1 Hybrid Composition Functions (F12-F19*)

The hybrid composition functions, F12-F19*, are built combining a non-separable function with other function. The considered functions are:

- **Non-Separable Functions:**
  - F3: Shifted Rosenbrock’s Function
  - F5: Shifted Griewank’s Function
  - NS-F9: Non-Shifted Extended f10
  - NS-F10: Non-Shifted Bohachevsky

- **Other Component Functions:**
  - F1: Shifted Sphere Function
  - F4: Shifted Rastrigin’s Function
  - NS-F7: Non-Shifted Schwefel’s Problem 2.22

The procedure used to hybridize a non-separable function $F_{ns}$ with other function $F'$ (function $F_{ns} \oplus F'$) is shown in Figure 1. Its main steps are: 1) to divide the solution into two parts, 2) to evaluate each one of them with a different function, and 3) to combine their results. The splitting mechanism uses a parameter, $m_{ns}$, which specifies the ratio of variables that are evaluated by $F_{ns}$. Using a higher value of $m_{ns}$, the hybrid function becomes more difficult to optimize dimension by dimension, because there is a greater interrelation between the variables and the fitness. With this procedure, we have defined the instances of hybrid functions shown in Table 1.
<table>
<thead>
<tr>
<th>Name</th>
<th>$F_{ns}$</th>
<th>$F'_{ns}$</th>
<th>$m_{ns}$</th>
<th>Range</th>
<th>Fitness</th>
<th>Optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>F12</td>
<td>NS-F9</td>
<td>F1</td>
<td>0.25</td>
<td>$[-100, 100]^D$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F13</td>
<td>NS-F9</td>
<td>F3</td>
<td>0.25</td>
<td>$[-100, 100]^D$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F14</td>
<td>NS-F9</td>
<td>F4</td>
<td>0.25</td>
<td>$[-5, 5]^D$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F15</td>
<td>NS-F10</td>
<td>NS-F7</td>
<td>0.25</td>
<td>$[-10, 10]^D$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F16*</td>
<td>NS-F9</td>
<td>F1</td>
<td>0.5</td>
<td>$[-100, 100]^D$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F17*</td>
<td>NS-F9</td>
<td>F3</td>
<td>0.75</td>
<td>$[-100, 100]^D$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F18*</td>
<td>NS-F9</td>
<td>F4</td>
<td>0.75</td>
<td>$[-5, 5]^D$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F19*</td>
<td>NS-F10</td>
<td>NS-F7</td>
<td>0.75</td>
<td>$[-10, 10]^D$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Hybrid composition functions

We should point out that the hybrid F15 and F19* functions were shifted.
Function $F_{ns} \oplus F'(S)$

1. $S$ is divided into two parts ($part_1$ and $part_2$):
   
   - If $m_{ns} \leq 0.5$ then
     - $part_1$ is composed by the first $D \cdot m_{ns}$ even variables.
       ($\text{length}(part_1) = D \cdot m_{ns}$)
     - $part_2$ is composed by the remaining variables.
       ($\text{length}(part_2) = D - \text{length}(part_1)$)
   
   - If $m_{ns} > 0.5$ then
     - $part_2$ is composed by the first $D \cdot (1 - m_{ns})$ odd variables.
       ($\text{length}(part_2) = D \cdot (1 - m_{ns})$)
     - $part_1$ is composed by the remaining variables.
       ($\text{length}(part_1) = D - \text{length}(part_2)$)

2. Return $F_{ns}(part_1) + F'(part_2)$.

Figure 1: Evaluation of a solution $S$ (with $D$ variables) by the hybrid function $F_{ns} \oplus F'$