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# HOMOGENEOUS LINEAR FUZZY FUNCTIONS AND RANKING METHODS IN FUZZY LINEAR PROGRAMMING PROBLEMS

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A general model for Fuzzy Linear Programming problem is studied. Fuzzy numbers generated by an homogeneous linear fuzzy function have been used for representing the imprecision of the parameters. A solution method is proposed using fuzzy numbers ranking procedures.

*Keywords:* Fuzzy linear programming, fuzzy numbers, ranking methods, homogeneous linear fuzzy function, parametric linear programming.

## 1. Introduction

A conventional Linear Programming (LP) problem can be represented by $(A, b, c)$, where $A$ is the technological matrix of the coefficients, $b$ the right hand side and $c$ the vector of costs. From this representation, several fuzzyfications of it can be proposed, and then different types of Fuzzy Linear Programming (FLP) problems considered.

In the literature can be found a lot of models dealing with this kind of imprecise LP problems [1,2,3,4,5,6,7,8,9]. Most of them assume that some of the elements describing the model are fuzzy, but no one deals with the case in which all of these elements, i.e. $A$, $b$ and $c$, are simultaneously fuzzy.

In this paper a general model of FLP problem with all the elements defining it being fuzzy is presented. From this model, each particular case of FLP problem can be easily derived and eventually solved according to their characteristics. A solution method for this general model is proposed and analyzed. The solution method is based on the use of Fuzzy Numbers Ranking Procedures (FNRP).

More concretely, in Section 2 the basic notations and definitions to be used are briefly introduced. Section 3 is devoted to introduce the general FLP problem. In section 4 the mentioned solution method is approached from a twice way: first by using Homogeneous Linear Fuzzy Functions to treat the fuzzy numbers assumed, and second with regard to a more general model than the above one which assumes different types of fuzzy numbers. Finally a numerical example is analyzed and some conclusions are pointed out.

## 2. Basic Notations and Definitions

As usual, the fuzzy set

$$\mu_A : \Re \to I = [0,1]$$

is a fuzzy number if

(i) $\mu_A$ is upper semi continuous, and

(ii) $Supp(A) = \{x \in \Re / \mu_A(x) > 0\}$ is a bounded set of $\Re$.

The set of these fuzzy numbers will be denoted $F(\Re)$.

Thus, fuzzy numbers are fuzzy subsets of $\Re$ whose $\alpha$-cuts are closed and bounded intervals on $\Re$ when $\alpha > 0$. If $\alpha = 0$, $A^0$ will denote the closure of $Supp(A)$. Hence, $\forall \alpha \in [0,1]$ the $\alpha$-cuts of $A$ will be represented by

$$A^\alpha = [a_1(\alpha), a_2(\alpha)]. \tag{1}$$

Extended sum and product by positive real numbers are considered to be defined in $F(\Re)$ by means of the Zadeh's Extension Principle. Hence, given any two fuzzy numbers $A, B \in F(\Re)$, $\forall \alpha \in [0,1]$, the following result concerned on their $\alpha$-cuts will be used,

$$(A+B)^\alpha = [a_1(\alpha) + b_1(\alpha), a_2(\alpha) + b_2(\alpha)] \tag{2}$$

$$(rA)^\alpha = [ra_1(\alpha), ra_2(\alpha)] \quad \forall r \in \Re \ r > 0. \tag{3}$$

In order to solve FLP problems involving fuzzy numbers, some ranking procedure in $F(\Re)$ will be needed. The problem of comparison of fuzzy numbers has been widely investigated in the literature, many FNRP can be found for instance in [10]. This paper will focus on those FNRPs which are defined by means of a ranking function, specially a linear ranking function, what is no too restrictive because many well known FNRP may be formulated by using linear ranking functions in some way.

Consider $A, B \in F(\Re)$, a simple method of comparison between them consists on the definition of a certain function $g : F(\Re) \to \Re$, if this function $g(\cdot)$ is known, then $g(A) > g(B)$, $g(A) = g(B)$, $g(A) < g(B)$ are equivalent to $A > B$, $A = B$, $A < B$ respectively. Usualy, $g$ is called a Linear Ranking Function (LRF) if

$$\forall A, B \in F(\Re);\ \forall r \in \Re \ r > 0;\ g(A+B) = g(A) + g(B) \text{ and } g(rA) = rg(A). \tag{4}$$

As it is well known, from this definition several FNRPs may be considered. In particular, properties (2) and (3) suggest that FNRP using linear functions of the

$\alpha$-cuts could be expressed by LRF. In [11] a good study of these LRF can be found. Here, some of them are shown in the following.

### 2.1. *Linear functions based on only one $\alpha$−cut*

Let $\alpha, \lambda \in [0,1]$ be and $A \in F(\Re)$. One defines

$$g(A) = \lambda a_1(\alpha) + (1-\lambda) a_2(\alpha).$$

The definition contains two parameters which depend on the decision-maker: the first, $\alpha$, is an accomplishment degree of the comparison, and the second, $\lambda$, is and optimism-pessimism level. Particular cases of this definition are,

(a) If $\alpha = 1$ and $A \in F(\Re)$ is any unimodal fuzzy number, then the first index of Yager, [12], is obtained.

(b) If $\alpha \in [0,1]$ is any fixed value and $\lambda = 0$, then the index of Adamo, [13], is deduced.

This definition is a particular case of the discrete method proposed in [14]. From the properties (2) and (3) is very easy to show as $g(\cdot)$ is linear in the sense of the above definition.

### 2.2. *Linear functions based on all the $\alpha$−cuts*

Let $A \in F(\Re)$, $\lambda \in [0,1]$ and $P(\cdot)$ be any additive measure on $[0,1]$. One defines

$$g(A) = \int_0^1 (\lambda a_1(\alpha) + (1-\lambda) a_2(\alpha))\, dP(\alpha).$$

As it can be seen, now one has only one parameter, $\lambda$, which acts as an optimism-pessimism degree. Particular cases of this definition are:

(a) If $P(\cdot)$ is the Lebesgue's measure and $\lambda = \frac{1}{2}$, then the fourth index of Yager, [15], is obtained.

(b) If $P(\cdot)$ is given by $P([a,b]) = b^2 - a^2$ and $\lambda = \frac{1}{2}$, then the index of Tsumura, [16], is deduced.

This general definition of $g(A)$ was proposed in [17], where also was shown as this ranking function is linear. In particular for triangular fuzzy number $g(A)$ takes the following expression:

$$g(A) = a_1(1) - \frac{(a_1(1) - a_1(0))}{(r+1)} + \lambda \frac{(a_2(0) - a_1(0))}{(r+1)}$$

where $r$ is a parameter according to which $g(A)$ can take values either close to the modal values ($r > 1$) or close to the values on the support ($r < 1$).

Therefore, as LRFs are good enough general tools to represent and treat many FNRP, in the following FLP problems in which is assumed the decision-maker can use those FRNP suitable of being represented in this way shall be considered.

From this point of view, a special class of fuzzy functions will be used to fuzzify the classical LP problem: Homogeneous Linear Fuzzy Functions, [6].

**Definition 2.1** *A function $f : \Re \rightarrow F(\Re)$ is called an Homogeneous Linear Fuzzy Function if*

$$f(e) = e\tau, \quad e \in \Re \tag{5}$$

*where $\tau \in F(\Re)$ is a fuzzy number. (Is is clear that $f(1) = \tau$).*

Consider $h : I \rightarrow \Re_+$ is a fixed function with the properties of a generator function of Arquimedean t-norms,

(a) $h$ is continuous and strictly decreasing,

(b) $h(1) = 0$, and

(c) $h^{(-1)}(x) = h^{-1}(x)\ \forall x \in [0, h(0)]$ and $h^{(-1)}(x) = 0\ \forall x > h(0)$.

Then, for all $\alpha \in \Re$, $d \in \Re_+ \cup \{0\}$, a fuzzy number $\tau$ may be defined as the fuzzy number with membership function, [7],

$$\mu(a) = \begin{cases} h^{(-1)}(\frac{|a-\alpha|}{d}) & \text{if } d > 0 \\ X_{\{\alpha\}}(a) & \text{if } d = 0. \end{cases} \tag{6}$$

This kind of fuzzy numbers are called quasitriangular fuzzy numbers generated by $h$, with center $\alpha$ and width $d$, and they are denoted by the pair $(\alpha,d)$. In our case we can consider $\tau$ as the pair $(1, d)$.

In particular, if we consider the generator function $h$, $h : [0, 1] \rightarrow [0, 1]$, $h(x) = 1 - x$, then the following linear membership function is obtained,

$$\mu(a) = \begin{cases} 1 - \frac{|a-\alpha|}{d} & \text{if } \alpha - d \leq a \leq \alpha + d \\ 0 & \text{otherwise.} \end{cases}$$

## 3. A General FLP Problem

Consider the following conventional LP problem

$$Max\{cx/Ax \leq b,\ x \geq 0\} \tag{7}$$

where $A$ is a $(m, n)$ matrix, $x, c \in \Re^n$ and $b \in \Re^m$.

A general fuzzification of this LP problem may be defined as

$$\begin{array}{ll} \underset{s.t.}{Max} & \sum_{j=1}^{n} \underset{\sim}{c_j} x_j \\ & \sum_{j=1}^{n} \underset{\sim}{a_{ij}} x_j \underset{\sim}{\leq} \underset{\sim}{b_i},\ i = 1, ..., m \\ & x_j \geq 0,\ j = 1, ..., n \end{array} \tag{8}$$

where the following fuzzy elements are to be considered:

(a) For each row, constraint, in (8)

$$\exists \mu_i \in F(\Re) \text{ such that } \mu_i : \Re \to [0,1],\ i = 1, ..., m \tag{9}$$

which defines the fuzzy number in the right hand side.

(b) For each $i = 1, ..., m$ and $j = 1, ..., n$,

$$\exists \mu_{ij} \in F(\Re) \text{ such that } \mu_{ij} : \Re \to [0,1] \tag{10}$$

defining the fuzzy numbers in the technological matrix.

(c) For each $j = 1, ..., n$

$$\exists \mu_j \in F(\Re) \text{ such that } \mu_j : \Re \to [0,1] \tag{11}$$

defining the fuzzy numbers in the objective function, and

(d) For each row of (8),

$$\exists \mu^i \in F[F(\Re)] : \mu^i : F(\Re) \to [0,1] \tag{12}$$

giving, for every $x \in \Re^n$, the accomplishment degree of the fuzzy number

$$\underset{\sim}{a_{i1}} x_1 + \underset{\sim}{a_{i2}} x_2 + ... + \underset{\sim}{a_{in}} x_n,\ i = 1, ..., m,$$

with respect to the $i$-th constraint, that is, the adequacy between this fuzzy number and the corresponding one $\underset{\sim}{b_i}$ with respect to the $i$-th constraint.

In the following, it will be supposed there exists a relation $\boxtimes$ between fuzzy numbers ranking them, that is

$$\underset{\sim}{a}, \underset{\sim}{b} \in F(\Re) \Rightarrow \underset{\sim}{a} \boxtimes \underset{\sim}{b}.$$

Now, let $\underset{\sim}{t_i}$ be a fuzzy number, fixed by the decision maker, giving his allowed maximum violation in the accomplishment of the $i$-th constraint. Then it makes sense to change that $i$-th constraint by the following one (between fuzzy numbers):

$$\underset{\sim}{a_i} x \boxtimes \underset{\sim}{b_i} + \underset{\sim}{t_i}(1-\alpha),\ i = 1, ..., m,\ \alpha \in (0,1]$$

which express for $\alpha = 1$ the constraint is completely verified with respect to the wishes of decision-maker. Moreover, smaller $\alpha$ smaller the accomplishment degree of the decision-maker will be. Then the fuzzy constraint set in (8) may be substituted by

$$\underset{\sim}{A} x \boxtimes \underset{\sim}{b} + \underset{\sim}{t}(1-\alpha),\ \alpha \in (0,1]$$

where $\underset{\sim}{A}$ is a $(m,n)$ matrix of fuzzy numbers and $\underset{\sim}{b}$ and $\underset{\sim}{t}$ column vectors of fuzzy numbers.

Thus, as auxiliary problem to solve (8), one can propose the next one:

$$\begin{aligned} \underset{s.t.}{Max} \quad & \sum_{j=1}^{n} \underset{\sim}{c_j} x_j \\ & \sum_{j=1}^{n} \underset{\sim}{a_{ij}} x_j \lesssim \underset{\sim}{b_i} + \underset{\sim}{t_i}(1-\alpha), \ i = 1, \ldots, m \\ & x_j \geq 0, \ \alpha \in (0,1], \ j = 1, \ldots, n. \end{aligned} \tag{13}$$

It is clear whether the objective function in (13) is a conventional one, that is, with costs defined by real numbers, the corresponding problem coincides with the model presented in [18]. Moreover, according to the characteristics of the relation $\lesssim$, different models of conventional LP problems are obtained.

## 4. Solving the FLP Problem

This section is devoted to approach a solution method for the FLP problem (8). The method will depend on the fuzzy numbers assumed in the problem.

### 4.1. *Fuzzy numbers generated by homogeneous linear fuzzy functions*

Consider the FLP problem (8). In the following it will be shown as (8) can be transformed into a special equivalent FLP problem if the fuzzy numbers are assumed defined as Homogeneous Linear Fuzzy Functions like (5).

As it is clear (8) may be written as

$$\begin{aligned} \underset{s.t.}{Max} \quad & \sum_{j=1}^{n} f(c_j) x_j \\ & \sum_{j=1}^{n} f(a_{ij}) x_j \underset{\sim}{\leq} f(b_i), \ i = 1, \ldots, m \\ & x_j \geq 0, \ \alpha \in (0,1] \ j = 1, \ldots, n \end{aligned} \tag{14}$$

from which one can consider the following equivalent model

$$\begin{aligned} \underset{s.t.}{Max} \quad & \sum_{j=1}^{n} f(c_j) x_j \\ & \sum_{j=1}^{n} f(a_{ij}) x_j \lesssim f(b_i) + f(t_i)(1-\alpha), \ i = 1, \ldots, m \\ & x_j \geq 0, \ \alpha \in (0,1], \ j = 1, \ldots, n. \end{aligned} \tag{15}$$

The next theorem permit us to solve (15).

**Theorem 4.1** *Let f be an Homogeneous Linear Fuzzy Function in the form (5), and let g be a Linear Ranking Function. Then the solution of (15)* ***is either***

*(i) the solution of the classical LP parametric problem*

$$\begin{array}{l} Max\ cx \\ s.t. \\ \quad Ax \leq b + t(1-\alpha) \\ \quad x \geq 0,\ \alpha \in (0,1] \end{array} \tag{16}$$

*if* $g(\tau) \geq 0$, *or*

*(ii) the solution of the classical LP parametric problem*

$$\begin{array}{l} Min\ cx \\ s.t. \\ \quad Ax \geq b + t(1-\alpha) \\ \quad x \geq 0,\ \alpha \in (0,1] \end{array} \tag{17}$$

*if* $g(\tau) < 0$.

**Proof.** If the Linear Ranking Function is applied to both the objective and the constraints, one has

$$\sum_{j=1}^{n} f(c_j)x_j = f(1)\sum_{j=1}^{n} c_j x_j = \tau \sum_{j=1}^{n} c_j x_j \Rightarrow g(\sum_{j=1}^{n} f(c_j)x_j) = g(\tau)\sum_{j=1}^{n} c_j x_j$$

$$\sum_{j=1}^{n} f(a_{ij})x_j \boxed{\leq} f(b_i) + f(t_i)(1-\alpha) \Leftrightarrow g(\sum_{j=1}^{n} f(a_{ij})x_j) \boxed{\leq} g(f(b_i) + f(t_i)(1-\alpha)) \Leftrightarrow$$

$$\Leftrightarrow g(\tau)\sum_{j=1}^{n} a_{ij}x_j \leq g(\tau)(b_i + t_i(1-\alpha))$$

which is true for all $x_j \geq 0$, $j = 1, ..., n$, $i = 1, ..., m$.

Therefore it follows that the nonfuzzy constrains set is one of the following sets:

$$C_1(\alpha) = \{x \in \Re^n / x \geq 0,\ \sum_{j=1}^{n} a_{ij}x_j \leq b_i + t_i(1-\alpha),\ i = 1, ..., m,\ \alpha \in (0,1]\} \text{ if } g(\tau) \geq 0$$

or

$$C_2(\alpha) = \{x \in \Re^n / x \geq 0,\ \sum_{j=1}^{n} a_{ij}x_j \geq b_i + t_i(1-\alpha),\ i = 1, ..., m,\ \alpha \in (0,1]\} \text{ if } g(\tau) < 0$$

being $x^*$ alternatively an optimal solution under these sets $C_1(\alpha)$ or $C_2(\alpha)$ if

$$\sum_{j=1}^{n} c_j x_j \leq \sum_{j=1}^{n} c_j x_j^* \ \forall x \in C_1(\alpha) \text{ if } g(\tau) \geq 0$$

or

$$\sum_{j=1}^{n} c_j x_j \geq \sum_{j=1}^{n} c_j x_j^* \ \forall x \in C_2(\alpha) \text{ if } g(\tau) < 0.$$

Which respectively coincide with problems (16) and (17). □

### 4.2. *Using general fuzzy numbers*

If one has fuzzy coefficients in the FLP problem, but they have not been obtained from Homogeneous Linear Fuzzy Functions, and Linear Ranking Functions $g$ are used, then to solve (8) the following associated parametric LP problem can be considered

$$\begin{array}{ll} \underset{s.t.}{Max} & \sum_{j=1}^{n} g(\underset{\sim}{c_j})x_j \\ & \sum_{j=1}^{n} g(\underset{\sim}{a_{ij}})x_j \leq g(\underset{\sim}{b_i}) + g(\underset{\sim}{t_i})(1-\alpha),\ i=1,...,m \\ & x_j \geq 0,\ \alpha \in (0,1],\ j=1,...,n. \end{array} \tag{18}$$

On other hand, to solve (8) one can use nonlinear Fuzzy Number Ranking Procedures. Then new models to solve the former problem would be obtained. This is the case for instance, when the index of Chang is assumed. Applying this ranking method to (8) it is easy of obtaining the corresponding particular auxiliary model to this case as

$$\begin{array}{l} \underset{s.t.}{Max}\ z = (\overline{c} - \underline{c})x(\overline{c} + \underline{c} + c)x \\ \qquad (\overline{a_i} - \underline{a_i})x(\overline{a_i} + \underline{a_i} + a_i)x \leq \\ \qquad \leq [(\overline{b_i} - \underline{b_i}) + (\overline{t_i} - \underline{t_i})(1-\alpha)][(\overline{b_i} + \underline{b_i} + b_i) + (\overline{t_i} + \underline{t_i} + t_i)(1-\alpha)] \\ \qquad x \geq 0,\ \alpha \in (0,1],\ i = 1,...,m. \end{array} \tag{19}$$

Evidently, this problem is nonlinear in both each of their constraints and the objective, which makes its solution a bit more complicated.

Also, may be possible to use fuzzy preference relations to deal with inequality constraints between fuzzy numbers. In [20], it is given a degree for which a normalized convex fuzzy number is greater than or equal to another fuzzy number. It is based on the compensation of areas determined by the membership functions. Using L-R fuzzy numbers it is proved that in this case, comparison of areas is reduced to the comparison of upper and lower bounds of $\alpha$-cuts. For trapezoidal fuzzy numbers are obtained inequalities which can be use to solve the problem (8).

## 5. Numerical Example

Let us consider the following problem

$$\begin{array}{l} \underset{s.t.}{Max}\ z = \underset{\sim}{3}\,x_1 + \underset{\sim}{1}\,x_2 \\ \qquad \underset{\sim}{2}\,x_1 - \underset{\sim}{1}\,x_2 \underset{\sim}{<} \underset{\sim}{4} \\ \qquad \underset{\sim}{5}\,x_1 + \underset{\sim}{2}\,x_2 \underset{\sim}{<} \underset{\sim}{15} \\ \qquad x_1, x_2 \geq 0 \end{array}$$

with margins $t_1 = \underset{\sim}{5}$ and $t_2 = \underset{\sim}{6}$ in each constraint respectively. Considering the fuzzy numbers defined by Homogeneous Linear Fuzzy Functions and using LRF, the associated parametric LP problem is

$$\begin{aligned} \underset{s.t.}{Max} \; & z = 3x_1 + x_2 \\ & 2x_1 - x_2 \leq 4 + 5(1-\alpha) \\ & 5x_1 + 2x_2 \leq 15 + 6(1-\alpha) \\ & x_1, x_2 \geq 0, \; \alpha \in (0,1] \end{aligned}$$

whose optimal solution is

$$x^* = \begin{cases} [4.2 - 1.2\alpha, 0] & \forall \alpha \in (0, 0.231] \\ [4.3333 - 1.7777\alpha, -0.3336 + 1.4443\alpha] & \forall \alpha \in (0.231, 1] \end{cases}$$

with the value of the objective function

$$z^* = \{12.6 - 3.6\alpha \; \forall \alpha \in (0, 0.231], \; 12.6669 - 3.8888\alpha \; \forall \alpha \in (0.231, 1]\}.$$

On the other hand, if fuzzy coefficients defined by triangular fuzzy numbers are supposed,

$$\begin{array}{lll} \underset{\sim}{c_1} = \underset{\sim}{3} = (1,3,5), & \underset{\sim}{c_2} = \underset{\sim}{1} = (0,1,2) & \\ \underset{\sim}{a_{11}} = \underset{\sim}{2} = (0,2,3.5), & \underset{\sim}{a_{12}} = \underset{\sim}{1} = (0,1,4), & \underset{\sim}{b_1} = \underset{\sim}{4} = (3,4,5) \\ \underset{\sim}{a_{21}} = \underset{\sim}{5} = (3,5,6), & \underset{\sim}{a_{22}} = \underset{\sim}{2} = (1.5,2,3), & \underset{\sim}{b_2} = \underset{\sim}{15} = (12,15,16) \\ \underset{\sim}{t_1} = \underset{\sim}{5} = (4,5,6), & \underset{\sim}{t_2} = \underset{\sim}{6} = (4,6,8) & \end{array}$$

then, by applying the fourth index of Yager, the following auxiliary problem is obtained

$$\begin{aligned} \underset{s.t.}{Max} \; & z = 3x_1 + x_2 \\ & 1.875x_1 - 1.500x_2 \leq 4 + 5(1-\alpha) \\ & 4.750x_1 + 2.125x_2 \leq 14.5 + 6(1-\alpha) \\ & x_1, x_2 \geq 0, \; \alpha \in (0,1] \end{aligned}$$

whose optimal solution is

$$x^* = \begin{cases} [4.3158 - 1.26318\alpha, 0] & \forall \alpha \in (0, 0.345] \\ [4.48943 - 1.76652\alpha, -0.38818 + 1.12517\alpha] & \forall \alpha \in (0.345, 1] \end{cases}$$

and the optimal value of the objective function is

$$z^* = \{12.9474 - 3.78954\alpha \; \forall \alpha \in (0, 0.345], \; 13.08011 - 4.17439\alpha \; \forall \alpha \in (0.345, 1]\}.$$

Now, applying the ranking function of Tsumura et al. it is obtained the following auxiliary problem:

$$\begin{aligned} \underset{s.t.}{Max} \; & z = 3x_1 + x_2 \\ & 1.833x_1 - 1.666x_2 \leq 4 + 5(1-\alpha) \\ & 4.666x_1 + 2.166x_2 \leq 14.333 + 6(1-\alpha) \\ & x_1, x_2 \geq 0, \; \alpha \in (0,1] \end{aligned}$$

whose optimal solution is

$$x^* = \begin{cases} [4.3577 - 1.0716\alpha, 0] & \forall\alpha \in (0, 0.383] \\ [4.5444 - 1.77346\alpha, -0.40221 + 1.05007\alpha] & \forall\alpha \in (0.383, 1] \end{cases}$$

and

$$z^* = \{13.0731 - 3.2148\alpha \ \forall\alpha \in (0, 0.383],\ 13.231 - 4.27031\alpha \ \forall\alpha \in (0.383, 1]\}.$$

## 6. Conclusion

In this paper a general FLP model involving simultaneously fuzzy objective, fuzzy constraints and fuzzy numbers in the set of constraints has been considered.

To solve this model a solution method using Fuzzy Numbers Ranking Procedures has been presented. If the fuzzy numbers are generated by an Homogeneous Linear Fuzzy Functions, then using a Linear Ranking Function it is obtained in a easy way an auxiliary parametric LP problem, Theorem 4.1. In general using some Linear Ranking Function, an auxiliary parametric LP problem is always obtained.

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