A Multi-Granular Linguistic Decision Model for Evaluating the Quality of Network Services

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Summary. The Fuzzy Linguistic Approach has been applied successfully to many problems dealing with qualitative aspects that are assessed by means of linguistic terms. The use of linguistic information implies in most cases the need for using fusion processes to obtain aggregated values that summarize the input information. One important limitation of the fuzzy linguistic approach appears when fusion processes are applied to problems in which the linguistic information is assessed in linguistic term sets with different granularity of uncertainty, i.e., different cardinality; this type of information is denoted as multi-granular linguistic information. This limitation consists of the difficulty in dealing with this type of information in fusion processes due to the fact that there is no standard normalization process for this type of information, as in the numerical domain.

In this contribution, taking as base the 2-tuple fuzzy linguistic representation model and its computational technique, we shall present a method for easily dealing with multi-granular linguistic information in fusion processes. Afterwards, we shall apply this fusion method to a decision process in a multi-expert decision-making (MEDM) problem with multi-granular linguistic information, that evaluates the quality of network services from different Operative Systems.

Keywords: Linguistic variables, aggregation, fusion processes, granularity of uncertainty, multi-granular linguistic information.

1 Introduction

On many occasions we find problems that present several sources of information to qualify their phenomena. When these phenomena present quantitative aspects they can be assessed by means of precise numerical values, however

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when the aspects presented by the phenomena are qualitative may be difficult to qualify using precise values. So, the use of the fuzzy linguistic approach [45] has shown itself as a good choice to model these phenomena, due to the fact that it represents qualitative aspects with qualitative terms by means of linguistic variables, i.e., variables whose values are not numbers but words or sentences in a natural or artificial language.

An important aspect when the fuzzy linguistic approach is used, is to determine the "granularity of uncertainty", i.e., the cardinality of the linguistic term set used to assess the linguistic variables. Depending on the uncertainty degree held by a source of information qualifying a phenomenon, the linguistic term set will have more or less terms. Then, in those problems with several sources of information each one may have a different uncertainty degree on the phenomena to qualify. Therefore, each source could express its knowledge by means of linguistic term sets with a different granularity of uncertainty from the other ones. In these situations we shall denote this type of information as multi-granular linguistic information.

The use of the fuzzy linguistic approach implies processes of "computing with words" (CW), in the specialized literature, three different linguistic computational techniques can be found [2, 10, 11, 20]. The first one is based on the Extension Principle [2, 11] that acts on the linguistic terms through computations on the associated membership functions, the second method or Symbolic one [10] acts by direct computations on the labels and the third method uses the 2-tuple fuzzy linguistic representation model [20] and acts on numerical values associated with the fuzzy linguistic 2-tuple. These computational techniques provide linguistic operators for CW.

When a problem presents multi-granular linguistic information, the fuzzy linguistic approach together with the first two linguistic computational techniques mentioned present an important limitation because in these computational methods, neither a standard normalization process nor fusion operators are defined for this type of information. Therefore, it is highly complex to solve this type of problems using these methods and the results obtained present loss of information during computing processes. Besides, they are expressed by values assessed in domains far removed from the initial expression ones, as occurs in the fusion method for multi-granular linguistic information that we presented in [18] for decision problems, introducing a fuzzy preference relation among the alternatives as a final step because it was not possible to manage the multi-granular information directly.

The aim of this paper is to develop an aggregation process, for multi-granular linguistic information, that overcomes the above limitations, i.e., it will be able to reduce the loss of information and express the final results in an expression domain near the initial one. To do so, we shall use the 2-tuple fuzzy linguistic representation model and its computational technique [20], together with the multi-granular linguistic information fusion ideas presented in [18]. For the development of a practical example of the multi-granular fusion method based on the 2-tuple linguistic representation we shall solve an

MEDM problem that evaluates quality of the network services from different Operative Systems.

In order to do so, this paper is structured as follows: in Section 2, we shall make a brief review of the fuzzy linguistic approach, of the 2-tuple fuzzy linguistic representation model and present a general scheme of an MEDM problem. In Section 3, we develop a fusion method for multi-granular linguistic information. In Section 4, we present an example of an evaluating services of Operative Systems. Finally some concluding remarks are pointed out in Section 5.

2 Preliminaries

In this section we briefly review the fuzzy linguistic approach and its computational models, afterwards we review the 2-tuple fuzzy linguistic representation model together its linguistic computational method and finally present a general scheme for MEDM problems.

2.1 Fuzzy Linguistic Approach

Usually, we work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [45]. This approach is adequate in some situations, for example, when attempting to qualify phenomena related to human perception, we are often led to use words in natural language. This may arise for different reasons. There are some situations where the information may be unquantifiable due to its nature, and thus, it may be stated only in linguistic terms (e.g., when evaluating the "comfort" or "design" of a car, terms like "bad", "poor", "tolerable", "average", "good" can be used [28]). In other cases, precise quantitative information may not be stated because either it is not available or the cost of its computation is too high, then an "approximate value" may be tolerated (e.g., when evaluating the speed of a car, linguistic terms like "fast", "very fast", "slow" are used instead of numerical values).

The fuzzy linguistic approach has been applied with very good results to different problems, such as, "information retrieval" [3, 39], "clinical diagnosis" [11], "marketing" [42], "risk in software development" [29], "technology transfer strategy selection" [7], "educational grading systems" [27], "scheduling" [1], "consensus" [36, 4], "materials selection" [8], "personnel management" [19], "decision-making" [9, 16, 17, 30, 31, 35, 43], etc.

We have to choose the appropriate linguistic descriptors for the term set and their semantics. In order to accomplish this objective, an important aspect to analyse is the *“granularity of uncertainty”*, i.e., the level of discrimination among different counts of uncertainty. The universe of the discourse over which the term set is defined can be arbitrary, in this paper we shall use linguistic term sets in the interval $[0, 1]$. In [2] the use of term sets with an odd cardinal was studied, representing the mid term by an assessment of “approximately 0.5”, with the rest of the terms being placed symmetrically around it and with typical values of cardinality , such as 7 or 9. These classical cardinality values seem to satisfy Miller’s observation about that human beings can reasonably manage to bear in mind seven or so items [32].

One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [16, 43]. For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 = none, s_1 = very\ low, s_2 = low, s_3 = medium, s_4 = high,$$
$$s_5 = very\ high, s_6 = perfect\}$$

In these cases, it is usually required that there exist:

1. A negation operator $\text{Neg}(s_i) = s_j$ such that $j = g$-i (g+1 is the cardinality).
2. A minimization and a maximization operator in the linguistic term set: $s_i \leq s_j \iff i \leq j$.

The semantics of the terms is given by fuzzy numbers defined in the [0,1] interval, which are usually described by membership functions. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function [2]. Since the linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since it may be impossible and unnecessary to obtain more accurate values [9]. This parametric representation is achieved by the 4-tuple (a, b, d, c), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function [2]. A particular case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, so we represent this type of membership function by a 3-tuple (a, b, c). For example, we may assign the following semantics to the set of seven terms:

$$P = Perfect = (.83, 1, 1) \quad VH = Very\_High = (.67, .83, 1)$$
$$H = High = (.5, .67, .83) \quad M = Medium = (.33, .5, .67)$$
$$L = Low = (.17, .33, .5) \quad VL = Very\_Low = (0, .17, .33)$$
$$N = None = (0, 0, .17).$$

which is graphically shown in Figure 1.
Other authors use a non-trapezoidal representation, e.g., Gaussian functions [3].

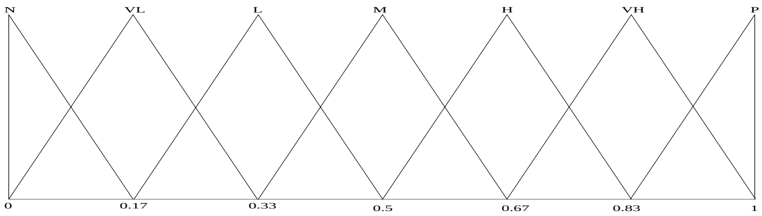

Fig. 1. A Set of Seven Terms with its Semantic

The linguistic variables are used in processes of CW that imply their fusion, aggregation, comparison, etc. To perform these computations with fuzzy linguistic approach have been developed two methods in the literature. (i) The model based on the Extension Principle, and (ii) the symbolic one. Here we briefly review the two models.

Linguistic Computational Model Based on the Extension Principle

The Extension Principle has been introduced to generalize crisp mathematical operations to fuzzy sets. The use of extended arithmetic based on the Extension Principle [12] increases the vagueness of the results. The results obtained by the fuzzy arithmetic are fuzzy numbers that usually do not match any linguistic term in the initial term set, so a linguistic approximation process is needed to express the result in the initial expression domain. In the literature we can find different linguistic approximation operators [2, 11].

A linguistic aggregation operator based on the Extension Principle acts according to:

$$S^n \xrightarrow{\tilde{F}} F(\mathcal{R}) \xrightarrow{app_1(\cdot)} S$$

where S^n symbolizes the n cartesian product of S, $\tilde{\mathrm{F}}$ is an aggregation operator based on the Extension Principle, $F(\mathcal{R})$ the set of all fuzzy subsets over the set of Real numbers $\mathcal{R}$, $app_1 : F(\mathcal{R}) \to S$ is a linguistic approximation function that returns a label from the linguistic term set S whose meaning is the closest to the obtained unlabelled fuzzy number and S is the initial term set.

Linguistic Computational Symbolic Model

A second approach used to operate on linguistic information is the symbolic one [10], that makes computations on the indexes of the linguistic labels. Usually it uses the ordered structure of the linguistic term sets, $S = \{s_0, \ldots, s_g\}$ where $s_i < s_j$ iff $i < j$, to perform the computations. The intermediate results are numeric values, $\alpha \in [0, g]$, which must be approximated in each step of the process by means of an approximation function $app_2 : [0, g] \to \{0, .., g\}$ that obtains a numeric value, such that, it indicates the index of the associated linguistic term, $s_{app_2(\alpha)} \in S$. Formally, it can be expressed as:

$$S^n \xrightarrow{C} [0,g] \xrightarrow{app_2(\cdot)} \{0,...,g\} \longrightarrow S$$

where C is a symbolic linguistic aggregation operator, $app_2(\cdot)$ is an approximation function used to obtain an index $\{0,...,g\}$ associated to a term in $S = \{s_0,...,s_g\}$ from a value in $[0,g]$.

2.2 The 2-tuple Fuzzy Linguistic Representation Model

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This model has been presented in [20, 21], where different advantages of this formalism to represent the linguistic information over classical models are shown, such as:

1. The linguistic domain can be treated as continuous, whilst in the classical models it is treated as discrete.
2. The linguistic computational model based on linguistic 2-tuple carries out processes of CW easily and without loss of information.
3. The results of the processes of CW are always expressed in the initial linguistic domain.

Due to these advantages, we shall use this linguistic representation model to accomplish our aim, to build an aggregation process for multi-granular linguistic information that expresses the aggregated values in the initial expression domain and with a lack of precision less than in before processes.

The 2-tuple representation model is based on the symbolic one and in a concept similar to the concept of *"Translation"* used in [5] to build linguistic adaptative modifiers. The 2-tuple fuzzy linguistic model uses a pair of values to represent the linguistic information, (s,α), where s is a linguistic label and α is a numerical value called *Symbolic Translation*.

Definition 1. *Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set $S = \{s_0,...,s_g\}$, i.e., the result of a symbolic aggregation operation. $\beta \in [0,g]$, being $g+1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0,g]$ and $\alpha \in [-.5,.5)$ then α is called a Symbolic Translation.*

From this concept in [20, 21] it was developed a linguistic representation model which represents the linguistic information by means of a 2-tuple (s_i,α_i), $s_i \in S$ and $\alpha_i \in [-.5,.5)$:

- s_i represents the linguistic label of the information, and
- α_i is a numerical value expressing the value of the translation from the original result β to the closest index label, i, in the linguistic term set (s_i), i.e., the Symbolic Translation.

Remark 1: In [5] the *"Translation"* is used to represent knowledge in Knowledge Based Systems and is computed according to the semantics of the linguistic labels that depends on the shape of their membership functions, whilst the *"Symbolic Translation"* is used in fuzzy preference modelling for processes of CW and depends on the order of linguistic terms and on the symbolic computations carried out over the linguistic labels.

This model defines a set of transformation functions between linguistic terms and 2-tuple, and between numeric values and 2-tuple.

Definition 2. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where round is the usual round *operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.*

Proposition 1. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.*

Proof.

It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Remark 2: From definitions 2 and 3 and from proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation:

$$s_i \in S \Longrightarrow (s_i, 0)$$

Together with the fuzzy linguistic 2-tuple representation model a wide range of 2-tuple aggregation operators were developed [20], such as, the extended LOWA, the extended weighted average, the extended OWA, ... The use of these extended aggregation operators is necessary for the development of our fusion method in order to combine the information.

2.3 A General Scheme of an MEDM problem

An MEDM problem can be defined as follows. Let $A = \{a_1, ..., a_n\}$ be a set of alternatives, each one assessed by a set of experts $\{e_1, ..., e_k\}$. This scheme is shown in Table 1.

Table 1. A general MEDM problem

Alternatives	*Experts*			
(a_i)	e_1	e_2	...	e_k
a_1	y_{11}	y_{12}	...	y_{1k}
...	...	...	...	...
a_n	y_{n1}	y_{n2}	...	y_{nk}

There exists wide literature on fuzzy MEDM problems [23, 41]. In the following, we focus in MEDM problems defined over multi-granular linguistic term sets, i.e., problems where their preference values y_{ij} can be assessed in linguistic term sets S_j that can have different granularity of uncertainty and/or semantics.

Decision-making problems that manage preferences from different experts follow a common resolution scheme [33] composed by two phases:

1. *Aggregation phase:* It combines the individual preferences to obtain a collective preference value for each alternative.
2. *Exploitation phase:* It orders the collective preference values according to a given criterion to obtain the best alternative/s.

In this paper we deal with MEDM problems defined in multi-granular linguistic contexts. In the literature, we can find approaches to accomplish the aggregation phase of the above resolution scheme in these types of contexts [18]. Those approaches carry out the aggregation phase in two processes:

- *Normalization process.* The multi-granular linguistic information is expressed in an unique linguistic expression domain.
- *Combination process.* The unified linguistic information expressed in an unique linguistic term set is aggregated.

3 Fusion Method for Multi-Granular Linguistic Information based on the 2-tuple Representation

This fusion method for multi-granular linguistic information is developed according to the following scheme.

1. **Making the information uniform** (Normalization process). The multi-granular linguistic input information is unified into "fuzzy sets" in a Basic Linguistic Term Set (BLTS).
2. **Transforming fuzzy sets into 2-tuple**. The fuzzy sets in the BLTS are transformed into 2-tuple assessed in the BLTS.
3. **Fusion of 2-tuple values**. We apply a 2-tuple aggregation operator in order to obtain aggregated values expressed by means of 2-tuple assessed in the BLTS.

4. **Backward step**. The values obtained by the aggregation method (2-tuple), assessed in the BLTS, can be distant from the expression domains used by the sources of information. Therefore, it may be interesting to offer the option to make an approximation of the aggregated values to the initial domains for a better comprehensiveness of them. This step is not necessary, it is simply convenient.

This scheme is shown graphically, in *Figure* 2:

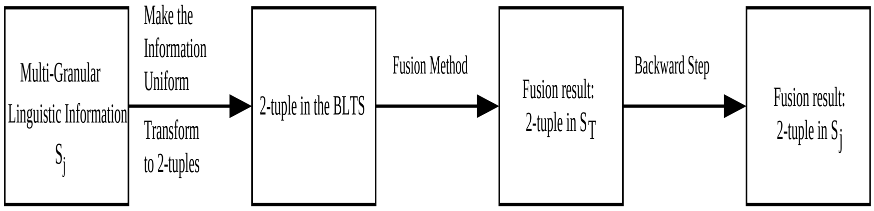

Fig. 2. Fusion of multi-granularity linguistic information

Subsequently, we shall develop each step in the above scheme in the following subsections over an MEDM problem dealing with multi-granular linguistic information.

We must remember that the aim of an MEDM problem is to compute a global evaluation of each alternative in order to determine the *"best"* one. To do so, the decision-making problems follows a decision model as the presented in subsection 2.3.

The scheme of *Figure* 2 develops a new process to carry out the processes of the aggregation phase, i.e., the normalization and combination processes.

3.1 Making the Information Uniform

With a view to manage the information we must make it uniform, i.e., the multi-granular linguistic information provided by all the sources must be transformed into a unified linguistic term set, called BLTS and denoted as S_T.

Before defining a transformation function into this BLTS, S_T, we have to decide how to choose S_T. We consider that S_T must be a linguistic term set which allows us to maintain the uncertainty degree associated to each expert and the ability of discrimination to express the performance values. With this goal in mind, we look for a BLTS with the maximum granularity. We take into consideration two possibilities:

- When there is only one term set with the maximum granularity, then, it is chosen as S_T.

- If we have two or more linguistic term sets with maximum granularity then, S_T is chosen depending on the semantics of these linguistic term sets, finding two possible situations to establish S_T:
  1. All the linguistic term sets have the same semantics, then S_T is any one of them.
  2. There are some linguistic term sets with different semantics. Then, S_T is a basic linguistic term set with a larger number of terms than the number of terms that a person is able to discriminate (normally 11 or 13, see [32]). We define a BLTS with 15 terms and the following semantics (see Figure 3):

s_0	$(0, 0, .07)$	s_1	$(0, .07, .15)$	s_2	$(.07, .15, .22)$
s_3	$(.15, .22, .29)$	s_4	$(.22, .29, .36)$	s_5	$(.29, .36, .43)$
s_6	$(.36, .43, .5)$	s_7	$(.43, .5, .57)$	s_8	$(.5, .57, .64)$
s_9	$(.57, .64, .71)$	s_{10}	$(.64, .71, .78)$	s_{11}	$(.71, .78, .85)$
s_{12}	$(.78, .85, .93)$	s_{13}	$(.85, .93, 1)$	s_{14}	$(.93, 1, 1)$

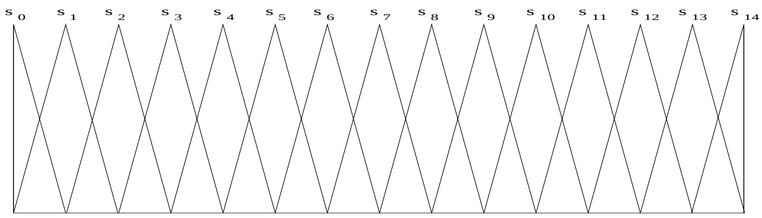

Fig. 3. Term set with 15 terms

Remark 3: We should point out that the justification on this choice is based on the use of linguistic term sets with odd granularity defined in the interval [0,1] and in the idea that the semantics is a parameter used by the conversion process, and thus, it has effect on the final result. We decide to use a symmetrical term set with a granularity bigger than the number of terms that an expert is able to discriminate (11 or 13, see [32]).

Once the BLTS has been chosen, the multi-granular linguistic information will be unified. The process of making the information uniform involves the comparison between fuzzy sets representing the semantics of the initial terms assessed in S_i and the fuzzy sets of the linguistic terms of the BLTS. Comparisons are usually carried out by means of a measure of comparison, that depending on the framework, the measure of comparison can have different forms [13, 14, 25, 37, 38, 47]. We focus on measures of comparison which evaluate the resemblance or likeness of two objects (fuzzy sets in our case). These type of measures are called *"measures of similitude"* [6, 34].

Measures of similitude are very general and different classes can be identify [6, 34]. For simplicity, in this paper we shall choose a measure of similitude based on a possibility function $S(A,B) = \max_x \min(\mu_A(x), \mu_B(x)$, where μ_A and μ_B are the membership functions of the fuzzy sets A and B respectively.

Therefore, to make the information uniform, we shall use the following function:

Definition 3 [18]. *Let $A = \{l_0, \ldots, l_p\}$ and $S_T = \{c_0, \ldots, c_g\}$ be two linguistic term sets, such that, $g \geq p$. Then, a multi-granular transformation function, τ_{AS_T} is defined as*

$$\tau_{AS_T} : A \longrightarrow F(S_T)$$

$$\tau_{AS_T}(l_i) = \{(c_k, \alpha_k^i) \ /k \in \{0, \ldots, g\}\}, \ \forall l_i \in A$$
$$\alpha_k^i = \max_y min\{\mu_{l_i}(y), \mu_{c_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_i}(y)$ and $\mu_{c_k}(y)$ are the membership functions of the fuzzy sets associated to the terms l_i and c_k, respectively.

The result of τ_{AS_T} for any linguistic value of A is a fuzzy set defined in the BLTS, S_T.We shall denote each $\tau_{S_iS_T}(y^{ij})$ as r^{ij}, and represents each fuzzy set of performance, r^{ij}, by means of its respective membership degrees, i.e.,

$$r^{ij} = (\alpha_0^{ij}, \ldots, \alpha_g^{ij}).$$

3.2 Transforming Fuzzy Sets into 2-tuple

So far, we have unified the multi-granular linguistic information transforming each linguistic term "y^{ij}" provided by the sources in a fuzzy set by means of $\tau_{S_iS_T}(y^{ij})$ over the BLTS S_T, such that, $\tau_{S_iS_T}(y^{ij}) = \{(c_0, \alpha_0^{ij}), ..., (c_g, \alpha_g^{ij})\}$. These fuzzy sets are far removed from the initial linguistic terms and are complex to manage [18]. To deal with this type of information it is usually transformed by means of ranking fuzzy methods to obtain crisp preference relations [13, 15, 24]. However, in this paper we try to reduce the loss of information produced in this process and its computational complexity. To do so, we shall transform each fuzzy set into a linguistic 2-tuple using a central value computed by means of a weighted average, where the weights are the membership degrees of the fuzzy set. We shall define the function χ that computes a value $\beta \in [0, g]$ that represents a central value of the information in the fuzzy set $\tau_{S_iS_T}(\mu^{ij})$.

Definition 4. *Let $\tau_{S_iS_T}(l_i) = \{(c_0, \alpha_0^i), \ldots, (c_g, \alpha_g^i)\}$ be a fuzzy set that represents a linguistic term $l_i \in S_i$ over the basic linguistic term set S_T. We shall obtain a numerical value, that supports the information of the fuzzy set, assessed in the interval $[0, g]$ by means of the following function:*

$$\chi : F(S_T) \longrightarrow [0, g]$$

$$\chi(\tau_{S_iS_T}(l_i)) = \frac{\sum_{j=0}^{g} j\alpha_j^i}{\sum_{j=0}^{g} \alpha_j^i} = \beta$$

This value β is easy to transform into a linguistic 2-tuple using the function Δ. Therefore, in the above step for the fusion process we have unified the input information with fuzzy sets in S_T and in this step we transform them into linguistic 2-tuple assessed in S_T by means of the functions χ and Δ:

$$\Delta(\chi(\tau_{S_iS_T}(\mu^{ij}))) = \Delta(\chi(r^{ij})) = (s_k, \alpha)^{ij}$$

where $s_k \in S_T$ and $\alpha \in [-.5, .5)$ is the value of the symbolic translation.

On this way, although a loss of information can appear in this process, it will be less than the presented by classical methods [13, 15, 24] since a 2-tuple is a fuzzy number with a symbolic translation that summarizes a bigger amount of information of a fuzzy set than a crisp value.

3.3 Fusion of 2-tuple values

Here we shall obtain the result we are looking for, an aggregated value from the multi-granularity linguistic information.

At this time, the input information is modelled by means of linguistic 2-tuple values assessed in S_T, $(s_k, \alpha)^{ij}$, and our objective is to aggregate this information. In [20] a wide range of 2-tuple linguistic aggregation operators were presented, therefore, to aggregate the 2-tuple values, $(s_k, \alpha)^{ij}$, we shall choose one of these linguistic 2-tuple aggregation operators and we shall apply it to combine them, obtaining as a result an aggregated linguistic 2-tuple assessed in S_T.

Formally, it can be expressed as:

$$FO((s_k, \alpha)^{1j}, ..., (s_k, \alpha)^{nj})) = (s_k, \alpha)^j$$

where FO is any 2-tuple fusion operator.

An example of this type of operator was shown in the subsection 2.2 where the arithmetic mean for 2-tuple values is defined as:

$$\overline{x}^e\{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\} = \Delta(\sum_{i=1}^{n} \frac{1}{n} \Delta^{-1}(r_i, \alpha_i)) = \Delta(\frac{1}{n} \sum_{i=1}^{n} \beta_i)$$

And applied to the set of 2-tuples $\{(M, 0), (L, 0), (VL, 0), (H, 0)\}$, obtained as result:

$$\overline{x}^e\{(M, 0), (L, 0), (VL, 0), (H, 0)\} = (M, -.5)$$

3.4 The Backward Step

This is an optional step in the fusion process. Depending on the problem we are dealing with, the aggregated 2-tuple may be expressed in a domain distant from the initial ones used by the sources of information. In these situations it might be appropriate to offer the possibility of making a transformation to the initial expression domains, for improving the comprehensiveness of the results. To accomplish the backward step we shall present a transformation function, that obtains 2-tuple in an initial expression domain $S_i = \{s_0, ..., s_{g_i}\}$ from a 2-tuple expressed in the BLTS, $S_T = \{s_0, ..., s_g\}$. This function will carry out the following processes:

1. In first place, it transforms each 2-tuple $(s_k, \alpha) \in S_T$ into a fuzzy set in S_T with an only two values of membership degree different from 0:

$$S_T \times [-.5, .5) \longrightarrow \{S_T \times [0, 1]\} \times \{S_T x[0, 1]\}$$

$$(s_k, \alpha) = \{(s_h, 1 - \gamma), (s_{h+1}, \gamma)\}$$

   where

$$h = trunc(\Delta{-}1(s_k, \alpha))$$
$$\gamma = \Delta{-}1(s_k, \alpha) - h$$

   trunc is the usual trunc operation.
   An example can be:
   Let $(s_8, .3)$ be a 2-tuple, with $s_8 \in S_T,\ S_T = \{s_0, ..., s_{14}\}$ and ".3" the value of the symbolic translation, its equivalent fuzzy set is:

$$(s_8, .3) = \{(s_8, .7), (s_9, .3)\}$$

2. Following, it is applied the measure of similitude $\tau_{S_T S_i}$ to the above fuzzy set, obtaining two fuzzy sets in S_i:

$$\tau_{S_T S_i}(s_h) = \{(s_0, \alpha_0^h), ..., (s_{g_i}, \alpha_{g_i}^h)\}$$
$$\tau_{S_T S_i}(s_{h+1}) = \{(s_0, \alpha_0^{h+1}), ..., (s_{g_i}, \alpha_{g_i}^{h+1})\}$$

3. The fuzzy sets in the initial expression domain, S_i, are converted into numerical values assessed in $[0, g_i]$ by means of the χ function, obtaining β_h and $\beta_{h+1} \in [0, g_i]$, such that,

$$\chi(\tau_{S_T S_i}(s_h)) = \beta_h$$
$$\chi(\tau_{S_T S_i}(s_{h+1})) = \beta_{h+1}$$

4. To achieve our objective, we need to obtain a value $\beta^i \in [0, g_i]$ that represents the amount of information of $\{(s_h, 1 - \gamma),\ (s_{h+1}, \gamma)\}$. We have β_h *and* $\beta_{h+1} \in [0, g_i]$, that represent the information supported by s_h *and* s_{h+1}, now we make a linear combination using the degrees of membership of the fuzzy set to obtain the value that we are looking for:

$$(\beta_h * (1 - \gamma)) + (\beta_{h+1} * \gamma) = \beta^i \in [0, g_i]$$

   Then applying Δ to β^i we shall obtain the linguistic 2-tuple assessed in S_i that we were looking for:

$$\Delta(\beta^i) = (s_k^i, \alpha)$$

Now we define the function $\mathbf{\Gamma}$ that accomplish the whole process of the backward step:

Definition 5: *Let* (s_k, α), $s_k \in S_T$ *be a 2-tuple assessed in the BLTS, therefore its equivalent 2-tuple in* S_i *is computed as:*

$$\Gamma : S_T \times [-.5, .5) \longrightarrow S_i \times [-0.5, 0.5)$$
$$\Gamma((s_k, \alpha)) = \Delta(\chi(\tau_{S_T S_i}(s_h)) \cdot (1-\gamma) + \chi(\tau_{S_T S_i}(s_{h+1}) \cdot \gamma)$$
$$h = trunc(\Delta^{-1}(s_k, \alpha))$$
$$\gamma = \Delta^{-1}(s_k, \alpha) - h$$

This process will be carried out for all source expression domains S_i, therefore each source can easily understand the results.

The backward step has sense only if the order of the alternatives is not altered during the process.

Proposition 2. Let $(s_k, \alpha_k) > (s_j, \alpha_j)$ be two 2-tuple assessed in S_T, then $\Gamma(s_k, \alpha_k) > \Gamma(s_j, \alpha_j)$, i.e., Γ satisfies the property of monotonicity.

Proof.

We have to prove that every process carried out for Γ is monotone.

- The transformation of (s_k, α) into a fuzzy set in S_T is monotone:

$$(s_k, \alpha_k) > (s_j, \alpha_j)$$

1. $k = j \Rightarrow \alpha_k > \alpha_j$

$$\left.\begin{matrix} s_{h_k} = s_{h_j} \\ s_{h_{k+1}} = s_{h-j+1} \end{matrix}\right\} \Rightarrow \gamma_k > \gamma_j \Rightarrow \{(s_{h_k}, 1-\gamma_k), (s_{h_{k+1}}, \gamma_k)\} > \{(s_{h_j}, 1-\gamma_j), (s_{h_{j+1}}, \gamma_j)\}$$

2. $k > j \Rightarrow s_{h_k} > s_{h_j} \Rightarrow \{(s_{h_k}, 1-\gamma_k), (s_{h_{k+1}}, \gamma_k)\} > \{(s_{h_j}, 1-\gamma_j), (s_{h_{j+1}}, \gamma_j)\}$

- In [6] we can see that $\tau_{S_T S_i}$ is a particular case of M-measure of similitude, therefore it satisfies the property of monotonicity. And it is obvious that χ and the linear combination are monotone.

Therefore Γ satisfies the property of monotonicity and hence the backward step has sense.

4 Evaluating the Quality of Network Services from Different Operative Systems

Here we shall apply the 2-tuple multi-granular fusion method in a decision process over the following MEDM problem.

A distribution company needs to evaluate the quality of the network services from the different Operative Systems to decide which to install in its

Information System. So it contracts a consulting company to carry out a survey of the different possibilities existing on the market, to decide which is the best option for his customer. The alternatives are the following ones:

UNIX	Windows-XP	Linux	VMS
x_1	x_2	x_3	x_4

The consulting company has a group of four consultancy departments (experts), that evaluate the network services from different viewpoints.

Cost Analysis	Systems Analysis	Risk Analysis	Technology Analysis
p_1	p_2	p_3	p_4

Each department (expert) provides a performance vector expressing its preferences for each alternative assessed in linguistic term sets with a different granularity and/or semantics:

- p_1 provides his preferences in the set of 9 labels, A.
- p_2 provides his preferences in the set of 7 labels, B.
- p_3 provides his preferences in the set of 5 labels, C.
- p_4 provides his preferences in the set of 9 labels, D.

Label set A		**Label set B**	
a_0	(0, 0, .12)	b_0	(0, 0, .16)
a_1	(0, .12, .25)	b_1	(0, .16, .33)
a_2	(.12, .25, .37)	b_2	(.16, .33, .5)
a_3	(.25, .37, .5)	b_3	(.33, .5, .66)
a_4	(.37, .5, .62)	b_4	(.5, .66, .83)
a_5	(.5, .62, .75)	b_5	(.66, .83, .1)
a_6	(.62, .75, .87)	b_6	(.83, 1, 1)
a_7	(.75, .87, .1)		
a_8	(.87, 1, 1)		

Label set C		**Label set D**	
c_0	$(0, 0, .25)$	d_0	$(0, 0, 0, 0)$
c_1	$(0, .25, .5)$	d_1	$(0, .01, .02, .07)$
c_2	$(.25, .5, .75)$	d_2	$(.04, .1, .18, .23)$
c_3	$(.5, .75, 1)$	d_3	$(.17, .22, .36, .42)$
c_4	$(.75, 1, 1)$	d_4	$(.32, .41, .58, .65)$
		d_5	$(.58, .63, .80, .86)$
		d_6	$(.72, .78, .92, .97)$
		d_7	$(.93, .98, .99, 1)$
		d_8	$(1, 1, 1, 1)$

The performance vectors provided by the experts are the following:

		alternatives			
	μ_{ij}	x_1	x_2	x_3	x_4
experts	p_1	a_4	a_6	a_3	a_5
	p_2	b_3	b_4	b_3	b_5
	p_3	c_2	c_3	c_2	c_1
	p_4	d_4	d_5	d_3	d_5

where $\mu_{ij} \in S_k$ is the performance value given by the expert p_i over the alternative x_j in the term set S_k.

We shall apply the decision process presented in section 2.3 to solve this MEDM problem with multi-granular linguistic information.

A. Collective Performance Vector.

1. **Making the Information Uniform**
   We have to choose the BLTS, $S_T = \{c_0, \ldots, c_g\}$. In this case, there are two term sets with the maximum granularity and different semantics, hence, we choose as S_T the special term set of 15 labels given in Figure 3. All the assessments must be converted to S_T by means of the set of multi-granular transformation functions $\{\tau_{AS_T}, \tau_{BS_T} \tau_{CS_T}, \tau_{DS_T}\}$. We obtain the following results:

$$
\begin{array}{l}
r^{11}\ (0,0,0,0,.05,.45,.8,.82,.48,.23,0,0,0,0,0)\\
r^{12}\ (0,0,0,0,.11,.45,.65,.95,.68,.39,.1,0,0,0,0)\\
r^{13}\ (0,0,0,.22,.35,.59,.8,.98,.75,.52,.32,.1,0,0,0)\\
r^{14}\ (0,0,0,0,.3,.77,1,1,1,.51,0,0,0,0,0)\\
r^{21}\ (0,0,0,0,0,0,0,0,.25,.99,.7,.31,.01,0,0)\\
r^{22}\ (0,0,0,0,0,0,0,.35,.63,.94,.76,.46,.2,0,0)\\
r^{23}\ (0,0,0,0,0,0,.01,.25,.5,.7,.9,.9,.65,.45,.2)\\
r^{24}\ (0,0,0,0,0,0,0,0,0,1,1,1,.55,0,0)\\
r^{31}\ (0,0,0,.18,.55,.95,.7,.35,0,0,0,0,0,0,0)\\
r^{32}\ (0,0,0,0,.1,.45,.65,.95,.68,.39,.1,0,0,0,0)\\
r^{33}\ (0,0,0,.22,.35,.59,.8,.98,.75,.52,.32,.1,0,0,0)\\
r^{34}\ (0,0,.41,1,1,.99,0,0,0,0,0,0,0,0,0)\\
r^{41}\ (0,0,0,0,0,0,0,.36,.71,.91,.56,.22,0,0,0)\\
r^{42}\ (0,0,0,0,0,0,0,0,0,.23,.54,.84,.86,.58,.3)\\
r^{43}\ (.25,.4,.7,.9,.87,.65,.4,.2,0,0,0,0,0,0,0)\\
r^{44}\ (0,0,0,0,0,0,0,0,0,1,1,1.55,0,0)
\end{array}
$$

2. **Transforming fuzzy sets (r^{ij}) into 2-tuple**
   To avoid dealing with fuzzy sets, we shall transform all the r^{ij} into 2-tuple based on the symbolic translation using the functions χ and Δ:

$$\begin{array}{ll}
\Delta(\chi(r^{11})) = (s_7, -.32)^{11} & \Delta(\chi(r^{12})) = (s_7, -.05)^{12} \\
\Delta(\chi(r^{13})) = (s_7, -.16)^{13} & \Delta(\chi(r^{14})) = (s_7, -.32)^{14} \\
\Delta(\chi(r^{21})) = (s_9, .48)^{21} & \Delta(\chi(r^{22})) = (s_9, .28)^{22} \\
\Delta(\chi(r^{23})) = (s_{10}, .25)^{23} & \Delta(\chi(r^{24})) = (s_{10}, .3)^{24} \\
\Delta(\chi(r^{31})) = (s_5, .17)^{31} & \Delta(\chi(r^{32})) = (s_7, -.05)^{32} \\
\Delta(\chi(r^{33})) = (s_7, -.15)^{33} & \Delta(\chi(r^{34})) = (s_4, -.25)^{34} \\
\Delta(\chi(r^{41})) = (s_9, -.15)^{41} & \Delta(\chi(r^{42})) = (s_12, -.43)^{42} \\
\Delta(\chi(r^{43})) = (s_3, .44)^{43} & \Delta(\chi(r^{44})) = (s_10, .3)^{44}
\end{array}$$

After this transformation, we manage 2-tuple values based on the symbolic translation assessed in the BLTS, S_T.

3. **Computing the collective performance values**
For each alternative x_i we compute its collective performance value using a 2-tuple linguistic aggregation operator, in this case we choose the 2-tuple mean operator. The collective performance values obtained for each alternative, x_i, are:

$$\begin{array}{l}
x_1 \longrightarrow \phi_Q^e((s_7, -.32)^{11}, (s_9, .48)^{21}, (s_5, .17)^{31}, (s_9, -.15)^{41}) = (s_8, -.46)^1 \\
x_2 \longrightarrow \phi_Q^e((s_7, -.05)^{12}, (s_9, .28)^{22}, (s_7, -.05)^{32}, (s_{12}, -.43)^{42}) = (s_9, -.32)^2 \\
x_3 \longrightarrow \phi_Q^e((s_7, -.16)^{13}, (s_{10}, .25)^{23}, (s_7, -.15)^{33}, (s_3, .44)^{43}) = (s_7, -.16)^3 \\
x_4 \longrightarrow \phi_Q^e((s_7, -.32)^{14}, (s_{10}, .3)^{24}, (s_4, -.25)^{34}, (s_{10}, .3)^{44}) = (s_8, -.25)^4
\end{array}$$

Then the collective performance vector is:

$$\{(s_8, -.46)^1, (s_9, -.32)^2, (s_7, -.16)^3, (s_8, -.25)^4\}$$

4. **The Backward step**
The collective performance vector obtained in the above step is sufficient to solve the decision process, but it is expressed in a different term set from the one used by the sources of information. Therefore, we can make the backward step to express the collective performance vector in the expression domains used by the experts, i.e., A, B, C, D. To do so, we shall use the Γ function:
   a) First, the collective values are transformed into fuzzy sets in S_T.

$$\begin{array}{l}
(s_8, -.46)^1 = \{(s_7, .46), (s_8, .54)\}^1 \\
(s_9, -.32)^2 = \{(s_8, .32), (s_9, .68)\}^2 \\
(s_7, -.16)^3 = \{(s_6, .16), (s_7, .84)\}^3 \\
(s_8, -.25)^4 = \{(s_7, .25), (s_8, .75)\}^4
\end{array}$$

   b) Following, we shall apply the functions $\tau_{S_T A}, \tau_{S_T B}, \tau_{S_T C}, \tau_{S_T D}$ to the above fuzzy sets:

$$\tau_{S_T A}(s_6) = \{(a_0,0)(a_1,0)(a_2,.02)(a_3,.69)(a_4,.66)(a_5,0)(a_6,0)(a_7,0)(a_8,0)\}$$
$$\tau_{S_T A}(s_7) = \{(a_0,0)(a_1,0)(a_2,0)(a_3,0.35)(a_4,1)(a_5,.38)(a_6,0)(a_7,0)(a_8,0)\}$$
$$\tau_{S_T A}(s_8) = \{(a_0,0)(a_1,0)(a_2,0)(a_3,0)(a_4,.64)(a_5,.75)(a_6,.1)(a_7,0)(a_8,0)\}$$
$$\tau_{S_T A}(s_9) = \{(a_0,0)(a_1,0)(a_2,0)(a_3,.0)(a_4,.25)(a_5,.92)(a_6,.49)(a_7,0)(a_8,0)\}$$
$$\tau_{S_T B}(s_6) = \{(b_0,0)(b_1,0)(b_2,.65)(b_3,.8)(b_4,0)(b_5,0)(b_6,0)\}$$
$$\tau_{S_T B}(s_7) = \{(b_0,0)(b_1,0)(b_2,.3)(b_3,.9)(b_4,.3)(b_5,0)(b_6,0)\}$$
$$\tau_{S_T B}(s_8) = \{(b_0,0)(b_1,.0)(b_2,.0)(b_3,.63)(b_4,.63)(b_5,0)(b_6,0)\}$$
$$\tau_{S_T B}(s_9) = \{(b_0,0)(b_1,.2)(b_2,0)(b_3,.35)(b_4,.85)(b_5,.21)(b_6,0)\}$$
$$\tau_{S_T C}(s_6) = \{(c_0,0)(c_1,.4)(c_2,.79)(c_3,0)(c4,0)\}$$
$$\tau_{S_T C}(s_7) = \{(c_0,0)(c_1,.22)(c_2,1)(c_3,.22)(c4,0)\}$$
$$\tau_{S_T C}(s_8) = \{(c_0,0)(c_1,.0)(c_2,.77)(c_3,.45)(c4,0)\}$$
$$\tau_{S_T C}(s_9) = \{(c_0,0)(c_1,0)(c_2,.55)(c_3,.67)(c4,0)\}$$
$$\tau_{S_T D}(s_6) = \{(d_0,0)(d_1,0)(d_2,0)(d_3,.5)(d_4,1)(d_5,0)(d_6,0)(d_7,0)(d_8,0)\}$$
$$\tau_{S_T D}(s_7) = \{(d_0,0)(d_1,0)(d_2,0)(d_3,0)(d_4,1)(d_5,0)(d_6,0)(d_7,0)(d_8,0)\}$$
$$\tau_{S_T D}(s_8) = \{(d_0,0)(d_1,0)(d_2,0)(d_3,0)(d_4,1)(d_5,.51)(d_6,0)(d_7,0)(d_8,0)\}$$
$$\tau_{S_T D}(s_9) = \{(d_0,0)(d_1,0)(d_2,0)(d_3,0)(d_4,.51)(d_5,1)(d_6,0)(d_7,0)(d_8,0)\}$$

c) Transforming the fuzzy sets into numerical values by means of the χ function:

$$\chi(\tau_{S_T A}(s_6)) = 3.47 \; \chi(\tau_{S_T A}(s_7)) = 4 \; \chi(\tau_{S_T A}(s_8)) = 4.63 \; \chi(\tau_{S_T A}(s_9)) = 5.15$$
$$\chi(\tau_{S_T B}(s_6)) = 2.55 \; \chi(\tau_{S_T B}(s_7)) = 3 \; \chi(\tau_{S_T B}(s_8)) = 3.5 \; \chi(\tau_{S_T B}(s_9)) = 3.9$$
$$\chi(\tau_{S_T C}(s_6)) = 1.66 \; \chi(\tau_{S_T C}(s_7)) = 2 \; \chi(\tau_{S_T C}(s_8)) = 2.28 \; \chi(\tau_{S_T C}(s_9)) = 2.54$$
$$\chi(\tau_{S_T D}(s_6)) = 3.66 \; \chi(\tau_{S_T D}(s_7)) = 4 \; \chi(\tau_{S_T D}(s_8)) = 4.33 \; \chi(\tau_{S_T D}(s_8)) = 4.66$$

d) Expressing the collective vector in all initial domains:

i. Domain A:

$$\{(a_4,.34)^1,(a_5,-.02)^2,(a_4,-.09)^3,(a_5,-.14)^4\}$$

where the collective value of x_1 is obtained as follows:

$$\Delta((4*.46)+(4.63*.54)) = (a_4,.34)$$

ii. Domain B:

$$\{(b_3,.3)^1,(b_4,-.23)^2,(b_3,-.08)^3,(b_4,-.33)^4\}$$

iii. Domain C:

$$\{(c_2,.15)^1,(c_2,.44)^2,(c_2,-.06)^3,(c_2,.21)^4\}$$

iv. Domain D:

$$\{(d_4,.17)^1,(d_5,-.46)^2,(d_4,.02)^3,(d_4,.24)^4\}$$

B. Selection Process

Finally, we shall apply a choice degree to the collective performance vector to obtain the solution set of alternatives. In this problem the solution set obtained is:

$$\{x_2\}$$

The Operative System with high quality network services according to the needs of the company after the survey of the experts is the Windows-XP based system.

If we apply the choice degree to the results expressed in the term set S_T the solution reached would be the same, since the backward step is monotone.

5 Concluding Remarks

In this paper we have presented a fusion method based on the 2-tuple fuzzy linguistic representation that allows us to easily deal with multi-granular linguistic information in fusion processes. The development of this method takes as base the 2-tuple linguistic representation model and its computational technique. This new fusion method is useful for problems with multiple sources of information that express their knowledge with linguistic information assessed in several linguistic term sets with different cardinality and/or semantics. We have applied this fusion method to an MEDM problem to evaluate the quality of the network services from different Operative Systems.

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