International Journal of Information Technology & Decision Making
Vol. 11, No. 4 (2012) 709–725

DOI: 10.1142/S0219622012500174

A CONSENSUS MODEL FOR GROUP DECISION-MAKING PROBLEMS WITH INTERVAL FUZZY PREFERENCE RELATIONS

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Interval fuzzy preference relations can be useful to express decision makers' preferences in group decision-making problems. Usually, we apply a selection process and a consensus process to solve a group decision situation. In this paper, we present a consensus model for group decision-making problems with interval fuzzy preference relations. This model is based on two consensus criteria, a consensus measure and a proximity measure, and also on the concept of coincidence among preferences. We compute both consensus criteria in the three representation levels of a preference relation and design an automatic feedback mechanism to guide experts in the consensus reaching process. We show an application example in social work.

Keywords: Group decision-making; consensus; interval fuzzy preference relations.

1. Introduction

Group decision-making (GDM) problems are characterized as a process of choosing the best alternative/s from a set of alternatives. In decision making a preference relation is the most common representation format used to represent the experts' preferences because it is very useful in expressing information about alternatives. We find there are many kinds of preference relations in the literatures, as binary preference relations,[1] fuzzy preference relations,[2–10] multiplicative preference relations,[11,12] interval fuzzy preference relations,[13–17] linguistic preference relations,[18–25] multi-granular preference relations,[26–28] etc.

In a usual fuzzy framework, there are a finite set of alternatives and a finite set of experts and each expert provides his/her opinions on the set of alternatives as a fuzzy preference relation.[1] During the last years fuzzy preference relations have received much attention. However, in a fuzzy preference relation an expert could have a vague knowledge about the preference degree of the alternative i over j and could not estimate his/her preference with an exact numerical value. In such cases, it is useful to use interval fuzzy preference relations.[13,15,16,29–32]

A usual resolution method for a GDM problem is composed of two different processes[20,22,33,34]:

(1) *Consensus process*: Clearly, in any decision process, it is preferable that the experts reach a high degree of consensus on the solution set of alternatives. Thus, this process refers to how to obtain the maximum degree of consensus or agreement among the experts on the solution alternatives.
(2) *Selection process*: This process consists in how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts.

In the literature, we can find some proposals of selection processes for GDM problems under interval fuzzy preference relations.[13–16] Up to date, however no investigation has been devoted to model the consensus in GDM problems under interval fuzzy preference relations. This paper is focused on the definition of a new consensus model for GDM problems with interval fuzzy preference relations.

In GDM problems, a group of experts initially can have disagreeing preferences and it is necessary to develop a consensus reaching process. Usually, a consensus reaching process can be viewed as a dynamic process where a moderator via exchange of information and rational arguments, tries the experts to update their opinions. In each step, the degree of actual consensus and the distance from an ideal consensus is measured. This is repeated until the distance to the ideal consensus is considered sufficiently small. Traditionally, the ideal consensus meant as a full and unanimous agreement of all experts' preferences. This type of consensus is a utopian consensus and it is very difficult to achieve it. This has led to the use and definition of a new concept called "soft" consensus degree[35–37] which assesses the consensus degree in a more flexible way. The soft consensus measures that allow to measure the closeness among experts' opinions are based on the concept of coincidence.[38–40] We can identify three different approaches to apply coincidence criteria to compute soft consensus measures[38]:

(1) *Consensus models based on strict coincidence among preferences*. In this case, similarity criteria among preferences provided by the experts are used to compute the coincidence concept. Only two possible results are assumed: the total coincidence (value 1) or null coincidence (value 0).[34,35,41]
(2) *Consensus models based on soft coincidence among preferences*. As stated above, similarity criteria among preferences are used to compute the coincidence concept. However, in this case, a major number of possible coincidence degrees are

considered. It is assumed that the coincidence concepts is a gradual concept, which could be assessed with different degrees defined in the unit interval $[0, 1]$. These are the most popular consensus models.[7,9,19,22,26,35–40,42,43]

(3) *Consensus models based on coincidence among solutions.* In this case, similarity criteria among the solutions obtained from the experts' preferences are used to compute the coincidence concept and different degrees assessed in $[0, 1]$ are assumed. Basically, we compare the positions of the alternatives between the individual solutions and the collective solution, which allows to know better the real consensus situation in each moment of the consensus process.[44–47]

The aim of this paper is to present a consensus model based on soft coincidence among preferences for GDM problems under interval fuzzy preference relations. As in Refs. 34, 40 and 41 this new consensus model is based on two consensus criteria to guide the consensus reaching process:

(1) *A consensus measure.* This measure evaluates the agreement of all the experts. It is used to guide the consensus process until the final solution is achieved.
(2) *A proximity measure.* This measure evaluates the agreement between the experts' individual opinions and the group opinion. It is used to guide the group discussion in the consensus process.

We compute both measures on the three levels of representation of an interval fuzzy preference relation: level of pair, level of alternative and level of relation. Then, we design an automatic feedback mechanism to guide experts in the consensus reaching process and substitute the moderator's activity.

This paper is set out as follows. The GDM problem based on interval fuzzy preference relations is described in Sec. 2. Section 3 presents the new consensus model. A practical example is given in Sec. 4. Finally, in Sec. 5 we draw our conclusions.

2. The GDM Problem Based on Interval Fuzzy Preference Relations

In this section we briefly describe the GDM problem based on interval fuzzy preference relations and the resolution process used to obtain the solution set of alternatives.

2.1. *The GDM problem*

Let $X = \{x_1, \ldots, x_n\} (n \geq 2)$ be a finite set of alternatives to be evaluated by a finite set of experts, $E = \{e^1, \ldots, e^m\} (m \geq 2)$. The GDM process consists to find the best alternative according to the experts' preferences $\{P^1, \ldots, P^m\}$.

In a usual GDM problem we assume that the experts provide their preferences on X by means of the fuzzy preference relations, $P^k \subset X \times X$, with membership function

$$\mu_{pk} : X \times X \to [0, 1],$$

where $\mu_{pk}(x_i, x_j) = p^k_{ij}$ denotes the preference degree of the alternative x_i over x_j:[48]

- $p^k_{ij} = 1/2$ indicates indifference between x_i and x_j,
- $p^k_{ij} = 1$ indicates that x_i is unanimously preferred to x_j, and
- $p^k_{ij} > 1/2$ indicates that x_i is preferred to x_j.

Furthermore, it is usual to assume that P^k is reciprocal,[4,8,10] i.e., $p^k_{ij} + p^k_{ji} = 1$ and $p^k_{ii} = -$ (undefined).

In this paper we assume that the experts' preferences on X are described by means of the interval fuzzy preference relation,[15,16] $P^k \subset X \times X$, with membership function

$$\mu'_{pk} : X \times X \to [0, 1],$$

where $\mu'_{pk}(x_i, x_j) = [p^{k-}_{ij}, p^{k+}_{ij}]$ denotes the interval fuzzy preference degree of the alternative x_i over x_j with $0 \leq p^{k-}_{ij} \leq p^{k+}_{ij} \leq 1/2$ or $1/2 \leq p^{k-}_{ij} \leq p^{k+}_{ij} \leq 1$, and $\mu'_{pk}(x_i, x_j)$ indicates that the preference degree of x_i over x_j is between p^{k-}_{ij} and p^{k+}_{ij} and

- if $p^{k-}_{ij} = p^{k+}_{ij} = 1/2$ indicates indifference between x_i and x_j,
- if $p^{k-}_{ij} = p^{k+}_{ij} = 1$ indicates that x_i is unanimously preferred to x_j, and finally
- if $(p^{k+}_{ij} > 1/2$ and $1/2 \leq p^{k-}_{ij})$ indicates that x_i is definitively preferred to x_j.

In this case, it is usual to assume that $p^{k-}_{ij} + p^{k+}_{ji} = p^{k+}_{ij} + p^{k-}_{ji} = 1$ and $p^{k+}_{ii} = p^{k-}_{ii} = -$.

2.2. *Resolution process of the GDM problem*

Usually, the resolution process of the GDM problem consists in obtaining a set of solution alternatives from the preferences given by the experts. As aforementioned, usually this resolution process is composed of two phases: consensus phase and selection phase. If we assume that the experts express their individual preferences by means of the interval fuzzy preference relations, then the resolution process would be as it is shown in Fig. 1.

The selection process is the last phase of the resolution process and allow us to obtain the solution set of alternatives. It is composed by two procedures[20,21,49,50]: (i) aggregation and (ii) exploitation.

(1) *Aggregation phase*
This phase defines a collective interval fuzzy preference relation obtained by means of the aggregation of all individual interval fuzzy preference relations. This collective relation, called U, indicates the global preference between every ordered pair of alternatives according to the majority experts' opinions. For example, a possibility to obtain U in the case of the interval fuzzy preference relations it would be to use the aggregation implemented by means of the *median* operator:

$$U = (U_{ij}) \quad \text{for } i, j = 1, \ldots, n \text{ with}$$

$$U_{ij} = U[p^-_{ij}, p^+_{ij}] = [median_k(p^{k-}_{ij}), median_k(p^{k+}_{ij})] \quad \text{for } k = 1, \ldots, m.$$

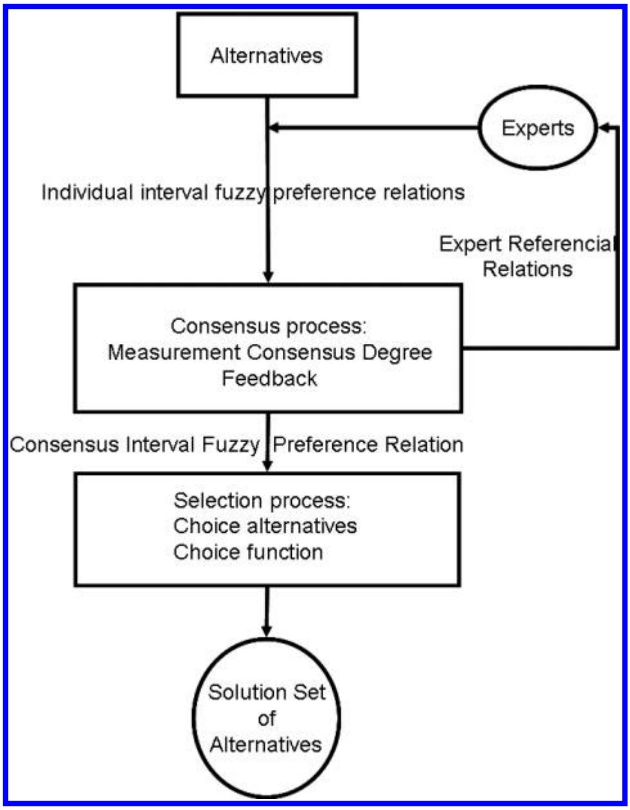

Fig. 1. Diagram of the GDM resolution process.

Example 1. Suppose that we want to invest some money and we have three possibilities: (i) buy a house, (ii) buy a plot of land and (iii) buy in stock exchange. Then we ask two experts and receive the following interval fuzzy preference relations:

$$e^1 = \begin{pmatrix} - & [0.2, 0.3] & [0.5, 0.7] \\ [0.7, 0.8] & - & [0.9, 1.0] \\ [0.3, 0.5] & [0.0, 0.1] & - \end{pmatrix},$$

$$e^2 = \begin{pmatrix} - & [0.3, 0.4] & [0.5, 0.5] \\ [0.6, 0.7] & - & [0.8, 0.9] \\ [0.5, 0.5] & [0.1, 0.2] & - \end{pmatrix}.$$

Therefore, using the previous aggregation tool we would obtain the following collective preference relation U:

$$U = \begin{pmatrix} - & [0.25, 0.35] & [0.50, 0.60] \\ [0.65, 0.75] & - & [0.85, 0.95] \\ [0.40, 0.50] & [0.05, 0.15] & - \end{pmatrix}.$$

(2) *Exploitation Phase*
This phase transforms the global and collective information about the alternatives into a global ranking of them, and then we choose the set of solution alternatives. To do so, it is usual choice functions of alternatives which applied on the collective preference relation allow us to obtain the ranking of alternatives.[51] For example, we could define choice functions using the dominance concept.[51] So, for each alternative x_i we could calculate its dominance degree px_i from the collective interval fuzzy preference relation as

$$px_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} (p_{ij}^{-} + p_{ij}^{+}).$$

In such a way, we obtain a classification of the alternatives:

$$\text{if } px_i > px_j \text{ then } x_i \text{ is preferable to } x_j.$$

Example 2. From the collective interval fuzzy preference relation obtained in Example 1 we could characterize each alternative with the following dominance degrees:

$$\begin{aligned} px_1 &= 0.25 + 0.35 + 0.50 + 0.60 = 1.7, \\ px_2 &= 0.65 + 0.75 + 0.85 + 0.95 = 3.2, \\ px_3 &= 0.40 + 0.50 + 0.05 + 0.15 = 1.1. \end{aligned}$$

So these alternatives can be classified from highest to lowest preference as:

$$x_2 > x_1 > x_3$$

and therefore, the alternative "buy a plot of land" is the recommended solution.

In Refs. 13–16 we can find different selection processes for GDM problems under interval fuzzy preference relations. As aforementioned, there does not exist consensus model to deal with GDM problems under interval fuzzy preference relations. In the following section, we present a consensus process for GDM problems with interval fuzzy preference relations.

3. Consensus Model

In this section we present a consensus model defined for GDM problems assuming that the experts express their preferences by means of the interval fuzzy preference relations. This model presents the following main characteristics:

(1) It is based on two soft consensus criteria: a consensus measure and a proximity measure.
(2) Both consensus criteria are defined using the coincidence among interval fuzzy preference relations provided by the experts.

(3) It incorporates a feedback mechanism that generates recommendations to the experts on how to change their interval fuzzy preference relations in the consensus reaching process.

Initially, we consider that in any nontrivial GDM problem the experts disagree in their opinions so that consensus has to be viewed as an iterate process, which means that the agreement is obtained only after many rounds of consultation. Then, in each round we calculate two consensus criteria, consensus measures and proximity measures.[34,40,41] The former evaluates the level of agreement among all the experts and it guides the consensus process, and the latter evaluates the distance between the experts' individual preferences and the collective one and it also supports the discussion phase of the consensus process. To do so, we compute the coincidence among interval fuzzy preference relations.

The main problem is how to find a way of making individual positions converge. To do this, a consensus level required for each decision situation is fixed in advance (A). When the consensus measure reaches this level then the decision-making session is finished and the solution is obtained applying a selection process. If that is not the case, the experts' opinions must be modified. This is done in a group discussion session in which a feedback mechanism is used to support the experts in changing their opinions. This feedback mechanism is defined using the proximity measures.[7,26,27,44] In order to avoid that the collective solution does not converge after several discussion rounds is possible to fix a maximum number of rounds. The scheme of this consensus model for GDM is presented in Fig. 2. In the following subsections we present the components of this consensus model in detail, i.e., the consensus criteria and the feedback mechanism.

3.1. *Consensus and proximity measures*

We calculate both consensus indicators in the following steps:

(1) First, we calculate the consensus relations of each expert e^k, called C^k, with respect to the collective preference relations as

$$C^k = (C^k_{ij}) \text{ with}$$

$$C^k_{ij} = |p^{k-}_{ij} - p^-_{ij}| + |p^{k+}_{ij} - p^+_{ij}| \quad \text{for } i, j = 1, \ldots, n.$$

In this consensus relation each value C^k_{ij} represents the agreement degree of the expert e^k with the group of experts on the preference p_{ij}.

(2) Then, we define the *consensus degree on a preference* p_{ij} as

$$CD_{ij} = 1 - \sum_{k=1}^{m} C^k_{ij}/m \quad \text{or} \quad CD_{ij} = \left(1 - \sum_{k=1}^{m} C^k_{ij}/m\right) * 100\%.$$

We have a total consensus in the preference p_{ij} if $CD_{ij} = 1$ or 100%.

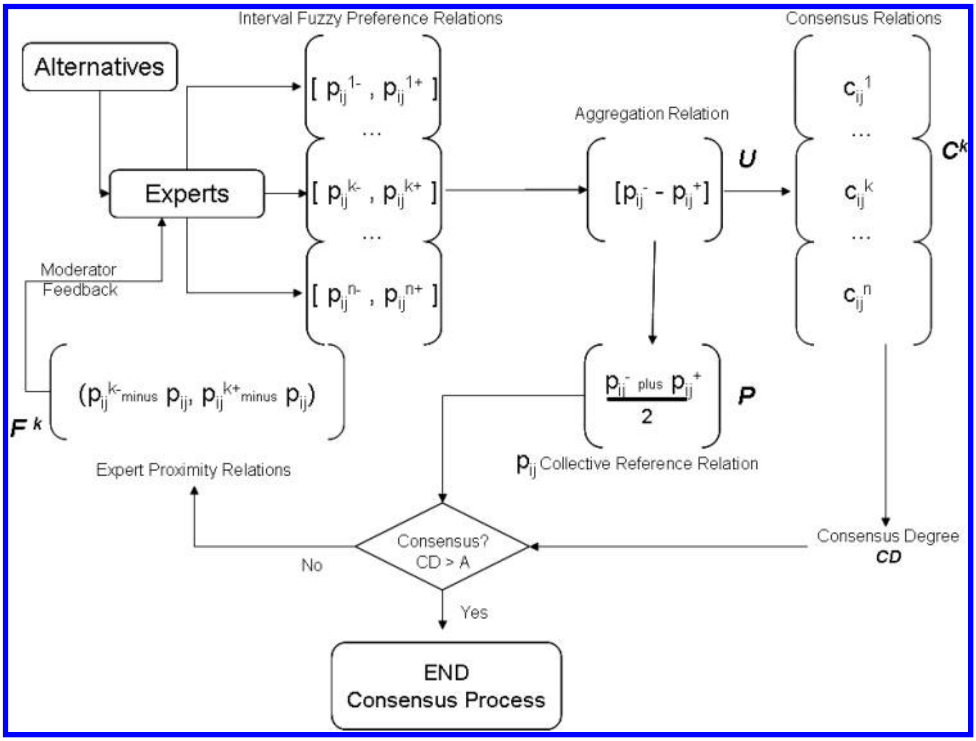

Fig. 2. Consensus model for GDM with interval fuzzy preference relations.

(3) We define the *consensus degree in the alternative* x_i as

$$CD_i = 1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{k=1}^{m} C_{ij}^k / ((n-1)m)$$

or

$$CD_i = \left(1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{k=1}^{m} C_{ij}^k / ((n-1)m) \right) * 100\% \,.$$

We have a total consensus in the alternative x_i if $CD_i = 1$ or 100%. So, we have:

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} CD_{ij}/(n-1) = CD_i.$$

(4) We define the *global consensus degree*, CD, as

$$CD = 1 - \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n^2-n)m)$$

or

$$CD = \left(1 - \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m} C_{ij}^{k}/((n^2-n)m)\right) * 100\%.$$

In this case, $0 \leq CD \leq 1$ or $0\% \leq CD \leq 100\%$. We have a total consensus in the process if $CD = 1$ or $CD = 100\%$.

Similarly, as stated in 3, we have:

$$\sum_{i=1}^{n} CD_i/n = CD.$$

Example 3. From the collective preference relation obtained in Example 1, we obtain the following two consensus relations

$$C^1 = \begin{pmatrix} - & 0.1 & 0.1 \\ 0.1 & - & 0.1 \\ 0.1 & 0.1 & - \end{pmatrix} \quad \text{and} \quad C^2 = \begin{pmatrix} - & 0.1 & 0.1 \\ 0.1 & - & 0.1 \\ 0.1 & 0.1 & - \end{pmatrix}$$

and therefore, the global consensus degree is $CD = 1 - 1.2/12 = 0.9$ or $CD = 90\%$, and for example, the consensus degree in the alternative x_1 is $CD_1 = 0.9$ or $CD_1 = 90\%$, and the consensus degree on the preference p_{23} is $CD_{23} = 0.90$ or $CD_{23} = 90\%$.

(5) Now, we continue the process to calculate the proximity measures. First, we calculate the expert proximity relations, called F^k, with respect to the collective preference relation U as

$$F^k = (F_{ij}^k) \text{ with}$$
$$F_{ij}^k = (p_{ij}^{k-} - p_{ij}, p_{ij}^{k+} - p_{ij}) = (f_{ij}^{k-}, f_{ij}^{k+}) \quad \text{for } i,j = 1,\ldots,n \quad \text{and}$$
$$p_{ij} = (p_{ij}^{-} + p_{ij}^{+})/2.$$

(6) Then, we define the *proximity measure of the expert e^k on a preference p_{ij}* as

$$PM_{ij}^k = (|f_{ij}^{k-}| + |f_{ij}^{k+}|)/2.$$

(7) Then, we define the *proximity measure of the expert e^k in an alternative x_i* as

$$PM_i^k = \sum_{\substack{j=1 \\ j\neq i}}^{n} PM_{ij}^k/(n-1).$$

(8) Then, we define the *global proximity measure of the expert e^k* as

$$PM^k = \sum_{i=1}^{n} PM_i^k/n.$$

Example 4. Using the data given in Example 1 we obtain the following expert proximity relations for experts e^1 and e^2, respectively:

$$F^1 = \begin{pmatrix} - & (-0.1, 0.0) & (-0.05, 0.15) \\ (0.0, +0.1) & - & (0.0, 0.1) \\ (-0.15, 0.05) & (-0.1, 0.0) & - \end{pmatrix},$$

$$F^2 = \begin{pmatrix} - & (0.0, 0.1) & (-0.05, -0.05) \\ (-0.1, 0.0) & - & (-0.1, 0.0) \\ (0.05, 0.05) & (0.0, 0.1) & - \end{pmatrix}.$$

We obtain proximity measures for experts for each alternative,

$$\begin{aligned} PM_1^1 &= 0.15/2, \quad PM_1^2 = 0.10/2, \\ PM_2^1 &= 0.10/2, \quad PM_2^2 = 0.10/2, \\ PM_3^1 &= 0.15/2, \quad PM_3^2 = 0.10/2, \end{aligned}$$

and for the set of preferences

$$PM^1 = 0.4/6, \quad PM^2 = 0.3/6.$$

3.2. *Moderator/feedback process*

As in Refs. 26, 27, 44 and 48, we can apply a feedback mechanism to guide the change of the expert's opinions with use proximity matrices F^k. This mechanism is able to help moderator in his/her tasks or even to substitute the moderator's actions in the consensus reaching process. In such a way, the feedback process helps experts to change their preferences in order to achieve an appropriate agreement degree. The main problem for the feedback mechanism is how to find a way of making individual positions converge and, therefore, how to support the experts in obtaining and agreeing with a particular solution.[44]

Usually, the feedback process is carried out in two phases: *Identification phase* and *Recommendation phase*.

(1) *Identification phase*: It is necessary to compare global consensus degree CD and a consensus threshold A, previously fixed. Then, if $CD > A$ or $CD = A$ the consensus process will stop, on the other hand, if $CD < A$ a new consensus round must be applied. If the agreement among all experts is low, then there exist a lot of experts' preferences in disagreement. In such a case, in order to bring the preferences closer to each other and so to improve the consensus situation, the number of changes in the experts' preferences should be high. However, if the agreement is high, the majority of preferences is close and only a low number of experts' preferences are in disagreement; it seems reasonable to change only these particular preferences. The procedure suggests modifying the preference values on all the pairs of alternatives where the agreement is not high enough. We finds

out the set of preferences to be changed as follows:

(a) First, the pairs of alternatives with a consensus degree smaller than a threshold value A defined at level of pairs of alternatives, $CD_{ij} < A$, are identified.

(b) Second, we identify the experts that will be required to modify the identified pairs of alternatives. To do that, we use the expert proximity measures PM^k and PM_i^k, and also we fix a threshold value B. The experts that are required to be modified are preferences whose $PM^k > B$.

(2) *Recommendation phase.* In this phase we recommend expert changes of their preferences according to some rules to change the opinions. Once the preferences to be changed and experts to send recommendations have been identified, we develop a recommendation phase. In this phase we apply recommendation rules that inform experts on the right direction of the changes in order to improve the agreement. We must find out the direction of change to be applied to the preference assessment p_{ij}^{k+} or p_{ij}^{k-} for each expert k on a preference. To do this, we define the following rules:

(a) If $(p_{ij}^{k-} - p_{ij}) = f_{ij}^{k-} > 0$ then expert e_k should decrease the assessment associated to the pair of alternatives (x_i, x_j).

(b) If $(p_{ij}^{k+} - p_{ij}) = f_{ij}^{k+} < 0$ then expert e_k should increase the assessment associated to the pair of alternatives (x_i, x_j).

(c) If $f_{ij}^{k-} < 0 < f_{ij}^{k+}$ then expert e_k should increase p_{ij}^{k-} and decrease p_{ij}^{k+} in the assessments associated to the pair of alternatives (x_i, x_j).

4. Example

Suppose that we have three experts in social work who want to find the best old people's home for an old person. Suppose that they have four possible old people's homes ($A = x_1, B = x_2, C = x_3, D = x_4$) and provide their preferences on them using the following interval fuzzy preference relations:

$$E^1 = \begin{pmatrix} - & [0.0, 0.1] & [0.6, 0.7] & [0.2, 0.3] \\ [0.9, 1.0] & - & [0.7, 1.0] & [0.5, 0.7] \\ [0.3, 0.4] & [0.0, 0.3] & - & [0.2, 0.3] \\ [0.7, 0.8] & [0.3, 0.5] & [0.7, 0.8] & - \end{pmatrix},$$

$$E^2 = \begin{pmatrix} - & [0.3, 0.4] & [0.5, 0.5] & [0.1, 0.4] \\ [0.6, 0.7] & - & [0.6, 0.8] & [0.7, 0.9] \\ [0.5, 0.5] & [0.2, 0.4] & - & [0.0, 0.2] \\ [0.6, 0.9] & [0.1, 0.3] & [0.8, 1.0] & - \end{pmatrix},$$

$$E^3 = \begin{pmatrix} - & [0.4, 0.5] & [0.2, 0.5] & [0.5, 0.5] \\ [0.5, 0.6] & - & [0.3, 0.5] & [0.8, 1.0] \\ [0.5, 0.8] & [0.5, 0.7] & - & [0.6, 0.8] \\ [0.5, 0.5] & [0.0, 0.2] & [0.2, 0.4] & - \end{pmatrix}.$$

Then, we obtain the following collective interval fuzzy preference relation:

$$U = \begin{pmatrix} - & [0.3, 0.4] & [0.5, 0.5] & [0.2, 0.4] \\ [0.6, 0.7] & - & [0.6, 0.8] & [0.7, 0.9] \\ [0.5, 0.5] & [0.2, 0.4] & - & [0.2, 0.3] \\ [0.6, 0.8] & [0.1, 0.3] & [0.7, 0.8] & - \end{pmatrix}.$$

Now, we calculate the consensus relations of each social worker

$$C^1 = \begin{pmatrix} - & 0.6 & 0.3 & 0.1 \\ 0.6 & - & 0.3 & 0.4 \\ 0.3 & 0.3 & - & 0.0 \\ 0.1 & 0.4 & 0.0 & - \end{pmatrix}, \quad C^2 = \begin{pmatrix} - & 0.0 & 0.0 & 0.1 \\ 0.0 & - & 0.0 & 0.0 \\ 0.0 & 0.0 & - & 0.3 \\ 0.1 & 0.0 & 0.3 & - \end{pmatrix},$$

$$C^3 = \begin{pmatrix} - & 0.2 & 0.3 & 0.4 \\ 0.2 & - & 0.6 & 0.2 \\ 0.3 & 0.6 & - & 0.9 \\ 0.4 & 0.2 & 0.9 & - \end{pmatrix}.$$

Therefore, consensus degrees on the preferences $[p_{ij}]$ are

$$\begin{pmatrix} - & 0.73 & 0.80 & 0.80 \\ 0.73 & - & 0.70 & 0.80 \\ 0.80 & 0.70 & - & 0.60 \\ 0.80 & 0.80 & 0.60 & - \end{pmatrix}$$

and the global consensus degree is $CD = 0.7333$ or $CD = 73.33\%$. If we fix a consensus threshold $A = 3/4 = 0.75$ then it seems unacceptable to finish the decision-making process.

Then, we calculate F^k for each expert

$$F^1 = \begin{pmatrix} - & (-0.35, -0.25) & (0.1, 0.2) & (-0.1, 0.0) \\ (0.25, 0.35) & - & (0.0, 0.3) & (-0.3, -0.1) \\ (-0.2, -0.1) & (-0.3, 0.0) & - & (-0.05, 0.05) \\ (0.0, 0.1) & (0.1, 0.3) & (-0.05, 0.05) & - \end{pmatrix},$$

$$F^2 = \begin{pmatrix} - & (-0.05, 0.05) & (0.0, 0.0) & (-0.2, 0.1) \\ (-0.05, 0.05) & - & (-0.1, 0.1) & (-0.1, 0.1) \\ (0.0, 0.0) & (-0.1, 0.1) & - & (-0.25, -0.05) \\ (-0.1, 0.2) & (-0.1, 0.1) & (0.05, 0.25) & - \end{pmatrix},$$

$$F^3 = \begin{pmatrix} - & (0.05, 0.15) & (-0.3, 0.0) & (0.2, 0.2) \\ (-0.15, -0.05) & - & (-0.4, -0.2) & (0.0, 0.2) \\ (0.0, 0.3) & (0.2, 0.4) & - & (0.35, 0.55) \\ (-0.2, -0.2) & (-0.2, 0.0) & (-0.55, -0.35) & - \end{pmatrix}.$$

The proximity measures for experts are:

$$\begin{aligned}
&PM_1^1 = 0.1667, \quad PM_1^2 = 0.0667, \quad PM_1^3 = 0.1500,\\
&PM_2^1 = 0.2167, \quad PM_2^2 = 0.0833, \quad PM_2^3 = 0.1667,\\
&PM_3^1 = 0.1167, \quad PM_3^2 = 0.0833, \quad PM_3^3 = 0.3000,\\
&PM_4^1 = 0.1000, \quad PM_4^2 = 0.1333, \quad PM_4^3 = 0.2500,
\end{aligned}$$

and

$$PM^1 = 0.1500, \quad PM^2 = 0.0917, \quad PM^3 = 0.2167.$$

Then applying the feedback mechanism we have:

- If we observe the preferences p_{12}, p_{23}, p_{34} and symmetrical ones do not present a reasonable consensus degree, i.e., they do not satisfy the threshold 0.75.
- If we fix a threshold value 0.15 for identifying those experts that should change their assessments, expert 3 and expert 1 would change in alternatives p_{12}, p_{23}, p_{34} and symmetrical ones at least.
- For example, some recommendations would be: expert 3 in the preference p_{34} would decrement his/her preferences.

Now, after some rounds, suppose that the social workers' preferences are:

$$E^1 = \begin{pmatrix} - & [0.2, 0.2] & [0.5, 0.5] & [0.4, 0.4] \\ [0.8, 0.8] & - & [0.5, 0.6] & [0.9, 0.95] \\ [0.5, 0.5] & [0.4, 0.5] & - & [0.2, 0.3] \\ [0.6, 0.6] & [0.05, 0.1] & [0.7, 0.8] & - \end{pmatrix},$$

$$E^2 = \begin{pmatrix} - & [0.15, 0.15] & [0.5, 0.5] & [0.39, 0.43] \\ [0.85, 0.85] & - & [0.7, 0.7] & [0.8, 0.85] \\ [0.5, 0.5] & [0.3, 0.3] & - & [0.18, 0.2] \\ [0.57, 0.61] & [0.15, 0.2] & [0.8, 0.82] & - \end{pmatrix},$$

$$E^3 = \begin{pmatrix} - & [0.25, 0.25] & [0.5, 0.6] & [0.35, 0.4] \\ [0.75, 0.75] & - & [0.6, 0.7] & [0.6, 0.8] \\ [0.4, 0.5] & [0.3, 0.4] & - & [0.25, 0.25] \\ [0.6, 0.65] & [0.2, 0.4] & [0.75, 0.75] & - \end{pmatrix}.$$

Then, we obtain the following collective interval fuzzy preference relation:

$$U = \begin{pmatrix} - & [0.2, 0.2] & [0.5, 0.5] & [0.39, 0.4] \\ [0.8, 0.8] & - & [0.6, 0.7] & [0.8, 0.85] \\ [0.5, 0.5] & [0.3, 0.4] & - & [0.2, 0.25] \\ [0.6, 0.61] & [0.15, 0.2] & [0.75, 0.8] & - \end{pmatrix}.$$

Now, we calculate the consensus relations of each expert

$$C^1 = \begin{pmatrix} - & 0.0 & 0.0 & 0.01 \\ 0.0 & - & 0.2 & 0.2 \\ 0.0 & 0.2 & - & 0.05 \\ 0.01 & 0.2 & 0.05 & - \end{pmatrix}, \quad C^2 = \begin{pmatrix} - & 0.1 & 0.0 & 0.03 \\ 0.1 & - & 0.1 & 0.0 \\ 0.0 & 0.1 & - & 0.07 \\ 0.03 & 0.0 & 0.07 & - \end{pmatrix},$$

$$C^3 = \begin{pmatrix} - & 0.1 & 0.1 & 0.04 \\ 0.1 & - & 0.0 & 0.25 \\ 0.1 & 0.0 & - & 0.05 \\ 0.04 & 0.25 & 0.05 & - \end{pmatrix}.$$

In this case, we obtain a global consensus degree $CD = 0.9275$ or $CD = 92.75\%$, which is acceptable.

So, from final collective interval fuzzy preference relations matrix U it is possible to obtain the following dominance degrees:

$$px_1 = 2.19, \quad px_2 = 4.55, \quad px_3 = 2.15, \quad px_4 = 3.11.$$

So, the old person's homes can be classified from highest to lowest preference as:

$$x_2 > x_4 > x_1 > x_3$$

and therefore, they would choose the old person's home B.

5. Conclusion

In this paper we have presented a new consensus model to deal with GDM with interval fuzzy preference relations. This consensus model is based on two consensus criteria, a consensus measures and proximity measures, and a feedback mechanism. This consensus model allows us to achieve adequate agreement degree among experts in an automatic way.

In the future we think to extend it to work in a fuzzy linguistic context.[52]

Acknowledgments

This paper has been developed with the financing of FEDER funds in FUZZYLING Project TIN200761079, FUZZYLING-II Project TIN201017876, PETRI Project PET20070460, Andalusian Excellence Projects TIC-05299 and TIC-5991, and project of Ministry of Public Works 90/07.

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