Computing with words in decision making: foundations, trends and prospects

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Abstract Computing with Words (CW) methodology has been used in several different environments to narrow the differences between human reasoning and computing. As Decision Making is a typical human mental process, it seems natural to apply the CW methodology in order to create and enrich decision models in which the information that is provided and manipulated has a qualitative nature. In this paper we make a review of the developments of CW in decision making. We begin with an overview of the CW methodology and we explore different linguistic computational models that have been applied to the decision making field. Then we present an historical perspective of CW in decision making by examining the pioneer papers in the field along with its most recent applications. Finally, some current trends, open questions and prospects in the topic are pointed out.

1 Introduction

CW (Zadeh 1996) deals with words and propositions from a natural language as the main objects of computation, for example: "small", "big", "expensive", "quite possi-

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ble" or even more complex sentences as "tomorrow will be cloudy but not very cold". The main inspiration of CW is the human ability of performing several different tasks (walk in the street, play football, ride a bicycle, understand a conversation, making a decision...) without needing an explicit use of any measurements nor computations. This capability is sustained by the brain's ability to manipulate different perceptions (usually imprecise, uncertain or partial perceptions), which plays a key role in human recognition, decision and execution processes (Herrera and Herrera-Viedma 2001; Herrera et al. 2006; Lawry et al. 2003; Rutkowska et al. 2002; Wang 2001; Zadeh and Kacprzyk 1999a,b).

CW is a methodology which contrasts with the usual sense of Computing, that is, manipulating numbers and symbols. Therefore, CW provides a methodology which can bring closer the gap between human's brain mechanisms and the machine's processes to solve problems providing computers with tools to deal with imprecision, uncertainty and partial truth.

Decision Making (Evangelos 2000) can be seen as the final outcome of some mental processes that lead to the selection of an alternative among several different ones. For example, typical decision making problems are to choose the best car to buy, deciding which candidate is more suitable for a certain position in a firm or choosing the most appropriate clothes for a meeting. It is interesting to note that decision making is an inherent human ability which is not necessarily rationally guided, which can be based on explicit or tacit assumptions and that it does not need precise and complete measurements and information about the set of feasible alternatives. This fact has led to many authors to apply fuzzy sets theory (Zadeh 1965) to model the uncertainty and vagueness in decision processes (Chen and Hwan 1992; Fodor and Roubens 1994; Kacprzyk and Fedrizzi 1990; Lu et al. 2007).

In recent years, many researchers have seen CW as a very interesting methodology to be applied in decision making (Herrera and Herrera-Viedma 2000). As it allows to model perceptions and preferences in a more human like style and it can provide computers some of the needed tools, if not to fully simulate human decision making, to develop complex decision support systems to ease the decision makers to reach a solution.

In this paper we present a review on the use of the CW methodology in decision making from the first attempts to integrate CW in decision making processes to the most recent trends, applications and prospects in the field.

To do so, in Sect. 2 we present a short overview of the CW methodology. In Sect. 3 we introduce the main steps for solving decision making problems using the CW methodology, present the currently developed computational linguistic models, and show an historical perspective on the use of CW in decision making examining the pioneer papers and its most recent applications. In Sect. 4 we present some current trends of the CW methodology with some prospects and open questions. Finally, in Sect. 5 we point out some concluding remarks.

2 A bird's eye view on CW

CW, that is, the methodology that uses words and propositions from a natural language as its main objects of computation, was introduced in the seminal paper (Zadeh 1973).
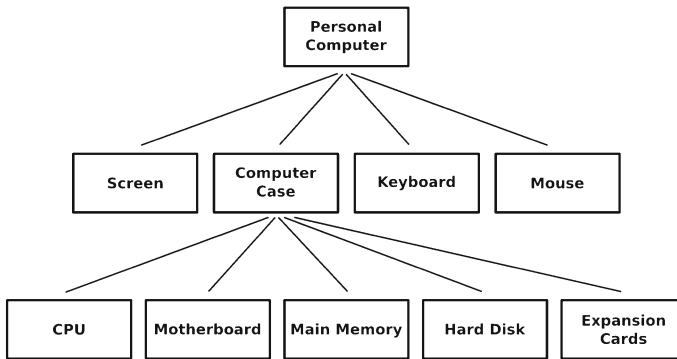
Fig. 1 Example of granulation of a personal computer

In that paper, the main concepts of CW were presented: granules (Zadeh 1997), and linguistic variables (Zadeh 1975a,b,c).

A granule is *"a clump of objects (or points) which are drawn together by indistinguishability, similarity, proximity or functionality"* (Zadeh 1997). As an example (see Fig. 1), the granules of a personal computer are the computer case, the screen, the keyboard and the mouse. Additionally, the granules of the computer case are the CPU, the motherboard, the main memory, the hard disk and the expansion cards. From the example, we can notice that granulation, that is, the decomposition of whole into parts is, in general, hierarchical in nature.

Granules can be seen from a double perspective. The first one, which is assumed in many computational methods and techniques as quantization, rough set theory, divide and conquer, decision trees and so on, corresponds to a crisp interpretation of granules. That is, the decomposition process is made into disjoint granules which can be easily differentiated. The second perspective appears specially in those processes that deal with human reasoning and concept formation: the granules are fuzzy rather than crisp. Following the above example, it is not difficult to find personal computers in which the computer case is integrated in the screen device or motherboards that incorporate several expansion cards (graphical and sound devices) making difficult to differentiate the granules and extending their meaning in an unclear and uncertain way. Moreover, the granules are usually associated with fuzzy attributes, e.g., it is possible to classify storage devices according to the attribute "storage capacity" with the fuzzy values "large", "medium" or "low". This fuzziness of granules, their attributes and their values is characteristic of the ways in which human concepts are formed, organized and manipulated.

Thus, in many situations a granule can be seen as a fuzzy set of points drawn together by similarity. In the CW methodology a granule g which is the denotation of a word, w is viewed as a fuzzy constraint on a variable. The concept of fuzzy granulation has been extensively studied in the literature: Pham (2000) discusses how fuzzy information granulation theory can be used as a base to build flexible formal software specifications. Bargiela and Pedrycz (2005) study granular mappings for both set and fuzzy set based granulation, and Dick et al. (2007) present a granular algorithm to translate information between two different granular worlds with different granulations.

Another basic assumption in CW is that information is conveyed by constraining the value of variables. Moreover, the information is assumed to consist of a collection of propositions expressed in a natural or synthetic language, that is, variables take as possible values linguistic ones.

A linguistic variable is *"variable whose values are not numbers but words or sentences in a natural or artificial language"* (Zadeh 1975a). The main purpose of using linguistic values (words or sentences) instead of numbers is that linguistic characterizations are, in general, less specific than numerical ones, but much more closer to the way that humans express and use their knowledge. For example, if we say "the building is tall" is less specific than "the building measures 300 m". In that case, "tall" can be seen as a linguistic value of the variable "height" which is less precise and informative than the numerical value "300". Despite its less informative nature, the value "tall" allows humans to naturally express and deal with information that may be uncertain or incomplete (the speaker may not know the exact building height). As this kind of situations where information is not precise are very common in real life, linguistic variables can be a powerful tool to model human knowledge.

Formally, a linguistic variable is a 5-tuple $\langle L, T(L), U, S, M \rangle$ (Zadeh 1975a) in which

- L is the name of the variable,
- $T(L)$ is a finite term set of labels or words (a collection of linguistic values),
- U is a universe of discourse,
- S is the syntactic rule which generates the terms in $T(L)$, and
- M is a semantic rule which associates with each linguistic value X its meaning $M(X)$, where $M(X)$ denotes a fuzzy subset of U.

In Fig. 2 we can see an example of the linguistic variable "Height", whose corresponding linguistic term set is $T(Height) = \{Very\ Low, Low, Medium, High, Very\ High\}$. We can see how the semantic rule associates each of the linguistic terms of X to its fuzzy subset $M(X)$ of U. It is clear that a crucial aspect that will determine the validity of a CW approach is the determination of correct membership functions for the linguistic term set.

Linguistic variables and the CW methodology have been a great field of research since its foundations: in Zadeh (1975a,b,c, 1996), Mendel (2007), Trillas (2006), Trillas and Guadarrama (2005) the tight relationship between fuzzy sets theory and CW has been described in more detail. In addition, they presented how CW can be used to transcend the current computation schemes that deal with measurements to a new kind of computing where data takes form of perceptions (Zadeh 1999). Thus, the concept of perceptual computer as an architecture for the CW methodology is also studied by Mendel (2001, 2002) and Wu and Mendel (2008). In Fig. 3 a diagram of a perceptual computer is shown: An Encoder transforms linguistic perceptions into fuzzy sets and leads to a codebook of words with their associated fuzzy sets. The outputs of the Encoder activate a CWW Engine, whose output is another fuzzy set, which is then mapped by a Decoder into a word most similar to a codebook word. In a recent contribution, Mendel (2008) tracks the origins of perceptual computing to Tong and Bonissone (1980) and remarks how Schmucker (1984) in his monography presented the essence of perceptual computing.
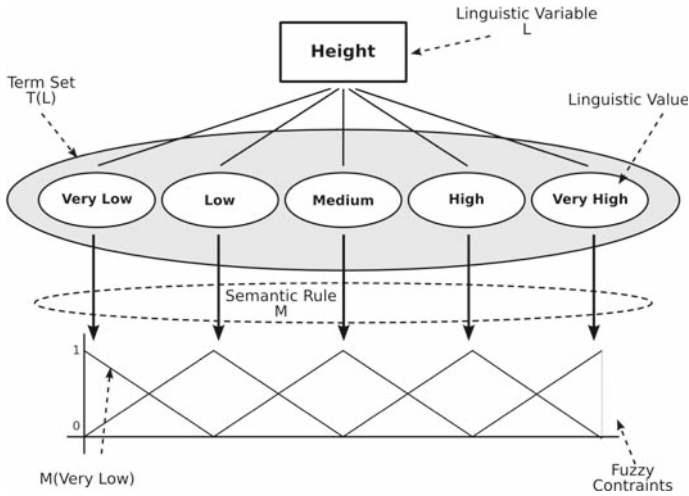
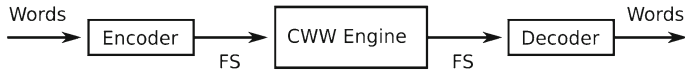Fig. 2 Example of the linguistic variable "Height"

Fig. 3 Perceptual computer for making subjective judgements (Mendel 2008)

Another important concept is the retranslation process in CW, that is, to approximately describe a given fuzzy set in natural language. Yager (2004) studied this problem and Martin and Klir (2006) presented two new different approaches to deal with this retranslation problem in the computing with perceptions field.

The high attention that CW has received in the last years can be noted from the publication of several new different approaches and methodologies for CW:

- Lawry (2001a,b, 2004) presents both an alternative approach to CW based on mass assignment theory and a new framework for linguistic modelling that avoid some of the complexity problems that arise by the use of the extension principle in Zadeh's CW methodology,
- Rubin (1999) defines CW as a symbolic generalization of fuzzy logic,
- Cao et al. (2007) and Ying (2002) propose a new formal model for CW based on fuzzy automatas whose inputs are strings of fuzzy subsets of the input alphabet,
- Wang and Qiu (2003) extend Ying's work considering CW via a different computational model, in particular, Turing machines,
- Tang and Zheng (2006) present a new linguistic modelling that can be applied in CW which does not directly rely on fuzzy sets to model the meaning of natural language terms but uses some fuzzy relations between the linguistic labels to model their semantics, and
- Türkşen (2007) proposes the use of meta-linguistic axioms as a foundation for CW as an extension of fuzzy sets and logic theory.

At this point we center our attention in the work of Dubois and Prade (1997) where they discuss and analyze the semantics of fuzzy sets. They say (p. 141): *"Three main*
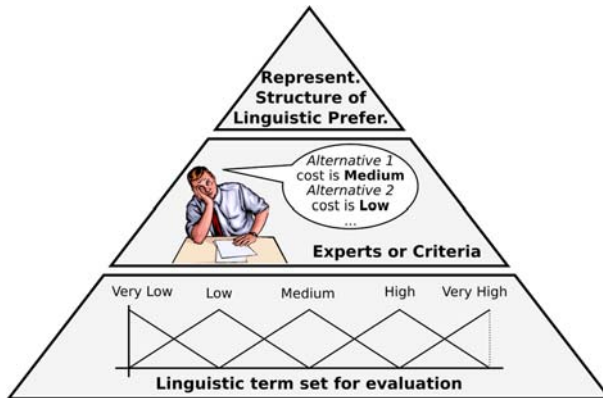
Fig. 4 Linguistic decision making

semantics for membership functions seem to exist in the literature: similarity, preference and uncertainty." Previously, we have presented the direct relation between similarity and granulation and the use of fuzzy sets in uncertain environments have been widely discussed in the literature (Dubois and Prade 1988; Klir and Folger 1988; Yager 1983; Zadeh 1983). It is natural to think that the third semantics of fuzzy sets, that is, the *preference* one, is used in decision making: a particular preference of a decision maker can be expressed by using fuzzy preferences or linguistic labels. In Fig. 4 we present a scheme of preference expression in a typical linguistic decision making environment: first, a set of linguistic terms is defined, which is then used by the expert to provide his preferences. Those preferences will be finally represented using a particular representation structure of linguistic preferences (for example, linguistic utility functions or linguistic preference relations).

3 CW in decision making

In this section we center our attention into the application of the CW methodology in decision making. To do so, we begin with a short explanation of the main steps to solve decision making problems in a linguistic environment followed by the main representation and computational models that have been developed in the literature for CW in decision making. Then we make a short historical perspective of the field by examining the pioneer papers in the field and finally, we present some of the recent applications in decision making.

3.1 Steps for solving decision making problems under linguistic information: the necessity of aggregation

Prior to the resolution of any decision making problem, several steps must be followed:

1. Defining the problem
2. Analyzing the problem

Fig. 5 Steps to solve a decision making problem

3. Identifying alternative solutions
4. Establishing criteria or experts by which they can be evaluated
5. Selecting the best one

In Fig. 5 we see an example of those steps. First, the problem of choosing a car is defined. The second step is to analyze the problem: finding the real necessities of a car, discarding other types of vehicles, making a preliminary list of possible models and so on. On the third step, after the initial analysis of the problem, three particular cars have been selected as the possible solution alternatives. On the fourth step two different experts have been chosen to provide their preferences about the alternatives. Finally, on the last step, the decision process has been carried out and the best alternative has been selected.

In classical fuzzy decision analysis, the last step of a multi-criteria or group decision making process basically consists of two different phases (Roubens 1997):

i An *aggregation phase* of the performance values with respect to all the criteria or decision makers to obtain a collective performance value for the alternatives.
ii An *exploitation phase* of the collective performance value to obtain a rank ordering, sorting or choice among the alternatives.

We can find a clear example of the application of the aggregation and exploitation phases in Dongrui and Mendel (2007) where a decision process about the evaluation scheme for a journal is presented (see Fig. 6): The associate editor asks some reviewers (the decision makers) to review a particular paper for a journal and to choose about several different alternatives, for example: to reject it, to accept it if the authors improve the paper or to directly accept it as is. To evaluate and give their final decision on the paper, the reviewers must summarize and aggregate different criteria about the paper as importance, content, depth, presentation, suitability for the journal and so on. Once all the reviewers finish their evaluations the editor must summarize (aggregate) again all the information received from the reviewers and must apply an exploitation phase in which from the aggregated information the final evaluation for the paper is chosen (to reject, to resubmit or to accept it).

Herrera and Herrera-Viedma (2000) analyze how in linguistic decision analysis it is necessary to introduce two prior steps before applying both the aggregation and exploitation phases, obtaining a three step solution scheme:

1. *The choice of the linguistic term set with its semantics*. It consists of establishing the linguistic expression domain used to provide the linguistic performance values about alternatives according to the different criteria. To do so, we have to choose the granularity of the linguistic term set, its labels and its semantics.
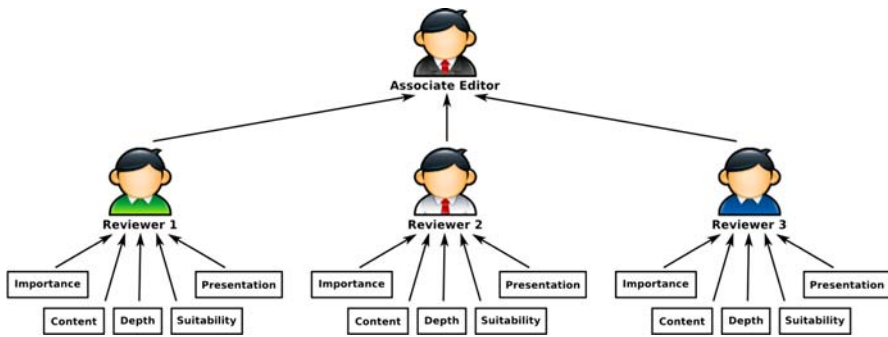
Fig. 6 Example of a decision process: the paper evaluation process for a journal

2. *The choice of the aggregation operator of linguistic information*. It consists of establishing an appropriate aggregation operator of linguistic information for aggregating and combining the linguistic performance values provided.
3. *The choice of the best alternatives*. It consists of choosing the best alternatives according to the linguistic performance values provided. It is carried out in two phases:
    i *Aggregation phase of linguistic information*: It consists of obtaining a collective linguistic performance value on the alternatives by aggregating the linguistic performance values provided according to all the criteria/decision makers by means of the chosen aggregation operator of linguistic information.
    ii *Exploitation phase*: It consists of establishing a rank ordering among the alternatives according to the collective linguistic performance value for choosing the best alternatives.

From this three step solution scheme, it is clear that it is necessary to develop linguistic computational models that allow the representation and the aggregation of linguistic information, respectively.

3.2 Computational models for CW in decision making

In the following we present the main computational models for linguistic aggregation that have been deployed in the decision making field.

3.2.1 Linguistic computational model based on membership functions

This computational model makes use of the Extension Principle from fuzzy arithmetic (Degani and Bortolan 1988): the result of an aggregation function $\tilde{F}$ over a set of n linguistic labels in the term set $T(L)$ is a fuzzy number $F(\mathcal{R})$ that usually does not have an associated linguistic label on $T(L)$. Therefore, it is necessary to apply an approximation function $app_1(\cdot)$ to associate it to a particular label on $T(L)$—the retranslation problem (Martin and Klir 2006; Yager 2004)—or to use fuzzy ranking procedures to obtain a final order of the alternatives:

Fig. 7 Retranslation problem
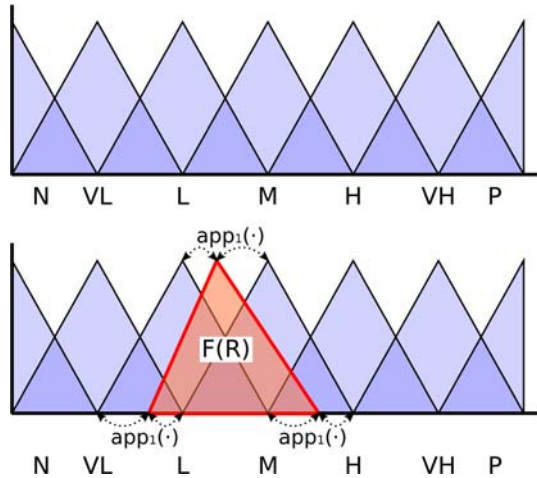
$$T(L)^n \xrightarrow{\tilde{F}} F(\mathcal{R}) \xrightarrow{app_1(\cdot)} T(L)$$

An example of the retranslation problem is presented in Fig. 7: Suppose that, as a result of the application of an aggregation function $\tilde{F}$ we obtain a fuzzy number $F(\mathcal{R})$ (highlighted in the figure) which does not have an associated linguistic label on a particular term set $S = \{N, VL, L, M, H, VH, P\}$. To solve the problem we need an approximation function $app_1(\cdot)$ that will assign one of the L or M labels to $F(\mathcal{R})$. From the example, it is clear that the use of the approximation function to solve the retranslation problem introduces some loss of information which is a critical aspect to be taken into account in decision models.

In this model the aggregation is carried out directly acting over the membership functions associated to the linguistic labels using classical fuzzy arithmetic and obtaining a fuzzy set that represent the aggregated value. These approaches usually use ranking functions to order the fuzzy numbers and to obtain a final numerical evaluation (Anagnostopoulos et al. 2008; Fu 2008).

3.2.2 Linguistic computational model based on type-2 fuzzy sets

This computational model makes use of type-2 fuzzy sets to model linguistic assessments (Mendel 2002, 2008; Türkşen 2002, 2007). Some of the reasons for using type-2 fuzzy sets are:

- From Türkşen (2002): *"Type-1 representation is a 'reductionist' approach for it discards the spread of membership values by averaging or curve fitting techniques and hence, camouflages the 'uncertainty' embedded in the spread of membership values. Therefore, Type-1 representation does not provide good approximation to meaning representation of words and does not allow computing with words a richer platform."*

- From Mendel (2007) *"Words mean different things to different people and so are uncertain. We therefore need a fuzzy set model for a word that has the potential to capture its uncertainties, and an interval type-2 fuzzy set should be used as a fuzzy set model of a word."*

The majority of the contributions in the field use interval type-2 fuzzy sets (a particular kind of type-2 fuzzy sets) which maintain the uncertainty modelling properties of general type-2 fuzzy sets but reduce the computational efforts that are needed to aggregate them. In Dongrui and Mendel (2007) the Linguistic Weighted Average operator is presented. It can be seen as an extension of the Fuzzy Weighted Aggregation operator where both weights and attributes are words modelled by interval type-2 fuzzy sets.

It is worth to note that as the previous linguistic model, this type-2 fuzzy sets based model also suffers from the retranslation problem, that is, the resulting type-2 fuzzy set from an aggregation operation must be mapped into a linguistic assessment at the end of a decision process.

3.2.3 Linguistic symbolic computational models based on ordinal scales

In the literature, we can find three different linguistic symbolic computational models which are based on ordinal scales: (i) a linguistic symbolic computational model based on ordinal scales and max-min operators, (ii) a linguistic symbolic computational model based on indexes and (iii) a linguistic symbolic computational model based on continuous term sets.

Linguistic symbolic computational model based on ordinal scales and max-min operators (Yager 1981): It uses an ordered linguistic scale $S = \{s_1, \ldots, s_g\}$ with a linear ordering as the only structure available in S.

To be able to aggregate information expressed as linguistic labels in that ordered linguistic scale the classical operators Max, Min and Neg are used, where:

- $\text{Max}(s_i, s_j) = s_i$ if $s_i \geq s_j$,
- $\text{Min}(s_i, s_j) = s_i$ if $s_i \leq s_j$ and
- $\text{Neg}(s_i) = s_{g-i+1}$ where g is the cardinality of S.

Yager (1993, 2007) studies several operators for ordinal information to be able to properly aggregate different linguistic evaluations. In addition, Yager (2007) studies several operators for ordinal information aggregation as weighted normed operators, uninorm operators, ordinal mean type operators, etc. Buckley (1984) uses different variations of the median, max and min operators to aggregate linguistic opinions and criteria.

Linguistic symbolic computational model based on convex combination (Delgado et al. 1993): In this model, the aggregation is made using a convex combination of linguistic labels which directly acts over the label indexes of the linguistic terms set $S = \{s_0, \ldots, s_g\}$ in a recursive way producing a real value on the granularity interval of the linguistic terms set S. Note that this model usually assumes that the cardinality of the linguistic term set is odd and that linguistic labels are symmetrically placed around a middle term. As the result of the aggregation is not usually an integer, that
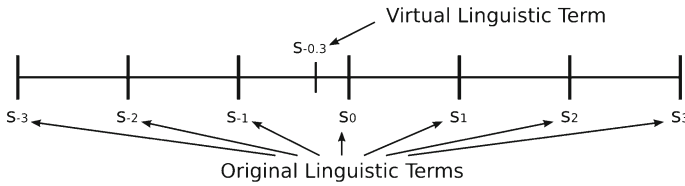
Fig. 8 Example of the linguistic model proposed by Xu (2004b)

is, does not correspond to one of the labels in S, it is also necessary to introduce an approximation function $app_2(\cdot)$ to obtain a solution on the S terms set:

$$S^n \xrightarrow{C} [0, g] \xrightarrow{app_2(\cdot)} \{0, \ldots, g\} \to S$$

For example, suppose that we want to aggregate the linguistic values M, L, VL and H from $S = \{N, VL, L, M, H, VH, P\}$. Then we have that $C\{M, L, VL, H\} = C\{S_3, S_2, S_1, S_4\} = 2.75$. Obviously 2.75 is not the label index of any of the terms in S. If we define $app_2(\cdot)$ as the usual *round* operation, then we would have that the final result of our aggregation process is $S_{round(2.75)} = S_3 = M$. From the example we can conclude that this model for linguistic aggregation also suffers from loss of information.

Aggregation operators that operate in this linguistic model are the Linguistic Ordered Weighted Averaging (LOWA) operator (Herrera et al. 1996a) (based on the OWA operator and the convex combination of linguistic labels), the Linguistic Weighted Disjunction (LWD), Linguistic Weighted Conjunction (LWC), the Linguistic Weighted Averaging (LWA) (Herrera and Herrera-Viedma 1997), the Linguistic Aggregation of Majority Additive (LAMA) operator (Peláez and Doña 2003) and the Majority Guided Induced Linguistic Aggregation Operators (Herrera-Viedma et al. 2006).

Linguistic symbolic computational model based on virtual linguistic terms (Xu 2004b): In this model, the discrete term set $S = \{s_{-\frac{g}{2}}, \ldots, s_0, \ldots, s_{\frac{g}{2}}\}$, with $g + 1$ being the cardinality of S, is extended into a continuous term set $\bar{S} = \{s_\alpha | \alpha \in [-t, t]\}$, where t $(t > q/2)$ is a sufficiently large positive integer. If $s_\alpha \in S$, then s_α is called an *original linguistic term*, otherwise, s_α is called a *virtual linguistic term*. In Fig. 8 we can see a discrete term set $S = \{s_{-3}, \ldots, s_3\}$ (original linguistic terms) which can be extended to a continuous term set in which virtual linguistic terms as $s_{-0.3} \in [-3, 3]$ can be obtained and manipulated to avoid loss of information.

With this extension we can preserve all given information in the problem (thus avoiding one of the biggest problems for the classical linguistic symbolic computational models). Xu states that, *"in general, the decision maker uses the original linguistic terms to evaluate alternatives, and the virtual linguistic terms only appear in operation"*.

It is important to note that this symbolic computational model uses a term set that changes during the decision process as new virtual terms are created in the aggregation processes. In addition, this model allows the use of arithmetic operators and the multiplication between a label and a real number which could lead to virtual terms being in a

quite different range than the original ones. These facts limit the interpretability of the decision models that implement this computational model. Therefore, this model also presents a retranslation problem if the results of the operations are virtual linguistic terms (and they will usually be virtual ones) and the final results must be expressed in the original linguistic term set. However, as the linguistic symbolic computational model based on linguistic terms is simple, as it avoids loss of information and as virtual linguistic terms can be used to rank alternatives and thus, to select the best of them, its use can be convenient in particular situations.

Several different operators have been defined in order to aggregate linguistic information using this model. For example, some extensions to the classical families of ordered weighted averaging and geometric operators have been presented in Xu (2004a, 2006b).

3.2.4 2-Tuple linguistic computational model: a symbolic model extending the use of indexes

The 2-tuple linguistic representation model (Herrera and Martínez 2000) is a symbolic model that improves the previous one in several ways:

- The linguistic computational model based on linguistic 2-tuples carries out processes of CW easily and without loss of information.
- The linguistic domain can be treated as continuous, whilst in the classical models it is treated as discrete.
- The results of processes of CW are always expressed in the initial linguistic domain extended to a pair of values that include the label and additional information.

To represent the linguistic information, this model uses a pair of values called *linguistic 2-tuple* (s, α), where s is a linguistic term and α is a numeric value representing a *Symbolic Translation*:

Definition (*Herrera and Martínez 2000*) "Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g + 1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [-0.5, 0.5)$ then α is called a Symbolic Translation."

In addition, the model defines a set of transformation functions between numeric values and linguistic 2-tuples:

Definition (*Herrera and Martínez 2000*) "Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), \, with \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases}$$
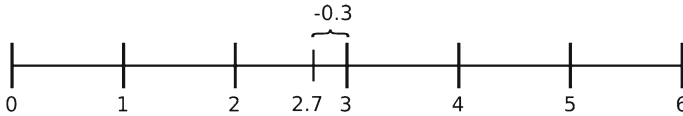
Fig. 9 Example of 2-tuple linguistic label

where round $(\cdot)$ is the usual round operation, s_i has the closest index label to β and α is the value of the symbolic translation. In addition we have:

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

It is obvious that the conversion between a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation: $s_i \in S \Longrightarrow (s_i, 0)$."

In Fig. 9 we can see an example of a 2-tuple linguistic label that express the equivalent information of the result of a symbolic aggregation operation: Suppose that $\beta = 2.7$ is a value representing the result of a symbolic aggregation operation on the set of labels $S = \{s_0, \ldots, s_6\}$, then the 2-tuple that express the equivalent information to β is $(s_3, -0.3)$.

Even being a quite recent model, the 2-tuple model has received a quite good acceptation in the specialized literature and some extensions to the 2-tuple linguistic model have been developed in which the underlying definitions of linguistic labels and linguistic variables are taken into account in the process of aggregating linguistic information by assigning canonical characteristic values of the corresponding linguistic labels (Wang and Hao 2006, 2007).

The aggregation of 2-tuple linguistic information can be achieved by applying some extensions of classical aggregation operators to the 2-tuple linguistic model that can be found in the literature as the Arithmetic Mean, the Weighted Average Operator, the Ordered Weighted Aggregation (OWA) operator, the LOWA operator (Herrera and Martínez 2000), the Lattice-based Linguistic-Valued Weighted Aggregation (LVWA) (Li et al. 2008) and the LAMA operator (Peláez and Doña 2003).

3.3 An historical perspective: pioneer papers

In this section we provide an historical perspective on the use of the CW methodology in decision making. To do so, we revise the pioneer journal papers in the field.

The first work in which a linguistic approach was applied to decision making was written by Tong and Bonissone (1980):

R.M. Tong and P.P. Bonissone. *A linguistic approach to decision making with fuzzy sets*. IEEE Transactions on Systems, Man and Cybernetics SMC-10(11): 716–723, 1980

The authors stated how a linguistic assessment of a decision can make explicit the subjective nature of any choice that is made using fuzzy information and how devel-

oping linguistic approaches can ease decision makers to interact with the decision process at every level. Thus, they propose a technique in which the preference of one alternative over the others can be expressed as a truth qualified proposition in a natural language in the form *"It is more or less true that a_k is preferred to all others"* (where a_k is one of the alternatives in the set of feasible alternatives $A = \{a_1, \ldots, a_n\}$). They used the linguistic computational model presented in Sect. 3.2.1 in which the computations are carried out directly acting over the membership functions associated to the linguistic labels.

Another seminal paper in which CW is applied to multiobjective decision making was written by Yager (1981):

> R.R. Yager. *A New Methodology for Ordinal Multiobjective Decisions Based on Fuzzy Sets*. Decision Sciences, 12:589–600, 1981

Yager emphasized the importance of using linguistic information, not only to provide the preferences of the decision maker, but also to express the importance of the individual objetives, as it is a much more natural approach for a decision maker than just using numbers: *"The values to be used for the evaluation of the ratings and importances will be drawn from a linguistic scale which makes it easier for the evaluator to provide the information."* In fact, Yager uses the linguistic symbolic model based on ordinal scales and max-min operators presented in Sect. 3.2.3 to solve multiobjective decision problems where not only the information about each alternative is given using linguistic terms but also where the importance of each objective is evaluated linguistically. It was later revisited and extended in Yager (1993).

Buckley (1984) also studied decision making problems in a linguistic context in which the best alternative from a feasible set has to be selected by an analyst according the the opinions of several judges which supply information about the alternatives for a set of different criteria:

> J.J. Buckley. *The multiple judge, multiple criteria ranking problem: A fuzzy set approach*. Fuzzy Sets and Systems, 13(1):25–37, 1984

The information supplied by the judges is given using an ordinal scale previously designed by the analyst. Buckley states several reasons for using the ordinal scale instead of other exact, ratio or interval scales, e.g. in problems with a large number of alternatives or criteria it might be easier to provide an element in the ordinal scale than concrete numbers or ratios of numbers, or, in addition, some of the criteria may be vaguely understood or imprecisely defined for the judges. To resolve these problems the author discusses the three main issues that have to be faced to obtain the final ranking of the alternatives: (1) when to pool, or average, the judges; (2) how to pool, or average, the judges; and (3) how to compute the final weight for each alternative. In this work the linguistic symbolic computational model based on ordinal scales and max–min operators presented in Sect. 3.2.3 is used with some variations of the median, max and min operators to aggregate the linguistic information.

Finally, two short contributions published in 1985 dealt with multicriteria optimization problems and selection of models.

> M.S. Shendrik and B.G. Tamm. *Approach to Interactive Solution of Multicriterial Optimization Problems with Linguistic Modeling of Preferences*. Automatic Control and Computer Sciences, 19(6):3–9, 1985

The authors extend a multicriteria optimization algorithm to accept fuzzy linguistic variables and thus, they convert it into a fuzzy linguistic algorithm. In particular they use different linguistic term sets to evaluate the degree to which a decision maker is satisfied with a particular solution and to control the rate of change between different criteria.

> V.B. Silov and D.V. Vilenchik. *Linguistic Decision-Making Methods in Multicriterial Selection of Models*. Soviet Journal of Automation and Information Sciences (English translation of Avtomatyka), 18(4):92–94, 1985

The authors make use of a linguistic model to characterize the preferences of the decision maker in form of rules that take linguistic variables as parameters. Both works use a linguistic computational model based on membership functions (Sect. 3.2.1).

3.4 Recent applications of CW in decision making

In this section we present some of the recent applications (published in the specialized literature in 2007 and 2008) based on a CW approach. The range of proposed applications is quite wide: from resource management and transfer, industry applications, evaluation of projects and processes to investment applications and risk assessment:

Application	References
Resource management and transfer	
Sustainable energy management	Doukas et al. (2007)
Water resources management	Fu (2008)
Human resources management	Yang et al. (2007)
	Genevois and Albayrak (2008)
	Sun et al. (2008)
	Tai and Chen (2008)
Knowledge management	Wang and Chang (2007)
Team situation awareness	Lu et al. (2008b)
Industry applications	
Supplier selection & evaluation	Chang et al. (2007)
	Li et al. (2007)
Location selection	Chou et al. (2008)
	Önüt and Soner (2008)
	Anagnostopoulos et al. (2008)
Material, stock & systems selection	Lin et al. (2007)
	Gharehgozli et al. (2008)
Manufacturing flexibility evaluation	Chuu (2007)

Application	References
Evaluation and recommendation	
Projects evaluation & selection	Büyüközkan and Ruan (2008)
Engineering evaluation	Martínez et al. (2007a)
Sensory evaluation	Zou et al. (2008)
	Martínez (2007)
	Martínez et al. (2008b)
Information retrieval	Herrera-Viedma and López-Herrera (2007)
	Herrera-Viedma et al. (2007a)
Recommender systems	Martínez et al. (2007b)
	Martínez et al. (2008a)
Web quality	Herrera-Viedma et al. (2007b)
Investments applications and risk assessment	
Investments evaluation	Shevchenko et al. (2008)
Situation Assessment	Lu et al. (2008a)
Investment improvement	Güngör and Arıkan (2007)
Bridge risk assessment	Wang and Elhag (2007)

4 Current trends and prospects of CW in decision making

In this section we present some current trends of the CW methodology applied in decision making, along with some open questions and prospects about them. We identify nine current trends: (i) to deal with different linguistic expression domains to represent preferences in decision making, i.e., using multi-granular linguistic modelling, (ii) to deal with non-symmetrical linguistic domains, i.e., unbalanced linguistic modelling, (iii) to deal with the integration of linguistic and numerical representations of preferences; (iv) to deal with the consistency of the linguistic preferences; (v) to deal with the missing linguistic information in decision making; (vi) to develop the linguistic computational model based on type-2 fuzzy sets; (vii) to develop web and mobile based linguistic decision making processes; (viii) to develop new linguistic computational models based on the paradigm of linguistic expressions; and (ix) to develop new linguistic consensus approaches.

4.1 Multi-granular linguistic information in decision making

In many real world situations it is not effective to define a unique linguistic term set to be used by all decision makers. Due to different cultural reasons or different points of view and knowledge about the problem it seems natural to allow decision makers to provide their preferences about the problem using different linguistic term sets, with different cardinalities and with different meanings for each label. For example, in a grading system a decision maker could choose to use a linguistic term set $S_1 = \{Low, Medium, High\}$ and another decision maker could prefer a linguistic term set with higher granularity as $S_2 = \{Very\ Low, Low, Medium, High, Very\ High\}$ (he might be able to better discriminate his preferences about the alternatives). In those cases where the information can be provided using linguistic term sets with different granularity we talk about *multi-granular approaches*.

In Herrera et al. (2000) an approach to fusion multi-granular information in decision making was presented. That model selects a *basic linguistic term set* (BLTS) (with
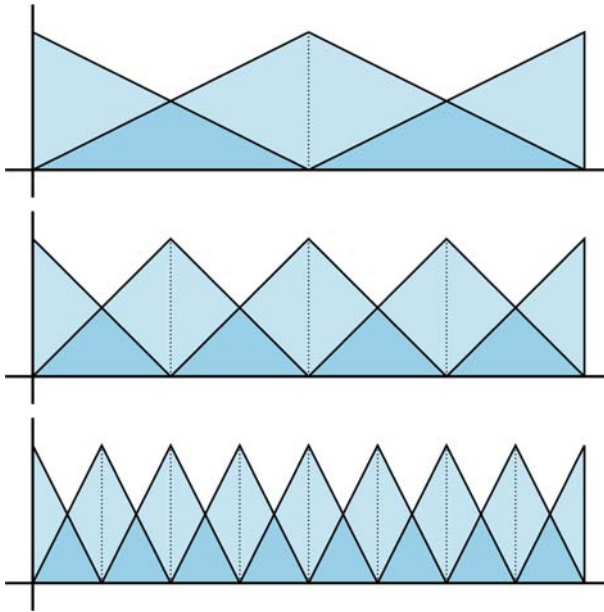
Fig. 10 Linguistic hierarchy of three, five, and nine labels

maximum granularity) and defines a transformation function that represents each linguistic performance value as a fuzzy set defined in the BLTS. Herrera and Martínez (2001) extend this methodology to deal with multi-granular hierarchical linguistic information using the 2-tuple linguistic model. In this model the linguistic term sets must have a hierarchical structure (see Fig. 10). This model has been used and extended in Chang et al. (2007), Chuu (2007), Huynh and Nakamori (2005) and Martínez et al. (2005).

In the most recent literature we can find some new multi-granular approaches such as Chen and Ben-Arieh (2006) where the authors propose a new fusion approach of multi-granularity linguistic information for managing information assessed in different linguistic term sets, and Jiang et al. (2008) where a new method is presented to solve group decision making problems with multi-granularity linguistic assessments in which the multi-granularity linguistic information provided by decision makers is expressed in the form of fuzzy numbers and a linear goal programming model is constructed to integrate the fuzzy assessment information and to directly compute the collective ranking values of alternatives without the need of information transformation.

There are still some open questions about this topic that must be addressed to build more effective multi-granular linguistic decision making models:

- To make the models more flexible to avoid the restrictions imposed in the term sets (such as the linguistic hierarchy or the even coverage of the universe of discourse by the labels) and thus, to be able to use linguistic terms with an arbitrary granularity.
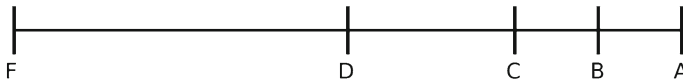
Fig. 11 Example of an unbalanced term set for grading evaluation

- Is it possible to minimize the computation efforts required to obtain the final choices using multigranular linguistic information?

4.2 Unbalanced linguistic information in decision making

Unbalanced information situations, that is, situations where the linguistic labels in the terms set are not evenly distributed around a central term, appear in many real world situations. For example, in grading systems, it is common to use unbalanced term sets where only one term represents a negative evaluation and several positive terms represent different grades of success. In Fig. 11 we present an example of an unbalanced term set for grading evaluation $S = \{F, D, C, B, A\}$ where only one term (F) is used for a negative evaluation and the rest of terms represent different degrees of success.

In Herrera et al. (2008) a model based on linguistic hierarchies was presented to deal with this kind of situations. The Jiang et al. (2008) model proposed for multi-granular environments is also able to deal with unbalanced term sets.

The use of unbalanced linguistic information in decision making is still in an early stage of development and several future challenges are still to be solved:

- Development of models that allow the use of any unbalanced term set.
- Development of models that integrate both unbalanced and multigranular linguistic approaches.
- To analyze the potential use of related work regarding antonyms in linguistic environments that has been developed (Torra 1996, 2001; Trillas and Guadarrama 2005).

4.3 Integration of linguistic and numerical preferences

In real decision making processes, where experts might have a different experience or perception about the alternatives or where the criteria might have a different nature, it is natural to think that the different evaluations about the alternatives can be expressed using different representation formats. In those cases with heterogeneous preference structures we need special mechanisms to allow the integration of evaluations (Chiclana et al. 1998).

In fact, it is not unusual to find decision problems in which the information about the alternatives is presented in both a linguistic and a numerical form. Thus, in the literature we can find several approaches that try to incorporate both qualitative and quantitative information to decision making models (Delgado et al. 1998; Herrera et al. 2005; Meesad and Yen 2003; Zarghami et al. 2008).

Some important points are still open:

- Is it possible to develop new transformation operators from numerical to linguistic information (and vice versa)?.
- To develop aggregation operators that simultaneously deal with both quantitative and qualitative data.

4.4 Analysis of consistency of linguistic preferences

In decision processes it is usually desirable to be able to measure the consistency (lack of contradiction) of the information provided by the decision makers. Non-contradictory information is usually considered more valuable than contradictory one and thus, to develop models to control consistency is usually a good idea in decision making environments. There have been efforts to characterize consistency for many types of preferences (usually by means of different transitivity properties) (García-Lapresta and Meneses 2003; Díaz et al. 2007; Herrera-Viedma et al. 2004).

Classical approaches to tackle and study of consistency are now being adapted to the linguistic framework. In Sahnoun et al. (1991) the authors propose a method to obtain a fuzzy measure of the consistency of linguistic information by applying some operators over the membership functions associated to the linguistic terms. In Dong et al. (2008) some consistency measures have been developed for linguistic preference relations.

There are still some interesting questions in the field to be solved:

- Design new consistency measures for different kinds of linguistic preference structures.
- Developed consistency measures are usually numerical ones, but it seems natural to think that they might have a qualitative nature. Thus, linguistic consistency measures need to be developed.
- Is it always interesting that decision makers comply with some consistency property?: Although consistent information is usually desired, in some environments some loss of consistency can introduce more diversity on the decision process (which could lead to more discussion and a possible better final solution). Consequently, it is interesting to study this kind of situations and to develop models that allow to correctly tackle inconsistent information.

4.5 Missing linguistic information in decision making

In real group decision making processes the information about the alternatives is not always complete, that is, the decision makers may not give all the information that they are requested (Kim and Ahn 1997). There are several possible reasons for a decision maker not giving complete preferences: he might not have a clear opinion about an alternative, he might not be able to compare some alternatives or he might prefer to avoid giving some preference values to avoid introducing inconsistency in his expressed preferences.

In some recent contributions, the issue of missing linguistic information in decision making environments has been addressed: in Li and Sun (2007) a linear programming approach for multiattribute linguistic group decision making which is able to deal with incomplete information has been developed, in Xu (2006a) incomplete linguistic preference relations and their fusion are studied and in Alonso et al. (2009) a group decision making approach with incomplete linguistic preference relations, based on consistency properties to estimate missing information, is presented.

Some issues still remain open in this field:

- To adapt existing linguistic decision models to deal with incomplete information situations.
- To study in greater detail how different consistency properties can be used to estimate missing information.

4.6 Type-2 fuzzy sets and CW in decision making

The use of the linguistic semantic model based on type-2 fuzzy sets is a current trend in decision making (Mendel 2002). Several recent works have developed new decision making models in which the linguistic information is computed and aggregated by means of interval type-2 fuzzy sets to maintain a higher (and more realistic) degree of uncertainty of the linguistic information: in Dongrui and Mendel (2007) the problem of aggregating linguistic information using a type-2 linguistic model is addressed and in Mendel and Wu (2006, 2007) how to transform linguistic perceptions into interval type-2 fuzzy sets has been considered.

However, several questions are still open:

- What is the limit of uncertainty for CW? Is it interesting to use different kinds of type-2 fuzzy sets? (not only interval ones).
- How to solve the retranslation problem for type-2 fuzzy sets in decision making?
- To develop complete decision models using the linguistic model based on type-2 fuzzy sets.
- To decrease the computational complexity of dealing with type-2 fuzzy sets in decision making.

4.7 Development of web and mobile based linguistic decision support systems

Decision Support Systems (Burstein and Holsapple 2008) have become a major area of interest in the decision making field. Nowadays, time and space constraints, the difficulty of assessing large numbers of alternatives and the necessity of carrying out decision making processes in distributed environments make the use of this kind of systems almost indispensable to solve complex decision making problems. Moreover, as information technologies evolve, there is a greater need to develop new decision support systems to be used in situations where decision makers cannot directly meet together to solve the decision problem. To do so, it is necessary to study current web and mobile technologies and adapt the existing decision models and decision support systems to carry this kind of distributed processes efficiently.

There have been some initial efforts in the field (Alonso et al. 2008) but there are many future developments still to be addressed:

- To adapt existing decision support systems to linguistic environments.
- To research on new linguistic interfaces to capture decision makers qualitative preferences.
- To adapt the linguistic decision support systems to distributed environments using web or mobile technologies.

4.8 On the use of complex expressions to evaluate alternatives in decision making

Almost all of the current approaches for CW in decision making use simple labels for alternatives evaluation. However, the use of linguistic expressions could probably provide a much richer environment to solve complex decision making problems. For example, Ma et al. (2007) introduce the possibility of using a set of labels for evaluating an alternative by experts. They analyze the determinacy of linguistic term sets and their consistency (in real applications, those linguistic terms used by experts in individual evaluations should be consistent enough). Tang and Zheng (2006) present a proposal that allows to compute the semantic similarity between a linguistic expression and basic linguistic labels, using the mass assignment to labels. Focused on linguistic decision problems, it allows to describe an object (alternative) using a linguistic expression (such as $\neg label$).

To do so, several questions still have to be answered:

- When should linguistic expressions be used in decision making?
- How can linguistic expressions be aggregated?
- How can linguistic expressions be used in multi-granular environments?

4.9 Consensus reaching

Consensus reaching is a transverse topic in the decision making field. To obtain a high degree of consensus or agreement among the decision makers is usually a crucial aspect of any group decision making problem (Saint and Lawson 1994). Many decision making approaches include an initial iterative consensus phase (prior to the selection of the best alternative) in order to reach a desired level of agreement. In Fig. 12 we can see a general scheme of a group decision making process in which a set of solution alternatives is presented to the decision makers (experts) which express their preferences about it. If the moderator detects that there is not enough consensus he will urge them to change their preferences (he might even give them recommendations about how to change them) and another consensus round starts. If the consensus level is high enough, the selection process is applied and the final solution is selected.

In the literature we can find several consensus models with linguistic information. Two classical papers by Herrera et al. (1996b) and Bordogna et al. (1997) developed consensus models for decision making with linguistic assessments. In addition, many of the current trends presented in Sect. 4 have been applied in conjunction with the
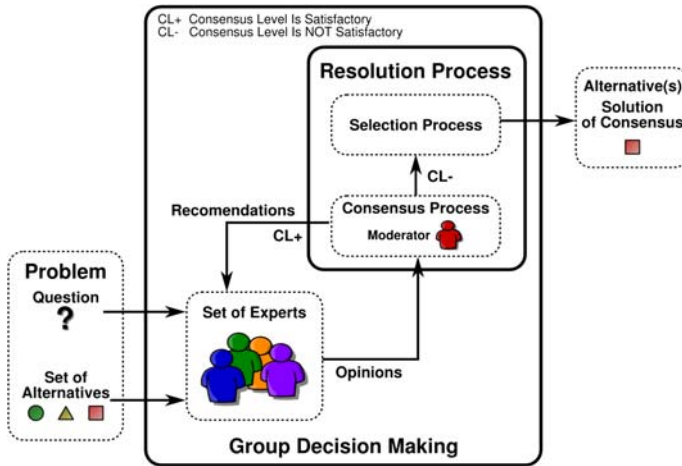
Fig. 12 General scheme of a group decision making process

consensus reaching topic. For example, Herrera-Viedma et al. (2005) developed a consensus support system with multigranular linguistic information, Ben-Arieh and Chen (2006) developed some consensus measures to guide a linguistic decision making process, Cabrerizo et al. (2009) presented a consensus model with linguistic unbalanced information, García-Lapresta (2006) studied the related concept of majority in decision linguistic environments and Cassone and Ben-Arieh (2005) adapted the classical Successive Proportional Additive Numeration (SPAN) consensus technique to use fuzzy linguistic labels.

There are still some open questions about this topic to be addressed:

- To develop new consensus models which incorporate feedback mechanisms to help decision makers to achieve a high level of consensus based on rules easy to understand and to apply by the experts.
- To create or adapt consensus models to consider new linguistic representation formats or expressions.
- To make use of new web and mobile technologies to implement consensus models and consensus support systems in distributed decision making environments.

5 Concluding remarks

An important use of the CW methodology is its application to decision making. In fact, CW can ease the resolution of decision processes when the decision makers can only provide qualitative information about the evaluated alternatives.

In this paper we have made a complete tour of CW in decision making. We have presented the foundations and developments in the CW paradigm along with several linguistic computational models that have been applied to the decision making context: two computational models (based on type-1 and type-2 fuzzy sets respectively) and two symbolic computational models (based on ordinal scales). An historical perspective

on the use of CW in decision making has been given by examining the pioneer papers in the field and a comprehensive list of the most recent developed applications in the specialized literature has been presented. Finally, several current trends and prospects about the topic have been introduced.

Note that this paper does not pretend to make an exhaustive list of the publications about CW in decision making. In http://sci2s.ugr.es/CWDM/ a comprehensive list of all the publications in the specialized literature about the topic is maintained, together with complementary material to this review.

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