INFLUENCE OF FUZZY IMPLICATION FUNCTIONS AND DEFUZZIFICATION METHODS IN FUZZY CONTROL

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ABSTRACT

The perfomance of a fuzzy system depends on the fuzzy reasoning method used. In this work we want to do a comparative study of the influence of different fuzzy implication functions and defuzzification methods proposed in specialized literature for their use on fuzzy controllers. We will carry out this study using the problem of Inverted Pendulum. We will study different measures of actuation from which we will present some measures of adaptation. Starting from the empirical results obtained from the application of the differents inference and defuzzification methods in the proposed problem, we will obtain an empirical classification of them through which it will be possible to distinguish those combinations that optimize the perfomance of the fuzzy controller.

1.- INTRODUCCION

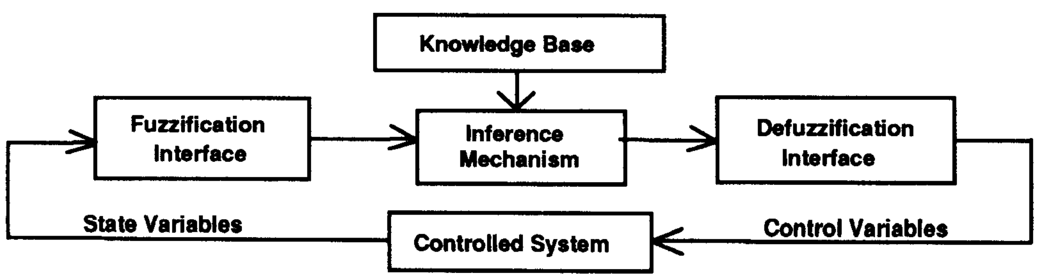
As it is known the perfomance of a fuzzy system depends on inference mechanisms, defuzzification methods, membership functions shape of the linguistic labels which constitute the control rules of the Knowledge Base, etc. (see [1,6,10,12]).

Many different fuzzy implication functions have been proposed in the Fuzzy Logic literature and some of them have been considered as specially suited for control problems (see [2,3,5,8,9]). In the same way, there is a wide range of defuzzification methods proposed, some of which have worked in a nice way in this kind of applications (see [4,10,11,15]).

The aim of this paper is to analize the influence of fuzzy implication functions and defuzzifications methods in fuzzy control. With this end, the work is organized as follows: in section 2 it is introduced the configuration of a fuzzy logic controller, in sections 3 and 4 the fuzzy implications functions and defuzzification methods selected are shown, in section 5 the measures of adaptation for the different combinations of controllers are presented and in 6 the problem of Inverted Pendulum is. The empirical results obtained working on it are shown in section 7. Finally, we show the obtained conclusions.

2.- CONFIGURATION OF A FUZZY LOGIC CONTROLLER

A fuzzy logic controller (FLC) is a control algorithm which is based on several linguistic control rules connected among them through a fuzzy implication and a compositional rule of inference, together with a defuzzification mechanism, that is to say, a mechanism that changes the action of the fuzzy control into one wich is not fuzzy. FLC's, as a focus for the analysis and design of control strategies, are obtaining good results, increasing considerably their aplication in the last years. Next figure shows the basic configuration of a fuzzy logic controller:



Knowledege Base is formed by a set of control rules R_i, i = 1, ... , m, in which are involved several linguistic labels of the shape: *If X_1 is A_1 and X_2 is A_2 and ...... and X_n is A_n then Y is B*

Inference Mechanism (IM) uses the Generalized Modus Ponens:

If X_1 is A_1 and X_2 is A_2 and ...... and X_n is A_n then Y is B
X_1 is A_1', ... , X_n is A_n'

Y is B'

Fuzzification Interface receives as an entry one or more state variables and gives a set of labels A', A' = (A_1 , ... , A_n). System acts on these entries applying to them the compositional rule of inference obtainig B' as a result. The generical expression of it is:

$$B'(y) = \underset{x}{\mathrm{Sup}} \; \{ T'(A'(x), I(A(x), B(y))), x \in R^n \}$$

where: • $A'(x) = T(A'_1(x1), A'_2(x2), \dots , A'_n(xn))$

• $A(x) = T (A_1 (x1), A_2(x2), \dots , A_n(xn))$
(T is a T-norm, in this work we use the Minimum T-norm)

• I is a fuzzy implication function

• T' is a connective or conjuntive operator

In fuzzy control, the entry is a crisp value, $x = x_0$. Due to it the function A'(x) has the following expression:

$$A'(x) = \begin{cases} 1 & \text{if } x=x_0 \\ 0 & \text{in other case} \end{cases}$$

where x_0 is the value associated to the state variables.

In this way, the expression of the compositional rule of inference is reduced to:

$$B'(y) = T'(1, I (A(x_0), B(y))) = I (A(x_0), B(y))$$

Fixed T as the Minimum T-norm, it is observed that the IM depends on the fuzzy implication function, I, used. When we are talking about a classification of IM we are referring to the importance that the different fuzzy implications functions have on the control system.

Since from each rule, R_i, it is obtained a fuzzy set B'_i (y), **Defuzzification Interface** uses an operator, G, which composes these. Finally, it applies a defuzzification method (DM), D, to convert fuzzy numbers obtained in this way into real values which correspond to the control variables of the system. So, calling S to FLC, x_0 to the value of the entries and y_0 to the numerical value obtained from the defuzzification, we have:

$$B'(y) = G \{B'_1(y), B'_2(y), \dots , B'_n(y)\} \quad ; \quad y_0 = S(x_0) = D(B'(y))$$

3.- INFERENCE MECHANISMS

We have worked with nine IM based on so many more fuzzy implication functions (see [1,3,5,6,9]). We shall distinguish those in terms of the kind of fuzzy implication used since the T-norm previously named T will be, in every case, the Minimum T-norm.

Being **T** a T-norm, **S** a T-conorm and **N** a negation function (in this work we use **N(a)** = 1-a). The models of implication studied are:

1. T-norms: The use of T-norms as fuzzy implication functions respond to a model of controllers based on T-norms and represented by the following structure:

$$B'(y) = T'(A(x_0), B(y))$$

where T and T' are T-norms that can or can't be the same.

As we have said before, we have established T as the Minimum T-norm to do this work. We shall work with the following T-norm as T':

• **Minimum** T-norm [**I1**]: $I(a,b) = a \wedge b$

The combination of different T-norms as fuzzy implications functions and connectives of conjuction over the entries (see [2,3]) can suppose an interesting extension of the study carried out here.

2. Strong Implications or S-Implications: They present the following form: I (a,b) = S (N(a) , b). Inside this group, we have choosen the following implications:

- **Diene** Implication **[I2]** : S (a,b) = Max (a,b) ; I (a,b) = Max (1-a , b)
- **Dubois y Prade** Implication **[I3]** :

$$S(a,b) = \begin{cases} a & \text{si } b = 0 \\ b & \text{si } a = 0 \\ 1 & \text{en otro caso} \end{cases} \quad ; \quad I(a,b) = \begin{cases} 1-a & \text{si } b = 0 \\ b & \text{si } a = 1 \\ 1 & \text{en otro caso} \end{cases}$$

- **Mizumoto** Implication **[I4]**: S (a,b) = a + b - a * b ; I (a,b) = 1 - a + a * b

3. Residual Implications or R-Implications: They presents the following form: I (a,b) = Sup { c : c ∈ [0,1] / **T** (c,a) ≤ b }. We work with the next implications of this kind:

- **Göguen** Implication **[I5]** :

$$T(a,b) = a * b \quad ; \quad I(a,b) = \begin{cases} \text{Max}(1 , b/a) & \text{si } a \neq 0 \\ 1 & \text{si } a = 0 \end{cases}$$

- **Gödel** Implication **[I6]**:

$$T(a,b) = \text{Min}(a,b) \quad ; \quad I(a,b) = \begin{cases} 1 & \text{si } a \leq b \\ b & \text{si } a > 0 \end{cases}$$

- **Gaines** Implication **[I7]** :

$$T(a,b) = \begin{cases} a & \text{si } b = 1 \\ b & \text{si } a = 1 \\ 0 & \text{en otro caso} \end{cases} \quad ; \quad I(a,b) = \begin{cases} 1 & \text{si } a \leq b \\ 0 & \text{en otro caso} \end{cases}$$

- **Lukaciewicz** Implication **[I8]** : T (a,b) = Max (0 , a+b-1) ; I (a,b) = Min (1 , 1-a+b)

This last implication is characterized by presenting the characteristics of the two models in the same way, since it is a S-implication for the T-conorm S (a,b) = Min (1,a+b).

4. Quantic Mechanics Implications or QM-Implicaciones: Are integred in this model the implications that presents the following form: I (a,b) = **S** (N(a) , T(a,b)). Inside it we will work with the **Early-Zadeh** Implication **[I9]**: S (a,b) = Max (a,b) ; **T** (a,b) = Min (a,b) ; I (a,b) = Max {1-a , Min (a,b) }

4.- DEFUZZIFICATION METHODS

Being C_i the fuzzy set obtained by means of making inference on the rule R_i and μ_{Ci} its membership function. We introduce the following terminology:

- S_i, to the **area** of the surface that contains the membership function μ_{Ci} with X axis.
- Y_i, to the **heigth** of C_i.
- H_i, to the **matching** of the fuzzy sets in the antecedent of the rule that gives rise to C_i when we make inference. This value is funtion of the T-norm T used, Minimum T-norm in our work, and so it has the following expression: $H_i = \text{Min}(\mu_{Ai}(x_i))$.

These three measures can constitute the **Importance Degrees** of the rule R_i.

- G_i, to the **maximum value (MV)** of the membership function μ_{Ci}. In case there is more than one, it will be equal to the average of all of them. So: $G_i = \text{Max}\ \mu_{Ci}(x), \forall x$.
- W_i , to the **centre of gravity (CG)** of C_i, whose value is obtained using the formula: $W_i = T_i / S_i$, where T_i is equal to the **area** of the surface which contains the function obtained of multplying μ_{Ci} **by the variable x**, with the X axis.

CG and MV can constitute the **Characteristic Values** of a fuzzy set C_i. In order to them, we shall distinguish two main groups of DM: **based on the CG** and **based on the MV**. Inside both of them, we distinguish two subgroups according to the Importance Degrees, one that contain the methods based on weighing by one of these and another with the methods based on the fuzzy set that presents the largest value of them. Besides, we consider two more methods according to the characteristic value MV. In this way, giving y0 to the numerical value from the deffuzzification process, we have:

- *Weighed by the different Importance Degrees:*

	Weighed by S_i :	Weighed by Y_i :	Weighed by H_i :
CG	**[D1]** $yo=\frac{\sum_i S_i \cdot W_i}{\sum_i S_i}$	**[D2]** $yo=\frac{\sum_i Y_i \cdot W_i}{\sum_i Y_i}$	**[D3]** $yo=\frac{\sum_i H_i \cdot W_i}{\sum_i H_i}$
MV	**[D4]** $yo=\frac{\sum_i S_i \cdot G_i}{\sum_i S_i}$	**[D5]** $yo=\frac{\sum_i Y_i \cdot G_i}{\sum_i Y_i}$	**[D6]** $yo=\frac{\sum_i H_i \cdot G_i}{\sum_i H_i}$

- *Based on the fuzzy set of largest Importance Degree:*

	Fuzzy set with Largest S_i :	Fuzzy set with Largest Y_i :	Fuzzy set with Largest H_i :
CG	**[D7]** $C_j = Max(S_i)$; $yo = W_j$	**[D8]** $C_j = Max(Y_i)$; $yo = W_j$	**[D9]** $C_j = Max(H_i)$; $yo = W_j$
MV	**[D10]** $C_j = Max(S_i)$; $yo = G_j$	**[D11]** $C_j = Max(Y_i)$; $yo = G_j$	**[D12]** $C_j = Max(H_i)$; $yo = G_j$

- Mean of Maximum (MOM) **[D13]**:

$$yo=\frac{\sum_i G_i}{m}$$; m is the number of fuzzy sets obtained when we make inference.

- Mean of Greater and Lower Maximum Value **[D14]**:

$$G_{min} = \underset{i}{Min}\ (G_i)\ ;\ G_{max} = \underset{i}{Max}\ (G_i)\ ;\ yo = \frac{G_{min} + G_{max}}{2}$$

5.- MEASURES OF COMPARATION

In order to define the different measures of comparation, we consider FLC as a function depending on the IM and DM used. S [i,j] represents the FLC formed by the IM i and the DM j, i = 1, .., 9 , j = 1, .. , 14. We shall represent the output of the controller when it receives the array of entries x_k by S [i,j] (x_k). Now we shall study some measures of actuation which let us establish the mentioned measures of comparation.

5.1.- Measures of Convergence:

This kind of measures are based on the speed of reply of the system. When there is a point of equilibrium we can define them trough the oscillations produced around it (for instance, in the inverted pendulum). We can define a **Measure of Convergence (MC)** as:

$$MC\,(S[i,j]) = \frac{\sum_{t_i=m}^{n} |e(t_i)|}{(n-m)/\Delta t}$$

where e (t_i) is the state of the system in the instant of time t_i; $\Delta t = |t_i - t_{i-1}|$, is the extent in seconds of the time nit of the system and m, n are the ends of the studied interval of time.

Point out that it is not posible to assign a value to this measure when the FLC S[i,j] loses the control of the system at issue during the interval of time between m and n. In that case it is considered that the Measure of Convergence takes infinity value.

5.2.- Measures of Error:

These measures are obtained trough a set of evaluation data of the system formed by N arrays of numerical data, Z_k, constituted by the values of the state variables, x_k, and the corresponding values of the associated control variables, y_k:

$$Z_k = (x_k\,,\, x_k) \quad ; \quad k = 1, ..., N$$

As measures of error, we can consider the following:

- **Maximum Punctual Error**: $MPE\ (S[i,j]) = \underset{k}{Max} \left| y_k - S[i,j](x_k) \right|$

- **Medium Linear Error** : $ME\ (S[i,j]) = \dfrac{\sum_{k=1}^{N} \left| y_k - S[i,j](x_k) \right|}{N}$

- **Medium Square Error** : $SE\ (S[i,j]) = \dfrac{\frac{1}{2} \sum_{k=1}^{N} (y_k - S[i,j](x_k))^2}{N}$

5.3.- Measures of Adaptation:

Because of the values represented by any of these measures are compared more easily, we can define an Adaptation Degree of the FLC S[i,j], **AD [i,j]**, as the quotient between the minimum value (MinVal) of those and the value obtained by that controller:

$$MinVal = \underset{i,j}{Min}\ (M(S[i,j])) \quad ; \quad AD[i,j] = \frac{MinVal}{M\ (S[i,j])}$$

where M represents any of the earlier measures (MC, MPE, ME, SE).

With this definition, the Adaptation Degree of S[i,j] is characterized for being included in the interval [AD_Min , 1], for any value of i and j,

$$MaxVal = \underset{i,j}{Max}\ (M(S[i,j])) \quad ; \quad AD\_Min = \frac{MinVal}{MaxVal}$$

And so, point out that, in case of MaxVal has the infinity simbolic value, AD_Min is equal to 0 and the interval in which the Adaptation Degree is included presents maximum extent: AD $\in$ [0,1].

It is possible to define a measure based on the combination of a measure of convergence and a meausre of error. In this way, this new measure will have the characteristics of this two groups discussed before and the value of adaptation that receives a determinated controller S [i,j] will be more suitable than that which it would receive from an only measure of one of the two groups.

We are going to combine the Measure of Convergence and the measure of Medium Square Error by means of an average function, obtaining then the Conjunctive Adaptation Degree (CAD), defined in the following way:

$$CAD\ (S[i,j]) = f\left(AD\_SE\ (S[i,j])\ ,\ AD\_MC\ (S[i,j])\right)$$

where Min (x,y) $\leq$ f (x,y) $\leq$ Max (x,y) ; $AD\_SE[i,j] = \dfrac{Min\_SE}{SE\ (S[i,j])}$ con $Min\_SE = \underset{i,j}{Min}\ (SE(S[i,j]))$

$$y \; AD\_MC[i,j] = \frac{Min\_MC}{MC(S[i,j])} \text{ con } Min\_MC = \underset{i,j}{Min}\,(MC(S[i,j])).$$

In this work we shall use:

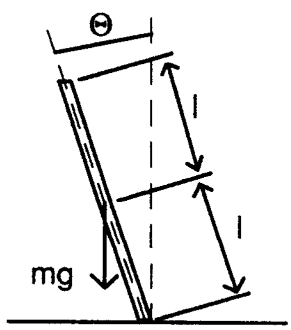
$$f(x,y) = \begin{cases} 1/2\,(x+y) & , \text{if } x \neq 0 \text{ and } y \neq 0 \\ 0 & , \text{in other case} \end{cases}$$

Finally, we define the concrete **Maximum, Minimum and Medium Adaptation Degree** for a IM or a MD of any of the earlier measures (AD can be equal to AD_MC, AD_SE o CAD):

	Inference Mechanism	Defuzzification Method
Maximum AD	$Max\_AD\_IM[i] = \underset{j}{Max}\,(AD[i,j])$	$Max\_AD\_DM[j] = \underset{i}{Max}\,(AD[i,j]))$
Mínimum AD	$Min\_AD\_IM[i] = \underset{j}{Min}\,(AD[i,j]))$	$Min\_AD\_DM[j] = \underset{i}{Min}\,(AD[i,j]))$
Medium AD	$Med\_AD\_IM[i] = \frac{1}{14}\sum_{j=1}^{14} AD[i,j]$	$Med\_AD\_DM[j] = \frac{1}{9}\sum_{i=1}^{9} AD[i,j]$

6.- THE INVERTED PENDULUM

As we have mentioned before, we are going to make the study in a practical way over a concrete application widely studied in control theory (see [5, 14]), the "Inverted Pendulum":



The behaviour of the pendulum is managed by the equation:

$$m\frac{l^2}{3}\frac{d^2\Theta}{dt^2} = \frac{1}{2}\left(-F + m\,g\,\text{sen}\Theta - k\frac{d\Theta}{dt}\right)$$

where $k\frac{d\Theta}{dt}$ is an approximation of the friction strength.

The chosen problem is appropiate for this study due to the important temporal restrictions that it presents, which force to the FLC to work with a high degree of precision. For this study we have implemented a simulation model of the pendulum which emulates its behaviour in real time following the instructions arisen by the different authors in [5, 7, 14]. The data used in the implementation are M = 5 Kilograms and L = 5 metres. The universes of discurse of the variables are the followings: $\Theta \in [-0.5283\,,\ 0.5283]$, $\omega \in [-0.8645\,, 0.8645]$, $F \in [-3003.8\,, 3003.8]$.

The simulation model allows us to apply the different variants of the FLC and take empirical data in order to make a correct comparison among the different combinations. The set of control rules that constitutes the Knowledge Base used by the controller to simulate the behaviour of the physical system is that provided by T. Yamakawa in [14]. We have considered the discretization of the universes presented in [7] to build the membership functions corresponding to the fuzzy sets of the control rules.

7.- EXPERIMENTS AND RESULTS

Using the empirical data obtained by the simulation model, we have calculated the values of the Measure of Convergence which correspond to each of the indicated combinations IM-DM. The parameters used to get these data have been:

- The ends of the temporal interval are: m = 0 s ; n = 20 s.
- The extent in seconds of the time unit of the system is: Δt = 100 ms.
- The fuzzy logic controller acts each 600 ms.
- The initial parameters of the system are Angle = 0.25 rad ; Angular Speed = 0.40 rad/s.

Experimentally we have obtained a set of evaluation data of the system constituted by 68 input-output arrays with values in the intervals [-0.275 , 0.275], [-0.454 , 0.454] and [-1576.681 , 1576.681], for Θ, ω and F respectively. We use these data to calculate the values of the Medium Square Error.

It has been obtained the Adaptation Degree associated to both measures and, using them, the Conjunct Adaptation Degree, data collected in the following tables:

ADAPTATION DEGREE ASSOCIATED TO THE MEASURE OF CONVERGENCE:

	Minimum	*S-Implications*			*R-Implications*			*R and S*	*QM*
	I1	**I2**	**I3**	**I4**	**I5**	**I6**	**I7**	**I8**	**I9**
D1	**1.00000**	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***
D2	0.30094	***0.00000***	***0.00000***	***0.00000***	***0.00000***	0.28486	***0.00000***	***0.00000***	***0.00000***
D3	0.30094	***0.00000***	***0.00000***	***0.00000***	0.30093	0.30067	0.29968	***0.00000***	***0.00000***
D4	**1.00000**	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***
D5	0.29968	***0.00000***	***0.00000***	***0.00000***	***0.00000***	0.27312	***0.00000***	***0.00000***	***0.00000***
D6	0.29968	0.29938	0.30120	0.29938	0.29968	0.29968	0.29968	0.29968	***0.00000***
D7	0.18713	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***
D8	0.18713	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***
D9	0.18713	***0.00000***	***0.00000***	***0.00000***	0.20662	0.18380	0.20548	***0.00000***	***0.00000***
D10	0.19060	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***
D11	0.19060	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***	***0.00000***
D12	0.19060	0.21463	0.19580	0.21463	0.19060	0.19060	0.19060	0.19060	***0.00000***
D13	0.27312	***0.00000***	***0.00000***	***0.00000***	***0.00000***	0.27312	***0.00000***	***0.00000***	***0.00000***
D14	0.27312	***0.00000***	***0.00000***	***0.00000***	***0.00000***	0.27312	***0.00000***	***0.00000***	***0.00000***

ADAPTATION DEGREE ASSOCIATED TO THE MEDIUM SQUARE ERROR:

	Minimum	*S-Implications*			*R-Implications*			*R and S*	*QM*
	I1	**I2**	**I3**	**I4**	**I5**	**I6**	**I7**	**I8**	**I9**
D1	0.93902	0.06310	0.06450	0.06337	0.05548	0.22114	0.05073	0.06364	0.06268
D2	**1.00000**	0.07052	0.07221	0.07097	0.06026	0.30104	0.06026	0.07137	0.06932
D3	**1.00000**	0.16167	0.20825	0.17300	**1.00000**	**1.00000**	**1.00000**	0.18418	0.14267
D4	0.93902	0.04998	0.05114	0.05021	0.05548	0.22114	0.05073	0.05043	0.06538
D5	**1.00000**	0.06026	0.06026	0.06026	0.06026	0.30104	0.06026	0.06026	0.07487
D6	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	0.68940
D7	0.48965	0.06026	0.06026	0.06026	***0.00415***	0.06899	***0.00415***	0.06026	0.06026
D8	0.48965	0.06026	0.06026	0.06026	***0.00415***	0.12094	***0.00415***	0.06026	0.06026
D9	0.48965	0.16496	0.20789	0.17581	0.48965	0.48965	0.48965	0.18627	0.14647
D10	0.48965	***0.00415***	***0.00415***	***0.00415***	***0.00415***	0.06899	***0.00415***	***0.00415***	0.06026
D11	0.48965	***0.00415***	***0.00415***	***0.00415***	***0.00415***	0.12094	***0.00415***	***0.00415***	0.06026
D12	0.48965	0.48965	0.48965	0.48965	0.48965	0.48965	0.48965	0.48965	0.48965
D13	0.30104	0.06026	0.06026	0.06026	0.06026	0.30104	0.06026	0.06026	0.07752
D14	0.30104	0.06026	0.06026	0.06026	0.06026	0.30104	0.06026	0.06026	0.16610

Considering that the minimum value of the Measure of Convergence obtained is 34.51 and the minimum Square Error is equal to 8940.85, it is possible to calculate the value of both measures for any of the combinations through the definition explained before.

CONJUNCTIVE ADAPTATION DEGREE:

	Minimum	*S-Implications*			*R-Implications*			*R and S*	*QM*
	I1	I2	I3	I4	I5	I6	I7	I8	I9
D1	**1.00000**	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*
D2	0.65047	*0.00000*	*0.00000*	*0.00000*	*0.00000*	0.29295	*0.00000*	*0.00000*	*0.00000*
D3	0.65047	*0.00000*	*0.00000*	*0.00000*	0.65047	0.65034	0.64984	*0.00000*	*0.00000*
D4	**1.00000**	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*
D5	0.64984	*0.00000*	*0.00000*	*0.00000*	*0.00000*	0.28708	*0.00000*	*0.00000*	*0.00000*
D6	0.64984	0.64969	0.65060	0.64969	0.64984	0.64984	0.64984	0.64984	*0.00000*
D7	0.33834	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*
D8	0.33834	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*
D9	0.33834	*0.00000*	*0.00000*	*0.00000*	0.34813	0.33672	0.34756	*0.00000*	*0.00000*
D10	0.34012	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*
D11	0.34012	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*	*0.00000*
D12	0.34012	0.35214	0.34272	0.35214	0.34012	0.34012	0.34012	0.34012	*0.00000*
D13	0.28708	*0.00000*	*0.00000*	*0.00000*	*0.00000*	0.28708	*0.00000*	*0.00000*	*0.00000*
D14	0.28708	*0.00000*	*0.00000*	*0.00000*	*0.00000*	0.28708	*0.00000*	*0.00000*	*0.00000*

The following tables show the values of the rest of the Medium Adaptation Degrees defined above:

MEDIUM ADAPTATION DEGREE FOR AN INFERENCE MECHANISM:

	Min	*S-Implications*			*R-Implications*			*R and S*	*QM*
	I1	I2	I3	I4	I5	I6	I7	I8	I9
MC	**0.34857**	0.03672	0.03550	0.03672	0.07127	0.14850	0.07110	0.03502	*0.00000*
SE	**0.67271**	0.16496	0.17166	0.16662	0.23914	0.35754	0.23846	0.16823	*0.15894*
C	**0.51064**	0.07156	0.07095	0.07156	0.14204	0.22366	0.14195	0.07071	*0.00000*

MEDIUM ADAPTATION DEGREE FOR A DEFUZZIFICATION METHOD:

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14
MC	0.111	0.065	0.133	0.111	0.063	**0.266**	*0.020*	*0.020*	0.086	0.021	0.021	0.175	0.060	0.060
SE	0.175	0.197	0.541	0.170	0.193	**0.965**	0.096	0.102	0.315	*0.071*	0.077	0.489	0.115	0.125
C	0.107	0.104	0.289	0.107	0.104	**0.577**	*0.037*	*0.037*	0.152	*0.037*	*0.037*	0.305	0.063	0.063

Now, we present conjunct results of the different groups of IM and DM:

ADAPTATION DEGREE OF THE DEFUZZIFICATION METHODS WEIGHED BY THE IMPORTANCE DEGREES:

		Min	R-Imp	S-Imp	R y S	QM-Imp
Si	MC	**0.99978**	*0.00000*	*0.00000*	*0.00000*	*0.00000*
	SE	**0.93902**	0.10912	0.05705	*0.05704*	0.06403
	C	**0.96940**	*0.00000*	*0.00000*	*0.00000*	*0.00000*
Yi	MC	**0.30031**	0.09300	*0.00000*	*0.00000*	*0.00000*
	SE	**1.00000**	0.14052	*0.06575*	0.06582	0.07210
	C	**0.65016**	0.09667	*0.00000*	*0.00000*	*0.00000*
Hi	MC	**0.30031**	0.30005	0.15000	0.14984	*0.00000*
	SE	**1.00000**	**1.00000**	0.59049	0.59209	*0.41604*
	C	**0.65016**	0.65003	0.32500	0.32492	*0.00000*

ADAPTATION DEGREE OF THE DEFUZZIFICATION METHODS BASED ON THE FUZZY SET OF LARGEST IMPORTANCE DEGREE:

		Mín	R-Imp	S-Imp	R y S	QM-Imp
Si	MC	**0.18881**	***0.00000***	***0.00000***	***0.00000***	***0.00000***
	SE	**0.48965**	***0.02576***	0.03221	0.03221	0.06026
	C	**0.33923**	***0.00000***	***0.00000***	***0.00000***	***0.00000***
Yi	MC	**0.18881**	***0.00000***	***0.00000***	***0.00000***	***0.00000***
	SE	**0.48965**	0.04308	***0.03221***	***0.03221***	0.06026
	C	**0.33923**	***0.00000***	***0.00000***	***0.00000***	***0.00000***
Hi	MC	0.18881	**0.19461**	0.10418	0.09530	***0.00000***
	SE	**0.48965**	**0.48965**	0.33627	0.33796	***0.31806***
	C	0.33923	**0.34213**	0.17450	0.17006	***0.00000***

Finally, we attach some comparative graphics which show the behaviour of the Inverted Pendulum under different combination of IM-DM:

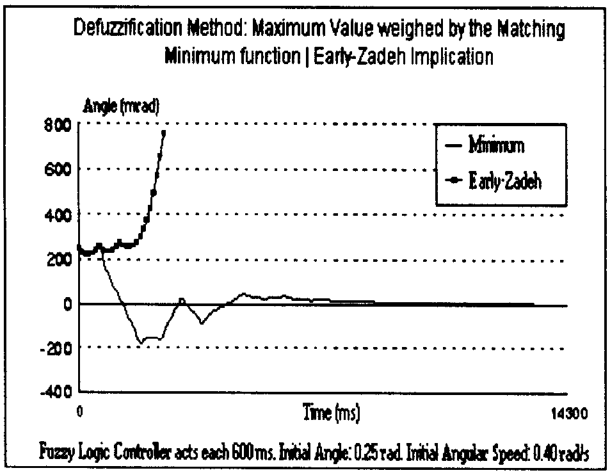


Fig. 1

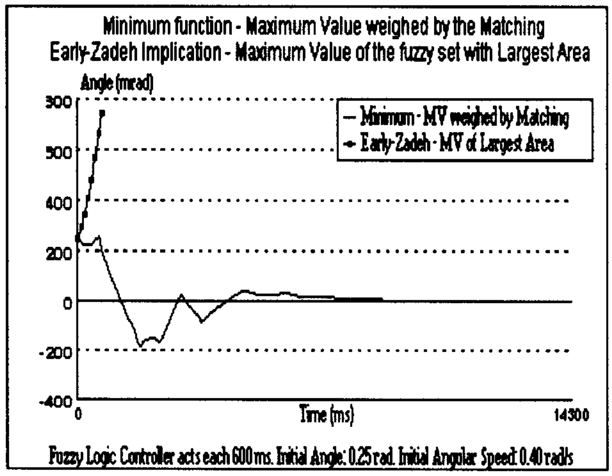


Fig. 2

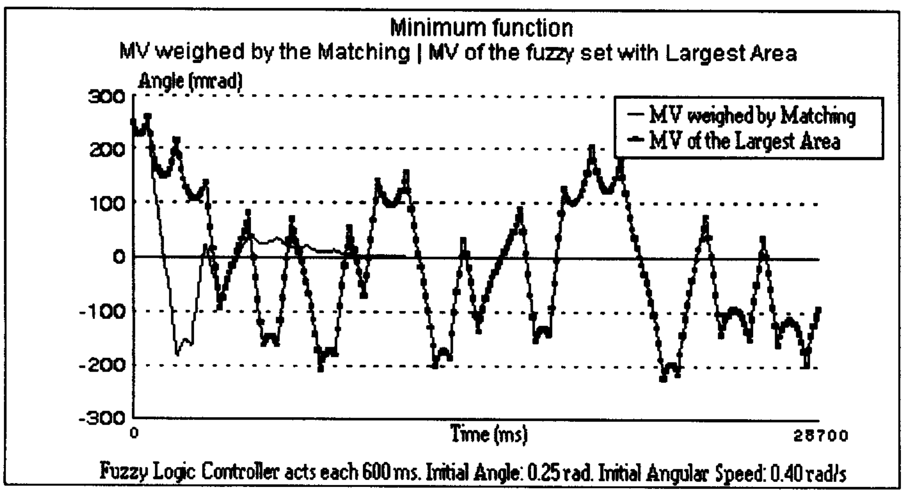


Fig. 3

Fig. 1.- Best DM
Best IM | Worst IM

Fig. 2.- Best DM-IM
Worst DM-IM

Fig. 3.- Best IM
Best DM | Worst DM

Fig. 4.- Minimum +
R-Implications
Best DM

Fig 5.- Minimum +
S-Implications
Best DM

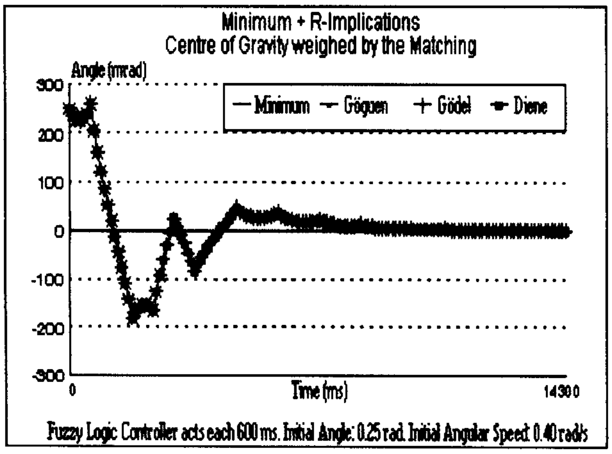


Fig. 4

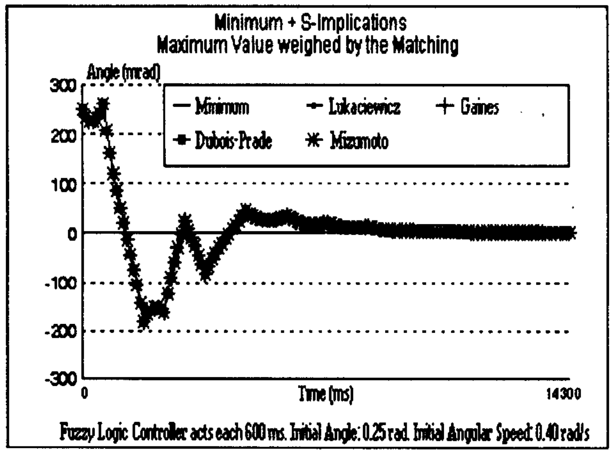


Fig. 5

8.- CONCLUSIONS

In this work we have studied the influence of fuzzy implication functions and defuzzification methods in fuzzy logic controllers using as a base the problem of the Inverted Pendulum. At the sight of the obtained results, we can underline the following:

- There is not an only combination that increases every one of the Adaptation Degrees, but we have several ones. All combinations that increase every single measure have exactly the same behaviour in the system (for example, see figures 4 and 5 for some of the combinations that increases the Adaptation Degree of the Medium Square Error).
- The fact that the membership function of the fuzzy sets that constitute the linguistic labels of the control rules is symmetrical, provokes that the defuzzification methods based on the MV and the CG give the same results when the defuzzification is made using implication functions as Minimum and R-Implications.
- Defuzzification Methods based on weighing are more efficient than those based on the fuzzy set with largest Importance Degree. Inside them, the Matching is the most important charasteristic for the Defuzzification because the four better adapted methods have it.
- The best defuzzification methods at the sight of the average results have been the Maximum Value weighed by the Matching and the Maximum Value of the fuzzy set with Largest Matching (D6 y D12, respectively) but none of them get increase the Conjuctive Adaptation Degree (although they increase the Adaptation Degree of the Medium Square Error).
- Minimum function (I1) has resulted to be the best fuzzy implication function seeing average results with a great advantage with regard to the others. The R-implications (maximum Gödel) have an acceptable behaviour, no in that way the S and QM-Implications. The implications of last model do not get optimum behaviour with any of the defuzzification methods.

Finally, we want to remark two points. First, the results obtained in the different measures depend directly of the Knowledge Base used. It would be interesting to do the same study using another Knowledge Bases obtained by mechanisms like clustering algorithms and learning methods. Anyway, this work can give an approximation to distinguish which combinations present better behaviour in fuzzy control. Second, this work can be continued in a more extensive study including factors as the influence of the shape of the membership functions of the linguistic labels that constitutes the control rules of the Knowledge Base (see [1]) and the study of the inference mechanisms based on T-norms (using them as conjunctive operators and fuzzy implication functions) (see [2, 3]), fact justified because the Minimum function has the best results among all the fuzzy implication functions used. And it is also interesting the study of dynamic methods of reasoning using as a base the obtained Adaptation Degrees (see [12, 13]).

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