

# Correspondence

## A New Consensus Model for Group Decision Making Problems With Non-Homogeneous Experts

Ignacio Javier Pérez, Francisco Javier Cabrerizo, Sergio Alonso, and Enrique Herrera-Viedma

**Abstract**—In the literature, we find that the consensus models proposed for group decision making problems are guided by consensus degrees and/or similarity measures and/or consistency measures [1]. When we work in heterogeneous group decision making frameworks, we have importance degrees associated with the experts by expressing their different knowledge levels on the problem. Usually, the importance degrees are applied in the weighted aggregation operators developed to solve the decision situations. In this paper, we study another application possibility, i.e., to use heterogeneity existing among experts to guide the consensus model. Thus, the main goal of this paper is to present a new consensus model for heterogeneous group decision making problems guided also by the heterogeneity criterion. It is also based on consensus degrees and similarity measures, but it presents a new feedback mechanism that adjusts the amount of advice required by each expert depending on his/her own relevance or importance level.

**Index Terms**—Consensus process, feedback mechanism, group decision making, heterogeneity.

### I. INTRODUCTION

Group decision making (GDM) consists of multiple individuals interacting to reach a decision. Each individual (expert) may have unique motivations or goals and thus, he/she may approach the decision process from a different angle, but all experts have a common interest in reaching eventual agreement on selecting the best option(s) [2]. To do so, experts have to express their preferences by means of a set of evaluations over a set of alternatives. Several authors have provided interesting results on GDM with the help of fuzzy sets theory [2], [3].

Usually, two processes are necessary to solve GDM problems [1]: a consensus process and a selection process. The consensus process is used to reach a final solution with a certain level of agreement among the experts. It is a dynamic and iterative process, composed of several rounds where the experts express, discuss, and modify their preferences. On the other hand, the selection process uses all individual preferences to obtain a collective solution. Clearly, it is preferable that the set of experts reach a high degree of consensus before applying the selection process. To achieve a high consensus level among the experts, it is useful to provide the whole group of experts with some advice (feedback information) on how far the

group is from consensus, what are the most controversial issues (alternatives), what preferences are in the highest disagreement with the rest of the group, how their change would influence the consensus degree, and so on.

Initially, GDM problems were defined in situations where all the experts' opinions were considered equally important. Different consensus models providing recommendations to the experts in order to increase the consensus level have been proposed in such situations [4]–[6]. In most cases these consensus models were guided by consensus degrees and/or similarity measures and/or consistency measures [1]. However, there are situations where the information handled by the experts is not equally important, and in such a heterogeneous decision context, it could be adequate to incorporate a heterogeneity criterion to guide the consensus model too.

To model such situations, the most usual approach in the literature consists of the assignment of weights to the experts, which reflect the relevance of the expert in the group, and the use of weighted aggregation operators in order to compute a weighted aggregation of their preferences [7]. However, with respect to modeling of the consensus process, we could assume that the most relevant experts are the main leaders of the discussion and therefore, they should be at the front of the negotiation to persuade the remaining experts in order to reach an agreement. Then, we could assume that the experts with highest importance degrees have deeper knowledge about the problem and they could require in the consensus reaching process a smaller quantity of recommendations than those with lowest ones. This would imply that the recommendation mechanism has to be adapted to provide a larger amount of recommendations to those experts with lowest importance degrees.

The aim of this paper is to propose a new consensus model to overcome this issue. To do so, this new consensus model takes into account the experts' importance weights not only to aggregate the experts' preferences but also when advising experts to change their preferences. In such a way, we design a consensus model that is also guided by heterogeneity criteria. As its main novelty, this consensus model incorporates a new feedback mechanism that adjusts the amount of advice provided to each expert depending on his/her own knowledge level about the problem.

The rest of the paper is set out as follows. Some general considerations about GDM and consensual processes are shown in Section II. The new consensus model is detailed in Section III. A practical example is illustrated in Section IV. Finally, in Section V, we point out our remarks and discuss the advantages and drawbacks of the new consensus model.

### II. PRELIMINARIES

In this section we show the main elements and features of GDM problems and consensus processes.

#### A. Group Decision Making

One of the reasons why decision making processes have been widely studied in the literature is the increasing complexity of the social-economic environment. It is obvious that the comparison of different actions according to their desirability in decision problems, in many cases, cannot be done by using a single criterion or a unique person. Hence, we focus on the decision processes in the framework of GDM.

In a classical GDM situation, there is a problem to solve, a set of feasible alternatives,  $X = \{x_1, x_2, \dots, x_n\}$ , ( $n \geq 2$ ), and a group of two or more experts,  $E = \{e_1, e_2, \dots, e_m\}$ , ( $m \geq 2$ ), characterized by

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their own ideas, attitudes, motivations, and knowledge, who express their opinions about this set of alternatives to achieve a common solution.

The usual resolution methods for GDM problems are composed by two different processes: consensus process and selection process [1]. Moreover, GDM problems can be defined in three different heterogeneous frameworks.

- 1) A first heterogeneous framework appears when we use different preference representation formats. Experts may represent their opinions about alternatives using different preference representation formats, such as orderings of alternatives or ranking of alternatives, utility functions (i.e., expressing a utility evaluation for each alternative), preference relations, (i.e., expressing a preference degree on each possible pair of alternatives), and so on [8], [9].
- 2) A second heterogeneous framework is focused on the expression domain used by experts to provide or express their particular preferences on each alternative or pair of alternatives. In some cases, experts may belong to distinct research areas and, therefore, they may express their preferences using different expression domains, such as numeric ones, linguistic ones, multigranular linguistic domains, unbalanced linguistic domains, expression domains based on interval numbers, hesitant fuzzy sets or intuitionistic fuzzy sets, and so on [5], [10]–[12].
- 3) A third heterogeneous framework appears when there are different backgrounds and levels of knowledge about the problem for each expert. Some classical models tackle this heterogeneity by assigning a weight value to each expert that is used in the aggregation phases to model their different importance levels or knowledge degrees [7]. The general procedure for the inclusion of the importance degrees in the aggregation process involves the transformation of the preference values under the importance degree to generate new values. Using induced ordered weighted aggregating operators [13] we can find an alternative way of implementing these importance degrees in the resolution process of a GDM problem.

In this paper, we focus on the third heterogeneity framework, i.e., we assume that each expert,  $e_k \in E$ , has an importance degree assigned. The importance is interpreted as a fuzzy subset,  $I$ , with a membership function,  $\mu_I : E \rightarrow [0, 1]$ , in such a way that  $\mu_I(e_k) \in [0, 1]$  denotes the importance degree of the opinion provided by the expert  $e_k$  [13]. Additionally, we assume that the experts provide their preferences using fuzzy preference relations because of their effectiveness as a tool for modeling decision processes and their utility and easiness of use when we want to aggregate experts' preferences into group ones. Moreover, pairwise preference relations provide users with more flexibility to express their real preferences than other representation formats.

*Definition 1:* A fuzzy preference relation  $P$  on a set of alternatives  $X$  is a fuzzy set on the product set  $X \times X$ , i.e., it is characterized by a membership function  $\mu_P : X \times X \rightarrow [0, 1]$ .

### B. Consensus Process

A consensus reaching process in a GDM problem is an iterative process composed by several discussion rounds in which experts are expected to modify their preferences according to the advice given by a moderator. The moderator plays a key role in this process. Normally, the moderator is a person who does not participate in the discussion but knows the preferences of each expert and the level of agreement during the consensus process. He/she is in charge of supervising and driving the consensus process toward success, i.e., to achieve the maximum possible agreement and reduce the number of experts outside of the consensus in each round.

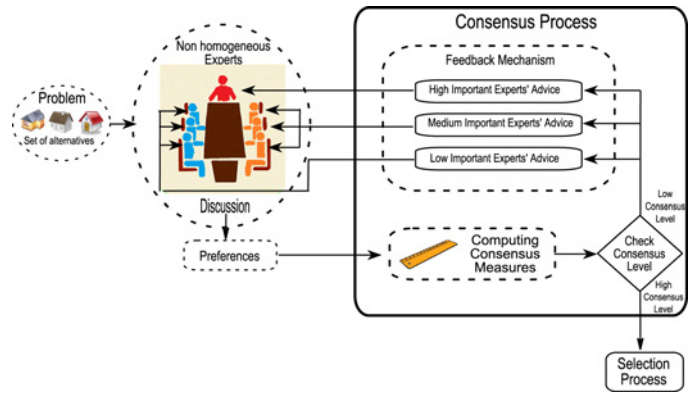


Fig. 1. Consensus reaching process with non-homogeneous experts.

The main tasks carried out by the moderator are: 1) computing the consensus measures, 2) checking the level of agreement, and 3) generating some advice for those experts that should change their minds.

In order to evaluate the agreement achieved among the experts, it is required to compute coincidence existing among them. According to [1], usual consensus models could use two kind of measures to guide the consensus processes: consensus measures to identify the preference values where the agreement is not sufficient and similarity measures to identify the experts who should change their preferences in the following rounds. In such a way, we could be able to automatize the moderator's activity [10].

### III. CONSENSUS MODEL FOR HETEROGENEOUS GDM

Different consensus models substituting the moderator's actions have been proposed with the aim of generating recommendations to the experts to increase the consensus level achieved among all the experts. Usually, these consensus models use consensus measures and/or consistency measures to guide the feedback mechanism that substitutes the moderator. However, these consensus models do not take into account the different levels of knowledge of the experts that participate in the GDM problem and they consider all the experts' opinions equally important. However, it is obvious that the heterogeneity of the experts should be taken into account in the consensus process to reach a desired global consensus degree in a more appropriate and realistic way. To deal with this issue, a feedback mechanism should be incorporated to the consensus model to give advice to the experts in function of their respective importance degrees. If the agreement among the experts is low, it seems reasonable to send more advice information to those experts with less importance or knowledge level. As the level of knowledge of the expert increases, less recommendations should be sent to him/her. In other words, the necessary amount of advice required by each expert depends on his/her own knowledge about the problem. Thus, a consensus model that implements this idea will improve GDM processes. In the following, we propose a consensus model incorporating a new feedback mechanism that replaces and automates the moderator's tasks by computing and sending customized recommendations to the experts according to their own importance degrees.

The new consensus model is composed of two different stages (see Fig. 1). In the following subsections, both stages are described in detail.

#### A. Computing Consensus Measures and Consensus Control Process

As previously mentioned in Section II-A, we assume that the experts provide their preferences about the alternatives using fuzzy

preference relations. Once the preferences have been given by the experts, we can compute the level of agreement achieved in the current consensus round. Consensus degrees are used to measure the current level of consensus in the decision process and they are given at three different levels of a preference relation: pairs of alternatives, alternatives, and relations.

- 1) For each pair of experts ( $e_k, e_l$ ) ( $k = 1, \dots, m-1, l = k+1, \dots, m$ ) a Similarity matrix,  $SM^{kl} = (sm_{ij}^{kl})$ , is defined

$$sm_{ij}^{kl} = 1 - |p_{ij}^k - p_{ij}^l|.$$

- 2) A consensus matrix,  $CM = (cm_{ij})$ , is calculated by aggregating all the Similarity matrices using the arithmetic mean as the aggregation function,  $\phi$ , although different aggregation operators could be used depending on the nature of the GDM problem to solve

$$cm_{ij} = \phi(sm_{ij}^{kl}, k = 1, \dots, m-1, l = k+1, \dots, m).$$

- 3) Once the consensus matrix is computed, the consensus degrees are obtained at three different levels.

- a) Consensus degree on pairs of alternatives

$$cp_{ij} = cm_{ij}$$

- b) Consensus degree on alternatives

$$ca_i = \frac{\sum_{j=1, j \neq i}^n (cp_{ij} + cp_{ji})}{2(n-1)}$$

- c) Consensus degree on the relation

$$cr = \frac{\sum_{i=1}^n ca_i}{n}.$$

Once the consensus measure,  $cr$ , is obtained, it is compared with the minimum required consensus level,  $cl \in [0, 1]$ , which will depend on the particular problem we are dealing with. When  $cr \geq cl$ , the consensus model finishes and the selection process is applied to obtain the solution. Otherwise, the feedback mechanism should be activated and a new consensus round is applied. Additionally, the consensus model should avoid situations in which the global consensus measure may not converge to the minimum required consensus level. To do that, a maximum number of rounds,  $MaxRounds$ , should be fixed [10].

## B. Feedback Mechanism

We present a new feedback mechanism to guide the change of the controversial experts' opinions with the aim of modeling those GDM situations in which the experts' knowledge level is quite different among them. To do so, we use the heterogeneity of the experts in a new way as the experts' importance is not only used in the computation of the global preference but also to generate recommendations to the experts according to their importance or level of knowledge.

This new feedback mechanism is based on the supposition that those experts with lower knowledge level on the problem will need more advice than others with higher importance.

We propose to compute a customized amount of advice that varies in accordance with the experts' weight values. To do so, the experts are classified according to their importance degrees,  $\mu_I(e_k)$ , into one of the three following groups: 1) high-importance experts,  $E_{high}$ , 2) medium-importance experts,  $E_{med}$ , and 3) low-importance experts,  $E_{low}$ . This classification is done by means of a fuzzy matching mechanism whose parameters depend on the problem dealt with. In such a way, each group of experts is a fuzzy set characterized by a membership function and we establish two parameters  $\lambda_1$  and  $\lambda_2$  as membership thresholds. Then, using the importance degree of each an expert, we can classify an expert in a particular group of experts.

We also define three different advising strategies to identify the preferences that each expert should modify to increase the consensus level in the next consensus round: 1) advising high-importance experts, 2) advising medium-importance experts, and 3) advising low-importance experts. For each group of experts we have a different search policy to identify the preferences with low agreement degree (controversial preferences).

To determine the degree of agreement between each individual and the group, similarity measures are used. To compute them for each expert, we need to obtain the collective fuzzy preference relation,  $P^c$ , which summarizes the preferences given by all the experts. The collective preference,  $P^c = (p_{ij}^c)$ , is computed by means of the aggregation of all individual preference relations,  $\{P^1, P^2, \dots, P^m\}$ :  $p_{ij}^c = \Phi(p_{ij}^1, p_{ij}^2, \dots, p_{ij}^m)$ , with,  $\Phi$ , an appropriate aggregation operator. The general procedure for the inclusion of importance weight values in the aggregation process involves the transformation of the preference values,  $p_{ij}^k$ , under the importance degree,  $\mu_I(e_k)$ , to generate a new value,  $\tilde{p}_{ij}^k$ , and then to aggregate these new values using an aggregation operator.

Once the collective preference matrix has been obtained, the similarity measures in each level of a fuzzy preference relation are computed.

- a) Similarity measure on pairs of alternatives

$$pp_{ij}^k = 1 - |p_{ij}^k - p_{ij}^c|.$$

- b) Similarity measure on alternatives

$$pa_i^k = \frac{\sum_{j=1, j \neq i}^n (pp_{ij}^k + pp_{ji}^k)}{2(n-1)}.$$

- c) Similarity measure on the relation

$$pr^k = \frac{\sum_{i=1}^n pa_i^k}{n}.$$

Once the similarity measures have been computed, the feedback mechanism may use them to generate personalized advice to the experts. This activity is carried out in two phases: 1) search for preferences, and 2) generation of advice.

1) *Search for Preferences*: As we consider three group of experts depending on their importance, we propose three different identification strategies to find the controversial preferences, respectively.

- a) Identify Low-Importance Experts' Controversial Preferences: Taking into account just the experts' subset,  $E_{low}$ , the feedback mechanism has to advise experts with low knowledge or confidence level. Thus, the consensus level should be improved by suggesting important changes in the experts' preferences. To do it, the procedure tries, for all the experts in this subset, to suggest modifications of the preference values on all the pairs of alternatives where the agreement is not high enough. In order to find the set of preferences to be changed by each expert,  $e_k \in E_{low}$ , this strategy acts as follows:

- i) The pairs of alternatives,  $P$ , with a consensus degree smaller than a threshold,  $\alpha_1$ , are identified

$$P = \{(i, j) \mid cp_{ij} < \alpha_1\}$$

$$\text{Where } \alpha_1 = \sum_{i=1}^n (\sum_{j=1, j \neq i}^n cp_{ij}) / (n^2 - n).$$

- ii) Finally, the set of controversial preferences,  $PCH_{low}^k$ , to be changed by each expert,  $e_k \in E_{low}$ , is

$$PCH_{low}^k = P.$$

- b) Identify Medium-Importance Experts' Controversial Preferences: In this case, where we consider just the experts' subset,  $E_{med}$ , it seems reasonable to reduce the number of changes and to modify the point of view for the analysis of the agreement.

While in the previous strategy we focused on all the pairs of alternatives in disagreement, now, the agreement is analyzed from the point of view of the alternatives. Thus, the system will only consider the preference values in disagreement of those alternatives where agreement is not high enough. Another important difference is the number of experts involved in the change of preferences. While in the previous strategy all experts were required to modify the identified preference values, in this case, the system just propose changes to those experts with a similarity value at level of alternatives, for those identified alternatives in disagreement, smaller than a similarity threshold  $\beta_1$ . Where  $\beta_1 = \sum_{k=1}^m pa_i^k / m$ . This strategy finds out the set of preferences to be changed by each expert,  $e_k \in E_{med}$ , as follows:

- i) Initially, alternatives to be changed,  $XCH$ , are identified. A new dynamic threshold at level of alternatives  $\alpha_2$  is suggested, in this case, as the average of the consensus degrees at level of alternatives, that is,  $\alpha_2 = \sum_i ca_i / n$ . Then

$$XCH = \{i \mid ca_i < \alpha_2\}.$$

- ii) Now, pairs of alternatives to be changed,  $P$ , are identified as

$$P = \{(i, j) \mid i \in XCH \wedge cp_{ij} < \alpha_1\}.$$

- iii) Finally, the set of preference values,  $PCH_{med}^k$ , that are required to be modified is

$$PCH_{med}^k = \{(i, j) \in P \mid pa_i^k < \beta_1\}.$$

- c) Identify High-Importance Experts' Controversial Preferences: In this situation, we are only dealing with the experts' subset,  $E_{high}$ , whose knowledge level is so high that expert preferences do not need to be strongly modified to get a well considered preference. Therefore, the agreement should be improved by suggesting fewer changes than in the previous two cases. We only need to change the mind of those experts who have similarity values, on the pairs of alternatives that are hindering the agreement, smaller than an specific similarity threshold at level of pairs of alternatives. To do so, we propose a new dynamic threshold  $\beta_2 = \sum_{k=1}^m pp_{ij}^k / m$ .

- a) Initially, alternatives to be changed,  $XCH$ , are identified

$$XCH = \{i \mid ca_i < \alpha_2\}.$$

- b) Now, pairs of alternatives to be changed,  $P$ , are identified as

$$P = \{(i, j) \mid i \in XCH \wedge cp_{ij} < \alpha_1\}.$$

- c) Finally, the set of preference values,  $PCH_{high}^k$ , that are required to be modified will be

$$PCH_{high}^k = \{(i, j) \in P \mid pa_i^k < \beta_1 \wedge pp_{ij}^k < \beta_2\}.$$

In short, the higher the knowledge level of an expert, the lower the number of changes that he/she is suggested, and the lower the knowledge level of an expert, the higher the number of changes that he/she is suggested.

2) *Generation of Advice*: Once the feedback mechanism has isolated the preferences to be changed by the experts depending on the importance degree of each one, the model shows the right direction of the changes to achieve the agreement. In this paper, we use a mechanism based on a set of direction rules to suggest the changes. For each preference value identified as controversial, the model will suggest increasing the current assessment if  $p_{ij}^k < p_{ij}^c$  or decreasing it if  $p_{ij}^k > p_{ij}^c$ .

It is worth noting that the changes suggested are just recommendations presented to show to the experts the most appropriate way

to narrow their positions. Then, each expert must decide, on his/her own, if and how to take the received advice into account.

Finally, we should point out that sometimes, when the whole ranking of alternatives is not important and experts just need to select the most valued alternatives, the consensus process could be optimized. To do so, some of the stages to reach agreement on the pairs of the remaining alternatives could be avoided.

#### IV. EXAMPLE OF APPLICATION

Suppose that university managers want to invest some money in improving some services of its academic library. These services are  $x_1 =$  increase library space,  $x_2 =$  hiring librarians,  $x_3 =$  improve web site,  $x_4 =$  increase number of online resources. In this way, the best position of the service in the ranking, the more money the service will receive. To do so, it is necessary to inquire some individuals (library staff or users) about their opinions  $\{e_1 = \text{LibraryManager}, e_2 = \text{Librarian}, e_3 = \text{User1}, e_4 = \text{User2}\}$ . This group is formed by individuals with different levels of knowledge about the services of the library. Due to this fact, we could assign them the following weight values:

$$\mu_1(e_1) = 0.35, \mu_1(e_2) = 0.25, \mu_1(e_3) = 0.20, \mu_1(e_4) = 0.20.$$

Initially, they provide the following fuzzy preference relations

$$P^1 = \begin{pmatrix} - & 0.9 & 0.9 & 0.9 \\ 0.1 & - & 0.7 & 0.8 \\ 0.2 & 0.3 & - & 0.5 \\ 0.2 & 0.2 & 0.5 & - \end{pmatrix} \quad P^2 = \begin{pmatrix} - & 0.1 & 0.3 & 0.3 \\ 0.9 & - & 0.8 & 0.9 \\ 0.7 & 0.2 & - & 0.5 \\ 0.8 & 0.1 & 0.5 & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & 0.5 & 0.3 & 0.3 \\ 0.5 & - & 0.2 & 0.2 \\ 0.8 & 0.8 & - & 0.5 \\ 0.7 & 0.7 & 0.5 & - \end{pmatrix} \quad P^4 = \begin{pmatrix} - & 0.5 & 0.3 & 0.3 \\ 0.5 & - & 0.2 & 0.2 \\ 0.8 & 0.8 & - & 0.5 \\ 0.7 & 0.7 & 0.5 & - \end{pmatrix}.$$

The suitable parameters applied in this example are:  $cl = 0.78$ ,  $MaxRounds = 10$ ,  $\lambda_1 = 0.25$ ,  $\lambda_2 = 0.35$ .

##### A. First Round

1) *Computing Consensus Measures and Controlling the Consensus Process*:

- a) Computing consensus degrees:  $cr = 0.68$ .
- b) Controlling the consensus process: As  $cr < cl$ , and  $1 < MaxRounds$ , the feedback mechanism is activated.

2) *Feedback Mechanism*:

- 1) Computing similarity measures

$$pr_1 = 0.74, pr_2 = 0.77, pr_3 = 0.77, pr_4 = 0.82.$$

- 2) Search for preferences: In order to compute customized recommendations, the experts are included by their own importance degree into three different subsets:  $E_{low} = \{e_3, e_4\}$ ,  $E_{med} = \{e_2\}$ ,  $E_{high} = \{e_1\}$ .

- a) Identify low-importance experts' controversial preferences.

- i) Identification of pairs of alternatives with low consensus degree  $P = \{(1, 2), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 2)\}$ .

- ii) Set of preferences to be changed by each expert  $e_k$  in  $E_{low}$ :  $PCH_{low}^k = P$ .

- b) Identify medium-importance experts' controversial preferences.

- i) Identification of the alternatives:  $XCH = \{1, 2\}$ .

- ii) Identification of the preference values in disagreement at the previous alternatives  $P = \{(1, 2), (1, 3), (2, 1), (2, 3), (2, 4)\}$ .

- iii) Set of preferences to be changed by each expert  $e_k$  in  $E_{med}$ :  $PCH_{med}^k = P$ .

- c) Identify high-importance experts' controversial preferences.

- i) Identification of the alternatives:  $XCH = \{1, 2\}$ .
  - ii) Identification of the preference values in disagreement at the previous alternatives  $P = \{(1, 2), (1, 3), (2, 1), (2, 3), (2, 4)\}$ .
  - iii) Set of preferences to be changed by each expert  $e_k$  in  $E_{high}$   $PCH_{high}^1 = \{(1, 2), (1, 3)\}$ .
- 3) Generation of advice: According to the direction rules, the experts are required to modify their preferences receiving the following recommendations:

$$R^1 = \begin{pmatrix} = & - & - & = \\ = & = & = & = \\ = & = & = & = \\ = & + & + & = \\ - & - & + & + \\ - & - & + & + \\ = & - & = & = \end{pmatrix} \quad R^2 = \begin{pmatrix} & + & + & = \\ = & = & - & = \\ = & = & = & = \\ = & + & + & = \\ - & - & + & + \\ - & - & + & + \\ = & - & = & = \end{pmatrix}$$

where  $R_{ij}^k = +/−$ , express the recommendation to the expert  $e_k$  to increase/decrease his/her preference  $p_{ij}^k$ .

After some consensus rounds, experts reached consensus enough following the virtual moderator recommendations. Finally, the consensual ranking of alternatives obtained in the selection process was  $x_1 > x_2 > x_3 > x_4$ .

## V. CONCLUSION

In this paper, we proposed a novel consensus approach that has been specially designed to model GDM frameworks with heterogeneous experts. Assuming experts with different levels of importance and fuzzy preference relations to express their opinions, we presented a consensus model incorporating a new feedback mechanism that computes different amount of advice according to the experts' importance level. To do so, different identification strategies to find out the controversial preferences furthest from the collective ones have been defined. Consequently, the most considerable experts' opinions never will be strongly modified during the consensus reaching process. Therefore, the computed advice is more suitable, pertinent, and customized.

In the following, regarding the previous published approaches, we analyze and discuss the main advantages and drawbacks of our proposal.

We should point out the following advantages.

- 1) With this proposal, we overcome the problem of the moderator [1], giving the way to use an automatic system to compute and send customized advice to the experts if there is not enough consensus.
- 2) This new model lets the importance be a new feature of the whole decision process. It seems reasonable and necessary, because the more capable experts can be better considered in the decision process as their opinions exert more influence than others. In such a way, experts receive personalized advice according to their own importance.
- 3) This new feature allows that the concept of importance guides the decision making process, obtaining the most appropriate collective solutions. In such a way, using the same data (experts, alternatives, and initial parameters) that in the example of the previous section and using three different previous approaches 1) a classical feedback mechanism [9], 2) an induced averaging operator (IOWA) [13], and 3) an adaptive consensus model [10], we have obtained the following results.
  - a)  $x_2 > x_3 > x_4 > x_1$ .
  - b)  $x_2 > x_3 > x_1 > x_4$ .
  - c)  $x_3 > x_2 > x_4 > x_1$ .

However, according to both high importance experts,  $e_1$  and  $e_2$ , the best alternatives should be  $x_1$  and  $x_2$ . This issue is considered in our feedback mechanism and therefore, with our

consensus model we would obtain a more realistic result, i.e.,  $x_1 > x_2 > x_3 > x_4$ .

- 4) Usually, the importance has been taken into account in some aggregation processes by means of weighted aggregation operators. However, we show a new way to drive a negotiation process guided by importance, filling this gap of classical GDM models.

On the other hand, we should point out the following drawbacks.

- 1) The final solution obtained by using this model tries to obey the fuzzy majority principle defined by Kacprzyk [14]. However, there could exist a limit scenario where the tyranny of the minority is accomplished if the excellence group is very small inside of the set of experts.
- 2) There exists the possibility of no convergence. That is, if some experts do not accept this kind of excellence guided model, they will not follow the recommendations or change their preferences to disrupt the consensus reaching process.
- 3) This model is not able to detect when a high importance expert is wrong or inconsistent. As a future work, we propose the incorporation of a mechanism to check the consistency of the experts [15] to reduce their importance degree and, therefore, their impact in the decision making process if the expressed preferences are inconsistent.

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