Visualizing Consensus in Group Decision Making Situations

S. Alonso, E. Herrera-Viedma, F.J. Cabrerizo, F. Chiclana, F. Herrera

***Abstract*— In the resolution of group decision making problems the consensus process, that is, the process where experts discuss about the alternatives to narrow their differences, is usually held with all the experts gathered together in a place where they can speak and discuss about the alternatives in the problem. However, in situations where it is not possible to bring them all together it is usually difficult for the experts to identify the closeness of the opinions of the rest of the experts, and thus, it can be difficult to have a clear view of the current state of the consensus process.**

In this paper we present a tool that creates consensus diagrams that help experts to easily comprehend the current consensus state in the decision problem. This tool is based on several consistency, consensus and similarity measures and with the application of a clustering algorithm, it identifies and represents the different groups of experts with similar opinions, identifies a possible candidate for spokesperson for each group and easily depicts the consistency level expressed by each one of the experts.

I. INTRODUCTION

Usually, to solve Group Decision Making problems, that is, problems where a set of experts $E = \{e_1, \ldots, e_m\}$ have to choose the best alternative or alternatives from a feasible set of alternatives $X = \{x_1, \ldots, x_n\}$, two different processes have to be carried out: the *consensus process* and the *selection process*. The former consists on obtaining the highest consensus level among experts, that is, to obtain a state were the opinions of the different experts are as close as possible one to another. The consensus process is usually guided by a special figure, called the *moderator*, whose main task is to guide experts towards a final solution with a high level of consensus. The latter process consists on obtaining the final solution to the problem from the opinions expressed by the experts in the last round of the consensus process.

In the literature, we can find several approaches to almost fully automatize the selection process by means of the application of different Soft Computing techniques [4], [5], [9], [10], [13], [18].

On the other side, the consensus process [2], [3], [12], [15], [17], [24] usually involves the communication and discussion among experts and between the experts and the moderator and thus, to fully automatize the consensus process is a more difficult task due to the high number of interactions involved. However, several new different approaches and tools which make use of new technologies (mainly web-based technologies) to adapt classical consensus processes and models to new environments can be found in the literature [1], [19], [20], [23].

The application of these new technologies allow to carry out consensus processes in situations which previously could not be correctly addressed. For example, nowadays it is possible to carry out consensus processes among several experts which are located in different countries around the world. Though, it is important to remark that even with the adoption of new communication technologies (video-conference, chat rooms, instant messaging, e-mail and so on) there is still an important need of new collaboration and information tools for the experts being able to solve decision making problems where they cannot meet together with the other experts.

In this work we center our attention in a particular problem that arises in many consensus processes for group decision making when experts do not have the possibility of gathering together: experts may not have a clear idea about the current consensus status among all the experts involved in the decision process. In usual decision making models, where experts gather together to discuss their opinions about the different alternatives, it is relatively easy to determine which experts have similar opinions just by attending to the discussions among experts, and thus, experts may join or form different groups to better discuss and to reason out about the pros and cons of every alternative. Additionally, when experts are able to determine the consensus state of the decision making process it is more easy for them to influence the other experts [8] and to detect if some experts are trying to bias the consensus process.

However, in the cases where direct communication is not possible, experts will probably need some assistance to stablish connections among them and to obtain a clear view of the consensus process progress.

To solve this problem we propose to use new techniques and tools to automatically generate high level information and simple consensus diagrams about the consensus state in the decision problem that is being solved. Among other information, we will be able to identify separate groups of experts with common opinions about the alternatives in the problem, we will be able to select a candidate for each of the groups to act -if necessary- as a spokesperson for the group and we will be able to identify isolated individuals (the ones whose preferences about the alternatives are very different from the preferences of the rest of experts). We will also generate some consensus diagrams in which the current consensus state will be drawn. In those consensus diagramas experts will be drawn as the nodes of a graph which are

Sergio Alonso is with the Software Engineering Department, University of Granada, Spain (email: salonso@decsai.ugr.es).

Enrique Herrera-Viedma, Francisco J. Cabrerizo and Francisco Herrera are with the Computer Science and Artificial Intelligence Department, University of Granada, Spain (email: viedma@decsai.ugr.es, cabrerizo@ugr.es, herrera@decsai.ugr.es).

Francisco Chiclana is with the Centre for Computational Intelligence, School of Computing, De Montfort University, Leicester, UK (email: chiclana@dmu.ac.uk).

separated according to the affinity of their preferences about the alternatives in the problem. Additionally, in those graphs we will introduce the relative position of the consensus solution and some other visual elements to easily recognize the consistency expressed in the preferences of each expert. As visual elements do have a great potential to influence experts in decision processes [22], these consensus diagrams, when presented to the experts, will allow them to have a more profound and clear view about the current consensus process and about which experts have similar or different opinions about the alternatives. Additionally, the use of these diagrams can help the experts to detect if other experts are trying to bias the consensus process. Those diagramas will be produced by a visualization tool which takes into account several factors as the consistency of the information expressed by each expert, the similarity of the opinions of the experts and some consensus measures at three different levels. It will also use a simple clustering algorithm (k-means [21]) to group the experts according to their opinions about the alternatives. This visualization tool can be easily integrated into existing consensus models.

The structure of the paper is as follows: In section II we present fuzzy preference relations as the representation model that the experts will use to provide their preferences about the alternatives and some consistency and consensus properties and measures about them. In section III we present some similarity masures that can be computed from the preferences expressed by the experts and the use of a clustering algorithm to identify groups of experts with similar opinions. Section IV describes a new visualization tool that using the previous similarity, consistency and consensus measures, along with the clustering algorithm, generates some consensus diagrams that can be used by the experts to obtain a clear picture of the current consensus state in the problem. Finally, some conclusions and future works are outlined in section V.

II. PRELIMINARIES

In this section we present fuzzy preference relations as the representation model that the experts will use to express their preferences about the alternatives in the problem. Additionally, some consistency measures for the preference relations at three different levels (pair of alternatives, alternatives and preference relation levels) are presented.

There exists many different representation formats that can be used by experts to provide their preferences about the alternatives in a group decision making problem. One of the most used formats is *fuzzy preference relations* due to their effectiveness as a tool for modelling decision processes and their utility and easiness of use when we want to aggregate experts' preferences into group ones [14], [16], [25]:

Definition 1: A fuzzy preference relation P^h given by expert e^h on a set of alternatives X is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_{P^h} : X \times X \longrightarrow [0,1]$.

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P^h = (p^h_{ik})$, being $p^h_{ik} = \mu_{P^h}(x_i, x_k)$ $(\forall i,k \in \{1,\ldots,n\})$ interpreted as the preference degree or intensity of the alternative x_i over x_k: $p^h_{ik} = 1/2$ indicates indifference between x_i and x_k $(x_i \sim x_k)$, $p^h_{ik} = 1$ indicates that x_i is absolutely preferred to x_k, and $p^h_{ik} > 1/2$ indicates that x_i is preferred to x_k $(x_i \succ x_k)$. Based on this interpretation we have that $p^h_{ii} = 1/2 \ \ \forall i \in \{1,\ldots,n\}$ $(x_i \sim x_i)$.

A. *Consistency of Fuzzy Preference Relations*

Consistency [14], that is, lack of contradiction, is usually a very desirable property for preference relations (information without contradiction is usually more valuable than contradictory information). In [13] we developed some consistency measures for fuzzy preference relations which are based on the additive consistency property, whose mathematical definition was provided by Tanino in [25]:

$$(p^h_{ij} - 0.5) + (p^h_{jk} - 0.5) = (p^h_{ik} - 0.5) \ \forall i,j,k \in \{1,\ldots,n\} \tag{1}$$

that can be rewritten as:

$$p^h_{ik} = p^h_{ij} + p^h_{jk} - 0.5 \quad \forall i,j,k \in \{1,\ldots,n\} \tag{2}$$

We consider a fuzzy preference relation P^h to be *additive consistent* when for every three alternatives in the problem $x_i, x_j, x_k \in X$ their associated preference degrees $p^h_{ij}, p^h_{jk}, p^h_{ik}$ fulfil (2).

Additionally, expression (2) can be used to calculate an estimated value of a preference degree using other preference degrees in a fuzzy preference relation. Indeed, the preference value p^h_{ik} $(i \neq k)$ can be estimated using an intermediate alternative x_j in three different ways:

1) From $p^h_{ik} = p^h_{ij} + p^h_{jk} - 0.5$ we obtain the estimate
$$(cp^h_{ik})^{j1} = p^h_{ij} + p^h_{jk} - 0.5 \tag{3}$$
2) From $p^h_{jk} = p^h_{ji} + p^h_{ik} - 0.5$ we obtain the estimate
$$(cp^h_{ik})^{j2} = p^h_{jk} - p^h_{ji} + 0.5 \tag{4}$$
3) From $p^h_{ij} = p^h_{ik} + p^h_{kj} - 0.5$ we obtain the estimate
$$(cp^h_{ik})^{j3} = p^h_{ij} - p^h_{kj} + 0.5 \tag{5}$$

The overall estimated value cp^h_{ik} of p^h_{ik} is obtained as the average of all possible $(cp^h_{ik})^{j1}$, $(cp^h_{ik})^{j2}$ and $(cp^h_{ik})^{j3}$ values:

$$cp^h_{ik} = \frac{\sum_{j=1; i \neq k \neq j}^{n} (cp^h_{ik})^{j1} + (cp^h_{ik})^{j2} + (cp^h_{ik})^{j3}}{3(n-2)} \tag{6}$$

When the information provided is completely consistent then $(cp^h_{ik})^{jl} = p^h_{ik} \ \forall j,l$. However, because experts are not always fully consistent, the information given by an expert may not verify (2) and some of the estimated preference degree values $(cp^h_{ik})^{jl}$ may not belong to the unit interval $[0,1]$. We note, from expressions (3–5), that the maximum value of any of the preference degrees $(cp^h_{ik})^{jl}$ $(l \in \{1,2,3\})$ is 1.5 while the minimum one is -0.5. Taking this into account, we define the error between a preference value and its estimated one as follows:

Definition 2: The error between a preference value and its estimated one in $[0, 1]$ is computed as:

$$\varepsilon p_{ik}^h = \frac{2}{3} \cdot |cp_{ik}^h - p_{ik}^h| \quad (7)$$

Thus, it can be used to define the consistency level between the preference degree p_{ik}^h and the rest of the preference values of the fuzzy preference relation.

Definition 3: The *consistency level associated to a preference value* p_{ik}^h is defined as

$$cl_{ik}^h = 1 - \varepsilon p_{ik}^h \quad (8)$$

When $cl_{ik}^h = 1$ then $\varepsilon p_{ik}^h = 0$ and there is no inconsistency at all. The lower the value of cl_{ik}^h, the higher the value of εp_{ik}^h and the more inconsistent is p_{ik}^h with respect to the rest of information.

Easily, we can define the consistency measures for particular alternatives and for the whole fuzzy preference relation:

Definition 4: The *consistency level associated to a particular alternative* x_i of a fuzzy preference relation P^h is defined as

$$cl_i^h = \frac{\sum_{\substack{k=1\\ i \neq k}}^{n} (cl_{ik}^h + cl_{ki}^h)}{2(n-1)} \quad (9)$$

with $cl_i^h \in [0, 1]$.

Definition 5: The *consistency level of a fuzzy preference relation* P^h is defined as follows:

$$cl^h = \frac{\sum_{i=1}^{n} cl_i^h}{n} \quad (10)$$

with $cl^h \in [0, 1]$.

B. Consensus Measures

In [12] we developed some consensus measures that can be obtained from the fuzzy preference relations expressed by the experts to solve a group decision making problem. In this section we briefly present them. In fact, as in [11], [15] we compute two different kinds of measures: consensus degrees and proximity measures. Consensus degrees are used to measure the actual level of consensus in the process, whilst the proximity measures give information about how close to the collective solution every expert is. Both kind of measures are given on three different levels for a fuzzy preference relation: pairs of alternatives, alternatives and relations.

Firstly, for each pair of experts (e_h, e_l) $(h < l)$ we define a similarity matrix $SM^{hl} = \left(sm_{ik}^{hl}\right)$ where

$$sm_{ik}^{hl} = 1 - |\overline{p}_{ik}^h - \overline{p}_{ik}^l| \quad (11)$$

Then, a collective similarity matrix, $SM = (sm_{ik})$ is obtained by aggregating all the $(m-1) \times (m-2)$ similarity matrices using the arithmetic mean as the aggregation function ϕ:

$$sm_{ik} = \phi(sm_{ik}^{hl}) \ ; \ \forall h, l = 1, ..., m \mid h < l. \quad (12)$$

Once the similarity matrices are computed we proceed to calculate the consensus degrees in the three different levels:

1) **Definition 6:** *Consensus degree on pairs of alternatives*. The consensus degree on a pair of alternatives (x_i, x_k), denoted cop_{ik}, is defined to measure the consensus degree amongst all the experts on that pair of alternatives:

$$cop_{ik} = sm_{ik} \quad (13)$$

2) **Definition 7:** *Consensus degree on alternatives*. The consensus degree on alternative x_i, denoted ca_i, is defined to measure the consensus degree amongst all the experts on that alternative:

$$ca_i = \frac{\sum_{k=1; k \neq i}^{n} (cop_{ik} + cop_{ki})}{2(n-1)} \quad (14)$$

3) **Definition 8:** *Consensus degree on the relation*. The consensus degree on the relation, denoted CR, is defined to measure the global consensus degree amongst all the experts' opinions:

$$CR = \frac{\sum_{i=1}^{n} ca_i}{n} \quad (15)$$

To compute proximity measures for each expert we need to obtain the collective fuzzy preference relation, P^c, which summarizes preferences given by all the experts. To obtain P^c we use an IOWA operator [26], [27], which uses both consensus and consistency criteria as inducing variable. In such a way, we obtain each collective fuzzy preference degree according to the most consistent and consensual individual fuzzy preference degrees. For more details check [12].

Once we have computed P^c, we can compute the proximity measures in each level of a fuzzy preference relation:

1) **Definition 9:** *Proximity measure on pairs of alternatives*. The proximity measure of an expert e_h on the pair of alternatives (x_i, x_k) to the group one, denoted pp_{ik}^h, is calculated as

$$pp_{ik}^h = 1 - |\overline{p}_{ik}^h - p_{ik}^c| \quad (16)$$

2) **Definition 10:** *Proximity measure on alternatives*. The proximity measure of an expert e_h on alternative x_i to the group one, denoted pa_i^h, is calculated as:

$$pa_i^h = \frac{\sum_{k=1; k \neq i}^{n} (pp_{ik}^h + pp_{ki}^h)}{2(n-1)} \quad (17)$$

3) **Definition 11:** *Proximity measure on the relation*. The proximity measure of an expert e_h on his/her preference relation to the group one, denoted pr^h, is calculated as:

$$pr^h = \frac{\sum_{i=1}^{n} pa_i^h}{n} \quad (18)$$

III. Similarity Measures and Clustering Algorithm for Grouping Experts

In this section we present some new similarity measures that can be computed from the fuzzy preference relations expressed by experts. These measures, as the consistency and consensus measures presented in section II, are computed in three different levels (pair of alternatives, alternatives and preference relations levels) for every pair of experts in the problem.

A. Similarity Measures

From the similarity matrix SM^{hl} presented in section II-B we define several similarity measures among experts at three diferent levels.

Definition 12: The measure of similarity of the preference experts e_h and e_l about the alternative x_i over x_k is sm_{ik}^{hl}.

The closer sm_{ik}^{hl} is to 1, the more similar is the preference of the experts e_h and e_l about the alternative x_i over x_k.

Following the same scheme we compute similarity measures at the alternatives and preference relation levels:

Definition 13: A similarity measure for experts e_h and e_l for a particular alternative x_i is computed as:

$$sm_i^{hl} = \frac{\sum_{\substack{k=1 \\ i \neq k}}^{n} (sm_{ik}^{hl} + sm_{ki}^{hl})}{2(n-1)} \tag{19}$$

The closer sm_i^{hl} is to 1, the more similar is the preference of the experts e_h and e_l about the alternative x_i.

Definition 14: A global similarity measure for experts e_h and e_l (taking into account the whole preference relations) is computed as:

$$sm^{hl} = \frac{\sum_{i=1}^{n} sm_i^{hl}}{n} \tag{20}$$

The closer sm^{hl} is to 1, the more similar are the preferences of the experts e_h and e_l as a whole.

As consistency of the information is also an important issue to take into account (inconsistent experts are usually far away from the opinions of the other experts) we have introduced the consistency measures presented in section II-A into the previous equations in order to alter the similarity measures according to the experts consistency, that is, for a pair of experts the similarity measures will be lower if they are not consistent:

$$\overline{sm}_{ik}^{hl} = sm_{ik}^{hl} \cdot \frac{(cl_{ik}^h + cl_{ik}^l)}{2}$$

$$\overline{sm}_i^{hl} = sm_i^{hl} \cdot \frac{(cl_i^h + cl_i^l)}{2}$$

$$\overline{sm}^{hl} = sm^{hl} \cdot \frac{(cl^h + cl^l)}{2}$$

B. Clustering Algorithm for Grouping Experts

In this section we describe the application of a clustering algorithm which allows to group the different experts in the problem according to their preferences. This grouping can be used to detect which experts in the problem have similar opinions. This kind of information can be very useful in group decision making problems where the experts are not able to gather together to discuss the alternatives to finally obtain a solution of consensus (for example, in web based consensus processes) because it can help experts to clearly differentiate which of the other experts have similar opinions and thus join them in order to effectively discuss with the rest of experts with different preferences. The algorithm can be applied to group the experts according to just a preference value, an alternative or according to the whole preference relations. To do so we just use the consistency, consensus and similarity measures at the desired level.

The algorithm that we have used is a variation of the k-means algorithm [21]. We must note that this algorithm, in adition to the distances between the points to cluster, takes as an input a parameter k which represents the number of clusters in which the points are going to be organized. It is the task of the moderator to provide a proper value for k to the clustering algorithm.

The algorithm begins randomly associating each expert to one of the k clusters. Once all the experts have assigned to a cluster the algorithm computes the centroid of the clusters in a similar way as we do with P^c in section II-B. Then a similarity measure is computed between every expert and the centroids and the cluster of each expert is changed to the most closer centroid. The process is repeated until there are no more changes in the clusters. When the clustering algorithm ends it returns a set of k experts groups:

$$EG = \{eg^1, \ldots, eg^k\} \mid \cup_{i=1}^{k} eg^i = E \ \wedge \ \cap_{i=1}^{k} eg^i = \emptyset$$

Additionally, we can compute the most representative expert for each of the groups, that is, the expert which is nearer from the centroid of his cluster. To compute which is the most representative expert for a group of experts that have similar opinions can be of great importance for group decision making situations where there are many experts involved. In those cases, when the consensus process is in an advanced stage, the most representative expert for the group can act as a spokesperson to accelerate and successfully end the process.

IV. A Tool to Visualize the Consensus State for Group Decision Making Problems

In this section we present a novel visualization tool that generates consensus diagrams in which the experts on the problem are drawn in different locations depending on the similarity of their opinions, that is, experts with similar opinions will be drawn near to each other, while the experts whose opinions differ greatly will be drawn far away from each other. This kind of diagrams can be a very simple and direct way to identify which is the current consensus state of the problem. Additionally, they allow to detect if some of the experts are trying to bias the consensus process: for example, if in every consensus diagram for each consensus round one expert is further away from the current consensus solution, this probably means that he/she is trying to bias the consensus status towards a non-consensus solution.

The tool has been programmed using the Java programming language and uses Scalable Vector Graphics (SVG) to draw the consensus diagrams. Both Java and SVG are current technologies which can be easily embedded in web and mobile systems.

To draw the consensus diagrams we use a spring model graph drawing algorithm [7] in which the experts correspond to the nodes of the graph and a similarity measure between each pair of experts act as the length of the spring associated to each edge of the graph. These kind of algorithms simulate a system of springs defined on the graph and output a locally minimum energy configuration.

As we have defined several different similarity measures, the tool can use different similarity measures depending on the information that we want to visualize. For example, if we need a general overview of the consensus state for the problem, we can choose to use the global similarity measures $\overline{sm}^{hl}$, but if we want to visualize the consensus state about a particular alternative x_i we can choose to use the similarity measures $\overline{sm}_i^{hl}$.

As we have previously said, consistency of the information is also an important issue to take into account and thus, we have programmed the tool to represent experts in different sizes according to their consistency expressed in their preference relations. Thus, it is very easy to recognize the most consistent experts from those who provide contradictory information.

Finally, the tool uses different colors to represent the different expert groups to easily recognize the main opinion factions in the consensus process. It also marks the relative positions of the current global consensus solution P^c, the centroids for each of the groups of experts and the experts which are nearer to the centroid for each group (the possible spokesperson for each group).

A. *Example of Consensus Diagram*

In figure 1 we have a snapshot of one of the consensus diagrams produced by the visualization tool for a toy group decision making problem. In the problem, 7 experts are requested to select the best of four possible alternatives. The experts have provided their preferences about the alternatives in form of fuzzy preference relations. As it can be seen, there exist two main groups of experts (the red and the blue ones) and two isolated experts (*Sergio* and *Javier*), that is, those whose opinions are quite different from the other experts (drawn in green).

The distances between experts depend on the similarity between their opinions, and so, just by looking at the diagram one can infere that the opinion of *Javier* it's quite different from the opinion of the other experts. Moreover, from the different sizes ofthe experts it is possible to know that *Sergio* has been the most inconsistent expert, that is, his preference relation introduces several inconsistencies, and that is probably why he is isolated from the other experts.

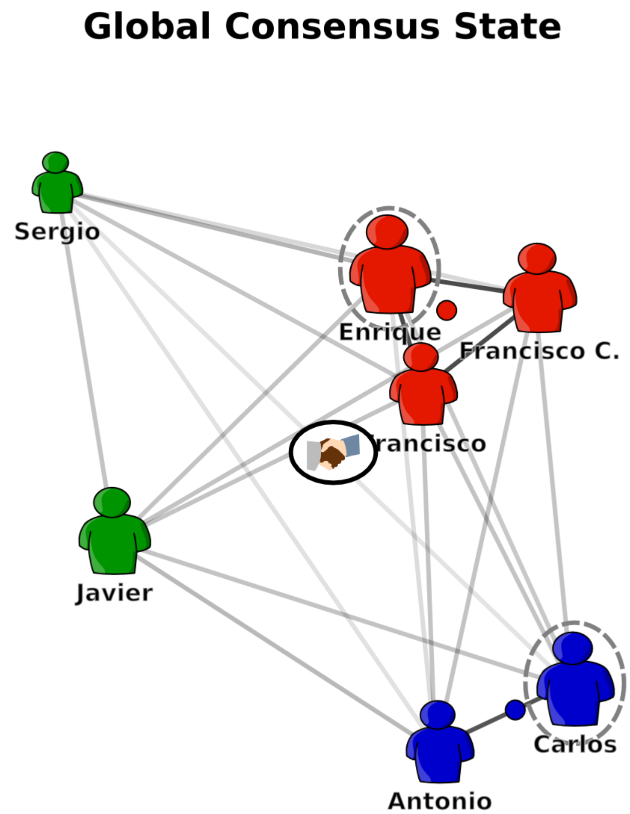

Fig. 1. **Snapshot of the Visualization Tool**

The centroids of the different groups have been marked with coloured spots. From that points we can say that although *Enrique*, *Francisco* and *Francisco C.* have similar opinions (they belong to the same group), *Enrique* is a better candidate to be the spokesperson of the group if he is required to. The same is reasoning is applied to select *Carlos* as a candidate for spokesperson of the blue group.

Finally, the current consensus solution, painted as a handshake, tells that the red group and *Javier* are the factions whose opinions are closer to the current consensus solution.

V. Conclusions

In this paper we have presented a tool that allows to visualize the status of a consensus process. It makes use of consistency, consensus and similarity measures along with a clustering algorithm in order to generate some consensus diagrams were experts are drawn nearer when their opinions are similar. The tool also identifies the main groups of experts (those who have similar opinions) and it is able to select a candidate for spokesperson for each group. Thus, we provide a powerful tool for experts that participate in consensus processes where there is no possibility of gathering together (for example if the consensus process is held by means of web technologies) and consecuently, where is difficult to obtain a clear overview of the current consensus state. Additionally, with the use of these consensus diagrams it is also possible to detect if some of the experts in the consensus process are trying to bias the consensus process.

In future works we will improve this visualiztion tool in order to be able to represent more useful information in the consensus diagrams, and add the possibility of get similar diagrams in situations where experts provide their preferences using other preference relations formats.

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