A classification method of alternatives for multiple preference ordering criteria based on fuzzy majority

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Abstract

The purpose of this paper is to present a classification method of alternatives for multiple preference ordering criteria based on the concept of fuzzy majority. The fuzzy majority is represented by a fuzzy quantifier, and applied in the aggregation by means of an OWA operator whose weights are calculated by the fuzzy quantifier.

For every preference ordering criterion we derive a preference ordering relation and from the set of preference ordering relations we derive a collective preference ordering relation, distinguishing between the two experts' opinions intensity possibilities, i.e. homogeneous group and heterogeneous group. We present two different choice degrees of alternatives acting over the collective preference ordering relation, a *dominance degree* and a *non-dominance degree* that generalizes Orlovski's non-dominated alternative concept. The application of the two alternative choice degrees is carried out according to two different selection processes, a sequential choice process and a conjunction choice process.

Keywords: Multiple criteria, preferece ordering, fuzzy majority, fuzzy quantifers.

1 Introduction

Decision making when more than one evaluation scheme exists has become a major concern of research in decision theory. The literature on multi-criteria decision making has grown tremendously in recent years [16].

There exist many opportunities to apply the theory of fuzzy sets in decision making. Fuzzy theories and methodologies can be used either to translate imprecise and vague information in the problem specification into fuzzy relationships (fuzzy objectives, fuzzy constraints, fuzzy preferences, ...) or to use fuzzy tools for designing a decision process trying to establish preference orderings of alternatives [11, 26, 5, 20].

The aim of this paper is to present a classification method of alternatives for multiple preference ordering criteria based on the concept of fuzzy majority for the aggregation and the exploitation in the decision process. The problem specifications are shown in the following.

It is assumed that there exists a finite set of alternatives $X = \{x_1, ..., x_n\}$ as well as a finite set of experts $E = \{e_1, ..., e_m\}$. Each expert $e_k \in E$ provides his opinion on X as an individual preference ordering $\{x_{o(1)}, ..., x_{o(n)}\}$, where $o(\cdot)$ is a permutation function over the index set $\{1, ..., n\}$, that is, every criterion classifies the alternatives according to a preference ordering from the best alternative to the worst alternative. Therefore, we have a multiple preference ordering criteria according to the expert preference orderings.

Sometimes, associated to the experts it is possible to consider their respective *importance degrees* as a fuzzy subset, such that, $\mu_E(k) \in [0, 1]$ denotes the importance degree of the expert e_k. When the experts' opinions are considered with the same intensity, it is called *homogeneous multiple preference ordering criteria*, othercase, it is called *heterogeneous multiple preference ordering criteria*.

The classification method of alternatives that we propose is developed according to the following four steps.

1. For every preference ordering we derive a preference ordering relation.

2. Using the concept of fuzzy majority represented by a fuzzy linguistic quantifier and applied in the aggregation operations by means of an ordered weighted averaging (*OWA*) operator [22], a collective preference ordering relation is obtained from the preference ordering relation set.

3. Using again the concept of fuzzy majority, two choice degrees of alternatives are defined: the *quantifier guided dominance degree* and the *quantifier guided nondominance degree*. The latter generalizes Orlovski's non-dominated alternative concept

[19]. These choice degrees will act over the collective preference ordering relation supplying a selection set of alternatives.

4. The application of the above choice degrees of alternatives over the collective preference ordering relation may be carried out according to different selection processes. We will present two selection processes that will be called *sequential selection process* and *conjunction selection process*.

On the other hand, we consider the two experts' opinions intensity possibilities, homogeneous groups and heterogeneous groups, and according to them we present the classification method of alternatives. The classification method of heterogeneous multiple preference ordering criteria will be a generalization of the other. We must first look at the homogeneous experts' opinions intensity case and then consider the problem in the heterogeneous expert's opinions intensity environment.

We note that in [15, 2, 21, 17, 18] this problem was also studied.

- The two initial proposals, [15, 2], present $Max - Min$ and $Min - Max$ criteria from a classification matrix obtained by counting the pairwise comparisons between the alternatives.

- In the two following [21, 18], the authors present a simple majority rule over the pairwise preference ordering between the pairs of alternatives obtaining a collective preference ordering matrix. Then ensuring fuzzy transitivity for revising the original relation matrix. Finally, they apply an algorithm to derive a nonfuzzy preference ordering, decomposing the fuzzy set of preference ordering into a union of the α-level sets based on the obtained transitive relation matrix.

- In [17], the authors assume that a collective fuzzy tournament matrix is obtained through pairwise comparisons between the alternatives, and obtain a selection set of alternatives applying either a strong covering relation or a weak fuzzy covering relation over the collective fuzzy tournament matrix.

These approaches present mathematical methods for obtaining the collective ordering matrix based on counting processes and independent from expert's criteria of aggregation about a quantity or fuzzy majority of experts sufficient for accepting a classification. It is important to state that we will use the fuzzy majority concept as a base of our aggregation and exploitation processes, managing the information under a fuzzy majority represented by the fuzzy quantifiers, aggregating the information by means of an OWA operator [22], whose weights are calculated by the fuzzy quantifiers. The difficulty of the

collective-choice decision problem is well known, the concept of democratic decision in a group leads us to impossibility theorems [1]. However, the application of the notion of fuzzy sets in the idea of the fuzzy majority concept is very appropriate in a multiple criteria decision making process. The fuzzy majority can provides a framework with more human-consistency to the aggregation and the choice process in an imprecise environment.

We also note that the *OWA* operator and the fuzzy quantifiers have already been also used in the context of decision making [3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 22, 23].

In the following, section 2 presents the fuzzy majority represented by the fuzzy quantifiers, using fuzzy quantifiers together with the *OWA* operators for aggregation. Section 3 presents the decision process, the classification method of alternatives for multiple preference ordering criteria. Then, and for the sake of illustrating the classification method, section 4 is devoted to the development of an example. At the end, in section 5 some conclusions are pointed out.

2   Fuzzy Majority

Traditionally, the majority is defined as a threshold number of individuals. Fuzzy majority is a soft majority concept, which is manipulated via a fuzzy logic based calculus of linguistically quantified propositions [10].

In this section we present the fuzzy quantifiers, used for representing the fuzzy majority concept, and the *OWA* operators, used for aggregating information. The *OWA* operator reflects the fuzzy majority calculating its weights by means of the fuzzy quantifiers.

2.1   Fuzzy linguistic quantifiers

Quantifiers can be used to represent the amount of items satisfying a given predicate. Classic logic is restricted to the use of the two quantifiers, *there exists* and *for all*, that are closely related respectively to the *or* and *and* connectives. Human discourse is much richer and more diverse in its quantifiers, e.g. *about 5, almost all, a few, many, most, as many as possible, nearly half, at least half.* In an attempt to bridge the gap between formal systems and natural discourse and, in turn, to provide a more flexible knowledge representation tool, Zadeh introduced the concept of linguistic quantifiers [27].

Zadeh suggested that the semantic of a linguistic quantifier can be captured by using fuzzy subsets for its representation. He distinguished between two types of linguistic quantifiers, *absolute* and *proportional or relative.* Absolute quantifiers are used to represent amounts that are absolute in nature such as *about 2* or *more than 5.* These absolute linguistic quantifiers are closely related to the concept of the count or number of elements.

He defined these quantifiers as fuzzy subsets of the non-negative real numbers, $\mathcal{R}^+$. In this approach, an absolute quantifier can be represented by a fuzzy subset Q, such that for any $r \in \mathcal{R}^+$ the membership degree of r in Q, Q(r), indicates the degree to which the amount r is compatible with the quantifier represented by Q. Proportional quantifiers, such as *most, at least half*, can be represented by fuzzy subsets of the unit interval, [0,1]. For any $r \in [0,1]$, Q(r) indicates the degree to which the proportion r is compatible with the meaning of the quantifier it represents. Any quantifier of natural language can be represented as a proportional quantifier or given the cardinality of the elements considered, as an absolute quantifier. Functionally, linguistic quantifiers are usually of one of three types, *increasing, decreasing,* and *unimodal.* An increasing type quantifier is characterized by the relationship

$$Q(r_1) \geq Q(r_2) \quad \text{if } r_1 > r_2.$$

These quantifiers are charecterized by values such as *most, at least half.* A decreasing type quantifier is characterized by the relationship

$$Q(r_1) \leq Q(r_2) \quad \text{if } r_1 < r_2.$$

The quantifiers characterize terms such as *a few, at most* α. Unimodal type quantifiers have the property that

$$Q(a) \leq Q(b) \leq Q(c) = 1 \geq Q(d)$$

for some $a \leq b \leq c \leq d$. These are useful for representing terms like *about q*.

An absolute quantifier $Q : R^+ \rightarrow [0,1]$ satisfies:

$$Q(0) = 0, \text{and } \exists k \text{ such that } Q(k) = 1.$$

A relative quantifier, $Q : [0,1] \rightarrow [0,1]$, satisfies:

$$Q(0) = 0, \text{ and } \exists r \in [0,1] \text{ such that } Q(r) = 1.$$

A non-decreasing quantifier satisfies:

$$\forall a, b \text{ if } a > b \text{ then } Q(a) \geq Q(b).$$

The membership function of a non-decreasing relative quantifier can be represented as

$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ \frac{r-a}{b-a} & \text{if } a \leq r \leq b \\ 1 & \text{if } r > b \end{cases}$$

with $a, b, r \in [0, 1]$.

Some examples of proportional quantifiers are shown in figure 1, where the parameters, (a, b) are $(0.3, 0.8)$, $(0, 0.5)$ and $(0.5, 1)$, respectively.
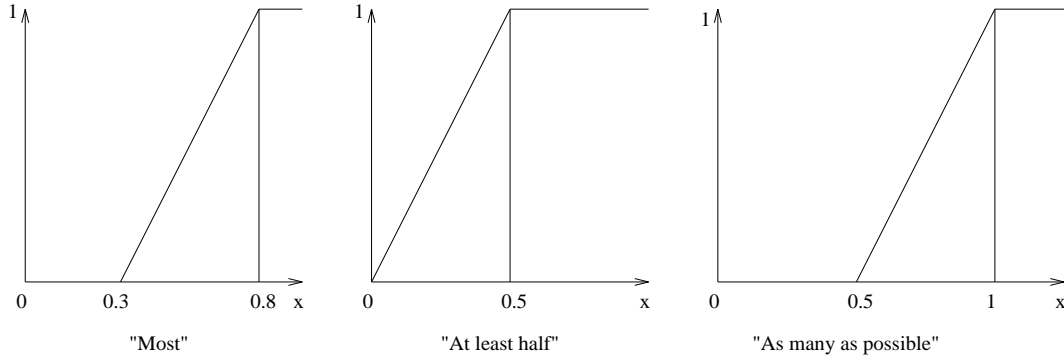

Fig. 1. Proportional fuzzy linguistic quantifiers

In [25] a formalism is described for evaluating the truth of linguistically quantified propositions based upon a logical interpretation that uses a generalization of the *and* and *or* operations via *OWA* operators. In this way the *OWA* operators reflect the fuzzy majority calculating their weights by means of the fuzzy quantifiers.

2.2 The ordered weighted averaging operator

The *OWA* operators were proposed by Yager in [22] and more recently characterized in [24] and provide a family of aggregation operators which have the *and* operator at one extreme and the *or* operator at the other extreme.

An *OWA* operator of dimension n is a function ϕ,

$$\phi : [0, 1]^n \to [0, 1],$$

that has associated with a set of weights. Let $\{a_1, \ldots, a_m\}$ be a list of values to aggregate, then the *OWA* operator ϕ is defined as

$$\phi(a_1, \ldots, a_m) = W \cdot B^T = \Sigma_{i=1}^m w_i \cdot b_i$$

where $W = [w_1, \ldots, w_m]$, is a weighting vector, such that, $w_i \in [0, 1]$ and $\Sigma_i w_i = 1$; and B is the associated ordered value vector. Each element $b_i \in B$ is the i-th largest value in the collection $a_1, \ldots, a_m$.

The *OWA* operators fill the gap between the operators *Min* and *Max*. It can be immediately verified that *OWA* operators are commutative, increasing monotonous and idempotent, but in general not associative.

A natural question in the definition of the OWA operator is how to obtain the associated weighting vector. In [22, 24], Yager proposed two ways to obtain it. The first approach is to use some kind of learning mechanism using some sample data; and the second approach is to try to give some semantics or meaning to the weights. The final possibility has allowed multiple applications on areas of fuzzy and multi-valued logics, evidence theory, design of fuzzy controllers, and the quantifier guided aggregations.

We are interested in the area of quantifier guided aggregations. Our idea is to calculate weights for the aggregation operations (made by means of the OWA operator) using linguistic quantifiers that represent the concept of *fuzzy majority*. In [22, 24], Yager suggested an interesting way to compute the weights of the OWA aggregation operator using linguistic quantifiers, which, in the case of a non-decreasing proportional quantifier Q, it is given by the expression:

$$w_i = Q(i/n) - Q((i-1)/n), i = 1, \ldots, n.$$

When a fuzzy linguistic quantifier Q is used to compute the weights of the OWA operator ϕ, it is symbolized by ϕ_Q.

3 Decision Process: Classification Method of Alternatives

As we have said before, we have a set of experts, E, and each expert $e_k \in E$ provides his opinions on X as an individual preference ordering $O^k = \{x_{o(1)}, \ldots, x_{o(n)}\}$, where $o(\cdot)$ is a permutation function over the index set $\{1, \ldots, n\}$. Every expert classifies the alternatives according to a weak order from the best to the worst alternative.

In the following we present the aggregation process where the collective preference ordering relation is obtained, and then we develop the choice process.

3.1 Aggregation: The collective preference ordering relation

For every preference ordering we derive a preference ordering relation, P^k, where p_{ij}^k reflects the pairwise preference ordering between the alternaties x_i and x_j for the expert e_k, $p_{ij}^k \in \{0, 1\}$. It takes the value 1 if x_i is prefered and 0 in other case. Therefore, we have a set of individual binary preference relations:

$$\{P^1, \ldots, P^m\}.$$

From the set of preference ordering relations we will derive the collective preference ordering relation, P, distinguishing between the two experts' opinion intensity possibilities, homogeneous and heterogeneous. Each value, $p_{ij} \in [0, 1]$, represents the degree to which the crisp weak preference "alternative x_i is at least as good as alternative x_j" is true.

3.1.1 Homogeneous multiple preference ordering criteria

In this case we suppose that all experts' opinions are considered with the same intensity. Then we aggregate the preference ordering relations to obtain p_{ij} from $\{p_{ij}^1, ..., p_{ij}^m\}$ for all i, j. We do that using the concept of *fuzzy majority*. Fuzzy linguistic quantifiers have provided tools to formally deal with fuzzy majority and can be used to define a weight vector for an OWA operator. We use the OWA operator to obtain the collective preference relation P as

$$P = \phi_Q(P^1, ..., P^m)$$

where $p_{ij} = \phi_Q(p_{ij}^1, ..., p_{ij}^m)$ and the weight vector, W, represents the fuzzy majority over the individuals.

3.1.2 Heterogeneous multiple preference ordering criteria

In this case, associated to the experts we have their respective *importance degrees* as a fuzzy subset, such that, $\mu_E(k) \in [0, 1]$ denotes the importance degree of the expert e_k.

Assuming that in our context each value $\mu_E(k)$ is a weight indicating its importance in the aggregation process, the general procedure for the inclusion of importance in the aggregation involves the transformation of the preference values under the importance degrees. This transformation follows the following expression:

$$\overline{p}_{ij}^k = g(p_{ij}^k, \mu_E(k)).$$

The Min aggregation operator plays a central role in fuzzy set theory when it is used for the default implementation of the fuzzy set intersection. We use it as an aggregation operator for the pairs $(p_{ij}^k, \mu_E(k))$,

$$\overline{p}_{ij}^k = Min\{p_{ij}^k, \mu_E(k)\}.$$

When experts are equally relevant, then $\overline{p}_{ij}^k$ is reduced to p_{ij}^k. We note that a class of functions that can be used for this transformation instead of the Min operator is

the general class of the t-norm operators. In this context their properties, $T(1, x) = x, T(0, x) = 0$, make all of them to have behaving the same way.

The collective preference relation P is obtained as

$$p_{ij} = \phi_Q(\overline{p}_{ij}^1, ..., \overline{p}_{ij}^m).$$

3.2 Exploitation: Choice process

We present two choice degrees of alternative acting over the collective preference ordering relation, a dominance degree and a non-dominance degree. The application of these two choice degrees is carried out according to two different selection processes that we will present, a *sequential selection process* and a *conjunction selection process*.

3.2.1 Choice degrees of alternatives

As we said earlier, we present two choice degrees based on the concept of fuzzy majority: a dominance degree based on the use of the OWA operators whose weights are calculated by means of the quantifier that represents the fuzzy majority, and a non-dominance degree, also based on the use of the OWA operators, that generalizes the Orlovski's non-dominated alternative concept [19]. They are described in the following paragraphs.

- **Quantifier guided dominance degree**

  We define the quantifier guided dominance degree, $QGDD(\cdot)$, used to quantify the dominance that one alternative has over all the others in a fuzzy majority sense. It acts on the set of alternatives as:

  $$QGDD(x_i) = \phi_Q(p_{ij}, j = 1, ..., n, j \neq i)$$

  where ϕ_Q is an OWA operator whose weights are defined using a relative quantifier Q, and whose components are the elements of the corresponding row of P, that is, for x_i the set of $n - 1$ values $\{p_{ij} | j = 1, ..., n \text{ and } i \neq j\}$.

  The elements of the set

  $$X^{QGDD} = \{x | x \in X, QGDD(x) = sup_{z \in X} QGDD(z)\}$$

  are called maximun dominance elements of the fuzzy majority of X quantified by Q.

- **Quantifier guided non-dominance degree**

  We define the quantifier guided non-dominance degree, $QGNDD(\cdot)$, also acting on the alternatives as a generalization of Orlovski's non-dominated alternative concept [19].

  In this context, the membership function $QGNDD(\cdot)$ gives the degree to which each alternative is not dominated by a fuzzy majority of the remaining alternatives. It is defined in the following expression:

  $$QGNDD(x_i) = \phi_Q(1 - p_{ji}^s, j = 1, ..., n, j \neq i)$$

  where

  $$p_{ji}^s = max\{p_{ji} - p_{ij}, 0\}$$

  represents the degree to which x_i is strictly dominated by x_j.

  We must denote that when the fuzzy quantifier represents the statement "*all*" whose algebraic aggregation representation corresponds to the conjunction operator, Min, then this dominance degree coincides with Orlovski's non-dominated alternative concept.

  It can be seen that if $QGNDD(x_i) = \alpha$ then the alternative x_i is dominated by a fuzzy majority of elements to a degree not higher than α.

  The elements of the set

  $$X^{QGNDD} = \{x | x \in X, QGNDD(x) = sup_{z \in X} QGNDD(z)\}$$

  are called maximal nondominated elements by the fuzzy majority of X quantified by Q.

3.2.2 Selection processes

We present two selection processes that will be based on the following ideas:

- Either selecting one of the two choice degrees according to the preference of the experts and applying it to obtain a selection set of alternatives. If there are more than one alternative in the selection set then the second choice degree may be applied selecting the alternative of the above set with best second choice degree. This process will be called *sequential selection process*.

- Or applying the two choice degrees over X and then obtaining the selection set of alternatives as the intersection of the two sets X^{QGDD} and X^{QGNDD}. This process will be called *conjunction selection process.*

The two above selection processes have the following mathematical representation:

- **Sequential selection process**

  The sequential process consists of applying the choice degrees, each one of them in sequence, according to a previously established order. There is no criterion to establish an order, therefore we can define two sequential processes according to a criterion either dominance or nondominance.

  - *Dominance based sequential selection process QG-DD-NDD*

    To aply the quantifier guided dominance degree over X, and obtain X^{QGDD}. If $\#(X^{QGDD}) = 1$ then End, and this is the solution set. Otherwise continue obtaining

    $$X^{QG-DD-NDD} = \{x | x \in X^{QGDD}, QGNDD(x) = sup_{z \in X^{QGDD}} QGNDD(z)\}.$$

    This is the selection set of alternatives.

  - *Nondominance based sequential selection process QG-NDD-DD*

    To apply the quantifier guided nondominance degree over X, and obtain X^{QGNDD}. If $\#(X^{QGNDD}) = 1$ then End, and this is the solution set. Otherwise continue obtaining

    $$X^{QG-NDD-DD} = \{x | x \in X^{QGNDD}, QGDD(x) = sup_{z \in X^{QGNDD}} QGDD(z)\}.$$

    This is the selection set of alternatives.

- **Conjunction selection process**

  The conjunction process consists of applying the two quantifier guided choice degrees obtaining the two partial selection sets of alternatives X^{QGDD} and X^{QGNDD}. The final selection set is the intersection of the two above sets,

  $$X^{QGCP} = X^{QGDD} \cap X^{QGNDD}.$$

We must note that the second selection process is more restrictive than the above sequential selection processes because in the second it is possible to obtain an empty selection set of alternatives. Therefore a complete process, called **selective selection process**, can be applied in two steps:

- first, to apply the *conjunction choice process*,

- second, if $X^{QGCP} \neq \emptyset$ then this is the selection set and End, otherwise continue applying one of the two *sequential choice processes*, according to a criterion either dominance or nondominance.

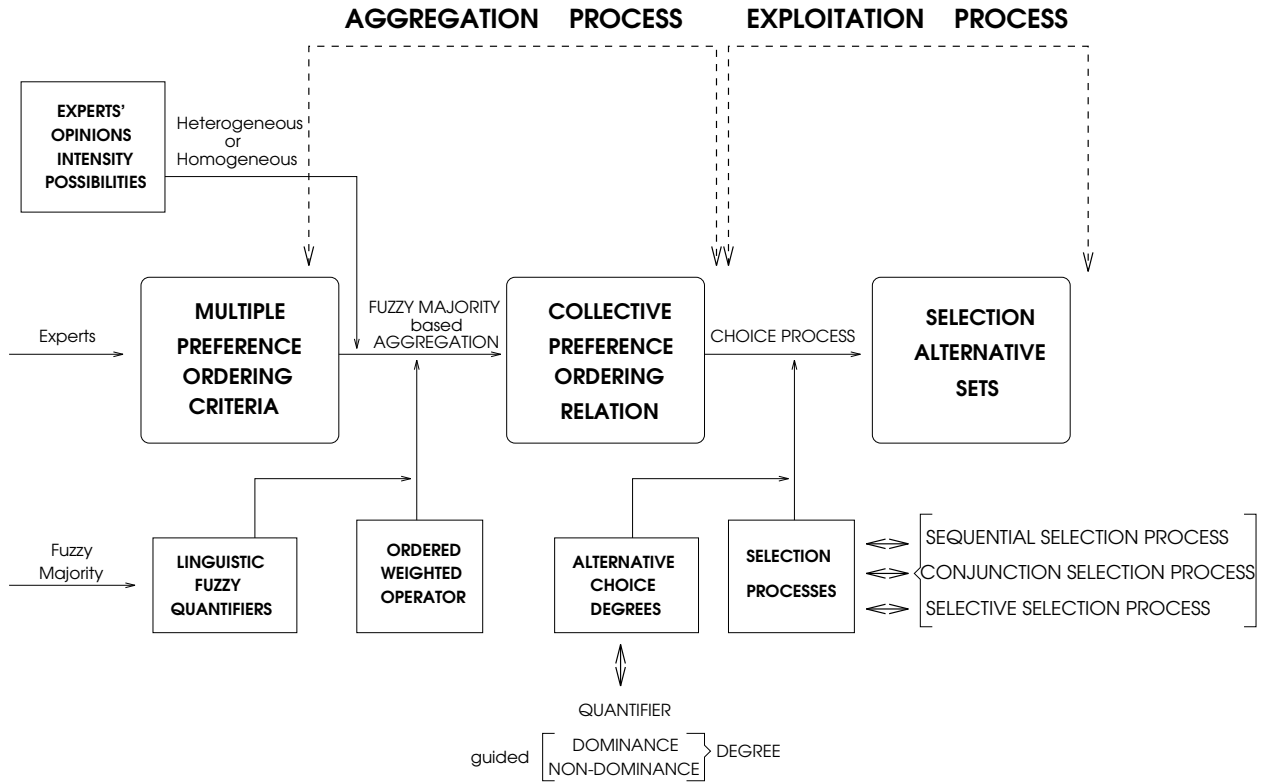Graphically, the classification method of alternatives can be seen in figure 2.

Fig. 2. Decision process: Classification method of alternatives

4 Example

Consider the following example presented in [21]. Suppose that $m = 20$, $X = \{x_1, x_2, x_3, x_4\}$, being an homogeneous multiple preference ordering criteria problem, and a score sheet for the alternative quaternary assessment is obtanied as follows:

$$O^1 = (x_1, x_2, x_3, x_4), \quad N(O^1) = 4$$
$$O^2 = (x_1, x_2, x_4, x_3), \quad N(O^2) = 2$$
$$O^3 = (x_2, x_1, x_3, x_4), \quad N(O^3) = 2$$
$$O^4 = (x_2, x_1, x_4, x_3), \quad N(O^4) = 1$$
$$O^5 = (x_3, x_1, x_2, x_4), \quad N(O^5) = 2$$
$$O^6 = (x_3, x_1, x_4, x_2), \quad N(O^6) = 1$$
$$O^7 = (x_1, x_3, x_2, x_4), \quad N(O^7) = 3$$
$$O^8 = (x_4, x_1, x_2, x_3), \quad N(O^8) = 2$$
$$O^9 = (x_4, x_1, x_3, x_2), \quad N(O^9) = 1$$
$$O^{10} = (x_2, x_4, x_3, x_1), \quad N(O^{10}) = 2$$

where $N(\cdot)$ indicates the number of experts according to the corresponding preference ordering criterion.

From the preference ordering criteria we derive their respective $4x4$ preference ordering relations. Using the fuzzy majority criterion with the linguistic quantifier "*As many as possible*" with the pair $(0.0, 0.5)$ and the corresponding OWA operator with the weight vector W as $w_i = 0, i = 1, \ldots, 10$ and $w_i = \frac{1}{10}, i = 11, \ldots, 20$, the collective preference ordering relation is

$$P = \begin{bmatrix} - & 0.5 & 0.5 & 0.5 \\ 0 & - & 0 & 0.3 \\ 0 & 0 & - & 0.2 \\ 0 & 0 & 0 & - \end{bmatrix}$$

We apply the choice process with the fuzzy quantifier "*most*" with the pair $(0.3, 0.8)$ and the corresponding OWA operator with the weight vector $W = (0.06, 0.672, 0.268)$. The choice degrees acting over the collective preference supply the following values:

	x_1	x_2	x_3	x_4
$QGDD$	0.5	0.018	0.012	0
$QGNDD$	1	0.812	0.812	0.652.

These values represent the dominance degree that one alternative has over "*most*" alternatives according to "*as many as possible*" experts, and the nondominance degree to which the alternative is not dominated by "*most*" alternatives according to "*as many as possible*" experts, respectively.

Clearly the maximal sets are:

$$X^{QGDD} = \{x_1\} \text{ and } X^{QGNDD} = \{x_1\},$$

therefore the selection set for all selection processes is $\{x_1\}$.

5   Conclusions

In this paper, we have presented a classification method of alternatives for multiple preference ordering criteria based on the concept of fuzzy majority for the aggregation and the exploitation of information in the decision process. The concept of fuzzy majority is represented by a fuzzy quantifier, and the aggregation of information is made by means of an OWA operator whose weights are calculated by the fuzzy quantifier.

We have presented two quantifier guided choice degrees, a dominance degree used to quantify the dominance that one alternative has over all the others in a fuzzy majority sense and a non-dominance that generalizes Orlovski's non-dominated alternative concept. The application of the above choice degrees over the collective preference ordering relation has been carried out according to the different selection processes.

Finally to point out that, as has been mentioned before, the fuzzy majority concept can provides a framework with more human-consistency to the rational aggregation and the choice process in a multi-person problem where there are not consensus in the criteria.

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