Combining Heterogeneous Information in Group Decision Making

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Abstract

Decision processes for solving group decision making problems are composed of two phases: (i) *aggregation* and, (ii) *exploitation.* When experts that participate in the group decision making process are not able to express their opinions using a same expression domain, then the use of information assessed in different domains, i.e., *heterogeneous information,* is necessary. In these cases, the information can be assessed in domains with different nature as linguistic, numerical and interval-valued. The aim of this contribution is to present an aggregation process to manage *heterogeneous information contexts* in the case of linguistic, numerical and interval-valued information. To do this, we take as representation base the 2-tuple fuzzy linguistic representation model [5].

Keywords: decision making, aggregation, linguistic 2-tuples, heterogeneous information.

1 Introduction

Group Decision Making (GDM) problems have a finite set of alternatives $X = \{x_1, ..., x_n\}$ $n \geq 2$, as well as a finite set of experts $E = \{e_1, ..., e_m\}$ $m \geq 2$. Usually, each expert e_k provides his/her preferences on X by means of a preference relation P_{e_k}, being $P_{e_k}(x_i, x_j) = p_{ij}^k$ the degree of preference of alternative x_i over x_j.

It seems difficult that the nature of the preference values, p_{ij}^k, provided by the experts be the same. Because it depends on the knowledge of them over the alternatives (usually it is not precise). Therefore, the preference values have been expressed in different domains. Early in DM problems, the uncertainty were expressed in the preference values by means of real values assessed in a predefined range [11, 16], soon other approaches based on interval valued [12, 15] and linguistic one [4, 17] were proposed. The most of the proposals for solving GDM problems are focused on cases where all the experts provide their preferences in a unique domain, however, the experts could work in different knowledge fields and could express their preferences with different types of information depending on their knowledge. We shall call this type of information as *Heterogeneous Information.* Hence, the GDM problem is defined in a heterogeneous information context.

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A solution for a GDM problem is derived either from the individual preference relations, without constructing a social preference relation, or by computing first a social fuzzy preference relation and then using it to find a solution [10]. In any of the above approaches called direct and indirect approaches respectively the process for reaching a solution of the GDM problems is composed by two steps [14]:

- *Aggregation phase*: that combines the expert preferences, and
- *Exploitation one*: that obtains a solution set of alternatives from a preference relation.

The main difficulty for managing GDM problems defined in heterogeneous information contexts is the *aggregation phase*, i.e., *how to aggregate this type of information?*. Because of, there not exist standard operators or processes for combining this type of information.

The 2-tuple fuzzy linguistic representation model presented in [5] has shown itself as a good choice to manage non-homogeneous information in aggregation processes [6, 8, 9]. In this paper, we propose an aggregation process based on the 2-tuple model that is able to deal with heterogeneous information contexts.

Our proposal for aggregating heterogeneous information follows a scheme comprised of three phases:

1. **Unification:** The heterogeneous information is unified in an unique expression domain by means of fuzzy sets. Different transformation functions will be defined to transform the input information into fuzzy sets.
2. **Aggregation:** The fuzzy sets will be aggregated by means of an aggregation operator to obtain collective preference values expressed by fuzzy sets.
3. **Transformation:** The collective preference values expressed by means of fuzzy sets will be transformed into linguistic 2-tuples.

The exploitation phase of the decision process is carried out over the collective linguistic 2-tuples, to obtain the solution for the GDM problem.

In order to do so, this paper is structured as follows: in Section 2 we shall review different basic concepts; in Section 3 we shall propose the aggregation process for combining heterogeneous information; in Section 4 we shall solve an example of a GDM problem defined in a heterogeneous information context and finally, some concluding remarks are pointed out.

2 Preliminaries

We have just seen that in GDM problems the experts express their preferences depending on their knowledge over the alternatives by means of preference relations. Here, we review different approaches to express those preferences. And afterwards, we shall review the 2-tuple fuzzy linguistic representation model.

2.1 Approaches for Modelling Preferences

2.1.1 Fuzzy Binary Relations

A valued (fuzzy) binary relation R on X is defined as a fuzzy subset of the direct product $X \times X$ with values in $[0,1]$, i.e, $R : X \times X \to [0,1]$. The value, $R(x_i, x_j) = p_{ij}$, of a valued relation R denotes the degree to which $x_i R x_j$. In preference analysis, p_{ij} denotes the *degree to which an alternative x_i is preferred to x_j*. These were the first type of relations used in decision making [10, 11].

2.1.2 Interval-valued Relations

About the fuzzy binary approach has been argued that the most experts are unable to make a fair estimation of the inaccuracy of their judgements, making far larger estimation errors that the boundaries accepted by them as feasible [2].

A first approach to overcome this problem is to add some flexibility to the uncertainty representation problem by means of interval-valued relations:

$$R : X \times X \to \wp([0, 1]).$$

Where $R(x_i, x_j) = p_{ij}$ denotes the interval-valued preference degree of the alternative x_i over x_j . In these approaches [12, 15], the preferences provided by the experts consist of interval values assessed in $\wp([0, 1])$, where the preference is expressed as $[\underline{a}, \overline{a}]_{ij},\ with\ \underline{a} \leq \overline{a}$.

2.1.3 Linguistic Approach

Usually, we work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [18].

To use the linguistic approach we have to choose the appropriate linguistic descriptors for the term set and their semantics. In the literature, several possibilities can be found (see [7] for a wide description). An important aspect to analyze is the *"granularity of uncertainty"*, i.e., the level of discrimination among different counts of uncertainty. The *"granularity of uncertainty"* for the linguistic term set $S = \{s_0, ..., s_g\}$ is $g + 1$, while its *"interval of granularity"* is $[0, g]$.

One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [17]. For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$$

Usually, in these cases, it is required that in the linguistic term set satisfy the following additional characteristics:

1. There is a negation operator: Neg$(s_i) = s_j$, with, $j = g - i$ (g+1 is the cardinality).
2. $s_i \leq s_j \iff i \leq j$. Therefore, there exists a *min* and a *max* operator.

The semantics of the linguistic terms are given by fuzzy numbers defined in the [0,1] interval. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function [1]. The linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments. The parametric representation is achieved by the 4-tuple (a, b, d, c), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function [1]. A particular case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, then we represent this type of membership functions by a 3-tuple (a, b, c). A possible semantics for the above term set, S, may be the following (Figure 1):

$$\begin{array}{llll} P = (.83, 1, 1) & VH = (.67, .83, 1) & H = (.5, .67, .83) & M = (.33, .5, .67) \\ L = (.17, .33, .5) & VL = (0, .17, .33) & N = (0, 0, .17) & \end{array}$$

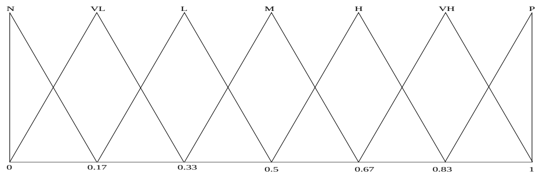

Figure 1: A set of seven linguistic terms with its semantics

2.2 The 2-Tuple Fuzzy Linguistic Representation Model

This model was presented in [5], for overcoming the drawback of the loss of information presented by the classical linguistic computational models [7]: (i) The model based on the Extension Principle [1], (ii) and the symbolic one [3]. The 2-tuple fuzzy linguistic representation model is based on symbolic methods and takes as the base of its representation the concept of Symbolic Translation.

Definition 1. *The Symbolic Translation of a linguistic term* $s_i \in S = \{s_0, ..., s_g\}$ *is a numerical value assessed in* $[-.5, .5)$ *that support the "difference of information" between a counting of information* $\beta \in [0, g]$ *and the closest value in* $\{0, ..., g\}$ *that indicates the index of the closest linguistic term in* $S(s_i)$*, being [0,g] the interval of granularity of* S.

¿From this concept a new linguistic representation model is developed, which represents the linguistic information by means of 2-tuples (r_i, α_i), $r_i \in S$ and $\alpha_i \in [-.5, .5)$. r_i represents the linguistic label center of the information and α_i is the Symbolic Translation.

This model defines a set of functions between linguistic 2-tuples and numerical values.

Definition 2. *Let* $S = \{s_0, ..., s_g\}$ *be a linguistic term set and* $\beta \in [0, g]$ *a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to* β *is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), \; with \; \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where $round(\cdot)$ *is the usual* round *operation,* s_i *has the closest index label to "*β*" and "*α*" is the value of the symbolic translation.*

Proposition 1.*Let* $S = \{s_0, ..., s_g\}$ *be a linguistic term set and* (s_i, α) *be a linguistic 2-tuple. There is always a* Δ^{-1} *function, such that, from a 2-tuple it returns its equivalent numerical value* $\beta \in [0, g]$ *in the interval of granularity of* S.

Proof. It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Remark 1. From Definitions 1 and 2 and Proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation: $s_i \in S \Longrightarrow (s_i, 0)$

3 Aggregation Process for Heterogeneous Information in a GDM Problem

In this section we propose a method to carry out the *aggregation step* of a GDM process defined in a heterogeneous information context. We focus on GDM problems in which the preference relations provided, can be:

- Fuzzy preference relations [11].
- Interval-valued preference relation [15].
- Linguistic preference relation assessed in a pre-established label set [4].

Our proposal for combining the heterogeneous information is composed of the following phases:

1. *Making the information uniform.* The heterogeneous information will be unified into a specific linguistic domain, called *Basic Linguistic Term Set* (BLTS) and symbolized as S_T. Each numerical, interval-valued and linguistic performance value is transformed into a fuzzy set in S_T, $F(S_T)$. The process is carried out in the following order:

   (a) Transforming numerical values in $[0, 1]$ into $F(S_T)$.

   (b) Transforming linguistic terms into $F(S_T)$.

   (c) Transforming interval-valued into $F(S_T)$.

2. *Aggregating individual performance values.* For each alternative, a collective performance value is obtained by means of the aggregation of the above fuzzy sets on the BLTS that represents the individual performance values assigned by the experts according to his/her preference.

3. *Transforming into 2-tuple.* The collective performance values (fuzzy sets) are transformed into linguistic 2-tuples in the BLTS and obtained a collective 2-tuple linguistic preference relation.

Following, we shall show in depth each phase of the aggregation process.

3.1 Making the Information Uniform

In this phase, we have to choose the domain, S_T, to unify the heterogeneous information and afterwards, the input information will be transformed into fuzzy sets in S_T.

3.1.1 Choosing the Basic Linguistic Term Set

The heterogeneous information is unified in a unique expression domain. In this case, we shall use fuzzy sets over a BLTS, denoted as $F(S_T)$. We study the linguistic term set S used in the GDM problem. If:

1. *S is a fuzzy partition,*
2. *and the membership functions of its terms are triangular,* i.e., $s_i = (a_i, b_i, c_i)$

Then, we select S as BLTS due to the fact that, these conditions are necessary and sufficient for the transformation between values in $[0, 1]$ and 2-tuples, being them carried out without loss of information [6].

If the linguistic term set S, used in the definition context of the problem, does not satisfy the above conditions then we shall choose as BLTS a term set with a larger number of terms than the number of terms that a person is able to discriminate (normally 11 or 13, see [1]) and satisfies the above conditions. We choose the BLTS with 15 terms symmetrically distributed, with the following semantics (graphically, Figure 2).

s_0	(0,0,0.07)	s_1	(0,0.07,0.14)	s_2	(0.07,0.14,0.21)
s_3	(0.14,0.21,0.28)	s_4	(0.21,0.28,0.35)	s_5	(0.28,0.35,0.42)
s_6	(0.35,0.42,0.5)	s_7	(0.42,0.5,0.58)	s_8	(0.5,0.58,0.65)
s_9	(0.58,0.65,0.72)	s_{10}	(0.65,0.72,0.79)	s_{11}	(0.72,0.79,0.86)
s_{12}	(0.79,0.86,0.93)	s_{13}	(0.86,0.93,1)	s_{14}	(0.93,1,1)

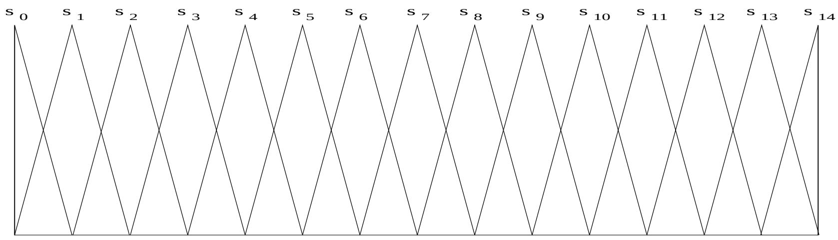

Figure 2: A BLTS with 15 terms symmetrically distributed

3.1.2 Transforming the Input Information Into $F(S_T)$

Once chosen the BLTS, we shall define the *transformation functions* that will be necessary to unify the heterogeneous information. The process of unifying the heterogeneous information involves in any occasions the comparison between fuzzy sets. These comparisons are usually carried out by means of a measure of comparison. We focus on measures of comparison which evaluate the resemblance or likeness of two objects (fuzzy sets in our case). These type of measures are called *measures of similitude* [13]. For simplicity, in this paper we shall choose a measure of similitude based on a possibility function $S(A, B) = \max_x \min(\mu_a(x), \mu_B(x))$, where μ_A and μ_B are the membership function of the fuzzy sets A and B respectively.

3.1.2.1. Transforming numerical values in $[0, 1]$ into $F(S_T)$.
Let $F(S_T)$ be the set of fuzzy sets in $S_T = \{s_0, \ldots, s_g\}$, we shall transform a numerical value $\vartheta \in [0, 1]$ into a fuzzy set in $F(S_T)$ computing the membership value of ϑ in the membership functions associated with the linguistic terms of S_T.

Definition 3. [6] *The function τ transforms a numerical value into a fuzzy set in S_T:*

$$\tau : [0,1] \to F(S_T)$$
$$\tau(\vartheta) = \{(s_0, \gamma_0), ..., (s_g, \gamma_g)\}, s_i \in S_T \, and \, \gamma_i \in [0,1]$$
$$\gamma_i = \mu_{s_i}(\vartheta) = \begin{cases} 0, & if \, \vartheta \notin Support(\mu_{s_i}(x)) \\ \frac{\vartheta - a_i}{b_i - a_i}, & if \, a_i \leq \vartheta \leq b_i \\ 1, & if \, b_i \leq \vartheta \leq d_i \\ \frac{c_i - \vartheta}{c_i - d_i}, & if \, d_i \leq \vartheta \leq c_i \end{cases}$$

Remark 2. We consider membership functions, $\mu_{s_i}(\cdot)$, for linguistic labels, $s_i \in S_T$, that achieved by a parametric function (a_i, b_i, d_i, c_i). A particular case are the linguistic assessments whose membership functions a triangular, i.e., $b_i = d_i$.

Example 1.

Let $\vartheta = 0.78$ be a numerical value to be transformed into a fuzzy set in $S = \{s_0, ..., s_4\}$. The semantics of this term set is:

$s_0 = (0, 0, 0.25)$ $\quad s_1 = (0, , 0.25, 0.5)$ $\quad s_2 = (0.25, 0.5, 0.75)$ $\quad s_3 = (0.5, 0.75, 1)$ $\quad s_4 = (0.75, 1, 1)$

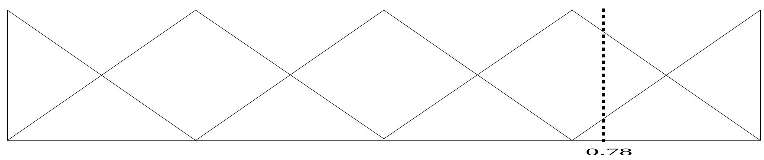

Figure 3: Transforming a numerical value into a fuzzy set in S

Then, the fuzzy set obtained is (See Fig. 3):

$$\tau(0.78) = \{(s_0, 0), (s_1, 0), (s_2, 0), (s_3, 0.88), (s_4, 0.12)\}.$$

3.1.2.2. Transforming Linguistic Terms in S into $F(S_T)$.

Definition 4.[9] *Let $S = \{l_0, \ldots, l_p\}$ and $S_T = \{s_0, \ldots, s_g\}$ be two linguistic term sets, such that, $g \geq p$. Then, a multi-granularity transformation function, τ_{SS_T}, is defined as:*

$$\tau_{SS_T} : A \to F(S_T)$$
$$\tau_{SS_T}(l_i) = \{(c_k, \gamma_k^i) \, / \, k \in \{0, ..., g\}\}, \, \forall l_i \in S$$
$$\gamma_k^i = \max_y \min\{\mu_{l_i}(y), \mu_{c_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_i}(\cdot)$ and $\mu_{c_k}(\cdot)$ are the membership functions of the fuzzy sets associated with the terms l_i and c_k, respectively.

Therefore, the result of τ_{SS_T} for any linguistic value of S is a fuzzy set defined in the BLTS, S_T.

Example 2.

Let $S = \{l_0, l_1, \ldots, l_4\}$ and $S_T = \{s_0, s_1, \ldots, s_6\}$ be two term sets, with 5 and 7 labels, respectively, and with the following semantics associated:

The fuzzy set obtained after applying τ_{SS_T} for l_1 is (see Fig. 4):

$$\tau_{SS_T}(l_1) = \{(s_0, 0.39), (s_1, 0.85), (s_2, 0.85), (s_3, 0.39), (s_4, 0), (s_5, 0), (s_6, 0)\}.$$

$l_0 = (0, 0, 0.25)$ $\quad l_1 = (0, , 0.25, 0.5)$ $\quad s_0 = (0, 0, 0.16)$ $\quad s_1 = (0, 0.16, 0.34)$
$l_2 = (0.25, 0.5, 0.75)$ $\quad l_3 = (0.5, 0.75, 1)$ $\quad s_2 = (0.16, 0.34, 0.5)$ $\quad s_3 = (0.34, 0.5, 0.66)$
$l_4 = (0.75, 1, 1)$ $\quad s_4 = (0.5, 0.66, 0.84)$ $\quad s_5 = (0.66, 0.84, 1)$
$s_6 = (0.84, 1, 1)$

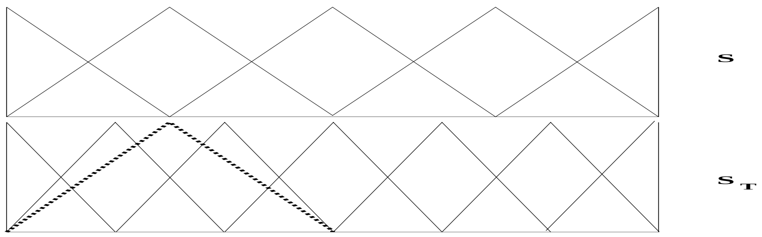

Figure 4: Transforming $l_1 \in S$ into a fuzzy set in S_T

3.1.2.3 Transforming Interval-Valued into $F(S_T)$.

Let $I = [\underline{i}, \overline{i}]$ be an interval-valued in $[0, 1]$, to carry out this transformation we assume that the interval-valued has a representation, inspired in the membership function of a fuzzy set [12] as follows:

$$\mu_I(\vartheta) = \begin{cases} 0, & if\, \vartheta < \underline{i} \\ 1, & if\, \underline{i} \leq \vartheta \leq \overline{i} \\ 0, & if\, \overline{i} < \vartheta \end{cases}$$

where ϑ is a value in $[0, 1]$. In Figure 5 can be observed the graphical representation of an interval.

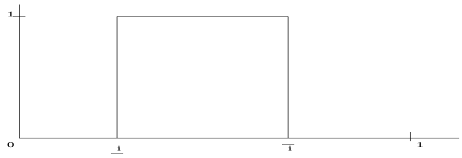
Figure 5: Membership function of $I = [\underline{i}, \overline{i}]$

Definition 5. *Let $S_T = \{s_0, \ldots, s_g\}$ be a BLTS. Then, the function τ_{IS_T} transforms a interval-valued I in $[0, 1]$ into a fuzzy set in S_T as follows*

$$\tau_{IS_T} : I \rightarrow F(S_T)$$
$$\tau_{IS_T}(I) = \{(c_k, \gamma_k^i) \,/\, k \in \{0, ..., g\}\},$$
$$\gamma_k^i = \max_y \min\{\mu_I(y), \mu_{c_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_I(\cdot)$ and $\mu_{c_k}(\cdot)$ are the membership functions associated with the interval-valued I and terms c_k, respectively.

Example 3.

Let $I = [0.6, 0.78]$ be an interval-valued to be transformed into $F(S_T)$. The semantic of this term set is the same of Example 1. The fuzzy set obtained applying τ_{IS_T} is (see Fig. 6) :

$$\tau_{IS_T} = \{(s_0, 0), (s_1, 0), (s_2, 0.6), (s_3, 1), (s_4, 0.2)\}$$

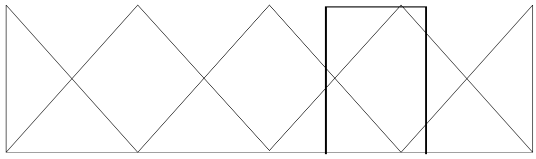

Figure 6: Transforming $[0.6, 0.78]$ into a fuzzy set in S_T

3.2 Aggregating Individual Performance Values

Using the above transformation functions we express the input information by means of fuzzy sets on the BLTS, $S_T = \{s_0, \ldots, s_g\}$. Now we use an aggregation function for combining the fuzzy sets on the BLTS to obtain a collective performance for each alternative that will be a fuzzy set on the BLTS.

For the heterogeneous GDM the preference relations are expressed by means of fuzzy sets on the BLTS, as in the Table 1. Where p^k_{ij} is the preference degree of the alternative x_i over x_j provides by the expert e_k.

Table 1: The preference relation

$$P_{e_k} = \begin{pmatrix} p^k_{11} = \{(s_0, \gamma^{11}_{k_0}), \ldots, (s_g, \gamma^{11}_{k_g})\} & \cdots & p^k_{1n} = \{(s_0, \gamma^{1n}_{k_0}), \ldots, (s_g, \gamma^{1n}_{k_g})\} \\ \vdots & \cdots & \vdots \\ p^k_{n1} = \{(s_0, \gamma^{n1}_{k_0}), \ldots, (s_g, \gamma^{n1}_{k_g})\} & \cdots & p^k_{nn} = \{(s_0, \gamma^{nn}_{k_0}), \ldots, (s_g, \gamma^{nn}_{k_g})\} \end{pmatrix}$$

We shall represent each fuzzy set, p^k_{ij}, as $r^k_{ij} = (\gamma^{ij}_{k_0}, \ldots, \gamma^{ij}_{k_g})$ being the values of r^k_{ij} their respective membership degrees. Then, the collective performance value of the preference relation according to all preference relations provided by experts $\{r^k_{ij}, \forall e_k\}$ is obtained aggregating these fuzzy sets. These collective performance values are denoted as r_{ij}, form a new preference relation of fuzzy sets defined in S_T, i.e.,

$$r_{ij} = (\gamma^{ij}_0, \ldots, \gamma^{ij}_g)$$

characterized by the following membership function:

$$\gamma^{ij}_v = f(\gamma^{ij}_{1_v}, \ldots, \gamma^{ij}_{k_v}),$$

where f is an "aggregation operator" and k is the number of experts.

3.3 Transforming into Linguistic 2-Tuples

In this phase we transform the fuzzy sets on the BLTS into linguistic 2-tuples over the BLTS. In [9] was presented a function χ that transforms a fuzzy set in a linguistic term set into a numerical value in the interval of granularity of S_T, $[0, g]$:

$$\chi : F(S_T) \to [0, g]$$

$$\chi(\tau(\vartheta)) = \chi(\{(s_j, \gamma_j),\, j = 0, ..., g\}) = \frac{\sum_{j=0}^{g} j\gamma_j}{\sum_{j=0}^{g} \gamma_j} = \beta.$$

Therefore, applying the Δ function to β we shall obtain a collective preference relation whose values are linguistic 2-tuples.

4 A GDM Problem Defined in a Heterogeneous Information Context

Let's suppose that a company want to renew its computers. There exist four models of computers available, {HP, IBM, COMPAQ and DELL} and three experts provide his/her preference relations over the four cars. The first expert expresses his/her preference relation using numerical values in $[0,1]$, P_1^n. The second one expresses the preferences by means of linguistic values in a linguistic term set S (see Figure 1), P_2^S. And the third expert can express them using interval-valued in $[0,1]$, P_3^I. The three experts attempt to reach a collective decision.

Table 2: Preference relations

$$\begin{array}{ccc} P_1^n & P_2^S & P_3^I \\ \begin{pmatrix} - & .5 & .8 & .4 \\ .5 & - & .9 & .5 \\ .8 & .9 & - & .4 \\ .4 & .5 & .4 & - \end{pmatrix} & \begin{pmatrix} - & H & VH & M \\ H & - & H & VH \\ VH & H & - & VH \\ M & VH & VH & - \end{pmatrix} & \begin{pmatrix} - & [.7,.8] & [.65,.7] & [.8,.9] \\ [.7,.8] & - & [.6,.7] & [.8,.85] \\ [.8,.9] & [.6,.7] & - & [.7,.9] \\ [.8,.9] & [.8,.85] & [.7,.9] & - \end{pmatrix} \end{array}$$

4.1 Decision Process

We shall use the following decision process to solve this problem:

A) Aggregation Phase

We use the aggregation process presented in Section 3.

1. **Making the information uniform**

   (a) *Choose the BLTS.* It will be S, due to the fact, it satisfies the conditions showed in Section 3.1.1.

   (b) *Transforming the input information into $F(S_T)$.* (e.g., see Table 3).

Table 3: Fuzzy sets in a BLTS

$$P_1^n = \begin{pmatrix} - & (0,0,0,1,0,0,0) & (0,0,0,0,.19,.81,0) & (0,0,.59,.41,0,0,0) \\ (0,0,0,1,0,0,0) & - & (0,0,0,0,0,.59,.41) & (0,0,0,1,0,0,0) \\ (0,0,0,0,.19,.81,0) & (0,0,0,0,0,.59,.41) & - & (0,0,.59,.41,0,0,0) \\ (0,0,.59,.41,0,0,0) & (0,0,0,1,0,0,0) & (0,0,.59,.41,0,0,0) & - \end{pmatrix}$$

   (c) *Aggregating individual performance values.* In this example we use as aggregation operator, f, the arithmetic mean obtaining the collective preference relation:

Table 4: The collective Preference relation.

$$\begin{pmatrix} - & (0,0,0,0,.6,.27,0) & (0,0,0,.04,.4,.67,0) & (0,0,.2,.47,.27,.33,.14) \\ (0,0,0,0,.6,.27,0) & - & (0,0,0,.14,.67,.26,.14) & (0,0,0,.33,.06,.67,.04) \\ (0,0,0,.04,.4,.67,0) & (0,0,0,.14,.67,.26,.14) & - & (0,0,.2,.14,.27,.67,.14) \\ (0,0,.2,.47,.27,.33,.14) & (0,0,0,.33,.06,.67,.04) & (0,0,.2,.14,.27,.67,.14) & - \end{pmatrix}$$

2. **Transforming into linguistic 2-tuples**. The result of this transformation is the following:

$$\begin{pmatrix} - & (H,.31) & (VH,-.43) & (H,-.18) \\ (H,.31) & - & (H,.33) & (H,.38) \\ (VH,-.43) & (H,.33) & - & (H,.29) \\ (H,-.18) & (H,.38) & (H,.29) & - \end{pmatrix}$$

B) Exploitation Phase

To solve the GDM problem, finally we calculate the dominance degree for the alternative x_i over the rest of alternatives. To do so, we shall use the following function:

$$\Lambda(x_i) = \frac{1}{n-1} \sum_{j=0 \,|\, j \neq i}^{n} \beta_{ij}$$

where n is the number of alternatives and $\beta_{ij} = \Delta^{-1}(p_{ij})$ being p_{ij} a linguistic 2-tuple. In this phase we shall calculate the dominance degree for this preference relation:

Table 5: Dominance degree of the alternatives

HP	IBM	**COMPAQ**	$DELL$
$(H,.23)$	$(H,.34)$	**(H,.4)**	$(H,.16)$

Then, dominance degrees rank the alternatives and we choose the best alternative(s) as solution set of the GDM problem, in this example the solution set is **{COMPAQ}.**

5 Concluding Remarks

We have presented an aggregation process for aggregating heterogeneous information in the case of numerical, interval-valued and linguistic values. This aggregation process is based on the transformation of the heterogeneous information into fuzzy sets assessed in a unique basic linguistic term set. And afterwards, these fuzzy sets are converted into linguistic 2-tuples. The aggregation process has been applied to a GDM problem defined in a heterogeneous information context.

In the future, we shall apply this aggregation process to other types of information used in the literature to express preference values as Interval-Valued Fuzzy Sets and Intuitionistic Fuzzy Sets.

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