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Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations

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Abstract

The purpose of this paper is to study a fuzzy multipurpose decision making problem, where the information about the alternatives provided by the experts can be of a diverse nature. The information can be represented by means of preference orderings, utility functions and fuzzy preference relations, and our objective is to establish a general model which cover all possible representations. Firstly, we must make the information uniform, using fuzzy preference relations as uniform preference context. Secondly, we present some selection processes for multiple preference relations based on the concept of fuzzy majority. Fuzzy majority is represented by a fuzzy quantifier, and applied in the aggregation, by means of an OWA operator whose weights are calculated by the fuzzy quantifier. We use two quantifier guided choice degrees of alternatives, a dominance degree used to quantify the dominance that one alternative has over all the others, in a fuzzy majority sense, and a non dominance degree, that generalises Orlovski's non dominated alternative concept. The application of the two above choice degrees can be carried out according to two different selection processes, a sequential selection process and a conjunction selection process.

Keywords: Multipurpose decision making; Preference orderings; Utility functions; Fuzzy preference relations; Fuzzy majority; Selection process

1. Introduction

A process of decision making, consisting in deriving the best option from a feasible set, is present in just about every conceivable human task. As a result, the study of decision making is necessary and very important not only in Decision Theory but also in areas such as Operations Research, Management Science, Politics, Social Psychology, Artificial Intelligence, etc.

The basic model of a decision in a classical normative Decision Theory has very little in common with real decision making because it portrays a decision as a clear-cut act of choice, in an environment in which the goals, constraints, information and consequences of possible actions are supposed to be precisely known. The only component in which uncertainty is permitted is the occurrence of the different states of nature, for which probabilistic descriptions are allowed. However, when the uncertainty is of a qualitative nature, the use of other techniques is necessary.

Fuzzy sets theory might provide the flexibility needed to represent the uncertainty resulting from the lack of knowledge. There exist many opportunities to apply fuzzy sets theory in decision making.

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Fuzzy tools and methodologies can be used either to translate imprecise and vague information in the problem specification into fuzzy relationships (fuzzy objectives, fuzzy constraints, fuzzy preferences,...) or to design a decision process trying to establish preference orderings of alternatives. Different fuzzy decision making problems were described in [13]. It is obvious that the comparison of different actions according to their desirability in decision problems, in many cases, cannot be done by using a single criterion or an unique person. We do not distinguish between "persons" and "criteria", and interpret the decision process in the framework of multipurpose decision making (MPDM) [14]. This has led to numerous evaluation schemes, and has become a major concern of research in decision making. Several authors have provided interesting results on group decision making or social choice theory and multicriteria decision making with the help of fuzzy theory [6–8, 12, 14, 15, 18, 26]. In all these decision making problems, procedures have been established to combine opinions about alternatives related to different points of view. Most procedures are based on pair comparisons, in the sense that processes are linked to some degree of credibility of preference of any alternative over another, even if the input data corresponds to an evaluation (utility, physical or monetary value) related to each alternative considered for each point of view. In this latter case a pair of alternatives can be compared in a transitive way on the basis of their evaluations.

A classical MPDM procedure follows two steps before to achieve a decision [3]: aggregation and exploitation. The aggregation phase defines an outranking relation which indicates the global preference between every ordered pair of alternatives, taking into consideration the different points of view. The exploitation phase transforms the global information about the alternatives into a global ranking of them. This can be done in different ways, the most common one being the use of a ranking method to obtain a score function.

We will consider MPDM problems where, for each purpose (expert or criterion), the information about the alternatives can be supplied in different ways. Usually, nonfuzzy preferences may be represented as the set of preferred alternatives (choice set), preference relations (orderings), or utility functions (cardinal) [22]. Analogously, the following three representations of fuzzy preferences may be considered: fuzzy choice sets, fuzzy preference relations, and fuzzy utility functions [22]. With a view to build a more flexible framework and to give more freedom degree to represent the preferences, we will assume a MPDM model in which the preferences can be provided in any of these three ways:

- *As a preference ordering of the alternatives.* In this case the alternatives are ordered from the best to the worst, without any other supplementary information.
- *As a fuzzy preference relation.* This is the usual case, i.e., when an expert supplies a fuzzy binary relation over the set of alternatives, reflecting the degree to which an alternative is preferred to another.
- *As an utility function.* In this case an expert supplies a real evaluation (physical or monetary value) for each alternative, i.e., a function that associates each alternative with a real number indicating the performance of that alternative according to his point of view.

Assuming this framework, our objective in this paper is to establish general MPDM models so that we can cover all those possible representations of the information, i.e., preference orderings, utility functions and fuzzy preference relations. With this objective in mind, firstly, we make the information uniform, using fuzzy preference relations as the main element of the uniform representation of the preferences. And, secondly, we design generic selection processes in MPDM (i) based on the concept of fuzzy majority [9], which is used to represent the concept of a social opinion, and (ii) using the OWA operator [23] as aggregation operator.

In order to do this, the paper is set out as follows. The MPDM problem is presented in Section 2. How to make the information uniform is discussed in Section 3, where we present a general model to relate preference orderings, utility values and fuzzy preference relations. Section 4 presents the selection processes of alternatives for multiple fuzzy preference relations. Then, and for the sake of illustrating the classification method of alternatives, Section 5 is devoted to develop an example. In Section 6 some conclusions are pointed out. Finally, the descriptions of fuzzy majority concept and the OWA operator are presented in the Appendices A and B, respectively.

2. Presentation of the problem

The problem we will deal with is that of choosing the best alternative(s) among a finite set, $X = \{x_1, x_2, \ldots, x_n\}$, $(n \geqslant 2)$. The alternatives will be classified from best to worst, using the information known according to a set of general purposes (experts or criteria). In the following, without lack of generality, we will use the term experts, i.e., $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geqslant 2)$. As each expert, $e_k \in E$, is characterised by their own ideas, attitudes, motivations and personality, it is quite natural to think that different experts will provide their preferences in a different way. Then, we assume that the experts' preferences over the set of alternatives, X, may be represented in one of the following three ways:

1. *A preference ordering of the alternatives.* In this case, an expert, e_k, provides his preferences on X as an individual preference ordering, $O^k = \{o^k(1), \ldots, o^k(n)\}$, where $o^k(\cdot)$ is a permutation function over the index set $\{1, \ldots, n\}$ for the expert e_k [3, 19]. Therefore, according to the viewpoint of each expert, an ordered vector of alternatives, from the best one to the worst one, is given.

2. *A fuzzy preference relation.* With this representation, the expert's preferences on X is described by a fuzzy preference relation, $P^k \subset X \times X$, with membership function, $\mu_{P^k}\colon X \times X \to [0,1]$, where $\mu_{P^k}(x_i, x_j) = p^k_{ij}$ denotes the preference degree of the alternative x_i over x_j [9, 11, 14, 20, 21]. We assume that P^k is reciprocal without loss of generality, i.e., by definition [17, 20, 21]: (i) $p^k_{ij} + p^k_{ji} = 1$ and (ii) $p^k_{ii} = -$ (undefined), $\forall i,j,k$.

3. *An utility function.* In this case, an expert e_k provides his preferences on X as a set of n utility values $U^k = \{u^k_i,\ i = 1, \ldots, n\}$, $u^k_i \in [0,1]$, where u^k_i represents the utility evaluation given by the expert e_k to the alternative x_i [16, 22].

In this context, the resolution process of the MPDM problem consists of obtaining a set of solution alternatives $X_{\text{sol}} \subset X$ from the preferences given by the experts. Since the experts provide their preferences in different ways, to obtain a uniform representation of the preferences must be the first step of the resolution process of the MPDM problem. Achieving this uniform representation, we can develop from it any known selection process [3, 7, 9]. In this sense, the resolution process of the considered MPDM problem presents the scheme given in Fig. 1. Therefore, the general model that we propose is developed in the following 2 steps:

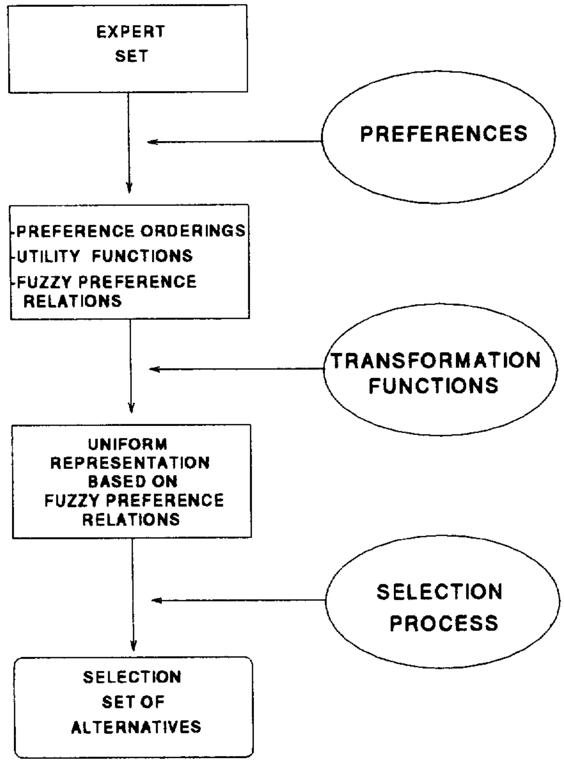

Fig. 1. Resolution process of the MPDM problem.

1. *Making the information uniform.* For every preference ordering and set of utility values we derive an individual fuzzy preference relation. To do this, several transformation functions, which are a generalisation of methods used in [3, 20, 21], are proposed.

2. *Application of a selection process.* As we said earlier, we apply selection processes in two steps [3]:
   (a) *Aggregation phase.* Using the concept of fuzzy majority represented by a linguistic quantifier and applied in the aggregation operations by means of an OWA operator [23], a collective fuzzy preference relation is obtained from all individual fuzzy preference relations.
   (b) *Exploitation phase.* Using again the concept of fuzzy majority, but in another sense, two choice degrees of alternatives are used: the quantifier guided dominance degree and the quantifier

guided non dominance degree [3]. These choice degrees will act over the collective preference relation supplying a selection set of alternatives.

In the next section we study the problem of the uniform representation and analyze different transformation functions to achieve an uniform representation, which are based on fuzzy preference relations.

3. Making the information uniform

In this general framework, where the information provided by a group of experts is supposed to be of a diverse nature (with these three different representations of the information), we need to make the information uniform. As we said at the beginning, due to their apparent merits, we propose to use fuzzy preference relations as the base element of the uniform representation. The use of fuzzy preference relations in decision making situations to represent an expert's opinion about a set of alternatives, appears to be a useful tool in modelling decision processes, overcoat when we want to aggregate experts' preferences into group preferences, that is, in the resolution processes of the MPDM problems [1, 2, 7–10, 17, 18, 20, 21]. Furthermore, preference orderings and utility values are included in the family of fuzzy preference relations [22] and most of the existing results on MPDM are obtained under fuzzy preferences relations [1, 2, 7–11, 14, 17, 20, 21].

Therefore, as it is shown in Fig. 1, we need some transformation functions to transform preference orderings and utility values into fuzzy preference relations. In the next subsections we analyse this aspect and present a generical transformation function which can be used to deal with preference orderings as well as with utility values.

3.1. Utility values and preference relations

The relationship between utility values, given on the basis of a positive ratio scale, and fuzzy preference relations will be studied in this subsection. It is assumed that each expert e_k provides his preferences on X by means of a set of utility values $U^k = \{u_i^k,\ i = 1,\ldots,n\}$, i.e., each alternative x_i is supposed to have associated a real number u_i^k indicating the performance of that alternative according to the expert e_k. For every set of utility values U^k, we will suppose, without loss of generality, that the higher the evaluation, the better the alternative satisfies the expert.

Any possible transformation function h to derive a fuzzy preference relation from a set of utility values, must obtain for an expert e_k his preference value of the alternative x_i over x_j, p_{ij}^k, depending only on the values of u_i^k and u_j^k, i.e. [4],

$$p_{ij}^k = h(u_i^k, u_j^k).$$

This transformation function h must satisfy that the more u_i^k the more p_{ij}^k, and the more u_j^k the less p_{ij}^k. Therefore, it must be a non-decreasing function of the first argument and a non-increasing function of the second argument [4].

An example of this type of transformation functions, defined from utility values which are given on the basis of a positive ratio scale, are those that obtain the credibility value of preference of any alternative over any other alternative depending on the value of the quotient between the respective utility values of the alternatives, i.e.,

$$h(u_i^k, u_j^k) = l\left(\frac{u_i^k}{u_j^k}\right),$$

where l is a non-decreasing function. This type of transformation functions l have been investigated by Luce and Suppes [16].

Interpreting u_i^k/u_j^k as a ratio of the preference intensity for x_i to that of x_j, that is, x_i is u_i^k/u_j^k times as good as x_j, and assuming a reciprocal fuzzy preference relation, a possible transformation function to obtain the intensity of preference of the alternative x_i over alternative x_j for expert e_k, p_{ij}^k, may be defined as [4]

$$p_{ij}^k = l^1\left(\frac{u_i^k}{u_j^k}\right) = \frac{u_i^k/u_j^k}{u_i^k/u_j^k + (u_j^k/u_i^k)} = \frac{(u_i^k)^2}{(u_i^k)^2 + (u_j^k)^2}, \quad i \neq j.$$

Other examples of this type of functions may be found in [20–22], as the following:

$$p_{ij}^k = l^2\left(\frac{u_i^k}{u_j^k}\right) = \frac{u_i^k}{u_i^k + u_j^k}, \quad i \neq j.$$

Obviously, these are not the only functions we can use to transform utility values given on the basis of a positive ratio scale into fuzzy preference relations. As we will show later, both functions are particular cases of a general family of transformation functions.

Without loss of generality, we suppose that utility values belong to $[0,1]$. Then the transformation function that we look for

$$h:[0,1]\times[0,1]\rightarrow[0,1]$$

has to verify the following properties:

1. $h(z,y)+h(y,z)=1\ \forall z,y\in[0,1]$.
2. $h(z,z)=\frac{1}{2}\ \forall z\in[0,1]$.
3. $h(z,0)=1\ \forall z\in[0,1]$.
4. $h(z,y)>\frac{1}{2}$ if $z>y\ \forall z,y\in[0,1]$.

- Property 1 is the reciprocity condition [17, 20, 21].
- Property 2 is a consequence of property 1, and indicates the indifference of an expert between two alternatives verifying his criterion with the same intensity.
- Property 3 means that if an expert has certain knowledge that an alternative does not satisfy his criterion, then any alternative satisfying his criterion with a positive value should be preferred with the maximum degree of preference.
- Finally, property 4 indicates that between two alternatives, the expert gives a definite preference to the alternative with higher evaluation over the other. This is a consequence of property 2 and the fact that function, h, has to be non-decreasing in the first argument and non-increasing in the second argument.

Without loss of generality, we assume that

$$h(z,y)=\frac{1}{1+t(z,y)},$$

where

$$t:[0,1]\times[0,1]\rightarrow\mathcal{R}^+,$$

is a non-increasing function of the first argument and a non-decreasing function of the second argument. To solve the above equation we assume t is a function able to be written so that variables are separated multiplicatively, i.e. that t is a separable function. This assumption is based on the fact that the set of utility values are given on the basis of a positive ratio scale, and it is necessary and relevant as we will show later. We then have

$$h(z,y)=\frac{1}{1+r(z)\cdot s(y)},$$

where r and s are functions with same domain $[0,1]$, same sign, non-increasing the first and non-decreasing the second, respectively. From property 2 we have

$$r(z)\cdot s(z)=1,\quad \forall z\in[0,1].$$

Therefore, $r=1/s$, with $s(n)\neq 0$ for any n belonging to the domain of definition of s. But, from property 3, we have that $s(0)=0$, so that

$$s:[0,1]\rightarrow\mathcal{R}^+,$$

and for $n=0$ $r(\cdot)$ is not defined, therefore, below we will consider the particular case $(0,0)$. The expression of h transforms into

$$h(z,y)=\frac{s(z)}{s(z)+s(y)}.$$

This expression is not defined when $(z,y)=(0,0)$ but from property 2 we can define it as

$$h(0,0)=\tfrac{1}{2}.$$

A desirable property to be verified by the defined fuzzy preference relation should be that if the valuations (u_i^k,u_j^k) of a pair of alternatives (x_i,x_j) change slightly, then the preference degree between them (p_{ij}^k) should change slightly too, i.e. it would be desirable for h to be *continuous*. This can be achieved if s is a continuous function.

To support the separable assumption we have made above, let us consider the following. We were trying to find functions

$$h:[0,1]\times[0,1]\rightarrow[0,1],$$

verifying

$$h(z,y)+h(y,z)=1,\quad \forall z,y\in[0,1].$$

Without loss of generality, we assume that

$$h(z,y)=[t'(z,y)]^2,\quad \forall z,y\in[0,1],$$

where $t'(z,y) \in [0,1]$. Then, we have

$$[t'(z,y)]^2 + [t'(y,z)]^2 = 1, \quad \forall z,y \in [0,1].$$

We can represent the above equation in the parametric form:

$$t'(z,y) = \cos\gamma, \quad t'(y,z) = \sin\gamma,$$

where $\gamma \in [0, \frac{1}{2}\pi]$ represents the value of the vector angle of a point with cartesian coordinates (z,y) or, in general, $(q(z), q(y))$, where

$$q\colon [0,1] \to \mathscr{R}^+$$

is any non-decreasing and continuous function, verifying $q(0) = 0$. We then have

$$\gamma = \arccos \frac{q(z)}{\sqrt{[q(z)]^2 + [q(y)]^2}},$$

and, therefore,

$$t'(z,y) = \frac{q(z)}{\sqrt{[q(z)]^2 + [q(y)]^2}}.$$

Finally, writing $s(z) = [q(z)]^2$, equation of h becomes

$$h(z,y) = \frac{s(z)}{s(z) + s(y)}.$$

Therefore, summarizing, we have the following results.

Proposition 1. *For every set of utility values, $U^k = \{u_1^k, \ldots, u_n^k\}$, over a set of alternatives, $X = \{x_1, \ldots, x_n\}$, given on the basis of a positive ratio scale, the preference of alternative x_i over x_j, p_{ij}^k, is obtained from the ratio u_i^k/u_j^k by the following transformation function h:*

$$p_{ij}^k = h^1(u_i^k, u_j^k) = \begin{cases} \dfrac{s(u_i^k)}{s(u_i^k) + s(u_j^k)} & \text{if } (u_i^k, u_j^k) \neq (0,0) \\ \frac{1}{2} & \text{if } (u_i^k, u_j^k) = (0,0) \end{cases} \quad (i \neq j),$$

where $s\colon [0,1] \to \mathscr{R}^+$ is any non-decreasing and continuous function, verifying $s(0) = 0$.

Corollary 1.1. *When $s(u_i^k) = u_i^k$ the transformation function, h^1, reduces to the transformation function l^2, proposed in* [20, 21]. *On the other hand, when $s(u_i^k) = (u_i^k)^2$, then it reduces to the transformation function l^1, proposed in* [4].

The relationship between utility values, given on the basis of a difference scale, and fuzzy preference relations is studied in the following subsection.

3.2. Preference orderings and preference relations

In this case, let us assume that each expert e_k provides his preferences on X by means of a preference ordering $O^k s = \{o^k(1), \ldots, o^k(n)\}$. For every preference ordering O^k, we will suppose, without loss of generality, that the lower the position of an alternative in a preference ordering, implies the better the alternative satisfies the expert, and vice versa. For example, suppose that an expert e_k supplies his preferences about a set of four alternatives $X = \{x_1, x_2, x_3, x_4\}$ by means of the following ordering preference $O^k = \{3,1,4,2\}$. This means that alternative x_2 is the best for that expert, while alternative x_3 is the worst.

We proposed in [3] a first approach to derive a fuzzy preference relation from a preference ordering. Clearly, an alternative satisfies an expert more or less depending on its position in his preference ordering. Therefore, in our approach, we considered that for an expert e_k his preference value of the alternative x_i over x_j, p_{ij}^k, depends only on the values of $o^k(i)$ and $o^k(j)$, i.e., we assert that there exists a transformation function f that assigns a credibility value of preference of any alternative over any other alternative, from any preference ordering,

$$p_{ij}^k = f(o^k(i), o^k(j)).$$

This transformation function f must satisfy that the more $o^k(i)$ the less p_{ij}^k, and the more $o^k(j)$ the more p_{ij}^k. Therefore, it must be a non-increasing function of the first argument and a non-decreasing function of the second argument [4].

An example of this type of transformation functions are those that obtain the credibility value of preference of any alternative over any other alternative depending on the value of the difference between the alternatives' positions, i.e.,

$$f(o^k(i), o^k(j)) = g(o^k(j) - o^k(i)),$$

where g is a non-decreasing function. For example, in [3] we use the following transformation function:

$$p_{ij}^k = g^1(o^k(j) - o^k(i)) = \begin{cases} 1 & \text{if } o^k(j) > o^k(i) \\ 0 & \text{if } o^k(i) > o^k(j) \end{cases} \quad (i \neq j).$$

This transformation function g^1 derives non-fuzzy preference relations, where p_{ij}^k reflects the degree in $\{0, 1\}$ to which x_i is declared not worse than x_j for the expert, e_k. In our example, the alternative x_2 is not worse than alternatives x_4, x_1, x_3; the alternative x_4 is not worse than alternatives x_1, x_3, and, finally, the alternative x_1 is not worse than alternative x_3. Therefore, we obtain the following non-fuzzy preference relation:

$$P^k = \begin{bmatrix} - & 0 & 1 & 0 \\ 1 & - & 1 & 1 \\ 0 & 0 & - & 0 \\ 1 & 0 & 1 & - \end{bmatrix}.$$

The simplicity and easy use are the only virtues of this particular transformation function, g^1. However, this preference relation does not reflect the case when an expert is not able to distinguish between two alternatives, that is when there is an indifference between two alternatives, although this can be achieved with an extension of this function g^1 as follows:

$$p_{ij}^k = g^2(o^k(j) - o^k(i)) = \begin{cases} 1 & \text{if } o^k(j) - o^k(i) > 0 \\ \frac{1}{2} & \text{if } o^k(j) - o^k(i) = 0 \\ 0 & \text{if } o^k(j) - o^k(i) < 0 \end{cases} \quad (i \neq j).$$

In any case, both functions g^1 and g^2 do not reflect any kind of intensity of preference between alternatives when we compare pairs of alternatives, that is, for example, they do not distinguish between the preference of alternative x_2 over x_4 and the preference of alternative x_2 over x_3. Therefore, to deal with these situations we need to use another type of function which reflects appropriately the different positions between alternatives, and for example, if $p_{24}^k = \frac{2}{3}$ then p_{41}^k and p_{13}^k should be equal to $\frac{2}{3}$, but p_{21}^k should be greater than or equal $\frac{2}{3}$ and less than or equal p_{23}^k.

This new type of function can be achieved, for example, by giving a value of importance or utility to each alternative, in such a way that the lower the position of an alternative, the higher the value of utility. We can assume that the preference of the best alternative over the worst alternative is the maximum allowed, that is 1. So if, for example, $o^k(i) = 1$ and $o^k(j) = n$, then we assume that $p_{ij}^k = 1$. In this case, the utility value u_i^k associated to alternative x_i depends on the value of its position $o^k(i)$, in such a way that the bigger the value of $n - o^k(i)$, the bigger the value of u_i^k, that is

$$u_i^k = v(n - o^k(i)),$$

where v is a non-decreasing function. As an example, we can assign the value

$$u_i^k = v(n - o^k(i)) = \frac{n - o^k(i)}{n - 1},$$

as a degree of importance or utility of the alternative x_i according to the preference ordering O^k, provided by an expert e_k. It is clear that the maximum utility value corresponds to the first alternative and the minimum utility value to the last alternative in the preference ordering. In this context, we have a normalised set of n utility values, that is,

$$MAX_i\{u_i^k\} - MIN_i\{u_i^k\} \leqslant 1.$$

Therefore, we have utility values given on the basis of a difference scale. For this type of utility values, Tanino proposed in [20, 21] a transformation function to obtain preferences between the alternatives p_{ij}^k which is defined from the difference $(u_i^k - u_j^k)$, as follows:

$$p_{ij}^k = g^3(u_i^k - u_j^k) = \tfrac{1}{2}(1 + u_i^k - u_j^k).$$

This transformation function g^3 has been investigated by Dombi [5]. Dombi defined g^3 as an universal preference function and showed that the utility based decision making gives the same result as the preference based using the universal preference function.

In our case, the transformation function of utility values (based on difference scale) given by Tanino [20, 21], becomes the following transformation function of preference orderings:

$$p_{ij}^k = g^4(o^k(j) - o^k(i)) = \frac{1}{2}\left(1 + \frac{o^k(j)}{n-1} - \frac{o^k(i)}{n-1}\right),$$

and therefore, the preference value p_{ij}^k is given depending on the difference $o^k(j) - o^k(i)$.

These two transformation functions of preference ordering g^2 and g^4 allow preference values, p_{ij}^k, to verify the following four relationships:

- $0 \leqslant p_{ij}^k \leqslant 1$, $\forall i,j$.
- $p_{ij}^k + p_{ji}^k = 1$, $\forall i,j$.
- When there is an indifference between two alternatives, that is, when $o^k(i) = o^k(j)$, then $p_{ij}^k = \frac{1}{2}$, $\forall i,j$.
- $p_{ij}^k > \frac{1}{2}$ if $o^k(i) < o^k(j)$, $\forall i,j$.

Obviously, these are not the only functions that can be used to transform preference orderings into fuzzy preference relations. As we will show later, g^2 and g^4 are particular cases of a general family of functions that can be used to transform preference orderings into fuzzy preference relations.

We are looking for a general expression of the transformation function of preference orderings into fuzzy preference relations f in such a way, that given a pair of alternatives (x_i, x_j) of which we only know their position numbers in a preference ordering $(o^k(i), o^k(j))$, then it gives us the preference of x_i over x_j according to expert e_k, p_{ij}^k. This transformation function f as was aforementioned, must be a non-increasing function of the first argument and a non-decreasing function in the second argument, and furthermore, has to verify the following properties:

1. $f(o^k(i), o^k(j)) \in [0,1]$, $\forall i,j$.
2. $f(o^k(i), o^k(j)) + f(o^k(j), o^k(i)) = 1$, $\forall i,j$.
3. $f(o^k(i), o^k(j)) = \frac{1}{2}$ if $o^k(i) = o^k(j)$, $\forall i,j$.
4. $f(o^k(i), o^k(j)) > \frac{1}{2}$ if $o^k(i) < o^k(j)$, $\forall i,j$.

These properties are equivalent to the above four relationships verified by the transformation functions g^2 and g^4.

As was aforementioned, there exists a function g such that, the credibility value of preference of any alternative over any other alternative p_{ij}^k is obtained depending on the value of the difference between the alternatives' positions, i.e.,

$$p_{ij}^k = f(o^k(i), o^k(j)) = g(o^k(j) - o^k(i)),$$

where g is a non-decreasing function. Furthermore, g must verify

1. $g(z) \in [0,1]$.
2. $g(z) + g(-z) = 1$.
3. $g(z) > \frac{1}{2}$ if $z > 0$.

Without loss of generality, it can be assumed that g presents the following:

$$g(z) = \tfrac{1}{2} + d(z),$$

where d is a non-decreasing function verifying

1. $d(z) \in [-\frac{1}{2}, \frac{1}{2}]$.
2. $d(z) + d(-z) = 0$.
3. $d(z) > 0$ if $z > 0$.

We then have

$$p_{ij}^k = \tfrac{1}{2} + d(o^k(j) - o^k(i)).$$

Property 2 implies

$$-d(z) = d(-z),$$

that is, d is an odd function. On the other hand, the following result is well known:

"*A function $d: D \to \mathfrak{R}$ with a symmetric domain is an odd function if and only if there exists a function $F: D \to \mathfrak{R}$ verifying*

$$d(z) = \frac{F(z) - F(-z)}{2}."$$

Applying this result to our situation, we have the following consequences:

Proposition 2. *Suppose we have a set of alternatives $X = \{x_1, \ldots, x_n\}$, and associated with it a preference ordering of any expert $O^k = \{o^k(1), \ldots, o^k(n)\}$. Then, the preference degree of alternative x_i over x_j, p_{ij}^k, is given by the following transformation function f^1:*

$$p_{ij}^k = f^1(o^k(i), o^k(j)) = \tfrac{1}{2}[1 + F(o^k(j) - o^k(i)) - F(o^k(i) - o^k(j))],$$

where F is any non-decreasing function.

Corollary 2.1. *Suppose that the preference degree of the best alternative over the worst alternative in a preference ordering is the maximum allowed, that is 1, then:*

1. *When the function F is presented in the following way:*

$$F(z) = \begin{cases} \frac{1}{2} & \text{if } z > 0, \\ 0 & \text{if } z = 0, \\ -\frac{1}{2} & \text{if } z < 0 \end{cases}$$

function f^1 reduces to expression of function g^2.

2. *When the function F is presented in the following way:*

$$F(z) = \frac{a \cdot z}{2},$$

where $a \in \mathcal{R}$, *if* $a = 1/(n-1)$ *then* f^1 *reduces to expression of function* g^4.

Suppose $o^k(i)$ and $o^k(j)$ are the position numbers of alternatives x_i and x_j according to the preference ordering O^k, provided by an expert e_k, respectively. In what follows, both $o^k(i)$ and $o^k(j)$ can be replaced by $u_i^k = v(n-o^k(i))$ and $u_j^k = v(n-o^k(j))$, respectively, or in general by two real numbers u_i^k and u_j^k given on the basis of a difference scale.

Proposition 3. *Suppose we have a set of alternatives,* $X = \{x_1, \ldots, x_n\}$, *and a set of n utility values,* $U^k = \{u_1^k, \ldots, u_n^k\}$ *associated to X, given on the basis of a difference scale, then the preference value* p_{ij}^k *is given by the following transformation function* f^2:

$$p_{ij}^k = f^2(u_i^k, u_j^k) = \tfrac{1}{2}[1 + F(u_i^k - u_j^k) - F(u_j^k - u_i^k)],$$

where F is any non-decreasing function.

Corollary 3.1. *Expression of function* f^2 *reduces to Tanino's transformation function* g^3, *when*

$$F(z) = \frac{z}{2}.$$

Corollary 3.2. *Suppose we have a set of alternatives, X, and associated with it any preference ordering* O^k. *Then, taking*

$$u_i^k = v(n - o^k(i)) = a \cdot [n - o^k(i)],$$

with $a = 1/(n-1)$ *(so function v is non-decreasing), then*

$$u_i^k = \frac{n - o^k(i)}{n-1},$$

and if, as above we did,

$$F(z) = \frac{z}{2},$$

then function f^2 *reduces to transformation function* g^4.

3.3. Preference orderings, utility values and preference relations

This section summarises everything we have seen in the previous subsections. The result we present includes all the results we have obtained, and in that sense can be considered as a general theorem to be used when we wish to make the information uniform.

Proposition 4. *Suppose we have a set of alternatives,* $X = \{x_1, \ldots, x_n\}$. *Suppose that* λ_i *represents an evaluation of alternative* x_i, *that is a function that associates each alternative,* x_i, *with a real number indicating the performance of that alternative,* x_i *according to a point of view (expert or criterion),* e_k. *Then, the intensity of preference of alternative* x_i *over alternative* x_j, p_{ij}^k *for that point of view is given by the following transformation function*

$$p_{ij}^k = \varphi(\lambda_i, \lambda_j) = \tfrac{1}{2}[1 + \psi(\lambda_i, \lambda_j) - \psi(\lambda_j, \lambda_i)],$$

where ψ *is a function verifying*

1. $\psi(z,z) = \frac{1}{2}$, $\forall z \in \mathcal{R}$.
2. ψ *is non-decreasing of the first argument and non-increasing of the second argument.*

Proof. Without loss of generality, we can assume that the higher the evaluation, the better the alternative satisfies the expert. The intensity of preference p_{ij}^k is given by

$$p_{ij}^k = \psi(\lambda_i, \lambda_j),$$

where ψ is a non-decreasing function of the first argument and a non-increasing function of the second argument. We are assuming a fuzzy preference relation being reciprocal, i.e.,

$$\psi(\lambda_i, \lambda_j) + \psi(\lambda_j, \lambda_i) = 1.$$

We then have

$$\psi(z,z) = \tfrac{1}{2}, \quad \forall z \in \mathcal{R},$$

and

$$\psi(\lambda_i, \lambda_j) = 1 - \psi(\lambda_j, \lambda_i).$$

Consequently,

$$p_{ij}^k = \psi(\lambda_i,\lambda_j) = \tfrac{1}{2}\cdot 2\psi(\lambda_i,\lambda_j)$$
$$= \tfrac{1}{2}[\psi(\lambda_i,\lambda_j) + \psi(\lambda_i,\lambda_j)]$$
$$= \tfrac{1}{2}[\psi(\lambda_i,\lambda_j) + 1 - \psi(\lambda_j,\lambda_i)].$$

Corollary 4.1 (Proposition 2). *Suppose we have a set of alternatives X, and associated with it any preference ordering O^k. Then, the preference degree of alternative x_i over x_j, p_{ij}^k, is given by the transformation function φ, where $\psi(z,y) = F(y-z)$, being F any non-decreasing function.*

Proof. In this case $\lambda_i = o^k(i)$, and then

$$\psi(\lambda_i,\lambda_j) = F(\lambda_i - \lambda_j) = F(o^k(j) - o^k(i)),$$

where F is a non-decreasing function. Therefore, φ reduces to the transformation function f^1.

Corollary 4.2 (Proposition 3). *Suppose we have a set of n utility values U^k associated to X, given on the basis of a difference scale, then the preference of x_i over x_j, p_{ij}^k, is given by the transformation function φ, where $\psi(z,y) = F(z-y)$, where F is any non-decreasing function.*

Proof. In this case $\lambda_i = n - o_i^k = u_i^k$, and then

$$\psi(\lambda_i,\lambda_j) = F(\lambda_i - \lambda_j) = F(u_i^k - u_j^k),$$

and therefore, φ reduces to the transformation function f^2.

Corollary 4.3 (Proposition 1). *Suppose we have a set of n utility values U^k associated to X, given on the basis of a positive ratio scale, then the preference p_{ij}^k is given by the transformation function φ, where*

$$\psi(z,y) = \begin{cases} \dfrac{s(z)}{s(z)+s(y)} & \text{if } (z,y) \neq (0,0),\\ \tfrac{1}{2} & \text{if } (z,y) = (0,0), \end{cases}$$

where $s : [0,1] :\longrightarrow \mathscr{R}^+$ is any non-decreasing and continuous function, verifying $s(0) = 0$.

Proof. Indeed, expression of Proposition 1 can be rewritten in the following way:

$$p_{ij}^k = \frac{s(u_i^k)}{s(u_i^k)+s(u_j^k)} = \frac{1}{2}\,\frac{2\cdot s(u_i^k)}{s(u_i^k)+s(u_j^k)}$$
$$= \frac{1}{2}\,\frac{2\cdot s(u_i^k) + s(u_j^k) - s(u_j^k)}{s(u_i^k)+s(u_j^k)}$$
$$= \frac{1}{2}\left[\frac{s(u_i^k)+s(u_j^k)}{s(u_i^k)+s(u_j^k)} + \frac{s(u_i^k)-s(u_j^k)}{s(u_i^k)+s(u_j^k)}\right]$$
$$= \frac{1}{2}\left[1 + \frac{s(u_i^k)}{s(u_i^k)+s(u_j^k)} - \frac{s(u_j^k)}{s(u_i^k)+s(u_j^k)}\right],$$

and therefore, φ reduces to transformation function h^1.

4. The decision process

In this section we will deal with choosing the alternative(s) which is (are) considered to be desirable for the group as a whole. For that reason, and after the information is uniformed into fuzzy preference relations, we have a set of m individual fuzzy preference relations. The selection processes we present here, as said at the beginning, has two steps: aggregation and exploitation. The aggregation phase defines a collective fuzzy preference relation, which indicates the global preference between every ordered pair of alternatives. The exploitation phase transforms the global information about the alternatives into a global ranking of them, supplying a selection set of alternatives.

4.1. Aggregation: The collective fuzzy preference relation

Once we have made the information uniform, we have a set of m individual fuzzy preference relations $\{P^1,\ldots,P^m\}$. From this set of relations we derive the collective fuzzy preference relation P^c. Each value $p_{ij}^c \in [0,1]$ represents the preference of alternative x_i over alternative x_j according to the majority experts' opinions. Traditionally, the majority is defined as a threshold number of individuals. *Fuzzy majority* is a soft majority concept expressed by a fuzzy quantifier, which is manipulated via a fuzzy-logic-based calculus of linguistically quantified propositions [25]. Then, we will compute each p_{ij}^{c} using an OWA operator [23], as the aggregation operator of information. The OWA operator reflects the fuzzy majority calculating its weighting vector by means of a fuzzy quantifier. Therefore, the collective fuzzy preference relation is obtained as follows:

$$p_{ij}^{c} = \phi_Q(p_{ij}^{1}, \ldots, p_{ij}^{m}),$$

where Q is the fuzzy quantifier used to compute the weighting vector of the OWA operator ϕ_Q.

4.2. Exploitation: Choosing the alternative(s)

At this point, in order to select the alternative(s) "best" acceptable to the group of individuals as a whole, we will use two quantifier guided choice degrees of alternatives, based on the concept of fuzzy majority: a dominance degree and a non-dominance degree. Both are based on the use of the OWA operator. The application of these two choice degrees of alternatives can be carried out according to two different selection policies: a sequential selection policy and a conjunction selection policy [3].

4.2.1. Choice degrees of alternatives

Concretely, we use the two following quantifier guided choice degrees:

1. *Quantifier guided dominance degree.* For the alternative, x_i, we compute the quantifier-guided dominance degree $QGDD_i$ used to quantify the dominance that one alternative has over all the others in a fuzzy majority sense as follows:

$$QGDD_i = \phi_Q(p_{ij}^{c},\ j = 1, \ldots, n,\ j \neq i).$$

2. *Quantifier guided non-dominance degree.* We also compute the quantifier guided non-dominance degree $QGNDD_i$ according to the following expression:

$$QGNDD_i = \phi_Q(1 - p_{ji}^{s},\ j = 1, \ldots, n,\ j \neq i),$$

where

$$p_{ji}^{s} = \max\{p_{ji}^{c} - p_{ij}^{c}, 0\},$$

represents the degree to which x_i is strictly dominated by x_j. In our context, $QGNDD_i$ gives the degree in which each alternative is not dominated by a fuzzy majority of the remaining alternatives. We note that when the fuzzy quantifier represents the statement "*all*", whose algebraic aggregation corresponds to the conjunction operator *Min*, then this non-dominance degree coincides with Orlovski's non-dominated alternative concept [17], i.e., it generalises Orlovski's concept.

4.2.2. Selection policies

The application of the above choice degrees of alternatives over X may be carried out according to two different policies.

1. *Sequential policy*: Selecting and applying one of them according to the preference of the experts, and thus obtaining a selection set of alternatives. If there is more than one alternative in that selection set, then the other choice degree may be applied to select the alternative of the above set with the best second choice degree. This policy defines a sequential selection process.

2. *Conjunctive policy*: Applying the two choice degrees to X, obtaining the final selection set of alternatives as the intersection of the two previous selection sets of alternatives. This policy defines a conjunction selection process.

We note that the latter conjunction selection process is more restrictive than the former sequential selection process because it is possible to obtain an empty selection set. Therefore, in a complete selection process the choice degrees can be applied in three steps [3]:

Step 1: The application of each choice degree of alternatives over X to obtain the following sets of alternatives:

$$X^{QGDD} = \left\{ x_i \,\middle|\, x_i \in X,\ QGDD_i = \sup_{x_j \in X} QGDD_j \right\},$$

$$X^{QGNDD} = \left\{ x_i \,\middle|\, x_i \in X,\ QGNDD_i = \sup_{x_j \in X} QGNDD_j \right\}$$

whose elements are called maximum dominance elements of the fuzzy majority of X quantified by Q and maximal non-dominated elements by the fuzzy majority of X quantified by Q, respectively.

Step 2: The application of the conjunction selection policy, obtaining the following set of alternatives:

$$X^{QGCP} = X^{QGDD} \cap X^{QGNDD}.$$

If $X^{QGCP} \neq \emptyset$, then End.

Otherwise continue.

Step 3: The application of the one of the two sequential selection policies, according to either a dominance or non-dominance criterion, i.e.,

- *Dominance based sequential selection process QG–DD–NDD.* To apply the quantifier guided dominance degree over X, and obtain X^{QGDD}. If $\#(X^{QGDD})=1$ then End, and this is the solution set. Otherwise, continue obtaining

$$X^{QG-DD-NDD} = \left\{x_i \,\middle|\, x_i \in X^{QGDD}, QGNDD_i = \sup_{x_j \in X^{QGDD}} QGNDD_j \right\}.$$

This is the selection set of alternatives.

- *Non-dominance based sequential selection process QG–NDD–DD.* To apply the quantifier guided non-dominance degree over X, and obtain X^{QGNDD}. If $\#(X^{QGNDD})=1$ then End, and this is the solution set. Otherwise, continue obtaining

$$X^{QG-NDD-DD} = \left\{x_i \,\middle|\, x_i \in X^{QGNDD}, QGDD_i = \sup_{x_j \in X^{QGNDD}} QGDD_j \right\}.$$

This is the selection set of alternatives.

5. Example

Consider the following illustrative example of the classification method of alternatives studied in this paper. Suppose that we have a set of six experts, $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$, and a set of four alternatives, $X=\{x_1,x_2,x_3,x_4\}$. Suppose that experts e_1, e_2 supply their opinions in terms of preference orderings, experts e_3, e_4 in terms of utility values, and experts e_5, e_6 in terms of fuzzy preference relations. Suppose the information is the following:

e_1: $O^1=\{3,1,4,2\}$,

e_2: $O^2=\{3,2,1,4\}$,

e_3: $U^3=\{0.5,0.7,1,0.1\}$,

e_4: $U^4=\{0.7,0.9,0.6,0.3\}$,

$$e_5:\ P^5=\begin{bmatrix} - & 0.1 & 0.6 & 0.7 \\ 0.9 & - & 0.8 & 0.4 \\ 0.4 & 0.2 & - & 0.9 \\ 0.3 & 0.6 & 0.1 & - \end{bmatrix},$$

$$e_6:\ P^6=\begin{bmatrix} - & 0.5 & 0.7 & 1 \\ 0.5 & - & 0.8 & 0.6 \\ 0.3 & 0.2 & - & 0.8 \\ 0 & 0.4 & 0.2 & - \end{bmatrix}.$$

Using transformation functions l^1 and g^4 to make the information uniform, we have

$$P^1=\begin{bmatrix} - & \frac{1}{6} & \frac{2}{3} & \frac{1}{2} \\ \frac{5}{6} & - & 1 & \frac{2}{3} \\ \frac{1}{3} & 0 & - & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \frac{5}{6} & - \end{bmatrix},\quad P^2=\begin{bmatrix} - & \frac{1}{3} & \frac{1}{6} & \frac{2}{3} \\ \frac{2}{3} & - & \frac{1}{3} & \frac{1}{6} \\ \frac{5}{6} & \frac{2}{3} & - & 1 \\ \frac{1}{3} & \frac{5}{6} & 0 & - \end{bmatrix},$$

$$P^3=\begin{bmatrix} - & \frac{25}{74} & 0.2 & \frac{25}{26} \\ \frac{49}{74} & - & \frac{49}{149} & 0.98 \\ 0.8 & \frac{100}{149} & - & \frac{100}{101} \\ \frac{1}{26} & 0.02 & \frac{1}{101} & - \end{bmatrix},$$

$$P^4=\begin{bmatrix} - & \frac{49}{130} & \frac{49}{85} & \frac{49}{58} \\ \frac{81}{130} & - & \frac{81}{117} & 0.9 \\ \frac{36}{85} & \frac{36}{117} & - & 0.8 \\ \frac{9}{58} & 0.1 & 0.2 & - \end{bmatrix}.$$

Using the fuzzy majority criterion with the fuzzy quantifier "*at least half*", with the pair $(0,0.5)$, and the corresponding OWA operator with the weighting vector, $W=[\frac{1}{3},\frac{1}{3},\frac{1}{3},0,0,0]$, the collective fuzzy preference relation is

$$P^c=\begin{bmatrix} - & 0.40492 & 0.65556 & 0.93546 \\ 0.8 & - & 0.86667 & 0.84889 \\ 0.68562 & 0.5485 & - & 0.96337 \\ 0.37778 & 0.61111 & 0.41111 & - \end{bmatrix}.$$

We apply the exploitation process with the fuzzy quantifier "*most*" with the pair $(0.3,0.8)$, i.e., the corresponding OWA operator with the weighting vector $W=[\frac{1}{15},\frac{3}{15},\frac{4}{15}]$. The quantifier guided choice degrees of alternatives acting over the collective fuzzy preference relation supply the following values:

	x_1	x_2	x_3	x_4
$QGDD_i$	0.60738	0.83703	0.66757	0.41556
$QGNDD_i$	0.87461	1	0.91515	0.46726

These values represent the dominance that one alternative has over the "most" alternatives according to "at least half" of the experts, and the non-dominance degree to which the alternative is not dominated by "most" alternatives according to "at least half" of the experts, respectively.

Clearly, the maximal sets are

$$X^{QGDD} = \{x_2\} \quad \text{and} \quad X^{QGNDD} = \{x_2\},$$

therefore, the selection set of alternatives for all selection procedures is the singleton $\{x_2\}$.

If we use the linguistic quantifier "*as many as possible*" instead of "*at least half*", with the pair $(0.5, 1)$, and the corresponding OWA operator with the weighting vector $W = [0, 0, 0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$, then the collective relation is

$$P^c = \begin{bmatrix} - & 0.2 & 0.31438 & 0.62222 \\ 0.59508 & - & 0.4515 & 0.38889 \\ 0.34444 & 0.13333 & - & 0.58889 \\ 0.06454 & 0.15111 & 0.03663 & - \end{bmatrix}.$$

Applying the selection process with fuzzy quantifier "most", then, in this case, the quantifier guided choice degrees of alternatives acting over the collective fuzzy preference relation supply the following values:

	x_1	x_2	x_3	x_4
$QGDD_i$	0.3044	0.44437	0.30444	0.06288
$QGNDD_i$	0.87461	1	0.91515	0.46726

The maximal sets are the above same ones, and therefore the selection set of alternatives for all selection procedures is the singleton $\{x_2\}$. The solution to our example is the same using different linguistic quantifiers in the aggregation of the individual fuzzy preference relations.

6. Conclusions

In this paper we have presented a general model for a MPDM problem, where the information supplied by the group of experts can be of a diverse nature, based on the concept of fuzzy majority for the aggregation and exploitation of the information in decision making.

It was necessary to make the information uniform, for which we used fuzzy preference relations according to their apparent merits. We have presented a general method to relate preference orderings, utility values and fuzzy preference relations. As we have seen, this general method generalises the procedures normally used, in particular those suggested in [4, 5, 20, 21].

We have used two quantifier-guided choice degrees of alternatives; a quantifier guided dominance degree used to quantify the dominance that one alternative has over all the others in a fuzzy majority sense, and a quantifier guided non-dominance degree that generalises Orlovski's non-dominated alternative concept. We have shown that the above choice degrees can be carried out according to two proposed selection policies.

Appendix A. Fuzzy majority

As we said before, the majority is traditionally defined as a threshold number of individuals. Fuzzy majority is a soft majority concept expressed by a fuzzy quantifier, which is manipulated via a fuzzy-logic-based calculus of linguistically quantified propositions.

In this appendix we present the fuzzy quantifiers, used for representing the fuzzy majority, and the OWA operators, used for aggregating information. The OWA operator reflects the fuzzy majority calculating its weights by means of the fuzzy quantifiers.

Quantifiers can be used to represent the amount of items satisfying a given predicate. Classic logic is restricted to the use of the two quantifiers, *there exists* and *for all*, that are closely related respectively to the *or* and *and* connectives. Human discourse is much richer and more diverse in its quantifiers, e.g. *about* 5, *almost all*, *a few*, *many*, *most*, *as many as possible*, *nearly half*, *at least half*. In an attempt to bridge the gap between formal systems and natural discourse and, in turn, to provide a more flexible knowledge representation tool, Zadeh introduced the concept of fuzzy quantifiers [25].

Zadeh suggested that the semantic of a fuzzy quantifier can be captured by using fuzzy subsets for its representation. He distinguished between two types of fuzzy quantifiers, *absolute* and *proportional or relative*. Absolute quantifiers are used to represent amounts that are absolute in nature such as *about* 2 or *more than* 5. These absolute linguistic quantifiers are closely related to the concept of

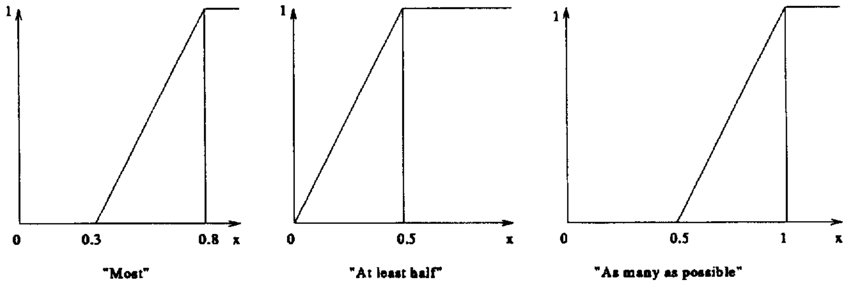

Fig. 2. Proportional fuzzy quantifiers.

the count or number of elements. He defined these quantifiers as fuzzy subsets of the non-negative real numbers, $\mathscr{R}^+$. In this approach, an absolute quantifier can be represented by a fuzzy subset Q, such that for any $r \in \mathscr{R}^+$ the membership degree of r in Q, $Q(r)$, indicates the degree to which the amount r is compatible with the quantifier represented by Q. Proportional quantifiers, such as *most*, *at least half*, can be represented by fuzzy subsets of the unit interval, $[0,1]$. For any $r \in [0,1]$, $Q(r)$ indicates the degree to which the proportion r is compatible with the meaning of the quantifier it represents. Any quantifier of natural language can be represented as a proportional quantifier or given the cardinality of the elements considered, as an absolute quantifier. Functionally, fuzzy quantifiers are usually of one of three types, *increasing*, *decreasing*, and *unimodal*. An increasing type quantifier is characterised by the relationship

$$Q(r_1) \geqslant Q(r_2) \quad \text{if } r_1 > r_2.$$

These quantifiers are characterised by values such as *most*, *at least half*. A decreasing type quantifier is characterised by the relationship

$$Q(r_1) \leqslant Q(r_2) \quad \text{if } r_1 < r_2.$$

An absolute quantifier $Q\colon \mathscr{R}^+ \to [0,1]$ satisfies

$$Q(0)=0 \text{ and } \exists k \text{ such that } Q(k)=1.$$

A relative quantifier $Q\colon [0,1] \to [0,1]$ satisfies

$$Q(0)=0 \text{ and } \exists r \in [0,1] \text{ such that } Q(r)=1.$$

A non-decreasing quantifier satisfies

$$\forall a,b \text{ if } a > b \text{ then } Q(a) \geqslant Q(b).$$

The membership function of a non-decreasing relative quantifier can be represented as

$$Q(r)=\begin{cases} 0 & \text{if } r < a, \\ \dfrac{r-a}{b-a} & \text{if } a \leqslant r \leqslant b, \\ 1 & \text{if } r > b \end{cases}$$

with $a,b,r \in [0,1]$.

Some examples of proportional quantifiers are shown in Fig. 2, where the parameters, (a,b) are $(0.3,0.8)$, $(0,0.5)$ and $(0.5,1)$, respectively.

Appendix B. The ordered weighted averaging operator

The OWA operator was proposed by Yager in [23] and more recently characterised in [24], and provide a family of aggregation operators which have the "and" operator at one extreme and the "or" operator at the other extreme.

An *OWA* operator of dimension n is a function ϕ,

$$\phi\colon [0,1]^n \to [0,1],$$

that has associated with a set of weights. Let $\{a_1,\dots,a_m\}$ be a list of values to aggregate, then the *OWA* operator ϕ is defined as

$$\phi(a_1,\dots,a_m)=W \cdot B^{\mathrm{T}}=\sum_{i=1}^{m} w_i \cdot b_i$$

where $W=[w_1,\dots,w_m]$ is a weighting vector, such that, $w_i \in [0,1]$ and $\sum_i w_i = 1$, and B is the associated ordered value vector. Each element $b_i \in B$ is the ith largest value in the collection $a_1,\dots,a_m$.

The *OWA* operators fill the gap between the operators *Min* and *Max*. It can be inmediately verified that *OWA* operators are commutative, increasing monotonous and idempotent, but in general not associative.

A natural question in the definition of the *OWA* operator is how to obtain the associated weighting vector. In [23, 24], Yager proposed two ways to obtain it. The first approach is to use some kind of learning mechanism using some sample data; and the second approach is to try to give some semantics or meaning to the weights. The final possibility has allowed multiple applications on areas of fuzzy and multi-valued logics, evidence theory, design of fuzzy controllers, and the quantifier guided aggregations.

We are interested in the area of quantifier guided aggregations. Our idea is to calculate weights for the aggregation operations (made by means of the *OWA* operator) using linguistic quantifiers that represent the concept of *fuzzy majority*. In [23, 24], Yager suggested an interesting way to compute the weights of the *OWA* aggregation operator using fuzzy quantifiers, which, in the case of a non-decreasing proportional quantifier Q, it is given by the expression:

$$w_i = Q(i/n) - Q((i-1)/n), \quad i=1,\dots,n.$$

When a fuzzy quantifier Q is used to compute the weights of the *OWA* operator ϕ, it is symbolized by ϕ_Q.

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