novel useful tools for pattern analysis and for extracting new kinds of useful information with a main advantage over other techniques, its interpretability in terms of fuzzy if-then rules. At the present we find interesting contributions focused on the genetic extraction of fuzzy association rules [59, 69, 70, 100].

We would like to pay attention to a subdivision of descriptive induction algorithms, which has recently received attention from researchers, called subgroup discovery. It is a form of supervised inductive learning of subgroup descriptions in which, given a set of data and having a property of interest to the user, attempts to locate subgroups which are statistically "most interesting" for the user. Subgroup discovery has the objective of discovering interesting properties of subgroups obtaining simple rules (i.e., with an understandable structure and with few variables), highly significant and with high support (i.e., covering many of the instances of the target class). The concept was initially formulated by Klösgen in [72] his rule learning algorithm EXPLORA and by Wrobel in [107] the algorithm MIDOS. Both use a rule-extraction model based on decision trees, in order to obtain the best subgroups among the population. In order to evaluate the subgroups, evaluation measurements are defined which determine the interest of an expression through a combination of unusualness and size. MIDOS tackles, within this same approach, the problem of discovery in multi-relational databases. A recent study on the topic can be found in [77]. In [36] we find a first approach to the use of GFSs for subgroup discovery.

The use of GFSs for association analysis is a topic that would provide interesting future contributions focusing the attention in the different research problems that we can find in the frequent pattern mining area [51].

Learning genetic models based on low quality data (3) (noise data and vague data). There are many practical problems requiring learning models from uncertain data. The experimental designs of GFSs learning from data observed in an imprecise way are not being actively studied by researchers. However, according to the point of view of fuzzy statistics, the primary use of fuzzy sets in classification and modelling problems is for the treatment of vague data. Using vague data to train and test GFSs we could analyze the performance of these classifiers on the type of problems for which fuzzy systems are expected to be superior. Preliminary results in this area involve the proposals of different formalizations for the definition of fuzzy classifiers, based on the relationships between random sets and fuzzy sets [90] and the study of fitness functions (with fuzzy values) defined in the context of GFSs [91].

This is a novel area that is worth being explored in the near future, and can provide interesting and promising results.

(4) Genetic learning of fuzzy partitions and context adaptation. The DB learning comprises the specification of the universes of discourse, the number of labels for each linguistic variable, as well as the definition of the fuzzy membership functions associated to each label. In [28] it was studied the influence of fuzzy partition granularity in the FRBS performance, showing that using an appropriate number of terms for each linguistic variable, the FRBS accuracy can be significantly improved without the need of a complex RB learning method.

On the other hand, the idea of introducing the notion of context into fuzzy systems comes from the observation that, in real life, the same basic concept can be perceived differently in different situations. In some cases, this information is related to the physical properties or dimensions of the system or process, including restrictions imposed due to the measurement acquisition or actuators. In the literature, context adaptation in fuzzy systems has been mainly approached as scaling of fuzzy sets from one universe of discourse to another by means of non-linear scaling functions whose parameters are identified from data.

Different approaches have been proposed to deal with the learning of membership functions, granularity, nonlinear contexts using GAs, ...[4, 11, 12, 29, 30, 50, 78].

Although there is an important number of contributions in the area of DB Learning, I think that this is an important research area where we can obtain important results, due to the importance of using the adequate membership functions and the adequate context. The use of GFSs is very important due to the flexibility for encoding DB components together with other fuzzy system components.

(5) Genetic adaptation of inference engine components. We know that it is possible to use parametric aggregation operators in the design of the inference system and the defuzzification method, trying to get the most appropriate parameter configuration in any application. The tuning of these components can be considered to get more accurate fuzzy models. We come across different GFS approaches for finding the most appropriate parameters [6, 31].

This is an interesting research area that can provide us with the opportunity to adapt the inference parameters to an FRBS and to design learning models that can coevolve the inference engine parameters together with the KB components.

(6) Revisiting the Michigan-style GFSs. The first description of a Michigan-style GFS was given in [101]. All the initial approaches in this area were based on the strength in the sense that a rule (classifier) gets strength during interactions with the environment (through rewards and/or penalties). This strength can then be used for two purposes: resolving conflicts between simultaneously matched rules during learning episodes; and as the basis of fitness for the GAs.

A completely different approach can be considered in which a rule's fitness, from the point of view of the GA, is based on its "accuracy", i.e., how well a rule predicts payoff whenever it fires. Notice that the concept of accuracy used here is different from the traditionally used in fuzzy modelling (i.e., capability of the fuzzy model to faithfully represent the modeled system). This accuracybased approach offers a number of advantages such as avoiding overgeneral rules, obtaining optimally general rules, and learning of a complete covering map. The first accuracy-based evolutionary algorithm, called XCS, was proposed in [105] and it is currently of major interest to the research community in this field.

Casillas et al. proposed in [15] a new approach to achieve accuracy-based Michigan-style GFSs. The proposal, Fuzzy-XCS, is based on XCS but properly adapted to fuzzy systems, with promising results for function approximation problems and for robot simulation online learning. In [84] it is proposed an extension of UCS algorithm, a recent Michigan-style genetic learning algorithm for classification [10].

These approaches build a bridge between the Michiganstyle genetic learning studies and the fuzzy systems models. This is an interesting research line that can provide interesting results in the near future.

#### 5.2 Some critical considerations

The question that I proceed to discuss in this section is related to some common features of a large number of recent publications in the topic.

In the last few years, we can find a lot of publications that present a "novel" proposal for evolving a KB or a specific component (RB or DB). When we read the abstract we feel a great interest to read the specific aspects of the proposal, and which is more important, to see the results and the comparative analysis against the well known approaches in the GFSs literature in particular, or fuzzy systems literature in general.

# What are the critical points for a large number of recent published papers?

Next, we discuss the critical issues that we can find in some recent publications. We focus our attention on two aspects, the EAs used in the GFSs and the experimental study.

- 1. On the EAs used in the GFSs. At this point, we discuss two questions, the use of a simple GA and the use of novel EAs.
- (a) Simple GAs. In an important number of contributions we find a description of the simplest GAs, with a classical parameter coding (binary alphabet) and simple components.

There exists a wide literature on GAs, in particular, and EAs in general, with important approaches that introduce important advances. Some examples are, the real coding for continuous variables, different parent replacement strategies, adaptive components, etc.

On the other hand, there are specific kinds of GAs for different tasks, such as niching GAs for multimodal functions, hybrid combinations of GAs and local search (memetic algorithms), etc.

Authors must really know GA components and models before applying a simple GA, and choose an adequate algorithm, if necessary, for getting a good GFS.

(b) On the use of novel EAs. Recently, it is usual to find evolutionary learning proposals that use a classical genetic representation for a KB and use a novel EA (such as particle swarm, differential evolution, etc.) but authors do not compare them with the classic GFSs that we can find in the literature, with the same coding approach and using GAs. Researchers claim that the novel EA provides very good results, but do not offer any justification for their use.

The use of a novel EA must be justified from whatever meaningful point of view: efficiency, efficacy/precision, interpretability, scalability, etc.

- 2. Experimental study. At this point, we focus our attention in four aspects to analyze: benchmark problems, comparison with the state of the art, reproducibility, and statistical analysis.
- (a) Benchmark problems. When we read the experimental analysis we usually find a different set of benchmark problems in every paper. In particular, we can find specific applications for learning from data without any possibility for getting the data set, therefore it is impossible to reproduce the same experimental study. It is necessary to manage adequate and unified sets of benchmark problems for learning from data, providing all the necessary information for reproducing the experimental study.

We are working in this sense and we are preparing a benchmark site for problems and data sets for unifying the experimentation, providing the data partitions used in the experimental studies. The set is called KEEL-dataset that can be found at our project site: http://www.keel.es. See [7] for a wide description of KEEL software tool.

(b) Reproducibility. In the same way, it is not possible to reproduce some algorithms due to the lack of the parameters values used by the authors in the experimental study. It is necessary to give a complete description of the algorithm components (coding approach, operators, parameters, ...) for allowing the reader to reproduce the algorithms.

- (c) Comparison with the state of the art. When authors propose a new approach, they must first justify its usefulness, indicating which is the objective and we must find a measure for evaluating it (precision, complexity, ...). Then, it is necessary to make an experimental study comparing with the best approach according to the same objective and same fuzzy system components that are considered. Unfortunately, we do not have a study determining the state of the art in every area of the taxonomy as the level to reach for a new proposal. However, this is a real need for the near future. In any case, authors must compare with the most well known approaches that exist in the abundant literature, discussing the advantages of the proposal. It is not enough to compare with a simple approach that is not the state of the art at the present.
- (d) Lack of experimental statistical analysis. Another critical point is related to the comparative study. Currently there is not a systematic evaluation methodology for GFSs.

Experimental results reported in the machine learning literature often use statistical tests of significance to support the claim that a new learning algorithm generalizes better. In fact, the performance analysis of learning algorithms has always centred the attention of investigators in the machine learning area, and different comparison proposals have been developed (cross validation, uses of  $5 \times 2cv$ , leave one out, etc.) in terms of their type I and type II errors, both on synthetic datasets, and standard benchmarks of machine learning [37, 38]). The use of statistical analysis tools is a peremptory necessity in the analysis of GFS models as it is in classical machine learning.

These last four critical points can be extended to other fuzzy rule learning approaches that use other learning techniques.

I consider these six points (organized into two groups) as critical points to advance towards the right issues, that is, to concentrate more on the strengths and distinctive features of the GFSs, providing an useful advance in the fuzzy systems theory.

# 6 Genetic fuzzy systems: prospects

Nowadays, I consider the field of GFSs as a mature area, that needs to advance towards new questions and problems. In what follows, I enumerate three research directions that are worth continuing the exploration, or initiate it in some aspects.

- 1. Interpretability quality.
- 2. New data mining tasks: frequent and interesting pattern mining, mining data streams, etc.
- 3. Dealing with high dimensional data sets.
- (1) Interpretability quality. There exists another important feature for measuring the FRBS quality, the model interpretability. We claim on the interpretability but without metrics for measuring it. We use it as an objective when we use MOEAs for extracting fuzzy models but only considering the size of the rule base or the number of variables that participate per rule.

Interpretability is considered to be the main advantage of fuzzy systems over alternatives like neural networks, statistical models, etc. As authors claim in [81], interpretability means that human beings are able to understand the fuzzy system's behaviour by inspecting the RB. Fuzzy systems constructed from expert knowledge—the traditional approach—are usually well understandable.

In the recent years, research has started to focus on the trade-off between interpretability and accuracy [18, 19]. Analysis of the model interpretability and comprehensibility is always convenient, and it is a necessity when accuracy is not a model feature.

The inclusion of novel interpretability measures in the fitness function of GFS models will provide novel and interesting approaches for getting a good balance between interpretability and precision.

(2) New data mining tasks: frequent and interesting pattern mining, mining data streams, etc. Many new problems have emerged and have been solved by the data mining researchers, but there are still a lot of problems that receive attention and new proposals are under development. We can find a lot of novel problems far from the classic classification and regression problems, problems such as frequent pattern mining open questions, data streams, sequential and time series data, adversary data mining, anomaly detection, non-static, imbalanced data... [109].

As we have already mentioned, linguistic variables with linguistic terms can contribute in a substantial way to advance in the design of data analysis approaches to establish relationships and identify patterns in some of the enumerated problems. The development of GFSs may be useful for providing algorithms and solutions to the mentioned problems.

(3) Dealing with high dimensional data sets. It is usual to find big databases, i.e., with high number of features and/or instances. Regarding the interpretability of linguistic FRBSs, the difficulty comes from the exponential growth of the fuzzy rule search space with the increase in the number of features/instances considered. Usually, human users do not want to check hundreds of fuzzy rules, the number of fuzzy rules is closely related to the interpretability of FRBSs. On the other hand, the rule length is also closely related to the interpretability of FRBSs.

This problem can be tackled in different ways:

- (a) Compacting and reducing the rule set. As a post-processing approach, this is done under an initial rule extraction process that provides a big number of rules. It appears the problem of the number of rules and the size of the coding representation, with the necessity of efficient and effective EAs.
- (b) Using data reduction techniques. Carrying out a feature selection process, that determines the most relevant variables before or during the inductive learning process of the FRBS, and removing irrelevant training instances prior to FRBS learning. The first approach has been already tried in the GFS specialized literature [16, 47, 48], but the latter, up to our knowledge in the topic, has not been used for learning FRBS. For example, it has been used for extracting decision trees, see [13, 92].

Feature and instance selection provide smaller training sets, which may get more accurate and more compact models. And in both cases, GAs are used frequently, because the selection problem may be defined as the problem of searching the optimal subset of features/instances.

The inclusion of genetic data reduction processes inside of a GFS model is a research direction that allows us to advance in the extraction of FRBSs with an appropriate balance between interpretability and accuracy in high dimensional problems.

- (c) Using genetic programming for learning compact FRBSs. Genetic programming is an extension to the inspiration of GA, where the main problem of GAs concerning the fixed problem definition is avoided by using variable-length trees instead of fixed-sized individuals. The definition of context-free grammars for rule construction has been revealed of special utility for this purpose [106]. The use of genetic programming in a GFS model can lead us to obtaining a reduced fuzzy rule set, with few antecedents conditions per rule and high-generalization capability, getting FRBSs with high interpretability for highdimensional problems.
- (d) Algorithm scalability. Another problem when we deal with high dimensional problems is the analysis of the algorithm scalability on big databases, emphasizing the training time and the convergence

towards compact and interpretable models. The balance between problem size and algorithm scalability is another important aspect for GFSs that are worth studying in depth. At this point, we must remark the existence of efficient parallel GAs [1] as a kind of GAs that would be evaluated for designing GFSs for large databases putting special emphasis on aspects of scalability and efficiency. Another interesting idea that has been advanced in [83] consisting of dividing training data sets and the population. They are divided into the same number of subpopulations and training data subsets, which is also the same as the number of client CPUs. Then each client CPU performs genetic learning (genetic rule selection in this contribution) using a single training data subset and a single sub-population given by the server CPU. It seems that each sub-population is likely to overfit the corresponding training data subset. To avoid that, the assignment of the training data subsets to the client CPUs change after a prespecified number of generations (i.e., every ten generation).

Of course, this is not a complete list of potential research directions for GFSs, but it is a set of personal reflections on some potential research lines for investigation together with the research lines that emerge from the current trends discussed in the previous section.

# 7 Concluding remarks

The hybridization between fuzzy systems and GAs in GFSs became an important research area during the last decade. GAs allow us to represent different kinds of structures, such as weights, features together with rule parameters, etc., allowing us to code multiple models of knowledge representation. This provides a wide variety of approaches where it is necessary to design specific genetic components for evolving a specific representation.

Nowadays, it is a mature research area, where researchers need to reflect in order to advance towards strengths and distinctive features of the GFSs, providing useful advances in the fuzzy systems theory.

Finally, I would like to finish with four considerations on this paper:

- This paper does not try to be a directory to guide the reader to a set of references, beginners in the topic can get three important list of references in [25, 27, 61].
- It presents a brief overview of the current trends and future directions of GFSs that I consider as "burning issues", but of course, it is not a complete list of potential research directions for GFSs.

- It pays special interest to propose a GFSs taxonomy. This can help to locate the new proposals into the existing literature, allowing to point out the novel contributions in comparison with the state of the art. It may also be useful for identifying critical problems related to GFSs and taxonomy areas.
- It calls the attention on critical points that need to be tackled for researchers working with GFSs and fuzzy knowledge extractions.

Finally, I would like to point out that the link http://sci2s.ugr.es/gfs/index.php will provide more information on the paper content (software and algorithm implementations, slides, more information on high cited papers, ...).

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