A Proposal of Visualization Method for Obtaining Interpretable Fuzzy Rules

Kosuke Yamamoto
Mie University
1515 Kamihama-cho, Tsu
Mie, 514-8507, Japan
yamamoto@pa.info.mie-u.ac.jp

Takeshi Furuhashi
Mie University
1515 Kamihama-cho, Tsu
Mie, 514-8507, Japan
furuhashi@pa.info.mie-u.ac.jp

Tomohiro Yoshikawa
Mie University
1515 Kamihama-cho, Tsu
Mie, 514-8507, Japan
tom@ip.elec.mie-u.ac.jp

***Abstract*— Interpretability of fuzzy models has become one of the major topics in the field of fuzzy modeling. Visualization that makes input-output relationships interpretable is effective in extracting useful knowledge from unknown data. This paper presents visualization method that considers the visibility of fuzzy models. This method identifys clusters that have different statistical features, and projects the data to the "fusion axes", which are linear combinations of the multiple input variables, considering the distribution of each cluster in the projected space. This paper applies the proposed method to artificial data and also to collected data from the mobile robot, and shows that the proposed method can extract useful knowldge from the obtained visible and interpretable models.**

I. Introduction

Interpretability of fuzzy model has become one of the major topics in the field of fuzzy modeling [1]. Many methods that construct interpretable fuzzy models from numerical data have been proposed [2]-[5]. So far, interpretability that has been discussed is related to shape and degree of overlapping of membership functions, the number of input variables, and/or the number of linguistic terms. These parameters are considered to be essential for the quality of linguistic expression of fuzzy models.

For the interpretability, we have emphasized the importance of visualization of the model. A visible model can give human a clear image of input–output relationships, and it helps to interpret the data. This technique is often used in the field of data-mining [6]. However, there had been few methods that sought an interpretable fuzzy model from the viewpoint of visualization. We have proposed a fuzzy modeling in a visible space with "fusion axes" which are linear combinations of multiple input variables [7].

Projection of high-dimensional data can reduce its dimensionality and make it visible. Criteria for the projection are vital for the visible and interpretable model. In [7], we defined the criteria of projection as follows: (i) features of the system should be preserved, and (ii) each feature in the projected space should be separated as wide as possible from others. We employed Fuzzy C-Means (FCM) [8][9] for finding the features of data, and Fuzzy Multiple Discriminant Analysis (FMDA) for identifying the fusion axes. The FMDA is able to identify the fusion axes that maximize the distance among clusters (features of data). That facilitates allocation of membership functions on the fusion axes. In [7], we showed that the proposed method was able to construct a visible and interpretable fuzzy model with a clarified structure of Box and Jenkins' gas furnaces data [10]. This method, however, does not always identify fusion axes that satisfy the criteria of interpretability, (i) and (ii). This is because the FCM can construct only clusters with uniform hyper sphere distributions, and the FMDA cannot consider the distribution of each cluster in the projected space, and it is not aimed at obtaining visible models.

This paper presents a new method to construct a visible and interpretable fuzzy model that avoids the above limitations. We employ two clustering methods for extracting different statistical features of clusters. One is a method with EM algorithm [11] and the other is kernel fuzzy clustering [12]. The hyper sphere distribution of each cluster can vary with EM Algorithm. The kernel fuzzy clustering is a method to construct clusters in a higher-dimensional feature space using a kernel function. This paper introduces new criteria of interpretability considering the correlation of the clusters to the fusion axes in a projected space as well as the distance among the clusters, and proposes a fuzzy modeling method which can construct an interpretable fuzzy model in a visible space formed by the identified fusion axes.

Some studies employed the linear combinations of input variables in the antecedent parts [13]-[15]. Kim *et al.* [13] proposed a fuzzy modeling method using Principal Component Analysis (PCA) and Fuzzy C-Regression Models (FCRM) [14] which could identify partially linear structures from input–output data. However, the objective of [13] is to improve the accuracy of model, not to obtain an interpretable model. Yam *et al.* [15] proposed a method that simplify a fuzzy model using orthogonal transformation by Singular Value Decomposition (SVD). Although this method was effective for reducing complexity of model, the interpretation of fuzzy model was not discussed.

A lot of methods for dimensionality reduction discussed so far in the field of data-mining have been based on statistical linear transform or projection. The linear projection that leads to the maximum variance in the projected space is obtained through PCA. Independent component analysis (ICA) [16] is a statistical technique by which observed random data is linearly

0-7803-8353-2/04/$20.00 © 2004 IEEE

transformed into components that are maximally independent of each other. Since these methods were devised for feature extraction of data, they were not intended to identify the input–output relationships and to visualize the data structure considering the data distribution in the projected space.

Self Organizing Map (SOM) [17] is a nonlinear method for extracting hidden features of data in 2 dimensional plane. Runkler's fuzzy nonlinear projection [18] is a method to visualize arbitrarily shaped multiple nonlinear manifolds based on Sammon mapping. These nonlinear projection methods can discover topological structures contained in data, but interpretation of nonlinearly projected data is difficult. The visualization for enhancing the interpretability of fuzzy model is the unique feature of the proposed method in this paper.

This paper shows through experiments with artificial data that it is possible to construct adaptive clusters and identify appropriate fusion axes. This paper also applies the proposed method to collected data of a mobile robot as one of multi-inputs systems while it was passing an aisle, and shows that it is possible to construct a visible and interpretable fuzzy model from this data. We can interpret actions of the mobile robot easily with the identified fuzzy rules.

II. Proposed Method

Table I shows a list of denotations to be used in this paper. A system with D input – 1 output non-linear relationships is assumed.

TABLE I

NOMENCLATURE

i, i', i''	Indexes related to i-th cluster (rule)
j, j', j''	Indexes related to j-th input variable x_j or X_j
k, k', k''	Indexes related to k-th data
l	Index related to l-th fusion axis
t	Index related to times of training
$\boldsymbol{x}_k$	Input vector of k-th data $\boldsymbol{x}_k = (x_{k1}, x_{k2}, \cdots, x_{kD})^t$
y_k	Output of k-th data
y^i	Output from i-th rule (singleton)
$\boldsymbol{X}_k$	$\boldsymbol{X}_k = (x_{k1}, x_{k2}, \cdots, x_{kD}, y_k)^t$
ξ_l	l-th fusion axis
$\boldsymbol{e}_l$	Weight vector of l-th fusion axis
N_C	Number of clusters (rules)
D	Dimension of input vectors
N_D	Number of data
μ_k^i	Degree of belonging of k-th data to i-th cluster
$\boldsymbol{m}^i$	Center of i-th cluster
$\hat{y}_k$	Inference value of k-th data

The rule structure is given by

$$\text{if}(\xi_1 \text{ is } A_1^i \text{ and } \xi_2 \text{ is } A_2^i) \text{ then } y = y^i, \tag{1}$$

where $\xi_l (l = 1, 2)$ is the "fusion axis" expressed as

$$\xi_l = \boldsymbol{e}_l^t \boldsymbol{x} = e_{l1}x_1 + e_{l2}x_2 \cdots + e_{lD}x_D. \tag{2}$$

(1) is the i-th rule. In this paper, we call the 2 dimensional projection plane constructed by the fusion axes "fused plane".

The unique feature of this rule is that every rule uses the same membership functions defined on the fusion axes ξ_1 and ξ_2, and this is the point different from Kim *et al.*'s method that uses different principal component axes for each rule.

A fuzzy model is constructed on the fused plane formed by the two fusion axes. Since each fusion axis is defined as a linear equation as in (2), it is easy to know influential input variables from its coefficients. If we could express the meanings of these fusion axes with appropriate words reflecting the coefficients and meanings of each input variable, the model would have a higher interpretability. The fuzzy model in this paper uses singletons in the consequent parts for simplicity. Though these simple rules sacrifice the accuracy of the model, we use this type of rules for obtaining an interpretable model.

The proposed method identifies the data structure by clustering and then derives the fusion axes based on the obtained clusters. This method has two choices for each of the clustering and the derivation of fusion axes depending on the characters of data. These methodologies and the fuzzy modeling on the fused plane are described below.

A. Clustering

1) EM Algorithm Clustering: Clustering is carried out in $\Re^{D+1}$ space including the output space. Clustering by EM algorithm uses multidimensional normal distribution as the basis function [11]. This method can construct hyper-ellipsoidal fuzzy clusters like Gustafson Kessel clustering [19]. This paper calls this clustering method EM Clustering (EMC). The EMC is possible to construct the clusters adaptive to the data by incorporating covariance matrices into clusters.

We can employ the singletons, which are the most simplified consequent parts in a fuzzy model, by obtaining the clusters that have no correlations between the input and the output. Then covariances between the input and the output in the covariance matrix of clusters are always kept zero.

2) Kernel Fuzzy Clustering: Since the EMC including covariance matrix described in the previous subsection increases the member of parameters, it often faces degradation of convergence or compute-intensive difficulty, e.g. covariance matrix having a probability of becoming singular. For avoiding such a case, the proposed method employs Kernel Fuzzy C-Means (K-FCM) [12] that identifies clusters in a higher-dimensional space using a kernel function. The data distribution mapped to the higher-dimensional space is different from that in the original space. Although the FCM can construct only hyper sphere clusters, adaptive clusters can be constructed even with the FCM when a proper kernel function is used. This paper uses the polynomial kernel function expressed with

$$K(\boldsymbol{X}_k, \boldsymbol{X}_{k'}) = (1 + \boldsymbol{X}_k^t \boldsymbol{X}_{k'})^H, \tag{3}$$

where H is a constant value that determines the dimensionality in the mapped higher-dimensional space.

B. Identification of Fusion Axes

The fused plane made by the fusion axes are for the reduction of the dimensionality of input space, and for the visualization of the input-output relationships. Thus, the output data in the obtained clusters is not used for the identification of fusion axes. If the identified two fusion axes are orthogonal, they can rotate without changing the projected data structure. It means that the designer can adjust the visible fused plane by rotating the fusion axes, and he/she can determine the proper axes for dividing each clusters by some straight lines that are parallel to the axes. In order to obtain the orthogonal fusion axes, however, the degree of freedom for the distribution of clusters on the fused plane has to be restricted. Thus this paper employs non-orthogonal fusion axes for the projection plane. The non-orthogonal ones cannot be rotated, because the projected distribution of clusters on the fused plane are changed with the rotation. The proposed method has to identify the fusion axes by taking the distribution of clusters on the fused plane into consideration. If the distribution of clusters on the fused plane is parallel to the fusion axes, the clusters can be divided by straight lines that are parallel to the axes. So the proposed method focuses on the correlations of data in each cluster on the fused plane. The criterion of the correlation is given by

$$C_F = \frac{\sum_i^{N_C}\left\{\sum_k^{N_D} \mu_k^i e_1^t(\boldsymbol{x}_k - \boldsymbol{m}^i)\cdot e_2^t(\boldsymbol{x}_k - \boldsymbol{m}^i)\right\}^2}{\sum_i^{N_C}\sum_k^{N_D}\mu_k^i}. \quad (4)$$

The center of clusters $\boldsymbol{m}^i$ in (4) is given by

$$\boldsymbol{m}^i = \frac{\sum_k^{N_D}\mu_k^i \boldsymbol{x}_k}{\sum_k^{N_D}\mu_k^i}. \quad (5)$$

Since each cluster represents a unique feature of the data structure, the projection of each cluster onto the fused plane should be well separated. This paper defines the distance among the clusters on the fused plane. This paper introduces two definitions of this distance. The designer can choose either of them depending on the needs of modeling.

C. Variance among Clusters

The variance among clusters on the fused plane V_B is given by

$$V_B = \sum_i^{N_C}\sum_l^{2}\{e_l^t(\boldsymbol{m}^i - \boldsymbol{m}^{all})\}^2. \quad (6)$$

(6) is used as the criterion of distance among clusters for the proposed method. $\boldsymbol{m}^{all}$ in (6) is the mean vector of data.

From (4) and (6), the quantitative criterion of interpretability is given by

$$F_1 = C_F - V_B. \quad (7)$$

The fusion axes identified by minimizing (7) can create the fused plane in which the clusters are separated well and the distribution of clusters are parallel to the axes.

If the number of fusion axes in (4) and (6) is changed from two to the original dimensionality of input data D and we define unit vectors of fusion axes e_l, i.e. $e_l^t e_l = 1$, (7) becomes an eigen value problem. The solution of this problem, however, does not always yield projection to 2 dimensional space. Since one of the major objectives in this paper is visualization of data structure, projection to 2 dimensionalities becomes essential and must be kept.

The gradient descent is employed for identifying the fusion axes. The differentiation of (7) with respect to each coefficient of fusion axes $e_{l'j'}$ yields

$$\frac{\partial F_1}{\partial e_{l'j'}} = \frac{2\sum_i^{N_C}\left(\sum_k^{N_D}\mu_k^i e_1^t \boldsymbol{x'}_k^i \cdot e_2^t \boldsymbol{x'}_k^i\right)\left(\sum_k^{N_D}\mu_k^i x'^i_{kj'} e_{l''}^t \boldsymbol{x'}_k^i\right)}{\sum_i^{N_C}\sum_k^{N_D}\mu_k^i} - 2\sum_i^{N_C} e_{l'}^t \boldsymbol{m'}^i \cdot m'^i_{j'}, \quad (8)$$

where $\boldsymbol{x'}_k^i = \boldsymbol{x}_k - \boldsymbol{m}^i$ and $\boldsymbol{m'}^i = \boldsymbol{m}^i - \boldsymbol{m}^{all}$. The indexes l' and l'' in (8) are as follows: if $l' = 1$ then $l'' = 2$, and if $l' = 2$ then $l'' = 1$. The fusion axis is normalized at every iteration so that its norm is always kept to be unity.

D. Distance Considering Features of Clusters

It is sometimes easier to interpret the model when a distance between particular clusters is emphasized. This section defines weighted distance among clusters considering some particular features of clusters as follows:

$$D_B = \sum_{i=1}^{N_C}\sum_{i'=i+1}^{N_C}\left[\eta^{ii'}\sum_l^{2}\{e_l^t(\boldsymbol{m}^i - \boldsymbol{m}^{i'})\}^2\right]. \quad (9)$$

Here, $\eta^{ii'}$ is the weight. If the mean value of outputs of data in each cluster is used, $\eta^{ii'}$ is given by

$$\eta^{ii'} = \frac{\left(\sum_k^{N_D}\mu_k^i y^i - \sum_k^{N_D}\mu_k^{i'} y^{i'}\right)^2}{\sum_{i''}^{N_C}\sum_k^{N_D}\mu_k^{i''}}. \quad (10)$$

If the designer would have some knowledge of the system, he/she could also determine another $\eta^{ii'}$.

The criterion of interpretability on the fused plane is quantified by using (4) and (9) as

$$F_2 = C_F - D_B. \quad (11)$$

Proper fusion axes are identified by minimizing (11). The differentiation of (11) with respect to $e_{l'j'}$ yields

$$\frac{\partial F_2}{\partial e_{l'j'}} = \frac{2\sum_i^{N_C}\left(\sum_k^{N_D}\mu_k^i e_1^t \boldsymbol{x'}_k^i \cdot e_2^t \boldsymbol{x'}_k^i\right)\left(\sum_k^{N_D}\mu_k^i x'^i_{kj'} e_{l''}^t \boldsymbol{x'}_k^i\right)}{\sum_i^{N_C}\sum_k^{N_D}\mu_k^i} - 2\sum_{i=1}^{N_C}\sum_{i'=i+1}^{N_C}\eta^{ii'} e_{l'}^t(\boldsymbol{m}^i - \boldsymbol{m}^{i'})(m_{j'}^i - m_{j'}^{i'}). \quad (12)$$

Two criteria F_1 and F_2 have been proposed in this paper. F_1 does not have any parameters that the designer should determine a priori. The fused plane can be determined only by (7). F_2 is more flexible with weights $\eta^{ii'}$. If the output values of data in clusters are taken into consideration as in (10), the obtained model that describes the input-output relationships in the visible space will probably be more interpretable. If you add $\lambda e_1^t e_2$ to F_1 or F_2 by using Lagrange multiplier λ, the fusion axes will be orthogonalized.

E. Determination of Antecedent Parts

The membership functions are to be allocated on each fusion axis for fuzzy modeling. Since the designer can see the data distribution on the fused plane, subjective allocation of membership functions is not difficult. Thus, in the case of K-FCM, it is up to the designer to define the membership functions by considering the visible data distributions in the clusters. Since the basis function of EMC, on the other hand, is multidimensional normal distribution, bell shaped membership functions would match the data distributions in the clusters. Hence, in the case of EMC, the proposed method uses a bell-shaped membership function.

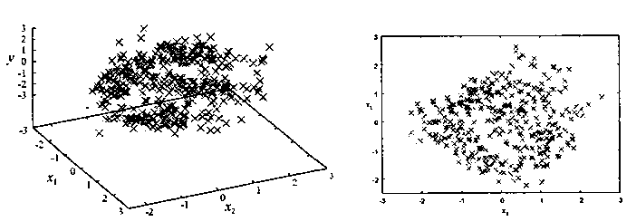
F. Determination of Consequent Parts

Since the main focus in this paper is interpretability of rules, the proposed method uses singletons in the consequent parts. The singleton of each rule is the mean of outputs of data belonging to each cluster. The proposed method identifies N_C fuzzy rules shown in (1) through these procedures. The obtained rules are interpolated using a simplified fuzzy inference.

III. Experiments & Discussions

A. Test with Artificial Data

1) Experiment: two inputs – one output test data shown in Fig. 1(a) were generated here for the examination of the proposed method. We used the test data that had only two inputs so that it is easy for us to visually understand the effectiveness of the proposed method. The test data had four ellipsoidal clusters as shown in Fig. 1(a). Figure 1(b) shows the projected test data in the input space. The data in the input space was not able to be divided properly by some lines that were parallel to the axes, x_1 and x_2. We applied the proposed method and the conventional method [7] to this data, and compared them.



(a) Distribution in input–output space

(b) Projected distribution in input space

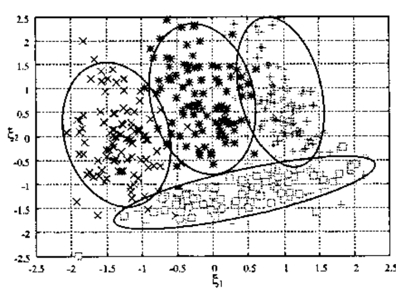
Fig. 1. Test data

2) Results: Figure 2(a)(b)(c) show the results of clustering by the FCM, the EMC and the K-FCM, respectively. (3) was used as the kernel function of the K-FCM, and H was set at three that was the number of dimensionalities of input-output variables. It is known from Fig. 2(a) that the clustering by the FCM was done by ignoring the data structure. The data was divided into four clusters with equal variance. The clustering by the EMC shown in Fig. 2(b), on the other hand, was constructed with ellipsoidal clusters with various variances. The clustering by the K-FCM shown in Fig. 2(c) yielded almost the same result as that by the EMC. Thus two clustering methods, the EMC and the K-FCM, used in the proposed method were able to construct clusters with adaptive variances. Next, we compared the fused plane obtained by the proposed method with that by the FMDA used in [7]. The clusters in this numerical experiment did not have any specific meanings. Thus, F_1 criterion in (7) was employed for identifying the fusion axes. Figure 3(a)(b) show the identified fused plane. The experiment was carried out by using the clusters shown in Fig. 2(b). (13) and (14) show the identified fusion axes by the FMDA and the proposed method, respectively.

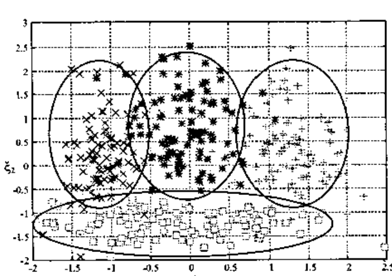
$$\textbf{FMDA:}\quad \begin{aligned} \xi_1 &= -0.88x_1 + 0.48x_2 \\ \xi_2 &= 0.49x_1 + 0.87x_2 \end{aligned} \tag{13}$$

$$\textbf{Proposed Method:}\quad \begin{aligned} \xi_1 &= 0.74x_1 - 0.67x_2 \\ \xi_2 &= 0.75x_1 + 0.66x_2 \end{aligned} \tag{14}$$

Each projected cluster by the FMDA shown in Fig. 3(a) was

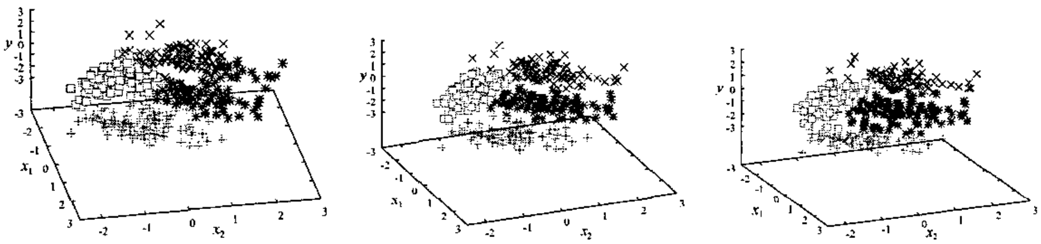


(a) FMDA



(b) Proposed method

Fig. 3. Results of projection to the fused plane



(a) FCM (b) EMC (c) K-FCM

Fig. 2. Results of clustering

not distributed in parallel to the fusion axes. On the other hand, the projected clusters by the proposed method shown in Fig. 3(b) were able to be divided with ease by some lines that were parallel to either of the fusion axes. Since each input variable did not have specific meanings in this experiment, we could not assign linguistic labels to the fusion axes. The coefficients of the identified fusion axes in (14) told that the data distribution in the original space was slanted to the original axes.

B. Modeling of Robot's Sensor Data

1) Collected Data: Figure 4 shows the sensing area of the robot. Morii *et al.* [20] constructed fuzzy rules of the mobile robot using human's assists. Since their rules based on human's assist contained few contradictions, we expected to extract their knowledge explicitly by using the proposed method.

Figure 5 shows the example of the track. The mobile robot was able to reach the exit without colliding with the walls. Then, we collected the data of five variables (x_1–x_5) obtained from the CCD cameras and rotation time (y) of the mobile robot in each step. Table II shows the meanings of each variable (x_1–x_5 and y). 47 samples were collected during the trials from the entrance to the exit.

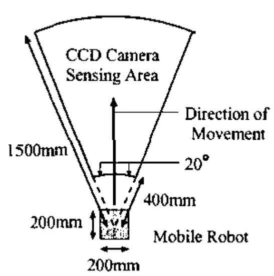


Fig. 4. Specification of the robot

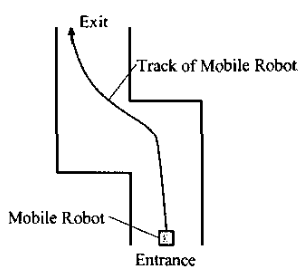


Fig. 5. Track of mobile robot

2) Experiment: We applied the proposed method to the data described above, and constructed fuzzy models of the mobile robot's actions. Since the number of data was small, it was difficult to apply the EMC which had a lot of parameters. So we constructed clusters by K-FCM of the proposed method. H of the kernel function in (3) was set to be equal to the number of input–output variables. The number of clusters was set at six. F_1 criterion was applied to the proposed method to identify the fusion axes.

TABLE II

MEANINGS OF THE INPUT AND OUTPUT VARIABLES

	Meaning of variable
x_1	Shortest distance to the wall
x_2	Angle from the front to the direction of the nearest wall
x_3	Distance to the wall in front
x_4	Distance to the wall at the right end of field of vision of the CCD camera
x_5	Distance to the wall at the left end of field of vision of the CCD camera
y	Time of rotation (right: positive value, left: negative value)

3) Results: Table III shows the result of clustering by K-FCM. For comparison, Table IV shows that by FCM. The 1st column is cluster number, the 2nd column is the symbol of data belonging to the cluster. These symbols are to be used later in Fig. 6. The 3rd–5th columns are the numbers of data belonging to one of the three actions, "Turn left", "Go ahead", "Turn right". The mobile robot's actions in the 3rd–5th columns were judged by the values of the outputs. The 6th column is the mean value of outputs in each cluster. The 7th column is the dominating action in each cluster. It is shown in Table III that the K-FCM clustering properly clustered the data by mobile robot's actions except for only one inconsistent data. The proposed method could incorporate the features of data represented with the robot's actions into the clusters. On the other hand, the result of clustering by FCM included many contradictory clusters as in TableIV, especially the data of "Turn right" was clustered with the data of "Go ahead" in the cluster No.5. The conventional clustering method was difficult to construct interpretable model from this data.

Figure 6 shows the identified fused plane with F_1 criterion. From Fig. 6, it is known that the projection methods with the new criterion F_1 make the visualized clusters parallel to the fusion axes. Grey scale regions of actions were drawn in Fig. 6. Table V shows the coefficients of fusion axes identified with F_2. The variables x_2, x_4, and x_5 were influential on the fusion axis ξ_1. Table II shows the meanings of these variables. x_2 is the angle to the nearest wall. It is positive when the nearest wall is on the right hand side, and vice versa. x_4 and x_5 are the normalized distances, and they become nagative if the wall is near. It is known from these meanings that "If the wall on the right hand side is near then ξ_1 becomes Positive Large", and "If it is opposite then ξ_1 becomes Negative Large". ξ_2 was

TABLE III

RESULT OF CLUSTERING BY K-FCM

No.	Symbol	Turn left	Go ahead	Turn right	Mean of output	Action
1	•	1	**27**	0	13.3	Go ahead
2	×	0	**3**	0	17.5	Go ahead
3	△	**2**	0	0	-100.0	Turn left
4	○	**5**	0	0	-110.0	Turn left
5	+	**6**	0	0	-133.3	Turn left
6	*	0	0	**3**	200.0	Turn right

TABLE IV

RESULT OF CLUSTERING BY FCM

No.	Symbol	Turn left	Go ahead	Turn right	Mean of output	Action
1	△	0	**9**	0	17.5	Go ahead
2	•	1	**7**	0	2.8	Go ahead
3	○	2	**5**	0	-16.1	Go ahead
4	*	2	**5**	0	-16.1	Go ahead
6	×	0	**4**	3	95.7	Go ahead
5	+	**9**	0	0	-127.8	Turn left

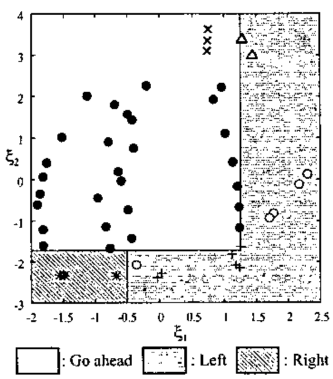


Fig. 6. Result of the projection using criterion F_2

dependent on the distance to the wall. The meanings of this axis could be "If the wall is near then ξ_2 becomes Negative Big", and "If it is far then ξ_2 becomes Positive Big". The fused

TABLE V

COEFFICIENTS OF FUSION AXES IDENTIFIED WITH CRITERION F_2

	x_1	x_2	x_3	x_4	x_5
ξ_1	-0.09	**0.82**	0.04	**-0.45**	**0.35**
ξ_2	**0.47**	0.28	**0.56**	**0.32**	**0.54**

plane shown in Fig. 6 could be described by the following four fuzzy rules:

if (*the nearest wall is on the right* **and** *it is not near*)
then *Go Ahead*
if (*the nearest wall is very near on the right*)
then *Turn Left*
if (*the nearest wall is on the right or fore* **and** *it is near*)
then *Turn Small Left*
if (*the nearest wall in on the left and* **it is near**)
then *Turn Right* (15)

These rules well explain the robot's actions in Fig. 5. It is also know from the visible clusters, that No. 1 cluster (•) and No. 2 cluster (×) can be merged. No. 3 (△) and No. 4 (○) can also merged.

IV. CONCLUSION

This paper discussed the interpretability of fuzzy modeling from viewpoint of visualization. We proposed fuzzy modeling in visible space with "fusion axes" which were the linear combinations of the multiple input variables. We employed two clustering methods for extracting the different statistical features of clusters and formulated new projection criteria for identifying fusion axes considering the data distribution in projected space. This paper applied the proposed method to the artificial data, and showed that this method could identify the proper fusion axes. This paper also applied the proposed method to the collected data of a mobile robot as one of the multi-inputs systems while it was passing an aisle. This paper showed the proposed method could construct the interpretable fuzzy model and extract the knowledge of mobile robot's actions from obtained fuzzy rules using linguistic expression.

REFERENCES

[1] J. Casillas, O. Cordón, F. Herrera and L. Magdalena, "Interpretability Issues in Fuzzy Modeling", Springer, 2003
[2] S. Horikawa, T. Furuhashi and Y. Uchikawa, "On Fuzzy Modeling Using Fuzzy Neural Networks with the Back-Propagation Algorithm", IEEE Trans. on Neural Networks, vol.3, No.5, pp.801-806, 1992
[3] M. Sugeno and T. Yasukawa, "A Fuzzy Logic based Approach to Qualitative Modeling", IEEE Trans. on Fuzzy System, Vol.1, No.1, pp.7-31, 1993
[4] D. Tikk, G. Biró, T. D. Gedeon, L. T. Kóczy and J. D. Yang, "Improvements and Critique on Sugeno's and Yasukawa's Qualitative Modeling", IEEE Trans. on Fuzzy Systems, Vol.10, No.5, pp.596-606, 2002
[5] M. Bikdash, "A highly interpretable form of Sugeno inference systems", IEEE Trans. on Fuzzy Systems, Vol.7 No. 6, pp.686-696, 1999
[6] A. K. Jain, R. Duin, and J. Mzo, "Statistical Pattern Recognition: A Review", IEEE Trans. on PAMI, Vol. 22, No.1, pp.4-37, 2000
[7] K. Yamamoto, T. Yoshikawa, T. Furuhashi, "A Proposal of Fuzzy Modeling on Fusion Axes Considering the Data Structure", FUZZ-IEEE 2003, pp.348-353, 2003
[8] J. C. Bezdek, "Pattern Recognition with Fuzzy Objective Function Algorithms", Plenum, New York, 1981
[9] S. Miyamoto and M. Mukaidono, "Fuzzy C-means as a Regularization and Maximum Entropy Approach", IFSA'97, Vol.2, pp.86-92, 1997
[10] G. E. P. Box and G. M. Jenkins, "Time Series Analysis", Forecasting and Control. San Francisco, CA, Holden Day, 1970
[11] S. Akaho, "EM Algorithm – Application to Clustering and Recent Development", Journal of Japan Society of Fuzzy Theory and Systems (In Japanese), Vol.12, No. 5, pp.594-602, 2000
[12] D. Suizu, S. Miyamoto, "Fuzzy c-Means Clustering using Mapping into High Dimensional Spaces", Proc. of the 18th Fuzzy System Symposium (In Japanese), pp.123-126, 2002
[13] E. Kim, M. Park, S. Kim and M. Park, "A Transformed Input-Domain Approach to Fuzzy Modeling", IEEE Trans. on Fuzzy Systems, Vol.6, No.4, pp.596-605, 1998
[14] R. J. Hathaway and J. C. Bezdek, "Switching Regression Models and Fuzzy Clustering", IEEE Trans. on Fuzzy Systems, Vol.1, No.3, 1993
[15] Y. Yam, P. Baranyi and C. T. Yang, "Reduction of Fuzzy Rule Base Via Singular Value Decomposition", IEEE Trans. of Fuzzy System. Vol.7, No.2, pp.120-132, 1999
[16] A. Hyvärinen and E. Oja, "Independent Compoment Analysis: A Tutorial", http://www.cis.hut.fi/aapo/papers/IJCNN99 _tutorialweb
[17] T. Kohonen, "Self Organizing Map", Heidelberg, Springer, 1995
[18] T. A. Runkler, "Fuzzy Nonlinear Projection", FUZZ-IEEE 2003, pp. 863-868, 2003
[19] D. E. Gustafson, W. C. Kessel, "Fuzzy Clustering with a Fuzzy Covariance Matrix", Proc. IEEE CDC, pp.761-766, 1979
[20] Y. Morii, T. Furuhashi, K. Morikawa, H. Itani, "A Study on Skill Acquisition from Teaching Information by Mobile Robot", Proc. of IEEE Int. Symposium on Computational Intelligence in Robotics and Automation (CIRA2003), pp.753-757, 2003