J. of Mult.-Valued Logic & Soft Computing, Vol. 14, pp. 371–385
Reprints available directly from the publisher
Photocopying permitted by license only

An Interactive Decision Support System Based on Consistency Criteria

S. ALONSO[1], F. J. CABRERIZO[1], F. CHICLANA[2], F. HERRERA[3] AND E. HERRERA-VIEDMA[3]

[1]*Department of Software Engineering, University of Granada, 18071, Granada, Spain*
E-mail: {zerjioi, cabrerizo}@ugr.es
[2]*Centre for Computational Intelligence, School of Computing,*
De Montfort University, Leicester LE1 9BH, UK
E-mail: chiclana@dmu.ac.uk
[3]*Department of Computer Science and Artificial Intelligence, University of Granada,*
18071, Granada, Spain
E-mail: {viedma, herrera}@decsai.ugr.es

Received: November 20, 2006. Revised: February 10, 2007. Accepted: April 5, 2007.

To achieve consistent solutions is an important goal in group decision making problems. However, the expression of preferences is often a very difficult task for the experts, specially in decision problems with a high number of alternatives and when experts use fuzzy preference relations to provide their opinions. This usually leads to situations where experts are not able to properly express all their preferences in a consistent way, that is, without contradiction. In this paper, to overcome this problem, we present a decision support system based on consistency criteria to aid experts to express their fuzzy preference relations in a more consistent way. The system works interactively with the expert by providing him/her recommendations on the preference values that he/she has not yet expressed. These recommendations are computed trying to maintain the expert's consistency level as high as possible.

Keywords: fuzzy preference relations, decision support systems, incomplete information, consistency property

1 INTRODUCTION

In Group Decision Making (GDM) problems we have a set of experts $E = \{e_1, \ldots, e_m\}$ that provide their preferences about a set of alternatives $X = \{x_1, \ldots, x_n\}$. Usually, GDM problems are faced by applying two different processes before a final solution of alternatives can be given [9, 10, 12, 15, 17, 18, 21]: *the consensus process* and *the selection process*. The consensus process refers to how to obtain the maximum degree of consensus or agreement between the set of experts. Usually, the consensus process is guided by a human figure called moderator [9, 10, 15]. The selection process obtains the final solution according to the preferences given by the experts. It involves two different steps [11, 12, 16, 23]: the aggregation of individual preferences and the exploitation of the collective preference. Clearly, it is preferable that the experts achieve a high level of consensus among their preferences before applying the selection process.

There exist different representation formats that experts can use to express their preferences, as for example [4,5]: preference orderings, utility functions, fuzzy preference relations or multiplicative preference relations. Fuzzy Preference Relations (FPRs) [4, 6, 20, 22] have been widely used because they are a very expressive format and also they present good properties that allow to operate with them easily [20, 22].

Although FPRs have a very high level of expressivity and present good properties to operate with them easily, however their use can also present some drawbacks. In order to use FPRs, a preference degree between all possible pairs of different alternatives is required, and therefore the amount of information that the experts have to provide increases exponentially with respect to the cardinality of the set of alternatives [18, 19]. This usually leads to situations where the expert is not able to express the required preferences in a consistent and complete way. This means that decision situations with inconsistent and incomplete FPRs need to be addressed.

In [1, 19], a procedure which computes the missing values of an incomplete FPR taking into account the expert's consistency level was developed. However, this procedure did not take into account the initial contradiction an expert could have introduced in his/her preferences. Also, the estimated values of those unknown ones this procedure computes might not be accepted by the expert even though they were computed in order to increase the overall consistency level of the expert; an scenario which is not covered by this procedure. Thus, it would be desirable to design a computer driven decision model to deal with GDM problems based on FPRs to avoid the appearance of inconsistent and incomplete information. This computer driven decision model should include software tools to aid the experts to express their preferences and to avoid the above the aforementioned problems. Because experts might not be familiar with FPRs, the aiding tools must be easy enough to use and should follow the general principles of interface design [3].

In this paper we present an *interactive decision support system* to aid experts to express their preferences in a consistent way. The system will give recommendations to each expert while he/she is providing the preference values of FPR in order to maintain a high level of consistency, as well as to avoid missing information. Also, the system will provide measures of the current level of consistency and completeness that the expert has achieved, which can be used to minimise situations of self contradiction. This system has been programmed using Java technologies and it allows its integration in web-based applications which are increasingly being used in group decision support environments [2, 28].

The rest of the paper is set out as follows. Section 2 presents concepts, notations, definitions and procedure necessary throughout the paper. In Section 3 the consistency based interactive decision support system is described in detail. Section 4 presents an example of application of the interactive decision support system. Finally, Section 5 draws our conclusions.

2 PRELIMINARIES

In this section the notion of Incomplete FPR, the Additive Consistency Property and its use in estimating missing values in a FPR, some Consistency Measures and Completeness Measures are presented.

2.1 Incomplete FPRs

Definition 1. A FPR P on a set of alternatives X is a fuzzy set on the product set $X \times X$ characterized by a membership function $\mu_P : X \times X \longrightarrow [0, 1]$.

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P = (p_{ik})$, being $p_{ik} = \mu_P(x_i, x_k)$ $(\forall i, k \in \{1, \ldots, n\})$ interpreted as the preference degree or intensity of the alternative x_i over x_k: $p_{ik} = 1/2$ indicates indifference between x_i and x_k $(x_i \sim x_k)$, $p_{ik} = 1$ indicates that x_i is absolutely preferred to x_k, and $p_{ik} > 1/2$ indicates that x_i is preferred to x_k $(x_i \succ x_k)$. Based on this interpretation we have that $p_{ii} = 1/2$ $\forall i \in \{1, \ldots, n\}$ $(x_i \sim x_i)$.

Usual models to solve GDM problems assume that experts are always able to provide all the preferences required, that is, to provide all p_{ik} values. This situation is not always possible to achieve. Experts could have some difficulties in giving all their preferences due to lack of knowledge about part of the problem, or simply because they may not be able to quantify some of their degree of preference. In order to model such situations, we define the concept of an *incomplete fuzzy preference relation* [19].

Definition 2. A function $f : X \longrightarrow Y$ is partial when not every element in the set X necessarily maps onto an element in the set Y. When every element from the set X maps onto one element of the set Y then we have a total function.

Definition 3. An incomplete FPR P on a set of alternatives X is a fuzzy set on the product set $X \times X$ that is characterized by a partial membership function.

As per this definition, we call a FPR complete when its membership function is a total one. Clearly, Definition 1 includes both definitions of complete and incomplete FPRs. However, as there is no risk of confusion between a complete and an incomplete FPR, in this paper we refer to the first type as simply FPR.

2.2 Additive Consistency Property and Some Consistency Measures

Obviously, an inconsistent source of information is not as useful as a consistent one. It would therefore be quite important to be able to measure the consistency of the information provided by experts so as to avoid inconsistent opinions. Consistency is related with rationality, which is associated with the *transitivity property* [8, 20]. Transitivity seems like a reasonable criterion of coherence for an individual's preferences: if x is preferred to y and y is preferred to z, common sense suggests that x should be preferred to z. Many properties have been suggested to model transitivity, amongst which we can cite [20]: *triangle condition, weak transitivity, max–min transitivity, max–max transitivity, restricted max–min transitivity, restricted max–max transitivity, multiplicative transitivity, additive transitivity.*

As shown in [20], additive transitivity for FPRs can be seen as the parallel concept of Saaty's consistency property for multiplicative preference relations [24]. The mathematical formulation of the additive transitivity was given by Tanino in [25]:

$$(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ik} - 0.5) \quad \forall i, j, k \in \{1, \ldots, n\} \qquad (1)$$

This kind of transitivity has the following interpretation: suppose we want to establish a ranking between three alternatives x_i, x_j and x_k, and that the information available about these alternatives suggests that we are in an indifference situation, i.e. $x_i \sim x_j \sim x_k$. When giving preferences this situation would be represented by $p_{ij} = p_{jk} = p_{ik} = 0.5$. Suppose now that we have a piece of information that says $x_i \prec x_j$, i.e. $p_{ij} < 0.5$. This means that p_{jk} or p_{ik} have to change, otherwise there would be a contradiction, because we would have $x_i \prec x_j \sim x_k \sim x_i$. If we suppose that $p_{jk} = 0.5$ then we have the situation: x_j is preferred to x_i and there is no difference in preferring x_j to x_k. We must then conclude that x_k has to be preferred to x_i. Furthermore, as $x_j \sim x_k$ then $p_{ij} = p_{ik}$, and so $(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ij} - 0.5) = (p_{ik} - 0.5)$. We have the same conclusion if $p_{ik} = 0.5$. In the case of $p_{jk} < 0.5$, then we have that x_k is preferred to x_j and this to x_i, so x_k should be preferred to x_i. On the other hand, the value p_{ik} has to be equal to or lower than p_{ij}, being equal only in the case of $p_{jk} = 0.5$ as we have already shown. Interpreting the value $p_{ji} - 0.5$ as the intensity of preference of alternative x_j over x_i, then it seems reasonable to suppose that the intensity of preference of x_i over x_k should be equal to the sum of the intensities of preferences when using an intermediate alternative x_j, that is, $p_{ik} - 0.5 = (p_{ij} - 0.5) + (p_{jk} - 0.5)$. The same reasoning can be applied in the case of $p_{jk} > 0.5$.

Additive transitivity implies additive reciprocity. Indeed, because $p_{ii} = 0.5 \; \forall i$, if we make $k = i$ in (1) then we have: $p_{ij} + p_{ji} = 1 \; \forall i, j \in \{1, \ldots, n\}$.

Expression (1) can be rewritten as:

$$p_{ik} = p_{ij} + p_{jk} - 0.5 \quad \forall i, j, k \in \{1, \ldots, n\} \tag{2}$$

We will consider a FPR to be *additive consistent* when for every three options in the problem $x_i, x_j, x_k \in X$ their associated preference degrees p_{ij}, p_{jk}, p_{ik} fulfil (2). An additive consistent FPR will be referred as consistent throughout the paper, as this is the only transitivity property we are considering.

Given a FPR we can define the following consistency measures [19]:

- *The consistency measure of a preference value p_{ik}:*

$$CL_{ik} = 1 - \varepsilon p_{ik} \tag{3}$$

where εp_{ik} represents the average deviation or error of all possible estimates cp_{ik}^{jl} ($l \in \{1, 2, 3\}$) with respect to the preference value p_{ik} defined as

$$\varepsilon p_{ik} = \frac{2}{3} \cdot \frac{\sum_{\substack{j=1 \\ j \neq i,k}}^{n} \left(|cp_{ik}^{j1} - p_{ik}| + |cp_{ik}^{j2} - p_{ik}| + |cp_{ik}^{j3} - p_{ik}|\right)}{3(n-2)} \tag{4}$$

- *The consistency measure of a FPR P^h:*

$$CL^h = \frac{\sum_{\substack{i,k=1 \\ i \neq k}}^{n} CL_{ik}^h}{n^2 - n} \tag{5}$$

2.3 Estimating Missing Values Using the Additive Consistency Property

Expression (2) can be used to calculate an estimated value of a preference degree using other preference degrees in a FPR. Indeed, the preference value p_{ik} ($i \neq k$) can be estimated using an intermediate alternative x_j in three different ways [19]:

- From $p_{ik} = p_{ij} + p_{jk} - 0.5$ we obtain the estimate

$$(cp_{ik})^{j1} = p_{ij} + p_{jk} - 0.5 \tag{6}$$

- From $p_{jk} = p_{ji} + p_{ik} - 0.5$ we obtain the estimate

$$(cp_{ik})^{j2} = p_{jk} - p_{ji} + 0.5 \tag{7}$$

- From $p_{ij} = p_{ik} + p_{kj} - 0.5$ we obtain the estimate

$$(cp_{ik})^{j3} = p_{ij} - p_{kj} + 0.5 \tag{8}$$

Then, we can estimate the value of a preference p_{ik} according to the following expression:

$$cp_{ik} = \frac{\sum_{\substack{j=1 \\ j\neq i,k}}^{n} (cp_{ik}^{j1} + cp_{ik}^{j2} + cp_{ik}^{j3})}{3(n-2)} \tag{9}$$

When working with an incomplete FPR, 9 cannot be used to estimate preference values.

If an expert e_h provides an incomplete FPR P^h, the following sets can be defined [19]:

$$\begin{aligned}
A &= \{(i,j) \mid i,j \in \{1,\ldots,n\} \wedge i \neq j\} \\
MV^h &= \{(i,j) \in A \mid p_{ij}^h \text{ is unknown}\} \\
EV^h &= A \setminus MV^h \\
H_{ik}^{h1} &= \{j \neq i,k \mid (i,j),(j,k) \in EV^h\} \\
H_{ik}^{h2} &= \{j \neq i,k \mid (j,i),(j,k) \in EV^h\} \\
H_{ik}^{h3} &= \{j \neq i,k \mid (i,j),(k,j) \in EV^h\}
\end{aligned}$$

- MV^h is the set of pairs of alternatives whose preference degrees are not given by expert e_h;
- EV^h is the set of pairs of alternatives whose preference degrees are given by the expert e_h;
- H_{ik}^{h1}, H_{ik}^{h2}, H_{ik}^{h3} are the sets of intermediate alternatives x_j $(j \neq i,k)$ that can be used to estimate the preference value p_{ik}^h $(i \neq k)$ using equations (6), (7), (8) respectively.

Expression 9 can be extended to compute missing preference values in an incomplete FPR as follows. The *final* estimated value of a missing preference degree $p_{ik}^h((i,k) \in EV^h)$ for an expert e_h is calculated as:

$$cp_{ik}^h = \frac{\sum_{j\in H_{ik}^{h1}} (cp_{ik}^h)^{j1} + \sum_{j\in H_{ik}^{h2}} (cp_{ik}^h)^{j2} + \sum_{j\in H_{ik}^{h3}} (cp_{ik}^h)^{j3}}{\#(H_{ik}^{h1}) + \#(H_{ik}^{h2}) + \#(H_{ik}^{h3})} \tag{10}$$

In [19] an estimation procedure of missing values for incomplete FPRs was designed and it was proved that an incomplete FPRs can be succssully completed when a set of $n-1$ non-leading diagonal preference values, where each one of the alternatives is compared at least once, is known. Consequently, when $(\#(H_{ik}^{h1}) + \#(H_{ik}^{h2}) + \#(H_{ik}^{h3})) = 0$ the preference value $p_{ik}^h((i,k) \in EV^h)$ cannot be estimated using the rest of known values.

2.4 Completeness Measures

In decision-making situations with incomplete information, the notion of completeness is also an important factor to take into account when analysing the consistency. Clearly, the higher the number of preference values provided by an expert the higher the chance of inconsistency. Therefore, a degree of completeness associated with the number of preference values provided should also be taken into account to produce a fairer measure of consistency of an incomplete FPR.

Given an incomplete FPR we can define two completeness measures [19]:

- *Completeness measure of a relation P^h:*

$$CP^h = \frac{\#(EV^h)}{n^2 - n}. \tag{11}$$

- *Completeness Measure of an alternative x_i:*

$$CP_i^h = \frac{\#EV_i^h}{2(n-1)} \tag{12}$$

  where $\#EV_i^h$ $(EV_i^h \subseteq EV^h)$ is the number of preference values known for x_i.

3 A DECISION SUPPORT SYSTEM TO AID EXPERTS TO EXPRESS CONSISTENT PREFERENCES

In this section we describe in detail the interactive decision support system to aid experts to express their FPRs in a consistent way. We will start by enumerating the design goals and requirements taken into account in its design, and aferwards we will describe the actual implementation of every requirement in the system will be described.

3.1 Design Goals and Requirements

Our design goals and requirements could be splitted in two different parts: *Interface Requirements*, and *Logical Goals*.

Interface Requirements: These requirements deal with the visual representation of the information and the different controls in the system. The system is desired to comply with the so called "*Eight Golden Rules*" [3] for interface design:

GR 1. Strive for consistency.
GR 2. Enable frequent users to use shortcuts.
GR 3. Offer informative feedback.

GR 4. Design dialogues to yield closure.
GR 5. Offer simple error handling.
GR 6. Permit easy reversal of actions (undo action).
GR 7. Support internal focus of control (user is at charge).
GR 8. Reduce short-term memory load of the user.

Logical Goals:

Goal 1. Offer recommendations to the expert to guide him toward a highly consistent and complete FPR.
Goal 2. Recommendations must be given interactively.
Goal 3. Recommendations must be simple to understand and to apply.
Goal 4. The user must be able to refuse recommendations.
Goal 5. The system must provide indicators of the consistency and completeness level achieved in every step.
Goal 6. The system should be easy to adapt to other type of preference relations.
Goal 7. The system should be easy to incorporate to Web-based GDM models and decision support systems [2, 28].

3.2 Actual Implementation

We will now detail how we have dealt with every requirement and goal that we have presented in the previous section. To do so, we will make use of a snapshot of the system (Figure 1) where we will point out every implementation solution.

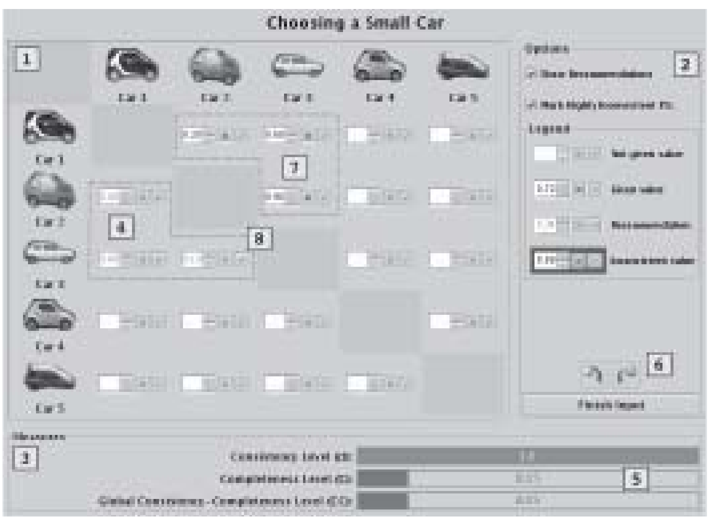

FIGURE 1
Snapshot of the Decision Support System.

Implementation of the Interface Requirements:

- **GR 1.** The interface has been homogenised in order to present easily a view of the process that is being carried out. We have introduced 3 main areas: In area number **(1)** we present the FPR that the expert is introducing, as well as a brief description of every alternative. Area number **(2)** contains several global controls to activate/deactivate certain functions, as well as to finish the input process. Area number **(3)** contains different measures that show the overall progress (see below).
- **GR 2.** Shortcuts have been added to the most frequent options, and the input text areas for the preference values have been ordered to access them easily using the keyboard.
- **GR 3.** Our system provides recommendations **(4)** and consistency and completeness measures **(5)** (see below). All controls have tooltips.
- **GR 4.** With every change that the user makes to his/her preferences the system provides new recommendations and measures.
- **GR 5.** Incorrect inputs are prompt with error messages.
- **GR 6.** We have introduced *undo* and *redo* buttons **(6)**.
- **GR 7.** The user can choose at every moment which preference value wants to give or update, as well as enabling/disabling options.
- **GR 8.** All information is presented in a single screen.

Logical Goals:

- **Goal 1.** To offer recommendations, the system computes all the missing values that could be estimated by using *equation 10* and it presents them in area **(1)**. As the values are computed taking into account the additive transitivity property, the recommendations should tend to increment the overall consistency level. They are presented in a different colour (gray) **(4)** to differentiate them from the expert values **(7)**.
- **Goal 2.** When the expert introduces or updates a preference value all possible recommendations are recomputed and presented.
- **Goal 3.** Recommendations are given in the same manner as the user inputs his/her preferences. There is also a button that enables the user to accept or validate a given recommendation **(8)**.
- **Goal 4.** A user can choose any value for a particular preference degree ignoring all the recommendations.
- **Goal 5.** The system provides consistency and completeness measures based on expressions (5) and (11), respectively. These two

measures re combined into a global consistency/completeness measure that informs the expert of his/her current degree of consistency and completeness:

$$CC^h = CL^h \cdot CP^h \tag{13}$$

- **Goal 6.** As the system is programmed following the principles of Object Oriented Programming, its adaptations to different type of preference relations is an easy task.
- **Goal 7.** As the system is Java based, it can be easily incorporated into a web-based environment.

4 EXAMPLE OF THE USE OF THE DECISION SUPPORT SYSTEM

In this section we present an example of the use of the interactive decision support system. In this example, an expert provides his/her preferences about four different cars in order to select the best one of them:

- Black, economic and slow car: *Black Car* (x_1).
- Red, very small, fast and comfortable car: *Red Car* (x_2).
- White and very fast car. It consumes little but it is very expensive: *White Car* (x_3).
- Blue, very small and very cheap car: *Blue Car* (x_4).

The expert will provide his preferences about the car in form of a FPR P, and thus, he can use the interactive decision support system to improve the consistency and completeness of his preference relation.

The expert starts by providing the following preference values: $p_{12} = 0.6$ and $p_{13} = 0.6$. The system estimates the following preference values $p_{23} = 0.5$ and $p_{32} = 0.5$ (see Figure 2). Note that as p_{12} and p_{13} are equal, it is logical to assume that x_2 and x_3 are equally preferred (if the consistency property is correctly applied). At this point, we observe that the consistency level is maximum ($CL = 1.0$), that is, there is no contradiction in the values that the expert has introduced, and the completeness level ($CP = 0.17$) is very low because he has provided just 2 values out of the 12 required.

At this point, the expert accepts the values estimated by the system. As he accepts those values, the system provides new estimations for the preference values $p_{21} = 0.4$ and $p_{31} = 0.4$. The completeness level increases ($CP = 0.33$) while the consistency level is maintained ($CL = 1.0$) (see Figure 3). These new estimated values have been computed based on totally consistent preference values and therefore are also totally consistent with them.

Now, the expert considers that the estimation for p_{21} does not really correspond with his/her opinion and changes that estimated value from 0.4 to 0.8.

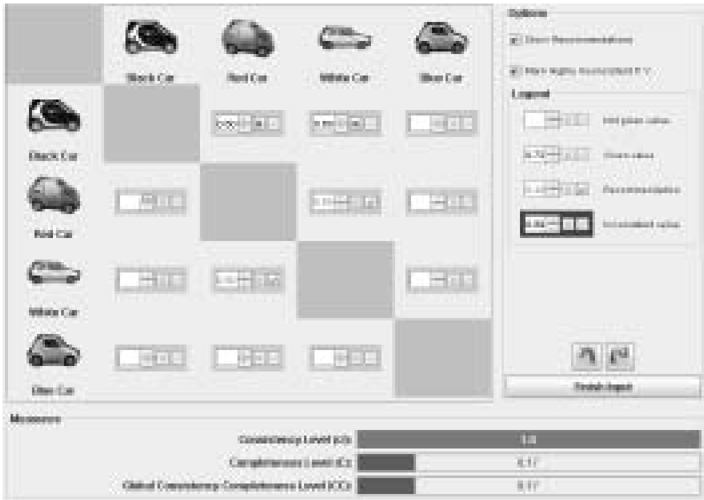

FIGURE 2
An expert gives his/her first preference values.

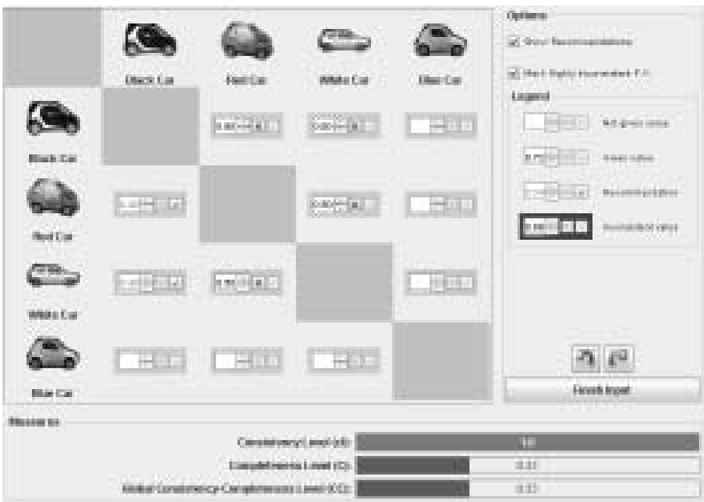

FIGURE 3
The expert accepts the values that the system suggestes.

The system detects that there is a notable contradiction with this new value, and consequently, it marks that value in red. Obviously, the global consistency level decreases ($CL = 0.85$) (see Figure 4). The expert realizes that the system estimation is indeed correct and that he is introducing a inconsistency in his/her preference relation ($p_{12} = 0.6 \Rightarrow x_1 \succ x_2$ and $p_{21} = 0.8 \Rightarrow x_2 \succ x_1$). In order to avoid this situation, the expert changes p_{21} to the value initially suggested by the system ($p_{21} = 0.4$).

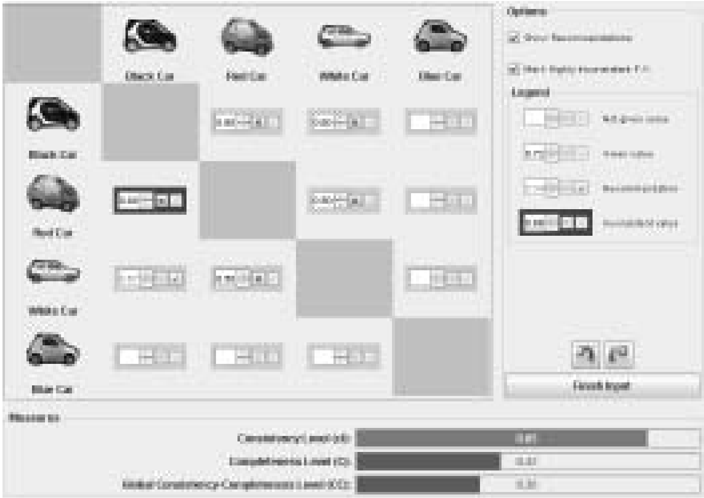

FIGURE 4
The expert introduces some inconsistency in his/her preferences.

The expert provides the following preference value: $p_{43} = 0.70$. In the following we suppose that the expert completes the preference relation P with values similar but not necessarily equal to the ones proposed by the system (in **bold** are the values provided by the expert and in *italic* the values estimated by the system):

$$\begin{pmatrix} - & \mathbf{0.60} & \mathbf{0.60} & x \\ \mathbf{0.40} & - & \mathbf{0.50} & x \\ \mathbf{0.40} & \mathbf{0.50} & - & x \\ x & x & x & - \end{pmatrix} \xrightarrow{p_{43}=0.70} \begin{pmatrix} - & \mathbf{0.60} & \mathbf{0.60} & \mathit{0.40} \\ \mathbf{0.40} & - & \mathbf{0.50} & \mathit{0.30} \\ \mathbf{0.40} & \mathbf{0.50} & - & x \\ \mathit{0.60} & \mathit{0.7} & \mathbf{0.70} & - \end{pmatrix} \xrightarrow{p_{14}=0.30}$$

$$\begin{pmatrix} - & \mathbf{0.60} & \mathbf{0.60} & \mathbf{0.30} \\ \mathbf{0.40} & - & \mathbf{0.50} & \mathit{0.23} \\ \mathbf{0.40} & \mathbf{0.50} & - & \mathit{0.20} \\ \mathit{0.60} & \mathit{0.73} & \mathbf{0.70} & - \end{pmatrix} \xrightarrow{p_{24}=0.20} \begin{pmatrix} - & \mathbf{0.60} & \mathbf{0.60} & \mathbf{0.30} \\ \mathbf{0.40} & - & \mathbf{0.50} & \mathbf{0.20} \\ \mathbf{0.40} & \mathbf{0.50} & - & \mathit{0.20} \\ \mathit{0.63} & \mathit{0.73} & \mathbf{0.70} & - \end{pmatrix} \xrightarrow{p_{34}=0.30}$$

$$\begin{pmatrix} - & \mathbf{0.60} & \mathbf{0.60} & \mathbf{0.30} \\ \mathbf{0.40} & - & \mathbf{0.50} & \mathbf{0.20} \\ \mathbf{0.40} & \mathbf{0.50} & - & \mathbf{0.30} \\ \mathit{0.62} & \mathit{0.72} & \mathbf{0.70} & - \end{pmatrix} \xrightarrow{p_{41}=0.62} \begin{pmatrix} - & \mathbf{0.60} & \mathbf{0.60} & \mathbf{0.30} \\ \mathbf{0.40} & - & \mathbf{0.50} & \mathbf{0.20} \\ \mathbf{0.40} & \mathbf{0.50} & - & \mathbf{0.30} \\ \mathbf{0.62} & \mathit{0.72} & \mathbf{0.70} & - \end{pmatrix} \xrightarrow{p_{42}=0.72}$$

$$\begin{pmatrix} - & \mathbf{0.60} & \mathbf{0.60} & \mathbf{0.30} \\ \mathbf{0.40} & - & \mathbf{0.50} & \mathbf{0.20} \\ \mathbf{0.40} & \mathbf{0.50} & - & \mathbf{0.30} \\ \mathbf{0.62} & \mathbf{0.72} & \mathbf{0.70} & - \end{pmatrix}$$

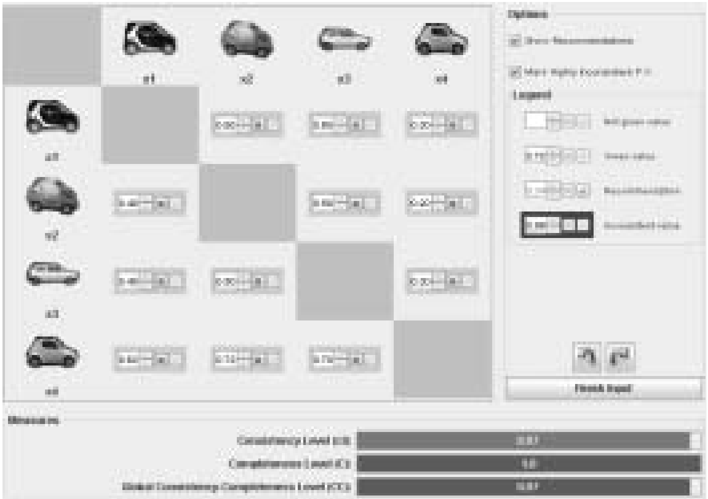

FIGURE 5
The expert introduces some inconsistency in his/her preferences.

Finally, the system provides (see Figure 5) a consistency level $CL = 0.97$ which means that the preference relation P is highly consistent, but not completely consistent as the expert slightly changed the values suggested by the system. The completeness level is $CP = 1.0$ which means that all the preference values have been provided by the expert.

5 CONCLUDING REMARKS

In this paper we have presented an interactive decision support system which aids experts to provide consistent and complete FRRs in decision-making contexts. The system reacts to an expert input of preference values by providing recommendations of future preference values. These are calculated based on the additive transitivity property and therefore the system tries to maintain the consistency of the expert.

In future works we will improve the system as to be capable of dealing with other type of preference relations (linguistic, multiplicative, interval-valued preference relations) [7,11–15,24,26,27]. Additionally, it will be incorporated into a more sophisticated consensus process and will be adapted to use other sources of information such as consensus measures. Finally, it will be deployed into mobile and distributed GDM environments where the experts will be able to provide their preferences about the alternatives using limited devices as mobile phones and PDAs.

ACKNOWLEDGMENTS

This work was supported by the Research Projects TIN2007-61079 and SAINFOWEB TIC2005-00602.

REFERENCES

[1] Alonso, S., Chiclana, F., Herrera, F., Herrera-Viedma, E. (2007). A consistency based procedure to estimate missing pairwise preference values. *International Journal of Intelligent Systems*, in press.

[2] Bhargava, H.K., Power, D.J., Sun, D. (2007). Progress in Web-based decision support technologies. *Decision Support Systems*, in press (Available online 8 August 2005).

[3] Chen, Z. (1995). Interacting with Software Components. *Decision Support Systems*, **14**, 349–357.

[4] Chiclana, F., Herrera, F., Herrera-Viedma, E. (1998). Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations. *Fuzzy Sets and Systems*, **97**(1), 33–48.

[5] Chiclana, F., Herrera, F., Herrera-Viedma, E. (2001). Integrating multiplicative preference relations in a multipurpose decision making model based on fuzzy preference relations. *Fuzzy Sets and Systems*, **112**(2), 277–291.

[6] Chiclana, F., Herrera, F., Herrera-Viedma, E. (2002). A Note on the Internal Consistency of Various Preference Representations. *Fuzzy Sets and Systems*, **131**(1), 75–78.

[7] Chiclana, F., Herrera-Viedma, E., Herrera, F. (2004). Induced ordered weighted geometric operators and their use in the aggregation of multiplicative preference relations. *International Journal of Intelligent Systems*, **19**(3), 233–255.

[8] Fodor, J., Roubens, M. (1994). *Fuzzy Preference Modelling and Multicriteria Decision Support*. Kluwert Academic Publichers, Dordrecht.

[9] Kacprzyk, J., Fedrizzi M., Nurmi, H. (1992). Group decision making and consensus under fuzzy preferences and fuzzy majority. *Fuzzy Sets and Systems*, **49**, 21–31.

[10] Kacprzyk, J., Nurmi, H., Fedrizzi, M., (Eds.). (1997). *Consensus Under Fuzziness*. Kluwer Academic Publishers, Massachusetts.

[11] Herrera, F., Herrera-Viedma, E. (2000). Choice functions and mechanisms for linguistic preference relations. *European Journal of Operational Research*, **120**, 144–161.

[12] Herrera, F., Herrera-Viedma, E. (2000). Linguistic decision analysis: steps for solving decisions problems under linguistic information. *Fuzzy Sets and Systems*, **115**, 67–82.

[13] Herrera, F., Herrera-Viedma, E., Chiclana, F. (2001). Multiperson decision making based on multiplicative preference relations. *European Journal of Operational Research*, **129**, 372–385.

[14] Herrera, F., Herrera-Viedma, E., Verdegay, J.L. (1996). A linguistic decision process in group decision making. *Group Decision and Negotiation*, **5**, 165–176.

[15] Herrera, F., Herrera-Viedma, E., Verdegay, J.L. (1996). A model of consensus in group decision making under linguistic assessments. *Fuzzy Sets and Systems*, **78**, 73–87.

[16] Herrera, F., Herrera-Viedma, E., Verdegay J.L. (1998). Choice processes for non-homogeneous group decision making in linguistic setting. *Fuzzy Sets and Systems*, **94**, 287–308.

[17] Herrera-Viedma, E., Herrera, F., Chiclana, F. (2002). A consensus model for multiperson decision making with different preference structures. *IEEE Transactions on Systems, Man and Cybernetics-Part A: Systems and Humans*, **32**(3), 394–402.

[18] Herrera-Viedma, E., Alonso, S., Chiclana, F., Herrera, F. (2007). A Consensus Model for Group Decision Making with Incomplete Fuzzy Preference Relations. *IEEE Transactions on Fuzzy Systems*, in press.

[19] Herrera-Viedma, E., Chiclana, F., Herrera, F., Alonso, S. (2007). A group decision making model with incomplete fuzzy preference relations based on additive consistency. *IEEE Transactions on Systems, Man and Cybernetics, Part B, Cybernetics*, **37**(1), 176–189.

[20] Herrera-Viedma, E., Herrera, F., Chiclana, F., Luque, M. (2004). Some issues on consistency of fuzzy preference relations. *European Journal of Operational Research*, **154**(1), 98–109.

[21] Herrera-Viedma, E., Mata, F., Martínez, L., Chiclana, F. (2005). A Consensus Support System Model for Group Decision-making Problems with Multi-granular Linguistic Preference Relations *IEEE Transactions on Fuzzy Systems*, **13**(5), 644–658.

[22] Kacprzyk, J. (1986). Group decision making with a fuzzy linguistic majority. *Fuzzy Sets and Systems*, **18**, 105–118.

[23] Roubens, M. (1997). Fuzzy Sets and Decision Analysis, *Fuzzy Sets and Systems*, **90**, 199–206.

[24] Saaty, Th.L. (1980). *The Analytic Hierarchy Process*. MacGraw–Hill, New York.

[25] Tanino, T. (1984). Fuzzy preference orderings in group decision making. *Fuzzy Sets and Systems*, **12**, 117–131.

[26] Xu, Z.S. (2004). On compatibility of interval fuzzy preference relations. *Fuzzy Optimization and Decision Making*, **3**, 217–225.

[27] Xu, Z.S. (2007). Intuitionistic preference relations and their application in group decision making. *Information Sciences*, **177**(11), 2363–2379.

[28] Zhang S., Goddard, S. (2007). A software architecture and framework for Web-based distributed Decision Support Systems. *Decision Support Systems*, in press (Available online 11 August 2005).