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Building and managing fuzzy ontologies with heterogeneous linguistic information

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ABSTRACT

Fuzzy ontologies allow the modeling of real world environments using fuzzy sets mathematical environment and linguistic modeling. Therefore, fuzzy ontologies become really useful when the information that is worked with is imprecise. This happens a lot in real world environments because humans are more used to think using imprecise nature words instead of numbers. Furthermore, there is a high amount of concepts that, because of their own nature, cannot be measured numerically. Moreover, due to the fact that linguistic information is extracted from different sources and is represented using different linguistic term sets, to deal with it can be problematic. In this paper, three different novel approaches that can help us to build and manage fuzzy ontologies using heterogeneous linguistic information are proposed. Advantages and drawbacks of all of the new proposed approaches are exposed. Thanks to the use of multi-granular fuzzy linguistic methods, information can be expressed using different linguistic term sets. Multi-granular fuzzy linguistic methods can also allow users to choose the linguistic term sets that they prefer to formulate their queries. In such a way, user-computer communication is improved since users feel more comfortable when using the system.

1. Introduction

Ontologies have become an important tool in the domain modeling field. Thanks to them, it is possible to carry out real world representations, stablish axioms and obtain conclusions of them [1,2]. Ontologies have been wide used in several fields. In biomedicine field [3–5], ontologies have been employed, for example, to build knowledge databases about genes and proteins characteristics that help researchers to classify and understand how the human body works. In semantic web field [6–8], ontologies have been used to classify concepts that can be referred through the web. This way, searches are improved and give better results to the users because a concept, instead of a word that can have different meanings, is used. In the artificial intelligence field [9–11], ontologies can also be applied to create knowledge databases to be used in systems that employ the provided information to carry out different tasks.

However, classical Crisp Ontologies have one important drawback, that is, their element descriptions can only be expressed using crisp membership values. Consequently, each described element has a set of fulfilled characteristics and another one with characteristics that do not describe the element. That is, membership value of each element to each concept is represented by the values {0,1} where 0 means that the element does not fulfill the concept and 1 means that the element has the characteristic expressed by the concept. In real world problems, this kind of scenario is not enough to describe correctly certain situations. For solving this issue and being able to provide a more flexible way of carrying out descriptions, Fuzzy Ontologies (FO) has been developed. Thanks to FOs, it is possible to provide membership values from the defined elements to the concepts using the interval [0,1]. Therefore, each described element can fulfill concepts totally (1 value), do not fulfill it (0 value) or partially fulfill it with a certain degree value (]0,1[). Thanks to this new representation, it is possible to model the uncertainty that is implicit in many real world environments and. Using fuzzy sets theory [12], it is possible for the ontology to deal with it using its associated mathematical environment. FOs is a field that is clearly present in the recent literature as it can be seen in [13–15].

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FOs also open the way for introducing linguistic modeling in this research field [16]. Thanks to it, elements can be described by using words instead of numbers. Linguistic modeling and linguistic term sets (LTSs) [17] in order to describe elements have one main advantage and one main drawback. The advantage is that words are more flexible that numbers. Consequently, this is the best way when trying to model concepts whose meaning is imprecise. They are also easier for humans to use that numbers making them a perfect choice when trying to model people opinions. On the other hand, the main drawback of using linguistic labels is the loss of precision that they produce when trying to represent precise data values.

FOs are used to create big knowledge stores whose data can come from different information sources, and therefore, source information is expressed using different representation methods. Due to the heterogeneity of the information, sometimes it is difficult to manage it. In such a way, it is extremely important to be able to work and combine different information expressed using different data types. Consequently, methods that are able to deal with data expressed using different representation models are needed. Thanks to them, data can be expressed in a way that it can be compared and managed together, without having to take into account the origin of the information.

In this kind of scenarios where data is heterogeneous and it is represented using fuzzy sets theory and linguistic modeling, multi-granular fuzzy linguistic methods (multi-granular FLM) [18–21] become essential. Thanks to them, it is possible to carry out conversion operations in order to homogenize the information. In such a way, the system can easily work with all the information. Multi-granular FLM can also allow users to select the LTSs that better fits them. Therefore, user-system communication is improved. In this paper, three new different ways of how multi-granular FLM processes can be applied when fuzzy ontologies are built and managed are proposed and analyzed. To do so, advantages, drawbacks and viability of the different processes depending on the type of information we are dealing with, are presented.

In Section 2, basis needed to understand the proposed methods are introduced. In Section 3, some new methods to solve the multi-granularity treatment problem that is present in FOs are proposed. In Section 4, examples of the exposed approaches described in Section 3 are showed. In Section 5, advantages and drawbacks of the proposed methods are analyzed. Finally, some conclusions are pointed out.

2. Preliminaries

To make this paper as self-contained as possible, this section introduces some concepts and methods to be referred to thorough this paper. In subSection 2.1, multi-granular FLMs are introduced. In subSection 2.2, Fuzzy Ontologies basis are exposed. In subSection 2.3, we describe some features of the Fuzzy Wine Ontology that we use for computing the example results.

2.1. Basis of multi-granular FLM

Linguistic Modeling [17] and the way that it allows people to communicate with computers using words has become an important improvement in human–computer communication. Thanks to it, humans can express themselves using imprecise information as it is the way that they are more used to provide it [22].

Traditional linguistic modeling usually force all the involved users to express themselves using the same LTS. This restriction can become a disadvantage since the selected LTS might not be the best choice for all of them. That is, there can be users that do not feel comfortable with certain LTSs. This situation usually appears when the LTS granularity does not fit the knowledge of the problem that the user has. Therefore, if the user has a wide knowledge of the dealt issue, he/she would prefer to use an LTS that have a high granularity. Then, user can provide more precise information to the system. On the other hand, if a user that does not have too much knowledge about the problem is given a set of words too big for him/her, then the user would get lost among all the possibilities that he/she is given. Consequently, he/she would have problems to provide the required information. In order to solve this kind of situations, it is mandatory that users are allowed to work with LTSs that are specifically designed for them.

In order to solve this problem, multi-granular fuzzy linguistic modeling [23,24] can be used. Thanks to multi-granular FLM, users that utilize the same computer system can provide information with the LTS that better fits them. Thus, user confidence and expressibility are increased and the provided information becomes more accurate and reliable. The usual process followed by multi-granular FLM methods in order to deal with different LTSs is showed below:

1. **Providing preferences**: Users provide the required information using the LTS that they prefer.
2. **Information uniformization**: All the information expressed using different sources is transformed into words expressed by the same LTS. This LTS is usually called the basic LTS (BLTS).
3. **Carrying out computations**: Once that all the information has been uniformed, it is possible for computers to carry out the required computations.

In Fig. 1, three LTSs are defined over the same space range. Vertical lines stablish correlations among them and can be used to define multi-granular transformation functions.

In the recent literature, there are several multi-granular FLMs methods. For instance, in [26], discrete fuzzy numbers are used in order to design a multi-granular FLM method. No membership functions are necessary in order to carry out the required operations. That is, all the computations are made using discrete fuzzy numbers environment. In [27], qualitative description spaces are used in order to carry out the required linguistic labels transformations. Distances in the space of qualitative assessments are used to carry out the required transformations. In [28], a multi-granular FLM method for dealing with multi-granularity uncertain linguistic group decision making problems with incomplete weight information is presented. In order to carry out computations, triangular fuzzy numbers are used. Operations are carried out using the membership function of the fuzzy numbers. In [29], a normalized numerical scaling method that is able to determine semantics of linguistic labels that belong to different LTSs is presented. This method works with either balanced or unbalanced LTSs. In [30], a multi-granular FLM for unbalanced LTSs is defined. For carrying out computations, linguistic distribution assessments with exact symbolic proportions are used. Aggregation and transforming operators are defined over this environment.

There are also several papers in the recent literature that applies multi-granular FLM methods to solve problems. For instance, in [31], multi-granular FLM methods are used to create a Project evaluation method. Non-formatted text information is used in the process. In [32,33], multi-granular FLM methods are applied to create a consensus based group decision making method.

In this paper, when transformations among labels from different LTSs want to be carried out, the multi-granular FLM exposed in [25] is used. This method is based on the concept of linguistic hierarchies (LHs) and the 2-tuple ordinal fuzzy linguistic modeling [34]. A linguistic 2-tuple is defined as a tuple (s, α) where s is an ordinal linguistic label and $\alpha \in [-0.5, 0.5)$ is called the symbolic

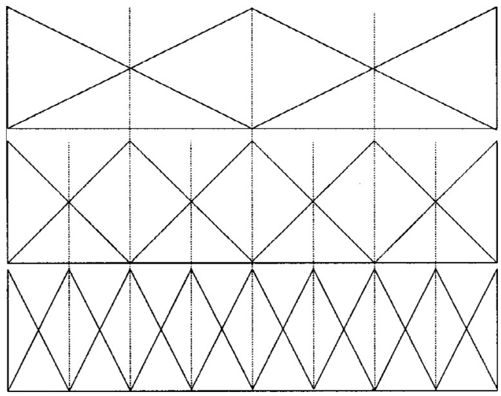

Fig. 1. LTSs of granularity values 3, 5 and 9 defined over the same real interval [25].

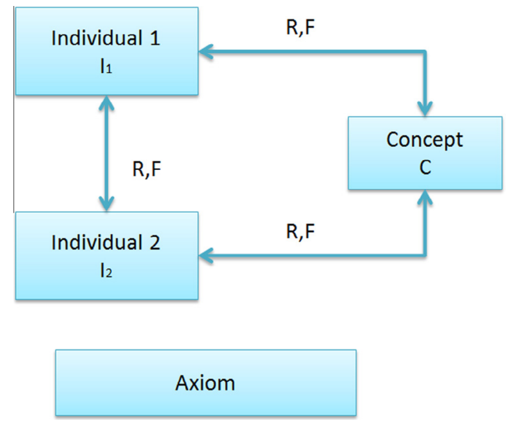

Fig. 2. Fuzzy ontology scheme.

translation. Let β be considered as the aggregation result of the indexes of a set of labels belonging to the same LTS and $i = round(\beta)$, then symbolic translation can be calculated as $\alpha = \beta - i$. Any aggregated numerical β can be trasnformed into (s, α) and viceversa, using the following transformation functions Δ and Δ^{-1} [34]:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha) \text{ with } \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases} \tag{1}$$

and

$$\begin{aligned} &\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g] \\ &\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \end{aligned} \tag{2}$$

A LH is a set of levels where each one is represented by a LTS with a unique granularity value. Formally, each level is defined as $l(t, n(t))$ where t is a number that indicates the level of the hierarchy and $n(t)$ is the granularity value of the associated LTS. Consequently, a LH is represented as the union of all its levels t as follows:

$$LH = \bigcup_t l(t, n(t)) \tag{3}$$

Using this structure, a multi-granular transformation function can be defined in order to carry out transformation among labels from different LTSs in the LH as follows:

$$\begin{aligned} &TF_t^{t'} : l(t, n(t)) \rightarrow l(t', n(t')) \\ &TF_t^{t'}\left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right) \cdot (n(t') - 1)}{n(t) - 1}\right) \end{aligned} \tag{4}$$

It should be pointed out that, in [35], Extended Linguistic Hierarchies (ELH) are presented. Their main purpose is to overcome some of the limitations that LHs have. Some applications of this tool can be seen in [36,37].

2.2. Fuzzy ontologies

A fuzzy ontology [16,38] can be defined as "an ontology which uses fuzzy logic to provide a natural representation of imprecise and vague knowledge and eases reasoning over it". Formally, a fuzzy ontology [39,40] can be represented as a quintuple $O_F = \{I, C, R, F, A\}$ where I is the set of individuals, C refers to the set of concepts, R represents the set of relations, F defines the set of fuzzy relations and A is the set of axioms. In Fig. 2, a fuzzy ontology scheme can be seen. Crisp concepts are related with individuals using relations from R while fuzzy concepts use relations from F. In the case of crisp concepts, individuals can fulfill them or, on the contrary, do not be represented by them, that is $\{0, 1\}$. In the case of fuzzy concepts, individuals fulfill them with a certain degree described using the interval $[0, 1]$. For example, if information want to be stored about the heigh of a set of people using the LTS $Heigh = \{Low, Medium, High\}$, if heigh is considered crisp, to describe each person, only one of the three values must be chosen. Nevertheless, if heigh is dealt as a fuzzy concept, a person can belong to several of the three values with different degrees in each case. For example, John heigh could be defined as $\{Medium = 0.8, High = 0.9\}$.

When information is stored in an FO, it is possible to formulate queries in order to retrieve some required information [41]. Let $C = \{c_1, \ldots, c_m\}$ be the set of fuzzy concepts from a fuzzy ontology and $IN = \{in_1, \ldots, in_k\}$ the set of individuals. Let $Q = \{q_1, \ldots, q_n\}$ be the indexes of the concepts that the user is interested in, let $W = \{w_1, \ldots, w_n\}$ be the importance given by the user to each of the selected concepts and $V = \{v_1, \ldots, v_n\}$ the desired values for each concept. Note that $\sum_i w_i = 1$ and $n \leqslant m$. For each individual in_j, that is associated to the concepts specified by the user, the matching value is calculated using the OWA operator [42] as follows:

$$MV = \sum_{l=1}^{n} w_l \cdot sim[v_l, (\mu_{in_j}(c_{q_l}))] \tag{5}$$

where MV is the calculated matching value and $\mu_{in_j}(c)$ indicates the value associated to the individual i_j for the concept c. Symbol $sim[\cdot]$ refers to a similarity measure.

Fuzzy ontologies have been widely used recently in the literature. For example, in [9], a fuzzy ontology is used to model the human behavior. In [43], fuzzy ontologies are used to design a simulator that helps vehicles in the marine environment to avoid collisions [44].

2.3. The fuzzy wine ontology

As an example of application of the proposed multigranular fuzzy linguistic methods for fuzzy ontologies, the Fuzzy Wine Ontology [13,41] will be used. Information about the stored wines has been collected from websites created and frequently visited by wine connoisseurs.[1] This ontology is selected for carrying out our tests because the knowledge that it represents is naturally imprecise and linguistic modeling is used in order to describe the wines. The most recent version of the Fuzzy Wine Ontology contains over 600 wines and 8 concepts to describe each one.

[1] e.g. www.alko.fi, www.winesfromspain.com, www.snooth.com.

The used concepts are listed below:

- **Alcohol:** Represents the alcohol level of the wine. An LTS of granularity 3 is used, that is $S_3 = \{Low, Medium, High\}$.
- **Acidity:** Represents how acid the wine is. S is used for its representation.
- **Price:** Price of the wine. It is a fuzzy concept represented also using S_3.
- **Year:** Wine year. It is represented as a fuzzy concept with an LTS whose granularity value is 4, that is,

  $$S_4 = \{Novello, Regular, Old, Exclusive\}$$

  From now on, x in S_x indicates the granularity of the LTS.
- **Country:** Country where the wine belongs. This concept has been considered as a crisp one.
- **Body:** Wine Body. Treated as a crisp concept. One of the values *Medium*, *Full* or *ExtraFull* can be chosen.
- **Sweetness:** Wine sweetness. It is also treated as a crisp concept whose possible values are *Dry*, *MediumDry* and *Sweet*.
- **Color:** Wine color (*White*, *Red* or *Rose*). It is stored as a crisp concept.

As it can be seen, both fuzzy concepts and crisp concepts are used in the Fuzzy Wine Ontology. This heterogeneity in the ontology definition makes the Fuzzy Wine Ontology a perfect example for us to carry out a better analysis of how to carry out multigranularity treatment in FOs.

3. Multi-granular FLM methods for building and managing fuzzy ontologies

In this section, several different ways of dealing with multi-granular information in the ontology creation process are exposed. Furthermore, a way for users to carry out queries using the LTS that better fits them will also be shown.

Generally, when an ontology is created, these steps are followed:

1. **Information search:** First of all, reliable information sources must be consulted and, afterwards, information is extracted in order to gather the necessary data for the ontology that is being created. When several information sources are consulted, the probability that the information is expressed using different means is very high. Information must be uniformed in order to be able to carry out comparisons.
2. **Information preprocessing step:** Transformation functions are applied over the extracted information in order to express them using the same representation method. This step is mandatory since it would be impossible to carry out any operation if the information is not homogeneous. Afterwards, data is stored in a way that can be used by queries. It should be taken into account in the design that the preprocessing step is carried out only once while queries are made repeatedly. This way, for the sake of efficiency, data computations that are always carried out in all the queries can be pre computed in this step. Consequently, time will be saved in the query process.
3. **Query design:** A method of user-system communication with the FO has to be developed. Depending on how the information has been stored in the preprocessing step, the building of possible queries differs. Therefore, the designing of a communication method with the ontology is a critical task. Depending on the representation, it could be possible to allow users to use different LTSs, that is, queries can become multi-granular if users can select the LTSs that better fits them when making a query.
4. **Validation:** After the ontology is created, a validation process must be carried out in order to confirm that the ontology works correctly and results are the expected ones. Since this paper deals with multi-granular FLMs application in the FO building and management processes, in the following, we focuss our efforts on the three previous steps. Fuzzy ontology validation processes can be further studied in the literature [45].

In Fig. 3, a scheme of how the ontology is created and used is shown. In the case of the Fuzzy Wine Ontology, the creation process is exposed below:

1. **Information search**: Well-known databases of wines were searched in order to gather all the wines information needed. Data recollected has different representations since several different sources were used.
2. **Preprocessing step**: Information is uniformed and expressed using LTSs or crisp values. More details about the final representation chosen can be seen in subSection 2.3.
3. **Query design**: Users can perform queries using labels of the LTS that have been used to represent the FO information or, in the case of the crisp values, users indicate the characteristics that they are interested in. For example, if a wine with low alcohol, high acidity and from Spain is needed, the search made by the user have the following form:

   $$\begin{aligned} Q_u = \{&Alcohol = LowAlcohol \wedge Acidity = HighAcidity \\ &\wedge Country = Spain\} \end{aligned} \tag{6}$$

   It can be seen that wine searchers are forced to carry out queries using the LTS that have been selected for representing the information in the FO. When the FO is going to be used by a high amount of people, it would be desirable to let them choose the way of expressing the query that better fits them.

In conclusion, there are two possible ways where it is possible to take advantage of multigranularity treatment methods:

- **Multi-granular source data treatment at the FO building process**: Linguistic data belonging to different sources may need multigranular treatment in order to be able to express the information using the same metrics and to work with it.
- **Multi-granular queries design for the FO management**: Users carrying out queries to the FO may need to have different LTSs for expressing themselves. In such a way, they can choose the most comfortable way to communicate with the system.

In this paper, both situations are analyzed and solutions are suggested. In subSection 3.1, how to apply multi-granular FLMs to the data recollected from different data sources in order to build a FO is studied. In subSection 3.2, methods to design queries for managing the FO are proposed.

3.1. Multi-granular source data treatment at the FO building process

In the first step of the creation of an ontology, data is extracted from different sources. Generally, each source has its own way of storing the information making it impossible to carry out comparisons among them directly. Consequently, data transformation operations must be carried out. Two types of data representations can be found in the information sources:

- **Numerical information**: It is the one referring to concepts that can be accurately measured. The main two operations that can be performed to uniform numerical information are defined below:

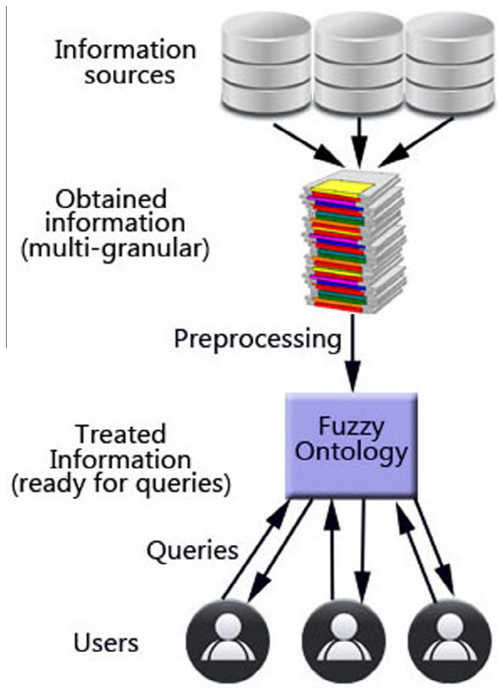

Fig. 3. Creation and use ontology scheme.

– *Domain change:* When a measure is carried out, it is usual to establish both minimum and maximum range values. Consequently, the minimum range value represents the lowest possible valid value while the maximun range value represents the highest one. It is usual that different information sources choose different range intervals for expressing the numerical information. Before being able to work with this type of information, a unique range value interval must be chosen and all the information must be normalized into it.

– *Number format:* Different numeric formats can be used to represent the numerical information, for example, real, integer, etc. Transformation rules must be established in order for numbers to use the same format. For example, if all the information must be expressed using integer values but there are values expressed using real numbers, rules of how to deal with real values must be defined. One possible way of dealing with this situation could be to apply the floor operator. It is important to point out that the best way of carrying out these operations is to express the information using the format that is able to represent more elements. This way, loss of precision is avoided. For example, when dealing with integer and real numbers, it is much better to express integer numbers using the real format.

- **Linguistic information**: It is the one referring to concepts whose definition entails imprecision and uncertainty. Concepts like beauty, tastiness and sympathy belong to this category. Nevertheless, it is also possible to express numerical nature information using words if an accurate value is not known or they do not want be expressed in a precise way. It is usual that different linguistic information sources use different LTSs with different granularities in order to represent the linguistic information. In order to carry out operations using all of this information, all labels must belong to the same LTS. Thanks to multi-granular linguistic FLMs [19,26,46], this task can be carried out without any trouble. Several possible options of dealing with multi-granular linguistic information are listed below:

  – *Symbolic multi-granular FLMs*: These FLMs carry out label translations belonging to different LTSs taking into account the indexes of the labels in each LTS. This way, computations become quite simple and no extra representation framework must be added to the labels. The main drawback of these FLMs is that they usually have restrictions, that is, these methods do not usually work with all the possible LTSs. Furthermore, they can produce loss of information. Inside this category, it can be found FLMs that use linguistic hierarchies [25,36], discrete fuzzy numbers [26] and qualitative descriptive spaces [27].

  – *Semantic multi-granular FLMs*: These FLMs associate a fuzzy set to each label. In such a way, the initial label representation is lost and all the transforming operations are carried out using the associated fuzzy sets and their mathematical environment. The main advantage of these methods is their flexibility, that is, they can operate with any LTS and do not have any restrictions as long as a fuzzy set is associated to every label in every LTS. Their main drawback is located in the results presentation. To associate a label to the resulting fuzzy set can become a troublesome task due to the fact that, after computations, it is possible that no label fits the result. Carrying out this process entails loss of precision in the process. Inside this category, it can be found FLMs that use triangular fuzzy numbers [28,47] and the ones that are based on a Basic Linguistic Term Set (BLTS) [23,48].

  – *Linguistic to numeric conversion*: If the FO designer considers that there is no need to work with linguistic information, it is possible to use semantic multi-granular FLMs. Thus, linguistic information can be converted into numeric one. The main advantage of this approach is to have all the advantages of semantic multi-granular FLMs and precision of numerical data without the consequences of having to translate fuzzy sets into labels.

There is not a best way of carrying out this task, depending on the desired results, the most suitable multi-granular FLM should be chosen. Using all the presented processes, it is possible to manage all the recollected heterogeneous information and transform it into what the designer needs for his/her FO design. Depending on how the user query is designed, information must be transformed and presented in an specific way. In the following subsection, several user query designs that allow users to select the LTS that they prefer are presented. Each design needs the information to be presented in an specific way.

3.2. Multi-granular queries design for the FO management

In regular FOs, users are forced to express themselves using, for each concept, a unique LTS. It would be desirable to allow users to choose the LTSs that they prefer. A FO query process using multi-granular FLM could be held as follows:

1. **LTS selection:** The user formulates his/her query using, for each characteristic, the LTS labels that better fits his/her expression capacity. Depending on the FO design and the multi-granular FLM used, there could be some restrictions, that is, it is possible that the set of chosen LTS must fulfill certain specifications in order to be valid.

2. **Query resolving process:** The FO support system carries out the necessary transformations to the user provided information in order to carry out the FO query resolution. For example, if an user provides his/her information using the LTS S_1 but in the FO the information is stored using the LTS S_2, a multi-granularity FLM must be applied. Thus, the labels from S_1 provided by the user are expressed using labels of S_2 and comparisons with the FO information can be carried out. In this paper, the method described in subSection 2.1 will be used.
3. **Result presentation:** The query results consist in a list of elements that are ordered according to their associated matching values. The FO user can select if he/she wants to see these results numerically or linguistically and, in the second case, he/she can select the target LTS. Transformation functions must be applied to the obtained results in order to fulfill the result representation requirements asked by the user.

Thanks to multi-granular FLMs, user-system communication is improved. Therefore, users have more means to formulate the FO queries because the FO support system adapts itself to the users communication needs. Users can express themselves better and, consequently, the system receives more reliable information.

In this subsection, several FO designs that allow users to express themselves linguistically using the LTS that they prefer are presented:

1. **Semantic approach**: All the gathered information is stored in its numerical value. Linguistic information is also expressed numerically using fuzzy sets mathematical environment. Membership values of the labels associated fuzzy sets are used to carry out this transformation. Linguistic queries provided by users are also expressed numerically in order to carry out computations.
2. **Duplicity approach**: Information is duplicated and stored using different representations. Users provide their queries in any of that representations.
3. **Symbolic approach**: Information is stored linguistically using the same LTS for each of the concepts. Users can provide their queries in any LTS and, in order to carry out comparisons, multi-granular FLMs are applied to it.

3.2.1. Semantic approach

This approach expresses all the gathered information in a numeric way. Therefore, semantic multi-granular FLMs [28,47] are applied to the user query in order for it to be also expressed numerically for computations to be carried out. To build an FO using this approach, the next steps must be followed:

1. **Selecting target numeric interval**: The numeric interval used to represent the information referring to each concept is chosen. The interval can be as wide as desired as long as it has a minimum and a maximum value. This restriction will allow us to transform the linguistic information into numerical one.
2. **Transforming linguistic information**: Gathered linguistic information is expressed using the chosen numerical interval associating a specific number inside the interval to each label. This way, a high linguistic value will be associated to numerical values close to the maximum interval value. On the other hand, low linguistic values will be associated to positions close to the minimum interval value. It should be pointed out that this process entails a loss of precision that will be traduced in less accurate results.
3. **Transforming numerical information**: It is possible that gathered numeric information is expressed using a different scale or measure that the chosen one. Depending on the case, a transformation function that let us express the numeric information using the chosen representation must be applied. For example, if some piece of information must represent the number of square meters of a house but the information gathered refers to square centimeters, information should be transformed and expressed using meters instead of centimeters. Also, if, for example, a student score in an specific subject is measured using the interval [0,100] but the interval [0,10] want to be used, it is possible to carry out a domain change as exposed in subSection 3.1.

Queries using this approach are formulated and resolved as follows:

1. **LTS selection**: User selects the LTS that he/she want to use for each of the characteristics that he/she will include on the search.
2. **Query providing step**: The user formulates the query linguistically using the LTSs that he/she have chosen.
3. **Transforming linguistic information**: Linguistic information provided is transformed into numeric one associating a fuzzy set to each of the labels. In order to carry out computations, the fuzzy set is defuzzified [49] in order to obtain a single number. One way of achieve this purpose is to calculate the gravity center, GV, of the fuzzy set as follows:

$$GV = \int_x \frac{x \cdot \mu(x)}{\mu(x)} \tag{7}$$

4. **Resolving the query:** Once that the query has been expressed numerically, the query is resolved using the following steps:
   (a) For each of the elements of the FO, the characteristics that the user has included in his/her query are retrieved.
   (b) For all the characteristics, distance value between the user specified value and the one of each element in the FO is measured.
   (c) Elements are sorted in a way that the elements whose proximity is closer to the one specified by the user are in high positions of the ranking.
   (d) Elements located in high positions of the ranking (or only the best element) are returned to the user.

In Fig. 4, a scheme of this approach can be seen graphically.

3.2.2. Duplicity approach

This approach stores the same information several times using different linguistic representations in order to allow the user to choose the representation that better fits him/her. This ontology building approach follows the next steps:

1. **Selecting target LTSs:** The set of LTSs that user will be able to choose in order to perform his/her queries are selected. It is important to select LTSs with different granularities in order for users to be able to select among a wide range of possibilities. This way, user-system communication will be increased. Otherwise, there can be experts that will not find a suitable LTS for them.
2. **Transforming the information:** All the gathered information that conforms the FO is replicated and expressed using, for each replication, a different LTS of the chosen ones in the previous step. Multi-granular FLMs can be used to carry out the necessary linguistic transformations. In the case of numeric information, membership function value to each of the labels from all the LTSs is calculated and stored.

Queries that use this FO approach are formulated and resolved as follows:

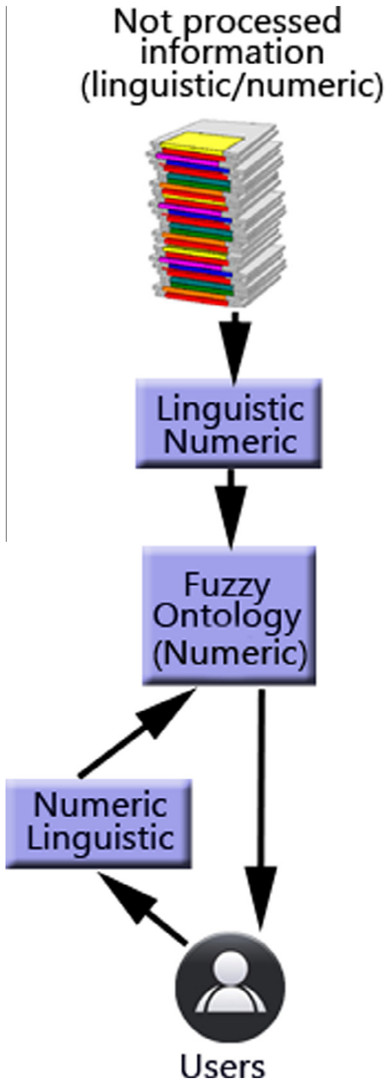

Fig. 4. Multi-granular ontology semantic approach scheme.

1. **LTS selection:** For each of the characteristics, user selects one of the LTSs that have been pre-selected in the FO building step.
2. **Query providing step:** The chosen set of LTSs are used to formulate the query.
3. **Resolving the query:** Query is resolved using the following steps:
   (a) For each element of the FO, characteristics expressed using the LTSs provided by the user are retrieved. The rest of redundant information is not taken into account.
   (b) Membership function values of each element to each of the labels provided by the user are aggregated. Any aggregation operator such as OWA [42] can be used for this purpose. A ranking of elements is made using the aggregation resulting values, that is, the matching values.
   (c) Elements with the best matching values of the raking (or only the best one) are returned to the user.

In Fig. 5, a graphical scheme of this approach is shown.

3.2.3. Symbolic approach

This approach stores all the information linguistically and uses symbolic multi-granular FLMs in order to express the query provided by the user into the FO used LTSs. This ontology building approach follows the following steps:

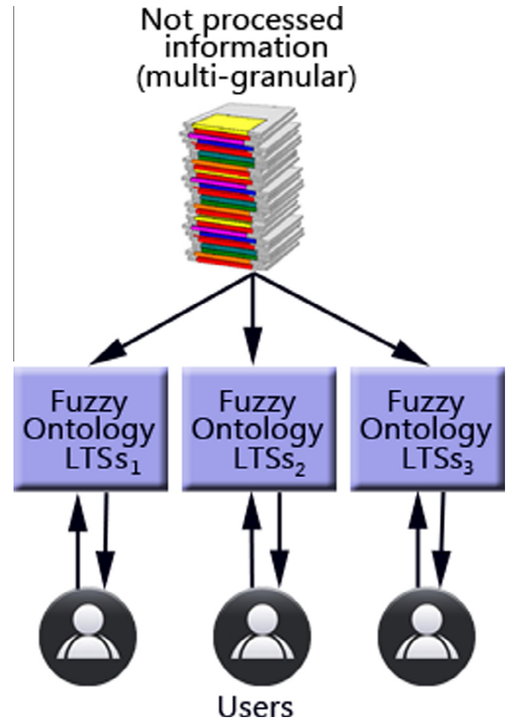

Fig. 5. Multi-granular ontology creation scheme.

1. **LTSs selection:** A LTS is chosen to represent the information of each of the characteristics.
2. **Transforming information:** Gathered information referring to each of the characteristics is expressed using the chosen LTSs. Multi-granular FLMs can be used to carry out this task. In the case of numeric information, membership function value to each of the labels of the chosen LTS are calculated and stored.

With this approach, queries are formulated and resolved as follows:

1. **LTS selection**: The user selects the LTSs that he/she wants to use to express his/her preferences for each characteristic.
2. **Uniforming linguistic information:** The user formulates his/her query using his/her chosen LTSs. Because user LTSs can be different from the FO LTSs chosen to represent the information, multi-granular FLMs are used to express the user information in terms of the FO stored one.
3. **Resolving the query**: Membership function values of each element to each of the query labels are aggregated. Finally, a ranking of elements is made using the obtained aggregated values. Better elements (or only the best one) are returned to the user.

In Fig. 6, a scheme of this approach is showed graphically.

4. Illustrative example

In this section, an example of each approach proposed in subSection 3.2 is exposed. Fuzzy Wine Ontology is used in order to test the different designed FO support system versions. Specifications of Fuzzy Wine Ontology can be seen in

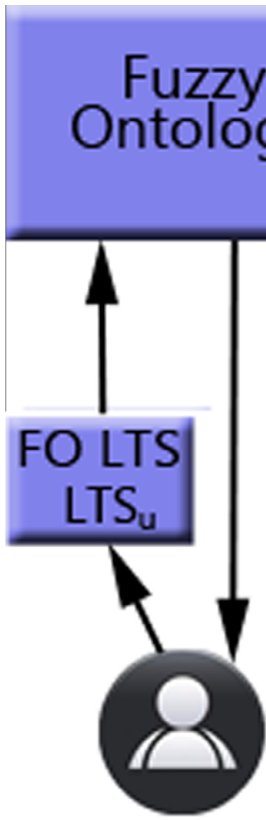

Fig. 6. Multi-granular ontology query maker scheme.

subSection 2.3. FOs for each example have been built using the techniques presented in subSection 3.2.

4.1. Example of semantic approach

A FO user wants to choose, among the 600 possibilities, the wine that better fits his/her desires. He/she focuses his/her query in the alcohol level, the acidity, the year and the price characteristics. For each one, the LTSs used by the user to express his/her desires and the selected label are shown in Table 1. Label s_x^y indicates a label whose index is x and belongs to a LTS of granularity value y. Because the information expressed by the user is linguistic and the one stored in the FO is numeric, it is necessary to carry out a linguistic to numeric conversion of the query. Because the user has provided labels and not precise values, imprecision and loss of information is produced during this step. For the alcohol, the label s_5^7 is transformed into a numeric value in the range $[0, 20]$ as follows:

$$\frac{5-1}{7-1} * 20 = 13.333$$

In Table 2, the numeric value associated to each query value after the transformation is showed. Range column indicates the minimum and maximum numerical values accepted by the FO as valid for each characteristic. Similarity measures among the wanted values and the characteristic values of each element of the FO are calculated. First four wines that obtained the lowest distance values are showed in Table 3. Distance value for, for example, Campo_Viejo_Reserva is calculated aggregating the distances between the characteristics of the wine and the numeric conversion of the query data. Mean operator of the distances is used to carry out this operation as follows:

Table 1
User selected LTSs.

Characteristic	LTS	Chosen label
Alcohol	$S_7 = \{s_1^7, s_2^7, s_3^7, s_4^7, s_5^7, s_6^7, s_7^7\}$	s_5^7
Acidity	$S_7 = \{s_1^7, s_2^7, s_3^7, s_4^7, s_5^7, s_6^7, s_7^7\}$	s_4^7
Year	$S_4 = \{s_1^4, s_2^4, s_3^4, s_4^4\}$	s_3^4
Price	$S_3 = \{s_1^3, s_2^3, s_3^3\}$	s_2^3

Table 2
Numeric conversion of the labels.

Characteristic	Chosen label	Numeric conversion	Range
Alcohol	s_5^7	13.333	[0,20]
Acidity	s_4^7	5	[0,10]
Year	s_3^4	2003	[1800,2012]
Price	s_2^3	14	[0,500]

Table 3
FO selected wines by semantic approach.

Wines	Distances
Campo_Viejo_Reserva	0.8916664
Chateau_Bonnin_Pichon	0.91416645
Cave_de_Tain_Crozes_Hermitage	0.93833363
Tiempo_Briego	0.9391664

$$\frac{|13.5 - 13.33| + |5.2 - 5| + |2006 - 2003| + |13.8 - 14|}{4} = 0.8916664 \quad (8)$$

Characteristics values stored in the FO for the best selected choice can be seen in Table 4. Distances vector of the numeric conversion of the query and the best choice selected in the FO is $\{0.167, 0.2, 3, 0.2\}$. Due to the low distances among the numeric version of the query and the FO element it can be stated that Campo_Viejo_Reserva is a great choice for the user. Nevertheless, it should not be forgotten that loss of information has been produced during the linguistic to numeric query conversion.

4.2. Example of duplicity approach

A user wants to retrieve a wine from the Fuzzy Wine Ontology that has specific characteristics. His/her preferences are described in Table 1. The FO process the query, and, without any further conversion, it selects, for each element of the FO, the characteristics values that are expressed using the user LTSs. Membership function values for each label provided by the user are aggregated into a one single value used for stablish comparisons among the different wines. The four best wines according to the query and their matching values are showed in Table 5. The wine with best matching value is Tiempo_Briego. Membership function values for each of the labels can be seen in Table 6. The aggregation of these membership function values results in the matching value. In this example, the mean operator has been used for carrying out computations. It is easy to see that membership function values to the labels provided by the user are quite high for Tiempo_Briego wine making it a excellent choice for the user. It should be also pointed out that Tiempo_Briego was also one of the better choices selected by the semantic approach. It can be estimated that, without taking into account any transformation process, this approach takes 21 times more time to execute this example than semantic approach. This is because, while semantic approach makes one comparison, in this approach a comparison per label is carried out.

Table 4
Campo_Viejo_Reserva characteristic values.

Alcohol	13.5
Acidity	5.2
Year	2006
Price	13.8

4.3. Example of symbolic approach

A user wants to use the Fuzzy Wine Ontology support system in order to retrieve a wine that has certain features. Characteristics that the user is interested in, LTSs used in the query and the selected labels are shown in Table 1. LTSs used by the FO for that characteristics are shown in Table 7. It can be seen that, although same LTS is used for representing year and price, the user and FO use different LTSs for representing linguistically the alcohol and acidity information of the wine. In such a way, a multi-granular FLM must be used in order to transform the query labels whose representations differ. It has to be pointed out that any multi-granular FLM is valid for carrying out this conversion. In this example, the membership function value of the gravity center of the query label to the FO labels is used to carried out the transformation of labels. For example, in the alcohol case, s_5^7 corresponds to $\{0.66 : s_2^3, 0.416 : s_3^3\}$. Then, wines that have a closer membership value for 0.66 in label s_2^3 and 0.416 for label s_3^3 will be selected as desired characteristics values. User query expressed using FO labels is exposed below:

$$\begin{aligned}
&Alcohol : \{0.666 : s_2^3, 0.416 : s_3^3\}\\
&Price : \{1.0 : s_2^3\}\\
&Year : \{1.0 : s_3^4\}\\
&Acidity : \{0.375 : s_1^3, 0.4285 : s_2^3\}
\end{aligned}$$

After performing the query, the first four better results and respective matching values obtained can be seen in Table 8. Tiempo_Briego is the most appropriate wine for the user. Its membership values for each label in alcohol and acidity concepts can be seen in Table 9. It can be seen that, for the alcohol, the distance values between the wine characteristic and the query are $\{0.06, 0.084\}$. For the acidity, the distance values are $\{0.125, 0.1428\}$. Furthermore, for the Year, distance value is 0 and for the price 0.41. First, an aggregation of all these distances in a similar way as it has been made in expression (8) is performed. Nevertheless, a weighted mean operator must be used in order to provide the same importance to all the features. Weight values used for each single distance value are exposed below:

Table 5
FO selected wines by duplicity approach.

Wines	Matching value
Tiempo_Briego	0.86962026
Marques_de_Arienzo_Reserva	0.75835556
Castillo_Montroy_Reserva	0.7372372
Beringer_Founders_Estate_Merlot_2	0.73692477

Table 6
Tiempo_Briego characteristic values.

Characteristic	Label	Membership value
Alcohol	s_5^7	0.9489
Acidity	s_4^7	0.94012
Year	s_3^4	1.0
Price	s_2^3	0.589412

Table 7
FO selected LTSs for symbolic approach.

Characteristic	LTS
Alcohol	$S_3 = \{s_1^3, s_2^3, s_3^3\}$
Acidity	$S_3 = \{s_1^3, s_2^3, s_3^3\}$
Year	$S_4 = \{s_1^4, s_2^4, s_3^4, s_4^4\}$
Price	$S_3 = \{s_1^3, s_2^3, s_3^3\}$

Table 8
FO selected wines for symbolic approach.

Wine	Matching value
Tiempo_Briego	0.8672779
Beringer_Founders_Estate_Merlot_2	0.8476215
Jean-Baptiste_Adam_Pinot_Gris_Reserve	0.820538

Table 9
Tiempo_Briego membership values for alcohol and acidity characteristics.

	s_1^3	s_2^3	s_3^3
Alcohol	0	0.6	0.5
Acidity	0.5	0.2857	0

$$\begin{aligned}
&Alcohol : \{s_2^3 : 0.125, s_3^3 : 0.125\}\\
&Price : \{s_2^3 : 0.25\}\\
&Year : \{s_3^4 : 0.25\}\\
&Acidity : \{s_1^3 : 0.125, s_2^3 : 0.125\}
\end{aligned}$$

After carrying out the weighted aggregation process the matching value is calculated as follows:

$$MV = 1 - 0.1327221 = 0.8672779 \tag{9}$$

where 0.1327 is the distances aggregated value.

Taking into account this results, it can be seen that Tiempo_Briego characteristics are quite close to the one desired by the user. Consequently, it is a good choice for the user to order.

It should be noticed that Tiempo_Briego is also the best choice selected by the duplicity approach and the fourth best wine chosen by semantic approach. Using this approach in this example and ignoring transformation information functions execution time, it can be estimated that the time consumed is 13 times higher than in semantic approach. As in duplicity approach, one comparison must be performed for each label used in the FO for the wanted characteristics.

In conclusion, it can be seen that the three approaches produce reliable results. Nevertheless, the obtained results by each one of them differ. This happens because of the use of heterogeneous information and the loss of precision that is present in the transformation functions used for making the information homogeneous.

5. Discussion

In this section, advantages and drawbacks of the presented FO designs that use multi-granular FLMs are exposed. Each proposed method has its own strengths and weaknesses and is not suitable for all the possible scenarios. To analyze the FO data environment and select a proper design method is extremely important if good results want to be obtained. The suitability of each proposed approach to every possible scenario is analyzed below:

- **Semantic approach:** Semantic approach stores all the gathered information from databases numerically, convert the linguistic labels provided by the user into numeric information and carry out computations numerically. The main advantage about this approach is that is the one requiring less disk space for storing information. This is due to the fact that only one numerical value per concept and element is stored. On the contrary, approaches that use linguistic labels need to store the membership function of each element to each of the labels, that is, several numerical values per concept and element. This approach is also quite efficient because only one number comparison and a unique linguistic to numeric conversion is made per query. The main drawback of this approach is the loss of accurateness that converting linguistic information into numeric one entails. All the imprecision and vagueness related to linguistic labels is lost in all the linguistic to numeric conversions carried out during the FO building and the query process. For example, if a *medium_alcohol* wine is searched, the numeric conversion just converts the linguistic value into a numeric one (or an interval) that is, indeed, medium. The problem is that alcohol values that do not belong with degree 1 to *medium_alcohol* values set are discarded, that is, all the imprecision representation capability that linguistic labels have is totally wasted. In conclusion, this approach is appropriate for environments where not much disk space is available or a high amount of information needs to be stored. Although it is the method that has the least number of comparisons per query, transformation functions need to be applied during the query process. On the other hand, it is not the best choice if linguistic nature information is being dealt because results will not be too accurate. This approach is also the best to choose when numerical nature information is dealt.

- **Duplicity approach:** This approach preselects a set of LTSs to represent each FO concept and stores the information several times using the selected LTSs. This way, the user can select one of the available LTSs for each concept and expresses his/her query using it. The main advantage of this method is that it does not need to carry out any information transformation during the query process. Nevertheless, it carries out more comparisons per query than the semantic approach. Therefore, it can be considered more efficient than the symbolic approach, which carry out information transformations in the query process, but less efficient than the semantic one. This is due to the fact that less comparisons per individual are carried out in the semantic approach. Another highlight of this method is that the information is stored linguistically making it able to take advantage of the imprecision nature of words. For example, if a *medium_alcohol* wine is searched, membership function values of each FO element to the label *medium_alcohol* are consulted. This way, no loss of information is produced. The two main disadvantages of this approach is the disk space requirement and that the LTSs that the user can use are preselected. Because information is replicated using different LTSs for its representation, FOs using this approach need a high amount of space. Let $\mathbb{S} = \{S_1, S_2, \ldots, S_a\}$ be the LTSs set used for representing the information and$G = \{g_1, \ldots, g_a\}$ represents the set of granularity values for each set. Then, for each concept and element, $\sum_{i=1}^{a} g_i$ numeric values are needed for a proper representation. Comparing to the unique numerical value used in the semantic approach, it can be seen that far more disk space is needed. Moreover, because the LTSs set $\mathbb{S}$ is fixed in the FO building step, only LTSs belonging to it can be used by the user in his/her queries. In conclusion, this approach is the best choice in environments where there is no disk space restrictions and information nature is linguistic.
- **Symbolic approach:** Symbolic approach stores the information linguistically and convert the user query labels into labels from the LTSs used to store the information in the FO. The main highlight of this approach is that it allows a proper management of linguistic information without the high requiring of disk space used by duplicity approach. a unique LTS with granularity g is used for representing each concept. Thus, only g numerical values are needed for each concept representation. It should be noticed that this approach still needs more space than semantic approach. The cost of having these advantages is that, in every query made, a multi-granular FLM must be applied to convert labels used by the user in the query to the FO labels. Consequently, this approach is the least efficient of the three exposed. In conclusion, symbolic approach is a good choice when a lot of information needs to be represented because it does not waste too much disk space. It also should be used when information nature is linguistic because it deals properly with the imprecision that is inherent to words. Although it is true that having to apply a multi-granular FLM in each query makes this approach the least efficient one, if the chosen method is efficient, then it is possible for this approach to work well in environments where there is a lot of information and a real time response is needed. Nevertheless, if a really high amount of elements are stored in the FO, like in big data problems [50,51], it is possible to experience a response delay.

Table 10
Characteristics summarizing table.

Characteristic	Semantic approach	Duplicity approach	Symbolic approach
Disk space required for storing	Very Low	High	Low
Efficiency in resolving queries	High	Medium	Low
Number of LTSs for the user to choose	Unlimited	Restricted	Unlimited
Deals properly with imprecision	No	Yes	Yes
Information stored nature	Numeric	Linguistic	Linguistic
Number of conversions	High	Low	High

A summary of this analysis can be seen in Table 10.

6. Conclusions

In this paper, several ways of taking advantage of multi-granular FLMs in the design of FOs have been presented. There are mainly two situations where multi-granular FLMs can help us:

- **Uniforming gathered information**: When retrieving information from different sources, it is usual that the information is not represented using the same means. When dealing with linguistic information, multi-granular FLMs can help us to unify and deal with all the heterogeneous gathered information. Therefore, it is concluded that FO designs can take advantage of multi-granular FLMs in order to improve the way that linguistic information is managed.
- **Allowing users to select the way of expressing their queries:** Due to the capacity of multi-granular FLMs to carry out transformations among labels from different LTSs, it is possible to use them in order for users to formulate their queries using the LTSs that they prefer. This way, users feel more comfortable when providing their queries and user-system communication is improved.

Thanks to multi-granular FLMs, it is possible to design flexible FOs that can manage linguistic information in a very easy and comfortable way.

In this paper, three possible FO designs that take advantage of multi-granular FLMs features are proposed. Each one has different advantages and drawbacks and is not suitable for all the possible situations. Designers should analyze the extracted data nature, the available resources and users requirements before selecting a proper design. In the future we plan to extend this proposal to the use of unbalanced linguistic information [30,52].

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