Fuzzy Linguistic Preference Modelling for Group Decision Making

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Resumen

The designer of a model to solve decision making problems usually have to face different aspects to finally be able to obtain the solution for a problem. Many of this aspects directly depend on the representation format and semantics of the information provided by the experts. In many occasions experts can not easily represent their preferences using artificial preference structures as fuzzy preference relations, but they can express those preferences by means of linguistic terms.

In this work we present four different fuzzy linguistic modelling techniques that allow the representation of experts' preferences by means of linguistic preference relations. The linguistic preference relations will be used to solve the group decision making problem by applying a resolution process capable of aggregating the linguistic information from all the experts to laterly exploit that information to find the best solution(s) for the problem.

Keywords: Preference Modelling, Fuzzy Linguistic Modelling, Group Decision Making

1 Introduction

Group Decision Making (GDM) is used to obtain the best solution(s) for a problem according to the information provided by some experts. Concretely, in a GDM problem we have a set of different alternatives to solve the problem and a set of experts which give their preferences about the alternatives in order to choose the best one(s). There exist many different models to solve GDM problems, and all of them to deal with some common issues, as how to reach a global solution according to all the information available, or how to manage that information. Concretely, the question of how can the experts express their preferences is a major issue to be faced.

There exist many different representation formats that can be used in each model, i.e., preference orderings, utility values, multiplicative preference relations, fuzzy preference relations and so on. Every representation format has its own advantages and disadvatages, like precision or easiness of use and understanding. Many of these preference formats use numerical data to represent experts preferences. For example, a fuzzy preference relation consists on a set of numerical evaluations over pairs of alternatives on the problem. The greater a number is, the most preferred an alternative is over another. It is almost clear that there are experts that do not find this kind of numerical representation formats easy to understand, and they can even find them difficult to use. An expert can be able to say that an option over the set of alternatives is *better* than anotherm whilst he is not capable to give an exact quantity on how better is that preference over the other.

There are tools that have been developed to solve this kind of issues. Concretely, the *fuzzy linguistic modelling* is a tool based on the concept of *linguistic variable* [16] that is capable to deal with qualitative assessments in problems, that is, assessments which are not precise ones, but contain a certain amount of vagueness.

In this work, we revise four different approaches of the fuzzy linguistic modelling: *Ordinal fuzzy linguistic modelling* [2, 3], *2-tuple fuzzy linguistic modelling* [4, 7], *Multi-granular fuzzy linguistic modelling* [5, 6], and *Unbalanced fuzzy linguistic modelling.* Those four approaches can be used by the experts to express linguistic preference relations about a particular GDM problem.

We will also introduce a resolution process that will act over the those linguistic preference relations, in order to identify the best alternative(s), and thus, to solve the GDM problem can be solved. The resolution process consist on two different steps:

1. *Aggregation Step*, in which all linguistic preference relations are aggregated into a collective one, and
2. *Exploitation Step*, where the best alternative(s) will be chosen from the collective preference relation by means of the application of a two quantifier guided choice degrees of alternatives.

In order to do this, the paper is set as follows. In *Section 2* we present four different approaches to the fuzzy linguistic modelling. In *Section 3* the resolution process for GDM problems with linguistic preference relations is presented. We also give an example of the resolution process for a particular GDM problem. Finally, in *Section 4* some conclusions are pointed out.

2 Fuzzy Linguistic Modelling

There are situations in which the information cannot be assessed precisely in a quantitative form but may be in a qualitative one. For example, when attempting to qualify phenomena related to human perception, we are often led to use words in natural language instead of numerical values, e.g. when evaluating the *comfort* or *design* of a car, terms like *good*, *medium* or *bad* can be used. In other cases, precise quantitative information cannot be stated because either it is unavailable or the cost for its computation is too high and an "approximate value" can be applicable, eg. when evaluating the speed of a car, linguistic terms like *fast*, *very fast* or *slow* can be used instead of numeric values.

The use of Fuzzy Sets Theory has given very good results for modelling qualitative information [16]. The *fuzzy linguistic modelling* is a tool based on the concept of *linguistic variable* [16] to deal with qualitative assessments in the problems. It has proven its useful in many problems, e.g., in decision making [2], quality evaluation [12], models of information retrieval [10, 11], etc.

In this section, we revise four different approaches of the fuzzy linguistic modelling which can provide a different support to represent the linguistic information managed in decision making models:

1. *Ordinal fuzzy linguistic modelling* [2, 3], which is defined to eliminate the excessive complexity of the traditional fuzzy linguistic modelling [16].
2. *2-tuple fuzzy linguistic modelling* [4, 7], which is is defined to improve the performance of the ordinal fuzzy linguistic approach.
3. *Multi-granular fuzzy linguistic modelling* [5, 6], which is defined to deal with situations in which the linguistic information is assessed on different label sets.
4. *Unbalanced fuzzy linguistic modelling* [8, 9], which is defined to deal with situations in which the linguistic information is assessed on an unbalanced label set, that is, a non-symmetrical and non-uniform label set.

2.1 The Ordinal Fuzzy Linguistic Modelling

The *ordinal fuzzy linguistic modelling* [2, 3] is a very useful kind of fuzzy linguistic approach proposed as an alternative tool to the traditional fuzzy linguistic modelling [16] which simplifies the computing with words process as well as linguistic aspects of problems. It is defined by considering a finite and totally ordered label set $S = \{s_i\}, i \in \{0, \ldots, g\}$ in the usual sense, i.e., $s_i \geq s_j$ if $i \geq j$, and with odd cardinality (7 or 9 labels). The mid term represents an assessment of "approximately 0.5", and the rest of the terms being placed symmetrically around it. The semantics of the label set is established from the ordered structure of the label set by considering that each label for the pair $(s_i, s_{\}-i})$ is equally informative. For example, we can use the following set of seven labels to represent the linguistic information:

$S = \{s_0 = N,\ s_1 = VL,\ S_2 = L,\ s_3 = M,\ s_4 = H,\ s_5 = VH,\ s_6 = P\}$.

Additionally, a fuzzy number defined in the $[0, 1]$ interval can be associated with each linguistic term. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function. The linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of such linguistic assessments. The parametric representation is achieved by the 4-tuple (a, b, c, d), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function. A particular case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, then we represent this type of membership functions by a 3-tuple (a, b, c). An example may be

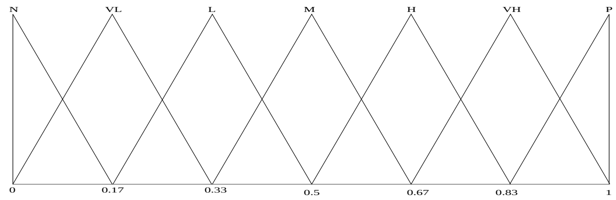

Figura 1: A set of seven linguistic terms with its semantics

the following set of seven terms (Figure 1):

$$
\begin{aligned}
s_0 &= Null(N) = (0, 0, .17)\\
s_1 &= VeryLow(VL) = (0, .17, .33)\\
s_2 &= Low(L) = (.17, .33, .5)\\
s_3 &= Medium(M) = (.33, .5, .67)\\
s_4 &= High(H) = (.5, .67, .83)\\
s_5 &= VeryHigh(VH) = (.67, .83, 1)\\
s_6 &= Perfect(P) = (.83, 1, 1).
\end{aligned}
$$

In any linguistic modelling we need management operators of linguistic information. An advantage of the ordinal fuzzy linguistic modelling is the simplicity and quickness of its computational model. It is based on the symbolic computation [2, 3] and acts by direct computation on labels by taking into account the order of such linguistic assessments in the ordered structure of labels. Usually, the ordinal fuzzy linguistic model for computing with words is defined by establishing i) a negation operator, ii) comparison operators based on the ordered structure of linguistic terms, and iii) adequate aggregation operators of ordinal fuzzy linguistic information. In most ordinal fuzzy linguistic approaches the negation operator is defined from the semantics associated to the linguistic terms as

$$NEG(s_i) = s_j \mid j = g - i;$$

and there are defined two comparison operators of linguistic terms:

1. *Maximization operator*: $MAX(s_i, s_j) = s_i$ if $s_i \geq s_j$; and
2. *Minimization operator*, $MIN(s_i, s_j) = s_i$ if $s_i \leq s_j$.

Using these operators it is possible to define automatic and symbolic aggregation operators of linguistic information, as for example the LOWA operator [2] and the LWA operator [3].

2.2 The 2-Tuple Fuzzy Linguistic Modelling

The *2-tuple fuzzy linguistic modelling* [4, 7] is a kind of fuzzy linguistic modelling that mainly allows to reduce the loss of information typical of the ordinal fuzzy linguistic modelling. Its main advantage is that the linguistic computational model based on linguistic 2-tuples can carry out processes of computing with words easier and without loss of information. To define it we have to establish the 2-tuple representation model and the 2-tuple computational model to represent and aggregate the linguistic information, respectively.

Let $S = \{s_0, ..., s_g\}$ be a linguistic term set with odd cardinality ($g + 1$ is the cardinality of S), where the mid term represents an assessment of approximately 0.5 and with the rest of the terms being placed symmetrically around it. We assume that the semantics of labels is given by means of triangular membership functions represented by a 3-tuple (a, b, c) and consider all terms distributed on a scale on which a total order is defined $s_i \leq s_j \iff i \leq j$. In this fuzzy linguistic context, if a symbolic method [2, 3] aggregating linguistic information obtains a value $\beta \in [0, g]$, and $\beta \notin \{0, ..., g\}$, then an approximation function is used to express the result in S.

Definition 1. [4] *Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation, $\beta \in [0, g]$. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-.5, .5)$ then α is called a Symbolic Translation.*

The 2-tuple fuzzy linguistic approach is developed from the concept of symbolic translation by representing the linguistic information by means of 2-tuples (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-.5, .5)$:

- s_i represents the linguistic label of the information, and
- α_i is a numerical value expressing the value of the translation from the original result β to the closest index label, i, in the linguistic term set ($s_i \in S$).

This model defines a set of transformation functions between numeric values and 2-tuples.

Definition 2. [4] *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha), \ with \left\{ \begin{array}{cc} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{array} \right.$$

where $round(\cdot)$ is the usual round *operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.*

For all Δ there exists Δ^{-1}, defined as $\Delta^{-1}(s_i, \alpha) = i + \alpha$. On the other hand, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a symbolic translation value of 0: $s_i \in S \Longrightarrow (s_i, 0)$.

The 2-tuple linguistic computational model is defined by presenting the comparison of 2-tuples, a negation operator and aggregation operators of 2-tuples.

1. Comparison of 2-tuples. The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order. Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a counting of information:

- If $k < l$ then (s_k, α_1) is smaller than (s_l, α_2).
- If $k = l$ then
  1. if $\alpha_1 = \alpha_2$ then (s_k, α_1) and (s_l, α_2) represent the same information,
  2. if $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2),
  3. if $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2).

2. Negation operator of 2-tuples: $Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))$.

3. Aggregation operators of 2-tuples. The aggregation of information consists of obtaining a value that summarizes a set of values, therefore, the result of the aggregation of a set of 2-tuples must be a 2-tuple. In the literature we can find many aggregation operators which allow us to combine the information according to different criteria. Using functions Δ and Δ^{-1} that transform without loss of information numerical values into linguistic 2-tuples and viceversa, any of the existing aggregation operator can be easily extended for dealing with linguistic 2-tuples. Some examples are:

Definition 3. (Arithmetic Mean). *Let $x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of linguistic 2-tuples, the 2-tuple arithmetic mean $\overline{x}^e$ is computed as,*

$$\overline{x}^e[(r_1, \alpha_1), \ldots, (r_n, \alpha_n)] = \Delta(\sum_{i=1}^{n} \frac{1}{n} \Delta^{-1}(r_i, \alpha_i)) = \Delta(\frac{1}{n} \sum_{i=1}^{n} \beta_i).$$

Definition 4. (Weighted Average Operator). *Let $x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of linguistic 2-tuples and $W = \{w_1, ..., w_n\}$ be their associated weights. The 2-tuple weighted average $\overline{x}^w$ is:*

$$\overline{x}^w[(r_1, \alpha_1), \ldots, (r_n, \alpha_n)] = \Delta(\frac{\sum_{i=1}^{n} \Delta^{-1}(r_i, \alpha_i) \cdot w_i}{\sum_{i=1}^{n} w_i}) = \Delta(\frac{\sum_{i=1}^{n} \beta_i \cdot w_i}{\sum_{i=1}^{n} w_i}).$$

Definition 5. (Linguistic Weighted Average Operator). *Let $x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of linguistic 2-tuples and $W = \{(w_1, \alpha_1^w), ..., (w_n, \alpha_n^w)\}$ be their linguistic 2-tuple associated weights. The 2-tuple linguistic weighted average $\overline{x}_l^w$ is:*

$$\overline{x}_l^w[((r_1, \alpha_1), (w_1, \alpha_1^w))...((r_n, \alpha_n), (w_n, \alpha_n^w))] = \Delta(\frac{\sum_{i=1}^{n} \beta_i \cdot \beta_{W_i}}{\sum_{i=1}^{n} \beta_{W_i}}),$$

with $\beta_i = \Delta^{-1}(r_i, \alpha_i)$ and $\beta_{W_i} = \Delta^{-1}(w_i, \alpha_i^w)$.

2.3 The Multi-Granular Fuzzy Linguistic Modelling

In any fuzzy linguistic approach, an important parameter to determinate is the "granularity of uncertainty", i.e., the cardinality of the linguistic term set S used to express the linguistic information. According to the uncertainty degree that an expert qualifying a phenomenon has on it, the linguistic term set chosen to provide his knowledge will have more or less terms. When different experts have different uncertainty degrees on the phenomenon, then several linguistic term sets with a different granularity of uncertainty are necessary (i.e. multi-granular linguistic information) [5, 6, 14]. The use of different label sets to assess information is also necessary when an expert has to assess different concepts, as for example it happens in information retrieval problems, to evaluate the importance of the query terms and the relevance of the retrieved documents [13]. In such situations, we need tools for the management of multi-granular linguistic information, i.e., we need to define a *multi-granular fuzzy linguistic modelling.* In [5] we define a proposal of multi-granular fuzzy linguistic modelling based on the ordinal fuzzy linguistic modelling and in [6] we define other one based on the 2-tuple fuzzy linguistic modelling. In this paper, we follow that defined in [6] which uses the concept of the *Linguistic Hierarchies* to manage the multi-granular linguistic information.

A *linguistic hierarchy* is a set of levels, where each level is a linguistic term set with different granularity from the remaining of levels of the hierarchy [1]. Each level belonging to a linguistic hierarchy is denoted as *l(t,n(t))*, being t a number that indicates the level of the hierarchy and *n(t)* the granularity of the linguistic term set of the level t.

Usually, linguistic hierarchies deal with linguistic terms whose membership functions are triangular-shaped, symmetrical and uniformly distributed in [0,1]. In addition, the linguistic term sets have an odd value of granularity representing the central label the value of *indifference*.

The levels belonging to a linguistic hierarchy are ordered according to their granularity, i.e., for two consecutive levels t and $t+1$, $n(t+1) > n(t)$. Therefore, each level $t+1$ provides a linguistic refinement of the previous level t.

A linguistic hierarchy, *LH*, is defined as the union of all levels t: $LH = \bigcup_t l(t, n(t))$. To build LH we must keep in mind that the hierarchical order is given by the increase of the granularity of the linguistic term sets in each level. Let $S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$ be the linguistic term set defined in the level t with $n(t)$ terms, then the building of a linguistic hierarchy must satisfy the following linguistic hierarchy basic rules [6]:

1. To preserve all *former modal points* of the membership functions of each linguistic term from one level to the following one.
2. To make *smooth transactions between successive levels*. The aim is to build a new linguistic term set, $S^{n(t+1)}$. A new linguistic term will be added between each pair of terms belonging to the term set of the previous level t. To carry out this insertion, we shall reduce the support of the linguistic labels in order to keep place for the new one located in the middle of them.

Generically, we can say that the linguistic term set of level $t+1$, $S^{n(t+1)}$, is obtained from its predecessor level t, $S^{n(t)}$ as: $l(t, n(t)) \rightarrow l(t+1, 2 \cdot n(t) - 1)$. Table 1 shows the granularity needed in each linguistic term set of the level t depending on the value $n(t)$ defined in the first level (3 and 7 respectively).

	Level 1	Level 2	Level 3
l(t,n(t))	l(1,3)	l(2,5)	l(3,9)
l(t,n(t))	l(1,7)	l(2,13)	

Table 1. Linguistic Hierarchies.

A graphical example of a linguistic hierarchy is shown in figure 2:

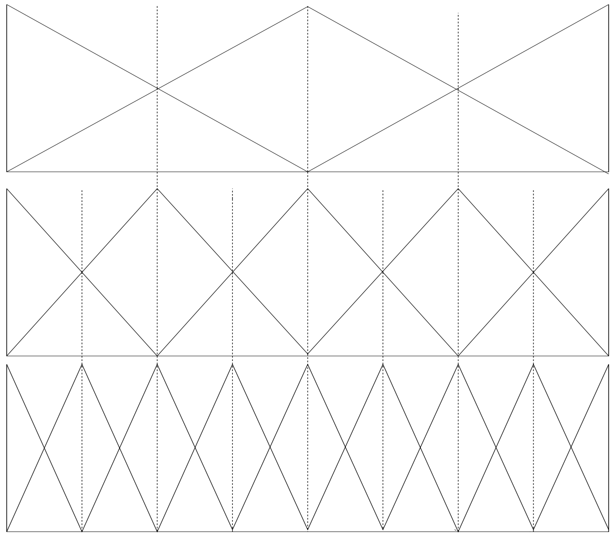

Figura 2: Linguistic Hierarchy of 3, 5 and 9 labels

In [6] was demonstrated that the linguistic hierarchies are useful to represent the multi-granular linguistic information and allow to combine multi-granular linguistic information without loss of information. To do this, a family of transformation functions between labels from different levels was defined:

Definition 6. *Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$. The transformation function between a 2-tuple that belongs to level t and another 2-tuple in level $t' \neq t$ is defined as:*

$$TF_{t'}^{t} : l(t, n(t)) \longrightarrow l(t', n(t'))$$

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t') - 1)}{n(t) - 1})$$

As it was pointed out in [6] this family of transformation functions is biyective.

2.4 The Unbalanced Fuzzy Linguistic Modelling

In any problem that uses linguistic information the first goal to satisfy is the choice of the linguistic terms with their semantics, for establishing the label set to be used in the problem. In the literature, we can find two different possibilities for choosing the linguistic terms and their semantics:

- We can assume that all the terms of the label set are equally informative, i.e., symmetrically distributed as it happens in the above fuzzy linguistic modelling.
- We can assume that all the terms of the label set are not equally informative, i.e., not symmetrically distributed. In this case, we need an *unbalanced fuzzy linguistic modelling* [8, 9] to manage the linguistic term sets with different discrimination levels on both sides of the mid term (see Figure 3). As was known in [8], in the information retrieval systems the use of unbalanced linguistic

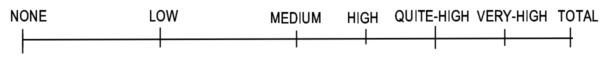

Figura 3: Unbalanced Linguistic Term Set of 7 Labels

term sets seems more appropriate than the use of symmetrical linguistic term sets, as to express the importance weights in the queries as to represent the relevance degrees of the documents.

To manage unbalanced linguistic term sets we propose a method based on the 2-tuple fuzzy linguistic modelling. Basically, this method consists of representing unbalanced linguistic terms from different levels of an LH, carrying out computational operations of unbalanced linguistic information using the 2-tuple computational model. The method consists of the following steps:

1. Represent the unbalanced linguistic term set $\mathcal{S}$ by means of a linguistic hierarchy, LH.

   1.1. Chose a level t^- with an adequate granularity to represent using the 2-tuple representation model the subset of linguistic terms of $\mathcal{S}$ on the left of the mid linguistic term.

   1.2. Chose a level t^+ with an adequate granularity to represent using the 2-tuple representation model the subset of linguistic terms of $\mathcal{S}$ on the right of the mid linguistic term.

2. Define an unbalanced linguistic computational model.

   2.1. Choose a level $t' \in \{t^-, t^+\}$, such that $n(t') = max\{n(t^-), n(t^+)\}$.

   2.2. Define the comparison of two 2-tuples $(s_k^{n(t)}, \alpha_1)$, $t \in \{t^-, t^+\}$, and $(s_l^{n(t)}, \alpha_2)$, $t \in \{t^-, t^+\}$, with each one representing a counting of unbalanced information. Its expression is similar to the usual comparison of two 2-tuples but acting on the values $TF_{t'}^{t}(s_k^{n(t)}, \alpha_1)$ and $TF_{t'}^{t}(s_l^{n(t)}, \alpha_2)$. We should point out that using the comparison of 2-tuples we can easily define the comparison operators Max and Min.

   2.3. Define the negation operator of unbalanced linguistic information. Let $(s_k^{n(t)}, \alpha)$, $t \in \{t^-, t^+\}$ be an unbalanced 2-tuple then:

   $\mathcal{NEG}(s_k^{n(t)}, \alpha) = Neg(TF_{t''}^{t}(s_k^{n(t)}, \alpha))$, $t \neq t''$, $t'' \in \{t^-, t^+\}$.

   2.4. Define aggregation operators of unbalanced linguistic information. This is done using the aggregation processes designed in the 2-tuple computational model but acting on the unbalanced linguistic values transformed by means of $TF_{t'}^{t}$. Then, once it is obtained a result, it is transformed to the correspondent level t by means of $TF_{t'}^{t}$ to express the result in the unbalanced linguistic term set.

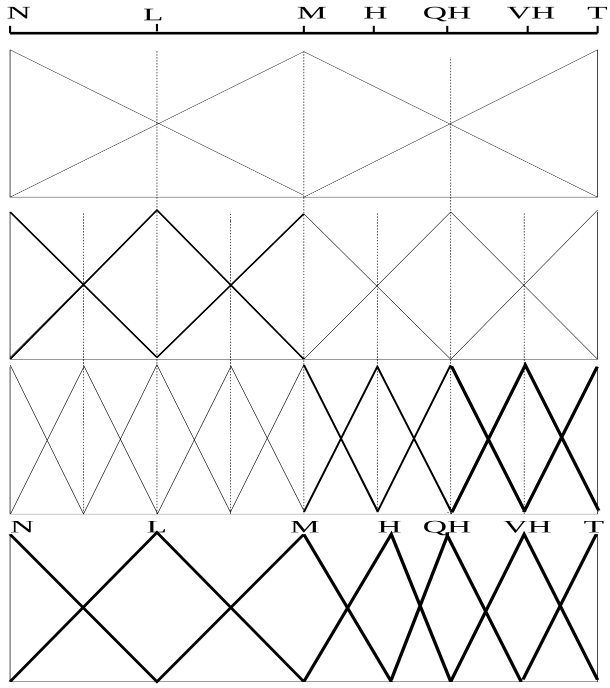

Figura 4: Unbalanced Linguistic Term Set of 7 Labels

Assuming the unbalanced linguistic term set shown in Figure 3 and the linguistic hierarchy shown in Figure 2, in Figure 4 we show how to select the different levels to represent the unbalanced linguistic term set.

3 GDM with Linguistic Preference Relations

A GDM problem consists on choosing the best alternative(s) among a finite set, $X = \{x_1, ..., x_n\}, (n \geq 2)$. The alternatives will be classified from best to worst, using the information known according to a set of experts, i.e., $E = \{e_1, ..., e_m\}(m \geq 2)$.

As we have said before, experts' preferences can be given in many different formats, and concretely, in this work we will make use of the 2-tuple fuzzy linguistic modelling approach to represent the experts' preferences (although any of the other fuzzy linguistic modelling approaches can be easily used).

Thus, to express their opinions about the alternatives, each expert $e_k \in E$, will provide his preferences by means of a *linguistic preference relation* P^k.

Definition: Let S be a set of labels which represents the linguistic domain where experts can express their preferences as it is defined in *Section 2.2*. A linguistic preference relation P on a set of alternatives X is a set of 2-tuples on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_P: X \times X \longrightarrow S \times [-0.5, 0.5)$.

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P = (p_{ij})$ being $p_{ij} = \mu_P(x_i, x_j) \quad \forall i, j \in \{1, \ldots, n\}$ interpreted as the preference degree or intensity of the alternative x_i over x_j. Note that every p_{ij} value is a 2-tuple in the form (s, α) with $s \in S$ and $\alpha \in [-0.5, 0.5)$.

3.1 Resolution Process for GDM Problems with Linguistic Preference Relations

Once the experts have provided their preferences by means of linguistic preference relations, a resolution process can be applied to obtain a final solution to the problem. The resolution process consist mainly in two different substeps:

1. *Aggregation phase.* The information provided by the different experts is aggregated into a collective linguistic preference relation. This collective preference relation summarizes all the information provided by the different experts.

2. *Exploitation phase.* Two different choice degrees of alternatives are applied to the collective preference relation to supply the set of best alternatives for the problem: the quantifier guided dominance degree (QGDD) and the quantifier guided non dominance degree (QGNDD).

3.1.1 Aggregation Phase

This phase consist on aggregating all the information provided by the experts, that is, to obtain a collective linguistic preference relation (CP) from the experts' individual linguistic preference relations (P^k). As it have been presented in *Section 2.2* there are several different aggregation operators that easily operate with linguistic terms expressed by the 2-tuple fuzzy linguistic model. For simplicity we will use the arithmetic mean aggregation operator (it gives equal preference to every expert):

$$CP = \sum_{k=1}^{m} \frac{1}{n} P^k = (cp_{ij})$$

where

$$cp_{ij} = \overline{x}^e[p_{ij}^1, \ldots, p_{ij}^m]$$

3.1.2 Exploitation Phase

At this point, in order to select the alternative(s) "best" acceptable to the group of individuals as a whole, we will use two quantifier guided choice of degrees of alternatives, both based on the OWA operator.

Definition [15]. An OWA operator of dimension n is a function $\phi : \Re^n \rightarrow \Re$, that has associated a set of weights or weighting vector $W = (w_1, ..., w_n)$ to it, so that $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, and is defined to aggregate a list of values $\{p_1, ..., p_n\}$ according to the following expression,

$$\phi_W(p_1, ..., p_n) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)} \qquad (1)$$

being $\sigma : \{1, ..., n\} \rightarrow \{1, ..., n\}$ a permutation such that that $p_{\sigma(i)} \geq p_{\sigma(i+1)}, \forall i = 1, ..., n-1$, i.e., $p_{\sigma(i)}$ is the i-highest value in the set $\{p_1, ..., p_n\}$.

Concretely, we use the following quantifier guided choice degrees:

1. *Quantifier guided dominance degree.* For the alternative x_i we compute the quantifier-guided dominance degree $QGDD_i$ used to quantify the dominance that one alternative has over all the others in a fuzzy majority sense as follows:

$$QGDD_i = \phi_Q(\Delta^{-1}(cp_{ij}), j = 1, ..., n, j \neq i). \qquad (2)$$

2. *Quantifier guided non-dominance degree.* We also compute the quantifier guided non-dominance degree $QGNDD_i$ according to the following expression:

$$QGNDD_i = \phi_Q(g - sp_{ji}, j = 1, ..., n, j \neq i), \qquad (3)$$

where $sp_{ji} = max\{\Delta^{-1}(cp_{ji}) - \Delta^{-1}(cp_{ij}), 0\}$, represents the degree to which x_i is strictly dominated by x_j. In our context, $QGNDD_i$ gives the degree in which each alternative is not dominated by a fuzzy majority of the remaining alternatives.

The application of the above choice degrees of alternatives over X may be carried out according to a sequential policy: Selecting and applying one of them according to the preference of the experts, and thus obtaining a selection set of alternatives. If there is more than one alternative in that selection set, then the other choice degree may be applied to select the alternative of the above set with the best second choice degree. This policy defines a sequential selection process.

3.2 Example

In this section we will give an example of the resolution process for a GDM problem using 2-tuple fuzzy linguistic modelling of the experts' preferences.

We have a GDM problem where three experts $E = \{e_1, e_2, e_3\}$ have to choose among 4 different alternatives, that is, $X = \{x_1, x_2, x_3, x_4\}$. They are asked to give their preferences using linguistic preference relations, with the following set of labels: $S = \{N, VL, L, E, H, VH, C\}$ (Null preference, Very Low preference, Low preference, Equal preference, High preference, Very High preference and Complete preference).

The experts provide the following linguistic preference relations:

$$P_1 = \begin{pmatrix} - & (VL,0) & (L,0) & (E,0) \\ (VH,0) & - & (H,0) & (VH,0) \\ (H,0) & (L,0) & - & (H,0) \\ (E,0) & (VL,0) & (L,0) & - \end{pmatrix}$$

$$P_2 = \begin{pmatrix} - & (L,0) & (H,0) & (VH,0) \\ (H,0) & - & (VH,0) & (VH,0) \\ (L,0) & (VL,0) & - & (E,0) \\ (VL,0) & (VL,0) & (E,0) & - \end{pmatrix}$$

$$P_3 = \begin{pmatrix} - & (VL,0) & (VL,0) & (L,0) \\ (VH,0) & - & (E,0) & (H,0) \\ (VH,0) & (E,0) & - & (H,0) \\ (H,0) & (L,0) & (L,0) & - \end{pmatrix}$$

It is interesting to remark that experts usually give their linguistic preference relations using only the linguistic element in the 2-tuple, and asuming that $\alpha = 0 \;\; \forall p_{ij}$. This happens because they are not able to discriminate with a high level of precision the preference that they have over the alternatives (this vagueness is precisely the reason to use fuzzy linguistic models instead of numerical ones).

Once we have the linguistic preference relations we have to apply the aggregation operator, in our example is the arithmetic mean one. Note that to apply the operator we have to use the Δ^{-1} and Δ functions to transform every 2-tuple into a numerical value and vice-versa:

$$\Delta^{-1}(P_1) = \begin{pmatrix} - & 1.0 & 2.0 & 3.0 \\ 5.0 & - & 4.0 & 5.0 \\ 4.0 & 2.0 & - & 4.0 \\ 3.0 & 1.0 & 2.0 & - \end{pmatrix}$$

$$\Delta^{-1}(P_2) = \begin{pmatrix} - & 2.0 & 4.0 & 5.0 \\ 4.0 & - & 5.0 & 5.0 \\ 2.0 & 1.0 & - & 3.0 \\ 1.0 & 1.0 & 3.0 & - \end{pmatrix}$$

$$\Delta^{-1}(P_3) = \begin{pmatrix} - & 1.0 & 1.0 & 2.0 \\ 5.0 & - & 3.0 & 4.0 \\ 5.0 & 3.0 & - & 4.0 \\ 4.0 & 2.0 & 2.0 & - \end{pmatrix}$$

$$\overset{Aggregation}{\Longrightarrow}$$

$$\Delta^{-1}(CP) = \begin{pmatrix} - & 1.3 & 2.3 & 3.3 \\ 4.7 & - & 4.0 & 4.7 \\ 3.7 & 2.0 & - & 3.7 \\ 2.7 & 1.3 & 2.3 & - \end{pmatrix}$$

$$\overset{\Delta}{\Longrightarrow}$$

$$CP = \begin{pmatrix} - & (VL,0.3) & (L,0.3) & (E,0.3) \\ (VH,-0.3) & - & (H,0) & (VH,-0.3) \\ (H,-0.3) & (L,0) & - & (H,-0.3) \\ (E,-0.3) & (VL,0.3) & (L,0.3) & - \end{pmatrix}$$

Finally we have to apply the exploitation process making use again of the fuzzy quantifier "*most*", whose weighting vector (for four alternatives) is $W = [0.5, 0.21, 0.16, 0.13]$. The quantifier guided dominance degree of alternatives acting over the collective fuzzy preference relation supply the following values:

	x_1	x_2	x_3	x_4
$QGDD_i$	2.8	4.4	3.4	2.6

Clearly the x_2 alternative is much better than the others, and as there is no other alternative with the same dominance degree, the process stops (there is no need to calculate the quantifier guided non-dominance degrees) giving $Sol = \{x_2\}$ as the solution of the problem.

4 Concluding Remarks

In this work we have presented four different approaches of the fuzzy linguistic modelling that can be used to solve GDM problems: *Ordinal fuzzy linguistic modelling*, *2-tuple fuzzy linguistic modelling*, *Multi-granular fuzzy linguistic modelling*, and *Unbalanced fuzzy linguistic modelling*.

We have also presented a resolution process for GDM problems where information given by the experts are expressed in a linguistic domain. This resolution process is carried out in two main steps: an aggregation phase where a collective linguistic preference relation is calculated and an exploitation phase where the best alternative(s) from the feasible set are calculated by means of the application of two quantifier guided choice of degrees of alternatives.

Finally we have provided an example of the application of the resolution process to a GDM problem where the experts expressed their preferences by means of linguistic preference relations, using the 2-tuple fuzzy linguistic model.

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