An Adaptive Consensus Support Model for Group Decision-Making Problems in a Multigranular Fuzzy Linguistic Context

Francisco Mata, Luis Martínez, and Enrique Herrera-Viedma

***Abstract*—Different consensus models for group decision-making (GDM) problems have been proposed in the literature. However, all of them consider the consensus reaching process a rigid or inflexible one because its behavior remains fixed in all rounds of the consensus process. The aim of this paper is to improve the consensus reaching process in GDM problems defined in multigranular linguistic contexts, i.e., by using linguistic term sets with different cardinality to represent experts' preferences. To do that, we propose an adaptive consensus support system model for this type of decision-making problem, i.e., a process that adapts its behavior to the agreement achieved in each round. This adaptive model increases the convergence toward the consensus and, therefore, reduces the number of rounds to reach it.**

***Index Terms*—Consensus, fuzzy preference relation, group decision making (GDM), linguistic modeling.**

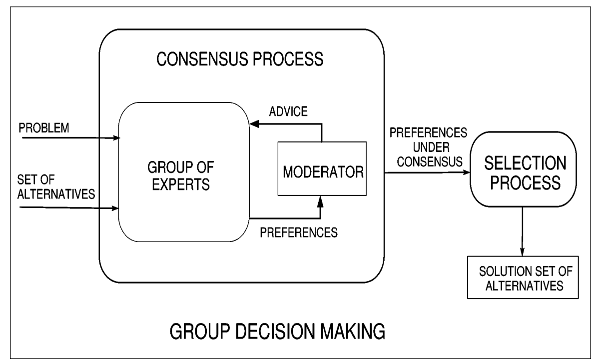

Fig. 1. Resolution process of a GDM problem.

I. Introduction

One of the reasons why decision-making processes have been widely studied in the literature is the increasing complexity of the social–economic environment [12], [19]. Many organizations have moved from a single decision maker to a group of experts to accomplish this task successfully. A group decision-making (GDM) problem may be defined as a decision problem with several alternatives and experts that try to achieve a common solution taking into account their opinions or preferences.

Our interest is focused on GDM problems in which the experts have to express their preferences on qualitative aspects that cannot be assessed by means of quantitative values. In these cases, the use of linguistic terms instead of precise numerical values seems to be more appropriate. For example, to evaluate the "comfort" of a car, linguistic terms like "good," "fair," or "poor" could be preferred by the experts instead of numerical values [48].

The use of the fuzzy linguistic approach [60]–[62] to assess qualitative aspects by using linguistic variables, i.e., variables whose values are not numbers but words or sentences in a natural or an artificial language, has proven successful in decision-making problems [1], [3], [5], [20], [27], [38], [42], [49], [58], [59].

In GDM problems, there are cases where experts have different levels of knowledge about the alternatives, and as a consequence, they might use linguistic term sets with different cardinality to express their preferences. In such cases, we say that the GDM problem is defined in a multigranular fuzzy linguistic context [11], [13], [21], [28], [32], [35], [50], [52], [57].

Usually, GDM problems are solved by carrying out *selection processes* to obtain a solution set of alternatives from the preferences given by the experts [19], [23], [25], [53]. However, it may happen that, after the selection process, some experts consider that their preferences have not been taken into account properly to obtain the solution, and hence, they might reject it. One way to avoid this situation would be the application of a *consensus process* (see Fig. 1) so that the experts discuss and modify their preferences in order to reach a sufficient agreement, before applying the selection process [5], [7], [9], [26], [31], [33], [41]. Selection processes for GDM problems defined in multigranular linguistic contexts were introduced in [21] and [28]; hence, here we focus on the consensus process.

Consensus modeling is an important area of research in decision analysis [5], [7]–[9], [14], [16]–[18], [26], [31], [33], [35], [37], [39], [40], [46], [47], [54], [55]. Consensus is defined as a state of mutual agreement among members of a group where all opinions have been heard and addressed to the satisfaction of the group [54]. A consensus reaching process is a dynamic and iterative process composed by several rounds where the experts express, discuss, and modify their preferences. Normally, this process is guided by the figure of a moderator, who helps the experts to make their preferences closer to each other [40], [54].

Manuscript received April 27, 2007; revised November 14, 2007, April 20, 2008, and December 2, 2008; accepted December 15, 2008. First published January 27, 2009; Current version published April 1, 2009. This work was supported in part by the Research Projects TIN2006-02121 and TIN2007-61079.

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Digital Object Identifier 10.1109/TFUZZ.2009.2013457

In each consensus round, the moderator evaluates the current agreement among the experts' preferences. If the agreement is not acceptable, i.e., it is lower than a specified threshold, the moderator would then recommend to modify the furthest preferences from the collective ones in order to make them closer. Otherwise, when the agreement is acceptable, the moderator would apply the selection process in order to obtain the final solution for the GDM problem.

The main interest in consensus research has been the development of new processes with different structures and methodologies to achieve its aim [7], [10], [33], [35], [44]. However, the enhancement of these processes has not been the focus in this research field yet. For instance, it is easy to check that if the agreement is "very low" (initial rounds), then the number of changes of preferences should be greater than when the agreement is "high" (final rounds). Thus, adapting the consensus reaching process to the level of agreement achieved in each discussion round could significantly improve its performance.

The aim of this paper is to propose an adaptive consensus support system (ACSS) model to support consensus processes in GDM problems with multigranular linguistic information, which improves the consensus reaching process by adapting the search for preferences in disagreement to the current level of consensus at each round. To do so, three different methods to identify the preferences that each expert should modify, in order to increase the agreement in the next consensus round, are defined. The result is a model that improves the convergence rate toward the consensus, and therefore decreases the number of rounds to achieve it.

The rest of the paper is set out as follows. In Section II, preliminaries about the multigranular fuzzy linguistic GDM problems and the consensus process are presented. The proposed ACSS model is described in detail in Section III. In Section IV, the application of the proposed ACSS model is given, while in Section V, we draw our conclusions. Finally, the Appendix introduces the meaning and features of the measurements used to evaluate the agreement.

II. Preliminaries

In order for this paper to be as self-contained as possible, we include in this section a brief review of the fuzzy linguistic approach, focusing on GDM problems defined in multigranular fuzzy linguistic contexts, and the main elements and features of the consensus processes.

A. Multigranular Fuzzy Linguistic GDM Problems

The fuzzy linguistic approach assesses qualitative attributes by using linguistic assessments by means of linguistic variables [60]–[62]. This approach has been successfully applied to different problems [2], [3], [6], [20], [22], [29], [30], [34], [36], [43], [56], [63].

In this approach, assessments of the preferences on pairs of alternatives are provided in the form of linguistic terms or labels of a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}, \#(S) = g + 1$. An important issue to analyze is the "granularity of uncertainty," i.e., the cardinality of the linguistic term set. The granularity of S should be small enough so as not to impose useless precision levels on the users but large enough to allow a discrimination of the assessments in a limited number of degrees. Additionally, the following properties are assumed.

1) The set S is ordered: $s_i \geq s_j$, if $i \geq$ j.
2) There is the negation operator: $Neg(s_i) = s_j$ such that $j = g - i$.

The semantics of S can be given by fuzzy numbers defined on the unit interval [0,1]. One way to characterize a fuzzy number is by using a representation based on parameters of its membership function [4]. For example, the following semantics can be assigned to a set of seven terms via triangular fuzzy numbers:

$$\begin{aligned} N &= \textit{None} = (0, 0, 0.17) \\ VL &= \textit{Very\_Low} = (0, 0.17, 0.33) \\ L &= \textit{Low} = (0.17, 0.33, 0.5) \\ M &= \textit{Medium} = (0.33, 0.5, 0.67) \\ H &= \textit{High} = (0.5, 0.67, 0.83) \\ VH &= \textit{Very\_High} = (0.67, 0.83, 1) \\ P &= \textit{Perfect} = (0.83, 1, 1). \end{aligned}$$

A GDM problem is classically defined as a decision situation where a set of experts, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$, express their preferences about a set of feasible alternatives, $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$. In many decision situations, it is assumed that each expert e_i provides his or her preferences by means of a fuzzy preference relation [19] $\mathbf{P}_{\mathbf{e}_\mathbf{i}} = [p_i^{lk}], l, k \in \{1, \ldots, n\}$, with $p_i^{lk} = \mu_{\mathbf{P}_{\mathbf{e}_\mathbf{i}}}(x_l, x_k)$ assessed in the unit interval [0,1] and being interpreted as the preference degree of the alternative x_l over x_k according to the expert e_i. In this paper, we use linguistic preference relations to represent the experts' preferences as in [23] and [24], i.e., with $p_i^{lk} = \mu_{\mathbf{P}_{\mathbf{e}_\mathbf{i}}}(x_l, x_k)$ assessed in a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$.

The ideal situation for GDM problems defined in linguistic contexts would be that all the experts use the same linguistic term set S to express their preferences about the alternatives. However, in some cases, experts may belong, e.g., to distinct research areas, and therefore could have different background and levels of knowledge. A consequence of this is that they need to express their preferences by using linguistic term sets with different granularity $S_i = \{s_0^i, \ldots, s_{g_i}^i\}, i \in \{1, 2, \ldots, m\}$. In these cases, the GDM problem is defined in a multigranular fuzzy linguistic context [11], [13], [21], [28], [32], [35], [50], [52], [57].

B. Consensus Process

A consensus reaching process in a GDM problem is an iterative process composed by several discussion rounds, in which experts are expected to modify their preferences according to the advice given by the moderator (see Fig. 1). The moderator plays a key role in this process. Normally, the moderator is a person who does not participate in the discussion but knows the preferences of each expert and the level of agreement during the consensus process. He/she is in charge of supervising and

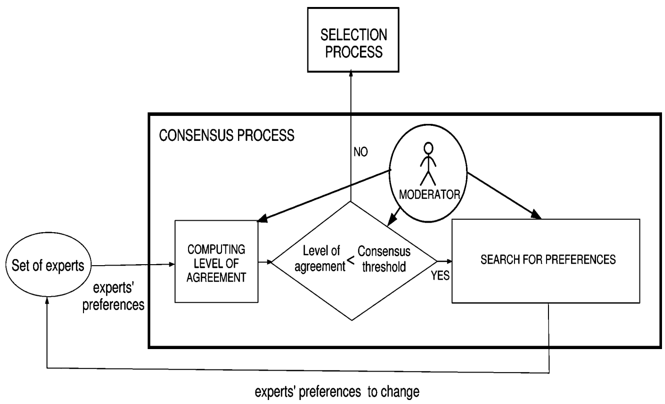

Fig. 2. Phases of the consensus process supervised by the moderator.

driving the consensus process toward success, i.e., to achieve the maximum possible agreement and reduce the number of experts outside of the consensus in each new consensus round.

An overall scheme of the different phases carried out in a consensus process guided by a moderator is shown in Fig. 2.

1) *Computing level of agreement:* The moderator computes the current agreement among all experts from their preferences.
2) *Checking level of agreement:* The moderator compares the current level of agreement with a consensus threshold fixed previously. If the consensus threshold is achieved, the selection process will be applied to obtain the final solution. Otherwise, the consensus process will continue its execution.
3) *Search for preferences:* The moderator searches for the experts' preferences furthest from the collective ones and suggests how to change them in order to improve the agreement in the next round.

In order to evaluate the agreement, it is required to compute similarity measures among the experts [7], [26], [27], [31], [35], [39]. Two types of measurements to guide the consensus reaching process were proposed in [26]:

a) *consensus degrees* to evaluate the level of agreement among all the experts. They will be used to identify the preference values where the agreement is not sufficient;
b) *proximity measures* to evaluate the distance between the experts' individual preferences and the group or collective ones. They will be used to identify the experts who should change their preferences in the next rounds.

These measurements are computed at the three different levels of representation of information of a preference relation: pairs of alternatives, alternatives, and relation.

Level 1: Pairs of alternatives. Given a pair of alternatives (x_l, x_k):

1) cp^{lk} is the agreement among all experts on the pair of alternatives (x_l, x_k);
2) pp_i^{lk} is the proximity between the preference value of expert e_i, p_i^{lk}, and the collective one on the pair of alternatives (x_l, x_k).

Level 2: Alternatives. Given the alternative $x_l \in X$:

1) ca^l is the agreement among all experts on x_l;
2) pa_i^l is the proximity between the preference values of expert e_i and the collective ones on x_l.

Level 3: Preference relation.

1) cr is the global agreement among all experts on all the pairs of alternatives of a preference relation.
2) pr_i is the global proximity between the preferences given by e_i and the collective ones of a preference relation.

A further detailed description of the meaning as well as a description of the computation of these measurements can be found in the Appendix.

III. Adaptive Consensus Support System Model for GDM in a Multigranular Fuzzy Linguistic Context

Several authors [7], [31], [35], [44] have proposed different models to carry out consensus processes where the human moderator's role is assumed by the own model. In all of them, the consensus reaching process is considered as a rigid or inflexible one because its behavior remains fixed in all rounds of the consensus process. However, it is obvious that when the level of agreement between the experts is "high", a few number of changes of opinions from some of the experts might lead to consensus in a few discussion rounds. On the contrary, when the level of agreement among the experts is "low," a high number of changes of opinions and many group discussion rounds might be necessary for consensus to be achieved. In this second case, it seems reasonable that many experts' preferences should be changed if they try to achieve a common solution. As the level of agreement increases, less and less experts might need to change their opinions. In fact, in these cases, it might be expected that only those experts whose preference values are furthest from the group ones should change them. In other words, the number of changes in different stages of a consensus process is clearly related to the actual level of agreement. A consensus model that implements this idea will improve the GDM processes.

In this section, following the aforementioned idea, we present an ACSS model for multigranular fuzzy linguistic GDM problems that improves the convergence rate toward the consensus, and therefore decreases the number of rounds to achieve it. It consists of four phases (see Fig. 3).

1) *Making the linguistic information uniform:* In this phase, all experts' multigranular linguistic preferences are unified into a single linguistic domain.
2) *Computing the consensus degree and control of the consensus process:* The consensus degree among all experts is calculated. If the consensus degree is high enough, the selection process is then applied. Otherwise, the consensus process keeps going.
3) *Adaptive search for preferences:* Different policies or procedures for searching the preferences to be changed in each consensus round are applied based on a broad classification of the global consensus level as *very low*, *low*, and *medium*. Each preference search procedure (PSp) will return the set of preferences each expert should change in

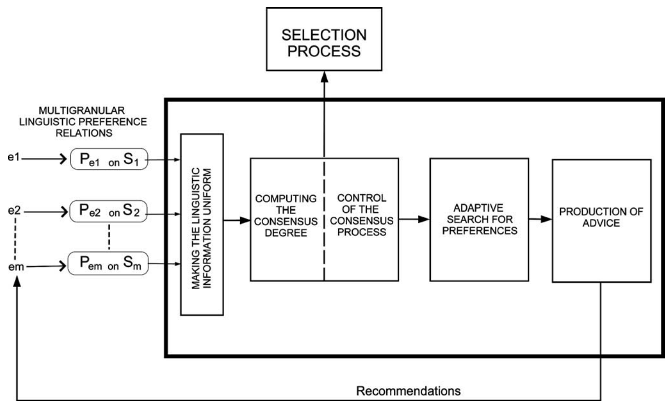

Fig. 3. ACSS model in a multigranular fuzzy linguistic context.

order to make his/her preferences closer to the collective opinion.

4) *Production of advice:* Once the sets of preferences in disagreement have been identified, an advice system suggests the direction of the changes to be recommended to the experts in order to improve the agreement in the next consensus round.

In the following sections, the aforementioned phases are described in detail.

A. Making the Linguistic Information Uniform

To manage multigranular fuzzy linguistic information, we need to make it uniform, i.e., experts' preferences have to be transformed (using a transformation function) into a single domain or linguistic term set that we call the *basic linguistic term set* (BLTS), denoted by S_T [21]. To do this, it seems reasonable to impose a cardinality high enough to maintain the uncertainty degrees associated with each one of the possible domains to be unified. This means that the cardinality of the BLTS has to be as high as possible. Therefore, in a general multigranular fuzzy linguistic context, to select S_T, we proceed as it was proposed in [21].

1) If there is only one linguistic term set, from the set of different domains to be unified, with maximum cardinality, then we choose that one as the BLTS, S_T.
2) If there are two or more linguistic term sets with maximum cardinality, then the selection of S_T will depend on the semantics associated with them.
   a) If all of them have the same semantics, i.e., the same fuzzy membership functions associated with the linguistic terms but with different syntax, then any one of them could be selected as S_T.
   b) If two or more of them have different semantics, then S_T is defined as a generic linguistic term set with a number of terms greater than the number of terms a person is able to discriminate, which is normally 7 or 9 [51].

Once S_T has been selected, the following multigranular transformation function is applied to transform every linguistic value into a fuzzy set defined on S_T.

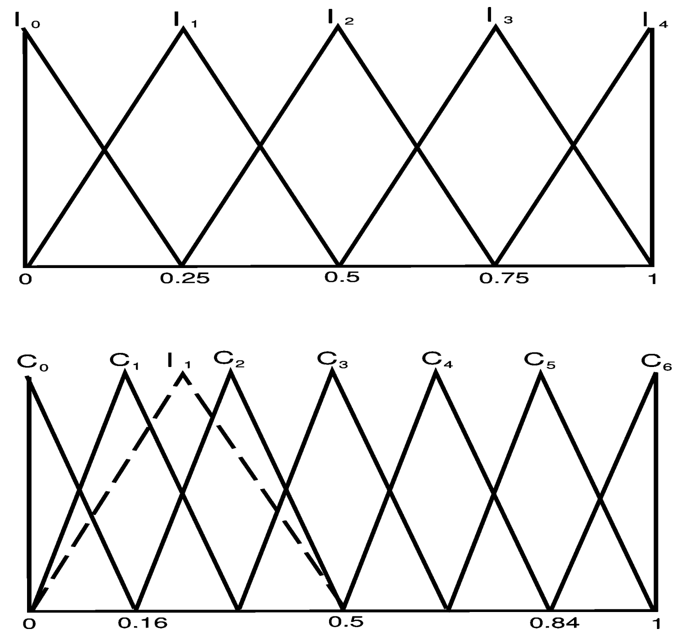

Fig. 4. Transforming $l_1 \in S$ into a fuzzy set on S_T.

Definition 1 [21]: If $S = \{l_0, \ldots, l_p\}$ and $S_T = \{c_0, \ldots, c_g\}$ are two linguistic term sets, with $g \geq p$, then a multigranular transformation function $\tau_{SS_T} : S \rightarrow F(S_T)$ is defined as follows:

$$\tau_{SS_T}(l_i) = \{(c_h, \alpha_h) \Big| \alpha_h = \max_y \min\{\mu_{l_i}(y), \mu_{c_h}(y)\}, \quad h = 0, \ldots, g\}$$

where $F(S_T)$ is the set of fuzzy sets defined on S_T, and $\mu_{l_i}(y)$ and $\mu_{c_h}(y)$ are the membership functions of the fuzzy sets associated with the linguistic terms l_i and c_h, respectively.

Example 1: Let $S = \{l_0, l_1, \ldots, l_4\}$ and $S_T = \{c_0, c_1, \ldots, c_6\}$ be two term sets with the following semantics:

$$\begin{array}{ll} l_0 = (0, 0, 0.25) & c_0 = (0, 0, 0.16) \\ l_1 = (0, 0.25, 0.5) & c_1 = (0, 0.16, 0.34) \\ l_2 = (0.25, 0.5, 0.75) & c_2 = (0.16, 0.34, 0.5) \\ l_3 = (0.5, 0.75, 1) & c_3 = (0.34, 0.5, 0.66) \\ l_4 = (0.75, 1, 1) & c_4 = (0.5, 0.66, 0.84) \\ & c_5 = (0.66, 0.84, 1) \\ & c_6 = (0.84, 1, 1). \end{array}$$

The fuzzy set obtained when applying τ_{SS_T} to l_1 is (see Fig. 4)

$$\tau_{SS_T}(l_1) = \{(c_0, 0.39), (c_1, 0.85), (c_2, 0.85), (c_3, 0.39), (c_4, 0), (c_5, 0), (c_6, 0)\}.$$

In order to unify all the experts' preferences, different *multigranular transformation functions* $\tau_{S_i S_T}$ are defined. Each linguistic preference value $p_i^{lk} \in S_i$ will be transformed in a fuzzy set $\tilde{p}_i^{lk} = \tau_{S_i S_T}(p_i^{lk}) = \{(c_h, \alpha_h^{lk}) | h = 0, \ldots, g\}$ on S_T. To simplify, we will use the membership degrees $(\alpha_0^{lk}, \ldots, \alpha_g^{lk})$

to denote each fuzzy set $\tilde{p}_i^{lk}$

$$\tilde{\mathbf{P}}_{\mathbf{e_i}} = \begin{pmatrix} \tilde{p}_i^{11} = (\alpha_0^{11}, \ldots, \alpha_g^{11}) & \cdots & \tilde{p}_i^{1n} = (\alpha_0^{1n}, \ldots, \alpha_g^{1n}) \\ \vdots & \ddots & \vdots \\ \tilde{p}_i^{n1} = (\alpha_0^{n1}, \ldots, \alpha_g^{n1}) & \cdots & \tilde{p}_i^{nn} = (\alpha_0^{nn}, \ldots, \alpha_g^{nn}) \end{pmatrix}.$$

B. Computing the Consensus Degree and Control of the Consensus Process

Once all the linguistic preferences have been unified by means of fuzzy sets on the BLTS, the following two steps are applied.

1) *Computing the consensus degree:* The level of agreement achieved in the current round is obtained. To do so, a global consensus degree, called consensus on relation $cr \in [0, 1]$, is computed (see the Appendix).
2) *Control of the consensus process:* In this phase, both the global consensus degree cr and the consensus threshold γ are compared such that:
   a) if $cr \geq \gamma$, the level of agreement is sufficient, the consensus process will stop, and the selection process will be applied;
   b) if $cr < \gamma$, a new consensus round is applied.

Note that γ is fixed in advance and represents the necessary level of agreement for a solution to be accepted by the group. A γ value too high may cause that the first condition will never be satisfied, and in consequence, we have a never-ending consensus process. In order to avoid this situation, we define a parameter *Max_rounds* that limits the maximum number of consensus rounds. This parameter has already been used by other authors in the control of consensus processes [7], [33].

The value of γ will obviously depend on the particular problem we are dealing with. When the consequences of the decision to be made are of utmost importance, the minimum level of consensus required to make that decision should be logically as high as possible, and it is not unusual if a minimum value of 0.8 or higher is imposed. At the other extreme, we have cases where the consequences are not so serious (but are still important), and it is urgent to obtain a solution to the problem, and thus, a minimum consensus value as close as possible to 0.5 could be required.

C. Adaptive Search for Preferences

If the agreement among all experts is low, then there exist a lot of experts' preferences in disagreement. In such a case, in order to bring the preferences closer to each other and so to improve the consensus situation, the number of changes in the experts' preferences should be high. However, if the agreement is high, the majority of preferences is close and only a low number of experts' preferences are in disagreement; it seems reasonable to change only these particular preferences. We distinguish three levels of consensus: *very low, low, and medium consensus*. Each level implies a different search policy to identify the preferences with low agreement degree. When the level of consensus is very low, all experts will be advised to modify all the preferences values identified in disagreement, while if the level of consensus is greater, the search will be limited to the preference values in disagreement of those experts furthest from the group. To do so, the system establishes three different PSps: "*PSp for very low consensus*," "*PSp for low consensus*," and "*PSp for medium consensus*." Each PSp will identify the preferences in disagreement in a different way. This fact defines the adaptive character of our model.

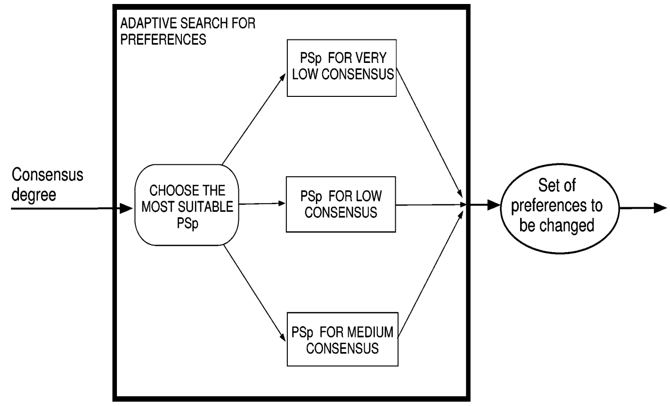

Fig. 5. Adaptive search for preferences.

The adaptive search for preferences consists of two processes (see Fig. 5).

1) *Choose the most suitable PSp:* Two parameters θ_1 and θ_2, whose values depend on the particular problem dealt with, are fixed at the beginning of the consensus process to differentiate the three consensus situations: very low, low, and medium consensus. Depending on both parameters, we choose the most appropriate PSp to apply to each particular consensus round: a) PSp for very low consensus if $cr \leq \theta_1$; b) PSp for low consensus if $cr \leq \theta_2$; and c) PSp for medium consensus otherwise.
2) *Apply the PSp:* Each PSp finds out a set of preferences, $PREFECH_i = \{(l, k), l, k \in \{1, 2, \ldots, n\}, l \neq k\}$, to be changed by each expert e_i in order to improve the agreement in the next round. In each PSp, the agreement is analyzed in a different preference representation level.
   a) In PSp for very low consensus, *the level of pairs of alternatives* is considered.
   b) In PSp for low consensus, *the level of alternative* is considered.
   c) In PSp for medium consensus, *the level of preference relation* is considered.

The three PSps are described in detail below:

1) PSp for Very Low Consensus (PSp^{VL}): Usually, at the beginning of the consensus process, experts' preferences are quite far from each other, and therefore, the agreement will be very low. In these situations, it seems reasonable to require many changes in order to make the preferences closer to one another. To do this, the procedure suggests modifying the preference values on all the pairs of alternatives where the agreement is not high enough. These changes may be carried out either by some experts, for example, the experts furthest from the group as proposed in [35], or by all experts. We consider the second option more appropriate because it prevents some experts imposing their preferences in the first rounds, and as a consequence, the

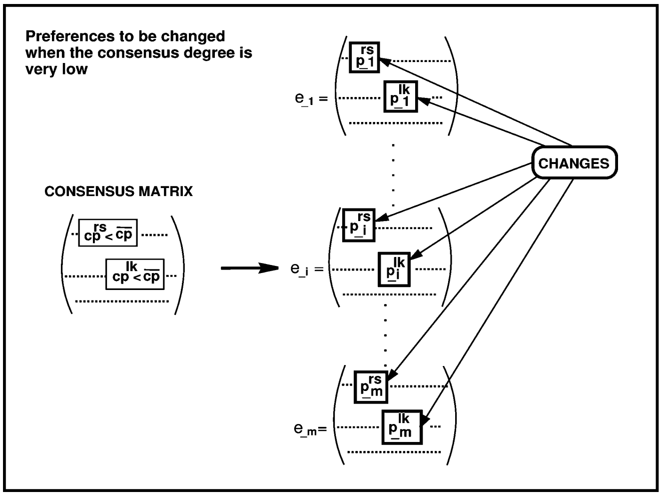

Fig. 6. Chosen preferences by PSp^{VL}.

consensus process could be guided toward their own opinions, which is known as "tyranny of the majority," and is a problem that should be avoided in consensus reaching processes [54]. Also, with the second option, all experts would be willing to share the final solution because their preferences were taken into account to obtain the solution.

PSp^{VL} finds out the set of preferences to be changed by $e_i, PREFECH_i^{VL}$, as follows.

1) First, the pairs of alternatives with a consensus degree smaller than a threshold ρ defined at level of pairs of alternatives, $P = \{(l,k)|cp^{lk} < \rho\}$, are identified. The value of ρ may be static and fixed before starting the consensus process or dynamic with respect to the level of consensus reached in each round. The selection of such a threshold plays a very important role in the identification process because a static value too high may imply many changes (leading to all experts having to change almost all their preference values), while a value too low may imply very few changes. We consider that a dynamic value that changes during the consensus process is better than static one fixed in advance. In this paper, we have assigned to ρ the average of the consensus degree at level of all pairs of alternatives, $\rho = \overline{cp}$, such that $\overline{cp} = \sum_{l=1}^{n}(\sum_{k=1,l\neq k}^{n} cp^{lk})/(n^2-n)$. Then, $P = \{(l,k)|cp^{lk} < \overline{cp}, l,k = 1,\ldots,n\}$.
2) The set of preference values $PREFECH_i^{VL}$ to be changed by each expert e_i will be

$$PREFECH_i^{VL} = P.$$

In Fig. 6, the characteristics and the behavior of this procedure are graphically described.

2) PSp for Low Consensus (PSp^L): After several discussion rounds, the agreement among all experts should be greater than at the beginning with $\theta_1 < cr \leq \theta_2$. In this situation, it seems logical to reduce the number of changes and modify the point of view for the analysis of the agreement. While in the PSp^{VL} we focused on all the pairs of alternatives in disagreement, in the PSp^L the agreement is analyzed from the point of view of the alternatives and only the preference values in disagreement of those alternatives where agreement is not sufficient will be considered.

Another important difference with respect to the PSp^{VL} is the number of experts involved in the change of preferences. While in the PSp^{VL}, all experts are required to modify the identified preference values, in the PSp^L, the following restriction is added: the experts required to modify the identified preference values will be those with proximity value at level of alternatives, for those identified alternatives in disagreement, smaller than an alternative proximity threshold β, i.e., $\{e_i|pa_i^l < \beta, \beta \in [0,1], i \in \{1,\ldots,m\}\}$. As in the previous case, the value of β may be static or dynamic. Again, we consider the second option more appropriate because it means that the restriction adapts to the proximity values obtained in each consensus round. A possible dynamic value in this case could be the arithmetic mean of all proximity on alternatives $\beta = \overline{pa}^l = \sum_{i=1}^{m} pa_i^l/m$.

PSp^L finds out the set of preferences to be changed by each $e_i, PREFECH_i^L$, as follows.

1) The consensus degrees at level of alternatives are obtained (see the Appendix): $\{ca^l | l = 1,\ldots,n\}$.
2) Alternatives to be changed X^{ch} are identified. A dynamic consensus threshold at level of alternatives is proposed in this case, such as the average of the consensus degrees at level of alternative $\overline{ca} = \sum_{l=1}^{n} ca^l/n$, and then, $X^{ch} = \{l|ca^l < \overline{ca}\}$.
3) Pairs of alternatives to be changed are identified: $P = \{(l,k)|l \in X^{ch} \wedge cp^{lk} < \overline{cp}\}$.
4) The proximity of the alternatives that should be changed is computed for all experts (see the Appendix): $\{pa_i^l | l \in X^{ch}\} \forall e_i \in E$.
5) The proximity threshold $\beta = \overline{pa}^l$ used to identify the experts that will be required to modify the identified pairs of alternatives is computed.
6) Then, the sets of preference values that are required to be modified are

$$PREFECH_i^L = \{(l,k) \in P \mid pa_i^l < \overline{pa}^l\}.$$

Clearly, the new restriction reduces the number of preferences and experts required to make changes. Consequently, we have

$$\#\left(\bigcup_i PREFECH_i^L\right) \leq \#\left(\bigcup_i PREFECH_i^{VL}\right).$$

Graphically, the behavior of this procedure is shown in Fig. 7. By comparing this figure with the previous one, we can check that, indeed, the number of changes required is reduced.

3) PSp for Medium Consensus (PSp^M): In the last consensus rounds, the agreement will be close to the desired consensus threshold, $\theta_2 < cr < \gamma$. Therefore, the agreement can be improved by suggesting fewer changes than in the previous two cases. Consequently, a new restriction is added to the PSp^M, which will reduce the number of experts required to modify their opinions: only those experts who have proximity values on the pairs of alternatives identified in disagreement smaller than a specific proximity threshold at level of pairs of

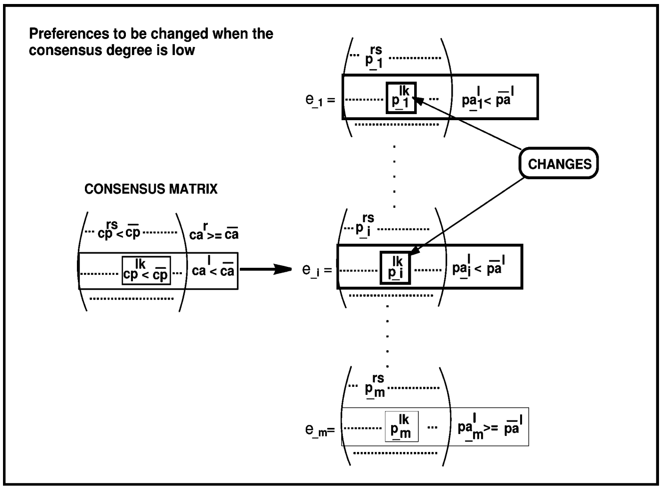

Fig. 7. Chosen preferences by PSp^L.

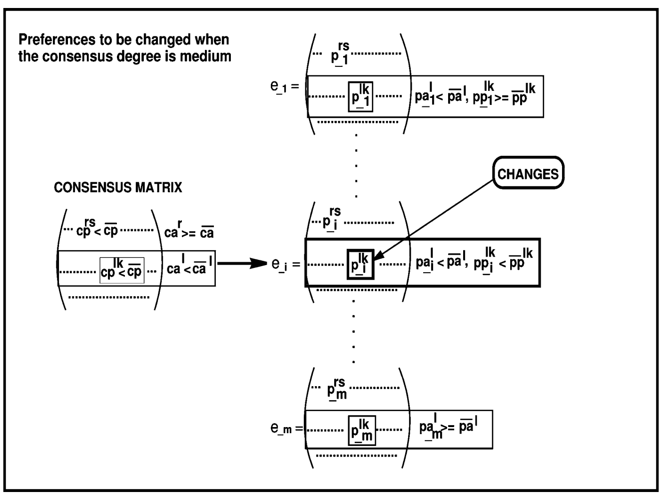

Fig. 8. Chosen preferences by PSp^M.

alternatives will have to change their opinions. This is illustrated in Fig. 8.

The computation of the set of preferences to be changed by e_i, $PREFECH_i^M$ in PSp^M, is as follows.

1) Operations enumerated from 1 to 5 in PSp^L are carried out.
2) The proximity threshold to be used in identifying the experts required to modify the identified pairs of alternatives in disagreement is computed: $\{\overline{pp}^{lk} = \sum_{i=1}^{m} pp_i^{lk}/m \mid (l,k) \in P\}$.
3) The sets of preference values that are required to be modified are

$$PREFECH_i^M = \{(l,k) \in P \mid pa_i^l < \overline{pa}^l \wedge pp_i^{lk} < \overline{pp}^{lk}\}.$$

Clearly, we have $\#(\bigcup_i PREFECH_i^M) \leq \#(\bigcup_i PREFECH_i^L)$.

Therefore, this adaptive search of preferences in disagreement reduces the number of changes as the consensus increases.

The main features of the PSps are shown in the Table I.

TABLE I
SUMMARY TABLE OF PSPS

	Level of agreement	Focus of attention	Experts involved
PSp^{VL}	Very Low	Pairs	All experts
PSp^L	Low	Alternatives	Furthest experts in alternatives in disagreement
PSp^M	Medium	Experts	Furthest experts in pairs and alternatives in disagreement

D. Production of Advice

Once the preferences to be changed have been identified, the model shows the right direction of the changes in order to improve the agreement. For each preference value to be changed, the model will suggest increasing or decreasing the current assessment.

A *guidance advice system* based on several direction parameters was proposed in [35] to increase the agreement. However, this system presented some difficulties. In this paper, we present a new mechanism based on a set of direction rules to identify and suggest the changes. These rules compare the central values of the fuzzy sets on S_T of the individual and collective preference assessments $cv(\tilde{p}_i^{lk})$ and $cv(\tilde{p}_c^{lk})$. The central value represents the center of gravity of the information contained in the fuzzy set (see the Appendix). The new direction rules, DR, in our case are as follows.

DR.1: If $(cv(\tilde{p}_i^{lk}) - cv(\tilde{p}_c^{lk})) < 0$, then the expert e_i should increase the assessment associated with the pair of alternatives (x_l, x_k).

DR.2: If $(cv(\tilde{p}_i^{lk}) - cv(\tilde{p}_c^{lk})) > 0$, then the expert e_i should decrease the assessment associated with the pair of alternatives (x_l, x_k).

DR.3: If $(cv(\tilde{p}_i^{lk}) - cv(\tilde{p}_c^{lk})) = 0$, then the expert e_i should not modify the assessment associated with the pair of alternatives (x_l, x_k).

IV. APPLICATION OF THE ACSS MODEL

In this section, we apply the ACSS model presented in Section III to a GDM problem with multigranular fuzzy linguistic information.

A. GDM Framework

Let us suppose that a supermarket wants to buy 10 000 bottles of Spanish wine from among four possible brands of wine or alternatives: $\{x_1 = \textit{Marques de Caceres}, x_2 = \textit{Los Molinos}, x_3 = \textit{Somontano}, x_4 = \textit{Rene Barbier}\}$.

The manager decided to inquire eight experts about their opinions $E = \{e_1, \ldots, e_8\}$. The experts have to reach a high level of agreement before choosing the best brand of wine. Due to the fact that the experts involved in the problem have different levels of knowledge about wine, three linguistic term sets with different cardinalities may be used to provide their preferences.

1) Experts e_3 and e_7 use label set A

$$a_0 = (0, 0, 0.13) \qquad a_1 = (0, 0.13, 0.25)$$
$$a_2 = (0.13, 0.25, 0.38) \qquad a_3 = (0.25, 0.38, 0.5)$$
$$a_4 = (0.38, 0.5, 0.63) \qquad a_5 = (0.5, 0.63, 0.75)$$
$$a_6 = (0.63, 0.75, 0.88) \qquad a_7 = (0.75, 0.88, 1)$$
$$a_8 = (0.88, 1, 1).$$

2) Experts e_4, e_5, and e_8 use label set B

$$b_0 = (0, 0, 0.17) \qquad b_1 = (0, 0.17, 0.33)$$
$$b_2 = (0.17, 0.33, 0.5) \qquad b_3 = (0.33, 0.5, 0.67)$$
$$b_4 = (0.5, 0.67, 0.83) \qquad b_5 = (0.67, 0.83, 1)$$
$$b_6 = (0.83, 1, 1).$$

3) Experts e_1, e_2, and e_6 use label set C

$$c_0 = (0, 0, 0.25) \qquad c_1 = (0, 0.25, 0.5)$$
$$c_2 = (0.25, 0.5, 0.75) \qquad c_3 = (0.5, 0.75, 1)$$
$$c_4 = (0.75, 1, 1).$$

Initially, the experts provide the following linguistic preference relations:

$$P_{e_1} = \begin{pmatrix} - & c_0 & c_0 & c_2 \\ c_4 & - & c_3 & c_4 \\ c_3 & c_0 & - & c_1 \\ c_2 & c_1 & c_3 & - \end{pmatrix} \qquad P_{e_2} = \begin{pmatrix} - & c_2 & c_0 & c_4 \\ c_1 & - & c_1 & c_1 \\ c_3 & c_3 & - & c_1 \\ c_0 & c_4 & c_3 & - \end{pmatrix}$$

$$P_{e_3} = \begin{pmatrix} - & a_1 & a_4 & a_3 \\ a_5 & - & a_8 & a_4 \\ a_4 & a_1 & - & a_2 \\ a_5 & a_5 & a_7 & - \end{pmatrix} \qquad P_{e_4} = \begin{pmatrix} - & b_0 & b_4 & b_5 \\ b_6 & - & b_1 & b_6 \\ b_3 & b_4 & - & b_2 \\ b_0 & b_1 & b_4 & - \end{pmatrix}$$

$$P_{e_5} = \begin{pmatrix} - & b_4 & b_1 & b_6 \\ b_2 & - & b_3 & b_2 \\ b_4 & b_3 & - & b_2 \\ b_0 & b_5 & b_3 & - \end{pmatrix} \qquad P_{e_6} = \begin{pmatrix} - & c_2 & c_3 & c_1 \\ c_2 & - & c_0 & c_1 \\ c_0 & c_4 & - & c_4 \\ c_4 & c_4 & c_0 & - \end{pmatrix}$$

$$P_{e_7} = \begin{pmatrix} - & a_0 & a_3 & a_7 \\ a_8 & - & a_0 & a_4 \\ a_4 & a_8 & - & a_5 \\ a_1 & a_4 & a_3 & - \end{pmatrix} \qquad P_{e_8} = \begin{pmatrix} - & b_6 & b_1 & b_3 \\ b_0 & - & b_0 & b_5 \\ b_6 & b_6 & - & b_5 \\ b_4 & b_1 & b_0 & - \end{pmatrix}.$$

The parameters applied to the ACSS model are

1) $\gamma = 0.75$;
2) $\theta_1 = 0.65$ and $\theta_2 = 0.72$;
3) *Max_rounds* $= 10$.

B. First Round

1) Making the Linguistic Information Uniform: According to conditions set out in Section III-A, $S_T = \mathbf{A}$. To unify the different linguistic term sets, three multigranular transformation functions $\{\tau_{AS_T}, \tau_{BS_T}, \tau_{CS_T}\}$ are used (see Tables II and III).

2) Computing the Consensus Degree and Control of the Consensus Process:

a) Computing consensus degree: The consensus degree obtained at the different levels is as follows (see the Appendix).

TABLE II

FUZZY SETS OBTAINED FOR τ_{AS_T} AND τ_{BS_T}

τ_{AS_T} :	τ_{BS_T} :
$a_0 \mapsto (1,0,0,0,0,0,0,0,0)$	$b_0 \mapsto (1,0.57,0.14,0,0,0,0,0,0)$
$a_1 \mapsto (0,1,0,0,0,0,0,0,0)$	$b_1 \mapsto (0.43,0.86,0.71,0.29,0,0,0,0,0)$
$a_2 \mapsto (0,0,1,0,0,0,0,0,0)$	$b_2 \mapsto (0,0.29,0.71,0.86,0.43,0,0,0,0)$
$a_3 \mapsto (0,0,0,1,0,0,0,0,0)$	$b_3 \mapsto (0,0,0.14,0.57,1,0.57,0.14,0,0)$
$a_4 \mapsto (0,0,0,0,1,0,0,0,0)$	$b_4 \mapsto (0,0,0,0,0.43,0.86,0.71,0.29,0)$
$a_5 \mapsto (0,0,0,0,0,1,0,0,0)$	$b_5 \mapsto (0,0,0,0,0,0.29,0.71,0.86,0.43)$
$a_6 \mapsto (0,0,0,0,0,0,1,0,0)$	$b_6 \mapsto (0,0,0,0,0,0,0.14,0.57,1)$
$a_7 \mapsto (0,0,0,0,0,0,0,1,0)$	
$a_8 \mapsto (0,0,0,0,0,0,0,0,1)$	

TABLE III

FUZZY SETS OBTAINED FOR τ_{CS_T}

τ_{CS_T} :
$c_0 \mapsto (1,0.67,0.33,0,0,0,0,0,0)$
$c_1 \mapsto (0.33,0.67,1,0.67,0.33,0,0,0,0)$
$c_2 \mapsto (0,0,0.33,0.67,1,0.67,0.33,0,0)$
$c_3 \mapsto (0,0,0,0,0.33,0.67,1,0.67,0.33)$
$c_4 \mapsto (0,0,0,0,0,0,0.33,0.67,1)$

i) *Pairs of alternatives*

$$CM = \begin{pmatrix} - & 0.6 & 0.69 & 0.68 \\ 0.58 & - & 0.58 & 0.66 \\ 0.71 & 0.58 & - & 0.69 \\ 0.61 & 0.61 & 0.63 & - \end{pmatrix}.$$

ii) *Alternatives*

$$\left(ca^1 = 0.642, ca^2 = 0.6, ca^3 = 0.645, ca^4 = 0.646\right).$$

iii) *Relation*: $cr = 0.633$.

b) Control of the consensus process: Because $cr = 0.63 < \gamma = 0.75$, the adaptive search of preference values in disagreement is activated.

3) Adaptive Search for Preferences:

a) Choose the most suitable PSp: Given that $cr = 0.633 \leq \theta_1 = 0.65$, the level of agreement is very low, and therefore, PSp^{VL} is applied.

b) Apply the PSp^{VL}:

i) Identification of pairs of alternatives in disagreement

$$P = \{(1,2), (2,1), (2,3), (3,2), (4,1), (4,2), (4,3)\}.$$

ii) Set of preferences to be changed by all experts

$$PREFECH_i^{VL} = P, \qquad i = 1, \ldots, 8.$$

4) Production of Advice:

a) According to rule *DR1*, the experts are required to *increase* the following preference assessments:

$$p_1^{12} = c_0 \rightarrow c_1 \qquad p_3^{12} = a_4 \rightarrow a_5$$
$$p_5^{41} = b_0 \rightarrow b_1 \qquad p_7^{23} = a_0 \rightarrow a_1$$
$$p_1^{32} = c_0 \rightarrow c_1 \qquad p_3^{32} = a_1 \rightarrow a_2$$
$$p_5^{43} = b_3 \rightarrow b_4 \qquad p_7^{41} = a_1 \rightarrow a_2$$
$$p_1^{42} = c_1 \rightarrow c_2 \qquad p_4^{12} = b_0 \rightarrow b_1$$
$$p_6^{21} = c_2 \rightarrow c_3 \qquad p_7^{42} = a_4 \rightarrow a_5$$
$$p_2^{21} = c_1 \rightarrow c_2 \qquad p_4^{23} = b_1 \rightarrow b_2$$
$$p_6^{23} = c_0 \rightarrow c_1 \qquad p_7^{43} = a_3 \rightarrow a_4$$

$$\begin{array}{ll} p_2^{23} = c_1 \rightarrow c_2 & p_4^{41} = b_0 \rightarrow b_1 \\ p_6^{43} = c_0 \rightarrow c_1 & p_8^{21} = b_0 \rightarrow b_1 \\ p_2^{41} = c_0 \rightarrow c_1 & p_4^{42} = b_1 \rightarrow b_2 \\ p_7^{12} = a_0 \rightarrow a_1 & p_8^{23} = b_0 \rightarrow b_1 \\ p_5^{21} = b_2 \rightarrow b_3 & p_8^{42} = b_1 \rightarrow b_2 \\ p_5^{32} = b_3 \rightarrow b_4 & p_8^{43} = b_0 \rightarrow b_1. \end{array}$$

b) According to rule *DR2*, the experts are required to *decrease* the following preference assessments:

$$\begin{array}{ll} p_1^{21} = c_4 \rightarrow c_3 & p_2^{43} = c_3 \rightarrow c_2 \\ p_4^{32} = b_4 \rightarrow b_3 & p_6^{41} = c_4 \rightarrow c_3 \\ p_1^{23} = c_4 \rightarrow c_3 & p_3^{23} = a_8 \rightarrow a_7 \\ p_4^{43} = b_4 \rightarrow b_3 & p_6^{42} = c_4 \rightarrow c_3 \\ p_1^{41} = c_2 \rightarrow c_1 & p_3^{42} = a_5 \rightarrow a_4 \\ p_5^{12} = b_4 \rightarrow b_3 & p_7^{21} = a_8 \rightarrow a_7 \\ p_1^{43} = c_3 \rightarrow c_2 & p_3^{21} = a_5 \rightarrow a_4 \\ p_5^{23} = b_3 \rightarrow b_2 & p_7^{32} = a_8 \rightarrow a_7 \\ p_2^{12} = c_2 \rightarrow c_1 & p_3^{41} = a_5 \rightarrow a_4 \\ p_5^{42} = b_5 \rightarrow b_4 & p_8^{12} = b_6 \rightarrow b_5 \\ p_2^{32} = c_3 \rightarrow c_2 & p_3^{43} = a_7 \rightarrow a_6 \\ p_6^{12} = c_2 \rightarrow c_1 & p_8^{32} = b_6 \rightarrow b_5 \\ p_2^{42} = c_4 \rightarrow c_3 & p_4^{21} = b_6 \rightarrow b_5 \\ p_6^{32} = c_4 \rightarrow c_3 & p_8^{41} = b_4 \rightarrow b_3. \end{array}$$

C. Second Round

1) Gathering Information (New Preferences): According to the previous advices, the experts implemented all suggested changes, and the new provided preferences are

$$P_{e_1} = \begin{pmatrix} - & \mathbf{c_1} & c_0 & c_2 \\ \mathbf{c_3} & - & \mathbf{c_2} & c_4 \\ c_3 & \mathbf{c_1} & - & c_1 \\ \mathbf{c_1} & \mathbf{c_2} & \mathbf{c_2} & - \end{pmatrix} \quad P_{e_2} = \begin{pmatrix} - & \mathbf{c_1} & c_0 & c_4 \\ \mathbf{c_2} & - & \mathbf{c_2} & c_1 \\ c_2 & \mathbf{c_3} & - & c_1 \\ \mathbf{c_1} & \mathbf{c_3} & \mathbf{c_2} & - \end{pmatrix}$$

$$P_{e_3} = \begin{pmatrix} - & \mathbf{a_2} & a_4 & a_3 \\ \mathbf{a_4} & - & \mathbf{a_7} & a_4 \\ a_4 & \mathbf{a_2} & - & a_2 \\ \mathbf{a_4} & \mathbf{a_4} & \mathbf{a_6} & - \end{pmatrix} \quad P_{e_4} = \begin{pmatrix} - & \mathbf{b_1} & b_4 & b_5 \\ \mathbf{b_5} & - & \mathbf{b_2} & b_6 \\ b_3 & \mathbf{b_3} & - & b_2 \\ \mathbf{b_1} & \mathbf{b_2} & \mathbf{b_3} & - \end{pmatrix}$$

$$P_{e_5} = \begin{pmatrix} - & \mathbf{b_3} & b_1 & b_6 \\ \mathbf{b_3} & - & \mathbf{b_2} & b_2 \\ b_4 & \mathbf{b_4} & - & b_2 \\ \mathbf{b_1} & \mathbf{b_4} & \mathbf{b_4} & - \end{pmatrix} \quad P_{e_6} = \begin{pmatrix} - & \mathbf{c_1} & c_3 & c_1 \\ \mathbf{c_3} & - & \mathbf{c_1} & c_1 \\ c_0 & \mathbf{c_3} & - & c_4 \\ \mathbf{c_3} & \mathbf{c_3} & \mathbf{c_1} & - \end{pmatrix}$$

$$P_{e_7} = \begin{pmatrix} - & \mathbf{a_1} & a_3 & a_7 \\ \mathbf{a_7} & - & \mathbf{a_1} & a_4 \\ a_4 & \mathbf{a_7} & - & a_5 \\ \mathbf{a_2} & \mathbf{a_5} & \mathbf{a_4} & - \end{pmatrix} \quad P_{e_8} = \begin{pmatrix} - & \mathbf{b_5} & b_1 & b_3 \\ \mathbf{b_1} & - & \mathbf{b_1} & b_5 \\ b_6 & \mathbf{b_5} & - & b_5 \\ \mathbf{b_3} & \mathbf{b_2} & \mathbf{b_1} & - \end{pmatrix}.$$

TABLE IV
PROXIMITY AT LEVEL OF ALTERNATIVES

x_1	x_3
$pa_1^1 = 0.85$	$pa_1^3 = 0.78$
$pa_2^1 = 0.84$	$pa_2^3 = 0.86$
$pa_3^1 = 0.87$	$pa_3^3 = 0.74$
$pa_4^1 = 0.8$	$pa_4^3 = 0.88$
$pa_5^1 = 0.82$	$pa_5^3 = 0.88$
$pa_6^1 = 0.692$	$pa_6^3 = 0.7$
$pa_7^1 = 0.84$	$pa_7^3 = 0.87$
$pa_8^1 = 0.71$	$pa_8^3 = 0.73$

Note: In the remaining rounds, we shall only show the most relevant information, the evolution of the consensus degrees and the operation of PSps.

2) Computing the Consensus Degree and Control of the Consensus Process:

a) Consensus degree:

i) *Pairs of alternatives*

$$CM = \begin{pmatrix} - & 0.77 & 0.69 & 0.68 \\ 0.73 & - & 0.73 & 0.66 \\ 0.72 & 0.71 & - & 0.69 \\ 0.77 & 0.8 & 0.78 & - \end{pmatrix}.$$

ii) *Alternatives*

$$\left(ca^1 = 0.725, ca^2 = 0.73, ca^3 = 0.719, ca^4 = 0.728\right).$$

iii) *Relation*: $cr = 0.726$.

b) Control of the consensus process: Because $cr = 0.726 < \gamma = 0.75$, the adaptive search of preference values in disagreement is activated.

3) Adaptive Search for Preferences:

a) Choose the most suitable PSp: Given that $\theta_2 = 0.72 < cr = 0.726 < \gamma = 0.75$, the level of agreement is medium, and therefore, PSp^M is applied.

b) Apply the PSp^M:

i) Identifying the alternatives with consensus degree not high enough

$$X^{ch} = \{l | ca^l < 0.726\} = \{x_1, x_3\}.$$

ii) For each one of the aforementioned alternatives, the preference values in disagreement are identified

$$P = \{(l,k) | l \in X^{ch}, cp^{lk} < 0.726\}$$
$$= \{(1,3), (1,4), (3,1), (3,2), (3,4)\}.$$

iii) Computing the proximity values at level of alternatives for these elements in X^{ch} (Table IV).

iv) Computing the proximity thresholds used to identify the experts required to modify their preferences

$$\overline{pa}^1 = 0.8 \qquad \overline{pa}^3 = 0.81.$$

v) Computing the proximity thresholds used to select the experts required to modify the identified pairs of alternatives in disagreement

$$\begin{array}{lll} \overline{pp}^{12}=0.83 & \overline{pp}^{13}=0.78 & \overline{pp}^{14}=0.76 \\ \overline{pp}^{21}=0.81 & \overline{pp}^{23}=0.82 & \overline{pp}^{24}=0.76 \\ \overline{pp}^{31}=0.82 & \overline{pp}^{32}=0.79 & \overline{pp}^{34}=0.77 \\ \overline{pp}^{41}=0.83 & \overline{pp}^{42}=0.86 & \overline{pp}^{43}=0.85. \end{array}$$

vi) Sets of preferences to be changed

$$PREFECH_1^M = \{(3,1),(3,2)\}$$
$$PREFECH_6^M = \{(1,3),(1,4),(3,1),(3,4)\}$$
$$PREFECH_3^M = \{(3,2)\}$$
$$PREFECH_8^M = \{(3,1),(3,2),(3,4)\}.$$

4) Production of Advice:

a) According to rule *DR1*, the experts are required to *increase* the following preference assessments:

$$p_1^{32} = c_1 \rightarrow c_2 \quad p_3^{32} = a_2 \rightarrow a_3$$
$$p_6^{31} = c_0 \rightarrow c_1 \quad p_6^{14} = c_1 \rightarrow c_2.$$

b) According to rule *DR2*, the experts are required to *decrease* the following preference assessments:

$$p_1^{31} = c_3 \rightarrow c_2 \quad p_6^{13} = c_3 \rightarrow c_2$$
$$p_6^{34} = c_4 \rightarrow c_3 \quad p_8^{31} = b_6 \rightarrow b_5$$
$$p_8^{32} = b_5 \rightarrow b_4 \quad p_8^{34} = b_5 \rightarrow b_4.$$

D. Third Round

1) Gathering Information (New Preferences):

$$P_{e_1} = \begin{pmatrix} - & c_1 & c_0 & c_2 \\ c_3 & - & c_2 & c_4 \\ \mathbf{c_2} & \mathbf{c_2} & - & c_1 \\ c_1 & c_2 & c_2 & - \end{pmatrix} \quad P_{e_3} = \begin{pmatrix} - & a_2 & a_4 & a_3 \\ a_4 & - & a_7 & a_4 \\ a_4 & \mathbf{a_3} & - & a_2 \\ a_4 & a_4 & a_6 & - \end{pmatrix}$$

$$P_{e_6} = \begin{pmatrix} - & c_1 & \mathbf{c_2} & \mathbf{c_2} \\ c_3 & - & c_1 & c_1 \\ \mathbf{c_1} & c_3 & - & \mathbf{c_3} \\ c_3 & c_3 & c_1 & - \end{pmatrix} \quad P_{e_8} = \begin{pmatrix} - & b_5 & b_1 & b_3 \\ b_1 & - & b_1 & b_5 \\ \mathbf{b_5} & \mathbf{b_4} & - & \mathbf{b_4} \\ b_3 & b_2 & b_1 & - \end{pmatrix}.$$

2) Computing the Consensus Degree and Control of the Consensus Process:

a) Consensus degree:

i) *Pairs of alternatives*

$$CM = \begin{pmatrix} - & 0.77 & 0.8 & 0.77 \\ 0.73 & - & 0.73 & 0.73 \\ 0.82 & 0.8 & - & 0.76 \\ 0.77 & 0.8 & 0.78 & - \end{pmatrix}.$$

ii) *Alternatives*

$$\left(ca^1 = 0.759, ca^2 = 0.746, ca^3 = 0.771, ca^4 = 0.749\right).$$

iii) *Relation*: $cr = 0.756$.

b) Control of the consensus process: Because $cr = 0.756 > \gamma = 0.75$, the desired level of consensus is achieved, and the selection process is applied.

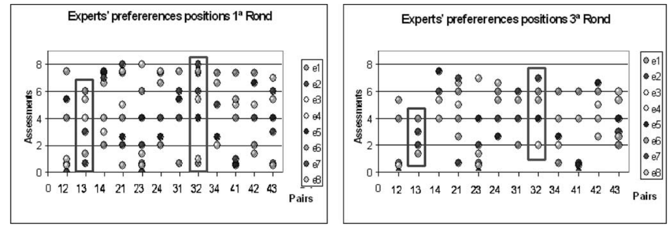

Fig. 9. Experts' preferences behavior during the consensus reaching process.

E. Graphic Description of the Experts' Preferences Behavior

Fig. 9 illustrates graphically the experts' preferences in the first and third rounds. Each point represents the preference value given by each expert (in a different color) on a pair of alternatives. In this figure, the movements of the experts' preferences on the pairs (1, 3) and (3, 2) are highlighted by means of a box. We note that the preferences come closer each other, forming a group (some experts' preferences cannot be seen because they are hidden by some other equal assessment), and therefore increases the level of agreement.

V. Conclusion

In this paper, we have analyzed the consensus processes in GDM problems under multigranular linguistic information, and have proposed an ACSS model to guide it and reduce the number of consensus rounds. To do so, different procedures to find out the experts' preference values furthest from the collective ones have been defined. These procedures are applied according to the level of agreement achieved in each consensus round. With this proposal, we overcome the convergence problems detected in other consensus approaches existing in the literature [3], [7], [26], [33], [35], [44].

Appendix

This appendix contains the measurements to evaluate the agreement, i.e., consensus degrees and proximity measures.

Experts might use linguistic term sets with different cardinality and semantics. To unify this information, each expert's linguistic preference p_i^{lk} is transformed in a fuzzy set $\tilde{p}_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$. Some drawbacks related to the use of traditional distance measurements were pointed out in [35], and an alternative similarity function $s(\cdot)$ was proposed to overcome them. The similarity function takes as its arguments the *central values* of the fuzzy sets to compare. Given a fuzzy set $\tilde{p}_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$, its central value defined as [15], [45], $cv(\tilde{p}_i^{lk}) = \sum_{h=0}^{g} h\alpha_{ih}^{lk} / \sum_{h=0}^{g} \alpha_{ih}^{lk}$ represents the center of gravity of the information contained in the fuzzy set.

The similarity between two preference values, $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk}) \in [0,1]$, is defined as $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk}) = 1 - \left|(cv(\tilde{p}_i^{lk}) - cv(\tilde{p}_j^{lk}))/g\right|$. The closer $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk})$ is to 1, the more similar $\tilde{p}_i^{lk}$ and $\tilde{p}_j^{lk}$ are, while the closer $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk})$ is to 0, the more distant $\tilde{p}_i^{lk}$ and $\tilde{p}_j^{lk}$ are.

Consensus degrees measure the agreement between experts' preferences. For each pair of experts i and j ($i < j$), a *similarity matrix* $SM_{ij} = \left(sm_{ij}^{lk}\right)$ is calculated with

$sm_{ij}^{lk} = s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk}), \quad l,k = 1,\ldots,n \ \wedge \ l \neq k$. The *consensus matrix*, $CM = (cm^{lk})$, is obtained by aggregating all the similarity matrices. This aggregation is carried out at level of pairs of alternatives: $cm^{lk} = \phi(sm_{ij}^{lk}),\ i,j = 1,\ldots,m,\ l,k = 1,\ldots,n \ \wedge \ i < j$. In this paper, we use the arithmetic mean as aggregation function ϕ, although different aggregation operators could be used according to the particular properties to implement.

Consensus degrees are obtained from the consensus matrix CM and in each one of the three different levels of the preference relation $\mathbf{P}_{\mathbf{e_i}}$.

Level 1 (Consensus on pairs of alternatives): The consensus degree on a pair of alternatives (x_l, x_k), called cp^{lk}, measures the agreement among all experts on that pair of alternatives: $cp^{lk} = cm^{lk} \forall l,k = 1,\ldots,n \wedge l \neq k$.

Level 2 (Consensus on alternatives): The consensus degree on an alternative x_l, called ca^l, measures the agreement among all experts on that alternative: $ca^l = (\sum_{k=1, l\neq k}^{n} (cp^{lk} + cp^{kl}))/2(n-1)$.

Level 3 (Consensus on the relation): The consensus degree on the relation, called cr, measures the global agreement among the experts' preferences: $cr = \sum_{l=1}^{n} ca^l/n$.

Proximity measurements evaluate the proximity between the individual experts' preferences and the collective ones. The collective preference $\tilde{\mathbf{P}}_{\mathbf{e_c}} = (\tilde{p}_c^{lk})$ is calculated by aggregating the set of (uniformed) individual preference relations $\{\tilde{\mathbf{P}}_{\mathbf{e_1}}, \ldots, \tilde{\mathbf{P}}_{\mathbf{e_m}}\}$: $\tilde{p}_c^{lk} = \psi(\tilde{p}_1^{lk}, \ldots, \tilde{p}_m^{lk})$, with ψ, an "aggregation operator." Then, for each expert e_i, we calculate a *proximity matrix* $PM_i = (pm_i^{lk}), pm_i^{lk} = s(\tilde{p}_i^{lk}, \tilde{p}_c^{lk})$, which measures the distance between $\tilde{P}_{e_i}$ and $\tilde{P}_{e_c}$.

The proximity measurements are computed in each one of the three levels of the relation, too.

Level 1 (Proximity on pairs of alternatives): Given an expert e_i, his/her proximity measure on a pair of alternatives (x_l, x_k), called pp_i^{lk}, measures the proximity between his/her preference and the collective one on that pair of alternatives: $pp_i^{lk} = pm_i^{lk} \forall l,k = 1,\ldots,n \wedge l \neq k$.

Level 2 (Proximity on alternatives): Given an expert e_i, his/her proximity measure on an alternative x_l, called pa^l, measures the proximity between his/her preference and the collective one on that alternative: $pa_i^l = (\sum_{k=1, k\neq l}^{n} (pp_i^{lk} + pp_i^{kl}))/2(n-1)$.

Level 3 (Proximity on the relation): Given an expert e_i, his/her proximity measure on the relation, called pr_i, measures the global proximity between his/her individual preferences and the collective one: $pr_i = (\sum_{l=1}^{n} pa_i^l)/n$.

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