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ORIGINAL PAPER

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A novel approach for combining fuzzy rules using mean operators for effective rule reduction

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Abstract This paper will present a novel method based on harmonic mean, geometric mean, arithmetic mean and root mean square to help reduce fuzzy rules. The objective of the new method proposed is to produce fuzzy models with both a small number of interpretable rules and sufficiently high precision. Comparisons will be made between systems utilizing reduced rules and original rules to verify efficacy of the new methods in terms of the defuzzified outputs. As a practical example of a nonlinear system, an inverted pendulum will be controlled by a minimal set of rules to illustrate the performance and applicability of the proposed method.

Keywords Fuzzy Membership function · Mean operators · Centroid defuzzification · Fuzzy rules

1 Introduction

In the last decades, fuzzy rule bases have been used to build models for control applications, function approximation, etc. [1–5]. Formally, a fuzzy model represents the relationships among components of the underlying system. These relationships are characterized by a set of if–then rules in a rule-based model. For developing a fuzzy logic controller first the problem must be identified. Then the membership functions, input/outputs of the system, inference method, defuzzification method and fuzzy rules of the system must be defined. The rules of the system are if–then statements that are defined according to the goals of the system. The rules are the key as they decide how the system would behave for different inputs. The fuzzy controller shown in Fig. 1 has three parts i.e., fuzzifier, fuzzy rule set and defuzzifier. In the first stage, the input to the controller is converted into fuzzy variables using membership functions. These fuzzy variables are then passed on to the fuzzy rule set which evaluates fuzzy rules and generates output fuzzy variables to be passed on to the defuzzifier. The defuzzifier, on the bases of output fuzzy variables, generates a crisp output for the system.

Validation of rule bases ensures correctness and robustness [6] by detection of anomalies. Anomalies in a rule base or a knowledge base can be detrimental in a rule-based system's performance. Anomalies like conflict of rules, existence of redundant rules, inconsistencies of rules, and a few more often occur in rule bases. Rules can be determined heuristically from domain experts or from the training data to build fuzzy models. Linguistic terms common to the problem domain are used by experts to describe the state of the system and the proposed action. A rule base [1] produced in this manner generally consists of a small number of rules with each rule covering a large number of situations. When learning, algorithms are used to generate a rule base from a set of known instances, fuzzy sets and the rules do not necessarily have a semantic interpretation. The fuzzy sets are often selected based on the distribution of the training data. Rule-learning algorithms have ability to generate precise rules with a limited range of applicability when sufficient training data is available [1].

It is possible to design more rules than those required by the actual process but still cannot stimulate the entire control process. As a result, it will decrease the effectiveness of control process due to the time required for firing all the rules and incomplete knowledge of the rules. Regardless of the manner in which a rule base is obtained, it is possible for rules whose regions of applicability are adjacent in the input domain to specify similar responses. When this occurs, these rules may be merged to produce a single rule. The possibility of the occurence of similar adjacent rules is accentuated when there is a large number of rules, as is frequently the case when a large number of rules are generated from training data. It is desired to use the most effective rule set instead of imple-

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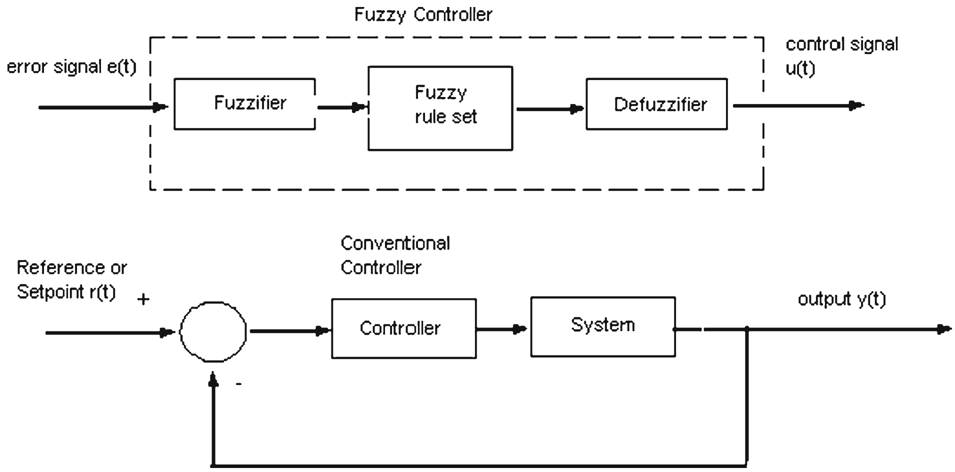

Fig. 1 To replace a conventional controller with fuzzy controller in a closed-loop system

menting all possible rules, and provide a set of compensation for incomplete knowledge.

Harmonic mean (HM) $\leq$ geometric mean (GM) $\leq$ arithmetic mean (AM) $\leq$ root mean square (RMS) is a well-known inequality constraint used in many areas like optimization, neural networks and many more; hence, it can be exploited here also. Given a set of rules for a system, the rules can be aggregated using any of the mean operators. The rules being reduced using mean operators is expected to have a similar performance as that of the original rule base. Therefore, the time required for computing the control signal will be decreased.

The rest of the paper is organized as follows. Section 2 presents an insight into fuzzy rule base. Section 3 gives a new procedure to reduce rules. In Sect. 4, an inverted pendulum is controlled by a set of minimal rules to illustrate the performance and applicability of the proposed methods followed by a discussion. Finally the paper is concluded in Sect. 6.

2 Fuzzy rule bases

An example of a fuzzy rule template is given as

If x_1 is μ_1 and x_2 is $\mu_2 \ldots$ and x_n is μ_n
Then y is μ

where $\mu, \mu_1, \mu_2, \ldots, \mu_n$ are fuzzy sets. The degree of fulfillment of a rule is calculated from the membership degrees of the antecedents by use of a t-norm, usually $T_{\min}$ or T_{prod}.

Since decisions are based on the testing of all the rules in a system, the rules must be combined in some manner in order to make a decision. Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set. Aggregation occurs only once for each output variable. For each fuzzy variable, the maximum or the weighted average [7] average of the rule activation is calculated. The output class is determined with the highest accumulated activation.

There are two basic approaches [7] for pruning of fuzzy rule bases: direct deletion versus expansion of only some rules. In direct deletion, the less important rules are identified by some measure and are removed. In the latter approach, the similar rules are merged to reduce the total number of rules. In this paper, a totally different and a new method is introduced for merging fuzzy rules.

3 New methodology for merging fuzzy rules

The characteristics of a fuzzy model are frequently determined by the manner in which the rules are constructed. All fuzzy rules contribute with some degree to the final decision or inference. However, some rules, which are fired weakly do not contribute significantly to the final decision and may even be merged. The aim is to minimize the rules in order to reduce the computation time to make a faster decision. The rule bases are simplified on the data level. The considered fuzzy rule bases are given in disjunctive normal form (DNF), i.e., each rule is a conjunction of its antecedents, and the rule base is a disjunction of its rules. For illustration purposes, assume a rule base consisting of the following two rules.

Original rules:

R1: If x is small and y is large then z is medium or
R2: If x is small and y is medium then z is small
If x is 0.65 (small) and y is 0.7 (large) then z is 0.65 (medium)
If x is 0.65 (small) and y is 0.4 (medium) then z is 0.4 (medium)

Those rules whose input regions overlap are merged by taking the mean of antecedent fuzzy values; so these two rules can be merged using any of the mean operators given in the inequality constraint earlier. Rules are merged as given below.

Reduced rules:

If x is RMS (0.65, 0.65) (small) and y is RMS (0.7, 0.4) (large, medium) then
z is min (0.65 (small, 0.57 (large_medium new membership function))
= 0.57 (large_ medium new membership function)

When two or more rules are identical, and if the identical rules are merged, the new methodology will give one single rule of same dimensions, as per the inequality constraint given earlier. Two rules become one, therefore the rules are reduced, and decision will be made faster. By merging rules using mean operators the system's defuzzified output remains similar to the original rule base. Rules may be merged using any of the mean operators, but HM gives a lower output as per the inequality.

4 Application

In this section, the control problem for the inverted pendulum is used to interpret how the mean operators can be used to reduce fuzzy rules. This control system has two inputs and one output. A fuzzy controller is designed and analyzed for the simplified version of the inverted pendulum problem. For detailed explanation one should refer to [8]. The differential equation governing the system is given as

$$-ml\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + m\lg(\sin\theta) = \tau = \mu(t).$$

where m is the mass of the pole located at the tip point of the pendulum, l is the length of the pendulum, θ is the deviation angle from vertical in the clockwise direction, $\tau = \mu(t)$ is the torque applied to the pole in the counterclockwise direction is time and g is gravitational acceleration constant.

If $x_1 = \theta$ and $x_2 = \frac{\mathrm{d}\theta}{\mathrm{d}t}$, as start variables, the state space representation for the nonlinear system is given by $\mathrm{d}x_1/\mathrm{d}t = x_2$ and $\mathrm{d}x_2/\mathrm{d}t = (g/l)x_1 - \left(1/ml^2\right)u(t)$.

If x_1 is measured in degrees and x_2 is measured in degrees per second (dps), by choosing $l = g$ and $m = 180/\pi \times g^2$, the linearized and discrete-time state-space equations can be represented as matrix difference equations,

$$x_1(k+1) = x_1(k) + x_2(k) \tag{1}$$

and

$$x_2(k+1) = x_1(k) + x_2(k) - u(k) \tag{2}$$

The universe of discourse for the two variables are assumed to be $-2^\circ \le x_1 \le 2^\circ$ and $-5\,\text{dps} \le x_2 \le 5\,\text{dps}$. Three membership functions for x_1 are constructed for the values positive (P), zero (Z) and negative (N), shown in Fig. 2. Then three membership functions for x_2 are constructed for the values P, Z and N, shown in Fig. 3. To partition the control space (output), five membership functions for *u(k)* are constructed on its universe, which is $-24 \le u(k) \le 24$, shown in Fig. 4.

Nine rules are constructed in a 3×3 FAM table and shown in Table 1. The entries in this table are control actions $u(k)$.

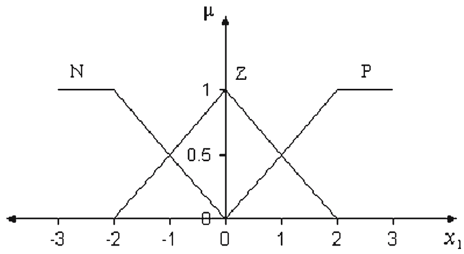

Fig. 2 Input x_1 partitioned

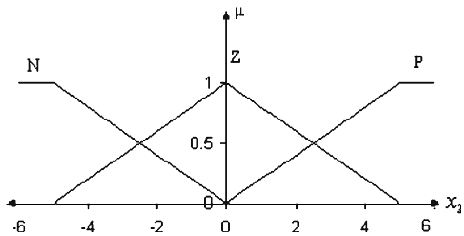

Fig. 3 Input x_2 partitioned

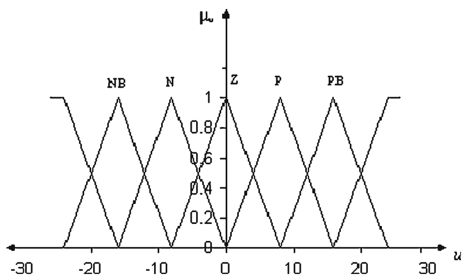

Fig. 4 Output $u(k)$, five membership functions

Table 1 Fam table

x_1 \ x_2	P	Z	N
P	PB	P	Z
Z	P	Z	N
N	Z	N	NB

4.1 Illustration

To start the simulation, the following crisp initial conditions are chosen: $x_1(0) = -0.5^\bullet$ and $x_2(0) = 1.5\,\text{dps}$. Only first cycle of simulation is conducted to show the effects of the new methodologies. The aggregated membership function for the original rules is shown in Fig. 5.

The original rules are given below:

R_1: If ($x_1 = Z$) and ($x_2 = P$) then (output = P) min (0.74, 0.3) = 0.3 (P)

R_2: If ($x_1 = Z$) and ($x_2 = Z$) then (output = Z) min (0.74, 0.7) = 0.7 (Z)

R_3: If ($x_1 = N$) and ($x_2 = P$) then (output = Z) min (0.26, 0.3) = 0.26 (Z)

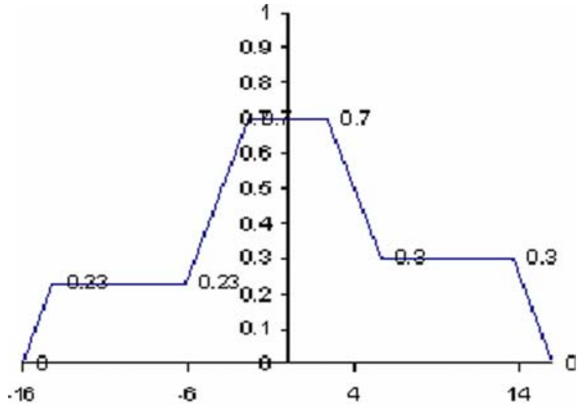

Fig. 5 Aggregated membership function

R_4: If ($x_1 = N$) and ($x_2 = Z$) then (output = N) min (0.26, 0.7) = 0.26 (N)

4.1.1 Rule Reduction using HM

R_1 and R_2 are merged into one rule using HM. Similarly R_3 and R_4 are combined into one rule.

R_{1-2}: If x_1 = HM ((0.74 (Z), 0.74 (Z)) and x_2 = HM (0.3 (P), 0.7 (Z)) then
Output = min (0.74 (Z), 0.42 (new function))
= 0.42 (positive_ zero membership function)
R_{3-4}: If x_1 = HM ((0.23 (N), 0.23 (N)) and x_2 =H.M (0.3 (P), 0.7 (Z)) then
output = min (0.26 (N), 0.42 (positive_ zero membership function)
= 0.26 (N)

The new membership function is the HM of the positive, zero membership function shown in Fig. 6. The new membership function and negative membership function for the above two rules are shown in Fig. 7. R_{1-2}trigger the negative membership function with the membership degree of 0.26, and R_{3-4} trigger the positive and zero membership function with a membership degree of 0.42. The consequent sets are shown in Fig. 6. The final aggregated membership function

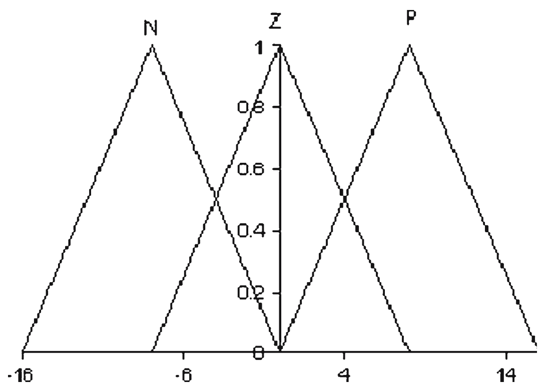

Fig. 6 Output sets of the consequent

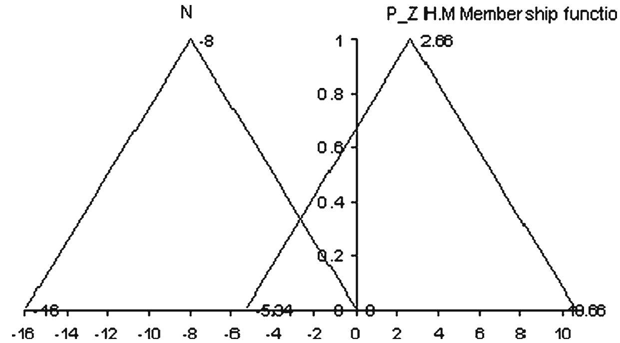

Fig. 7 Output sets with modified harmonic mean (HM) membership function

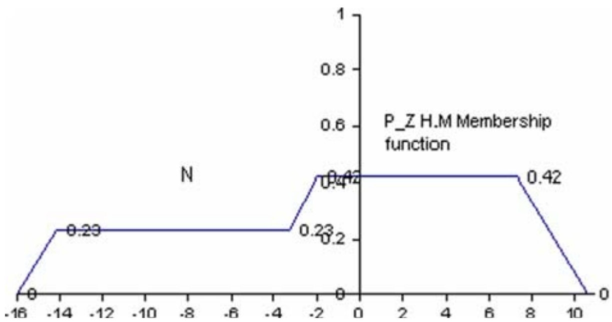

Fig. 8 Aggregated membership function using HM

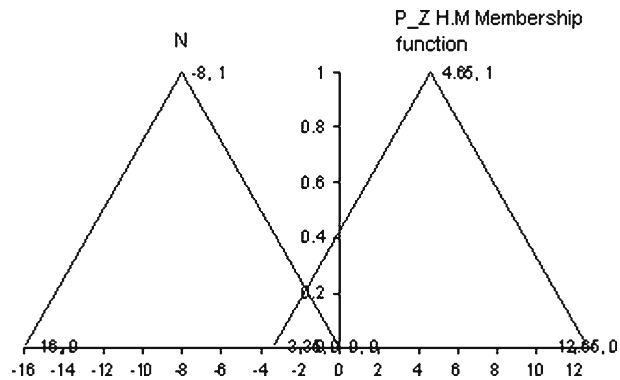

Fig. 9 Output sets with modified root mean square (RMS) membership function

with the membership degrees 0.23 and 0.42 are shown in Fig. 8.

4.1.2 Rule Reduction using RMS

R_1 and R_2 are merged to one rule using RMS. Similarly R_3 and R_4 are combined as one rule.

R_{1-2}: If x_1 = RMS (0.74 (Z), 0.74 (Z)) and x_2 = RMS (0.3 (P), 0.7 (Z)) then
output = min(0.74 (Z), 0.54 (positive_zero new membershipfunction))
= 0.54 (positive_zero membershipfunction)
R_{3-4}: If x_1 = RMS (0.26 (N), 0.26 (N)) and x_2 = RMS (0.3 (P), 0.7 (Z)) then

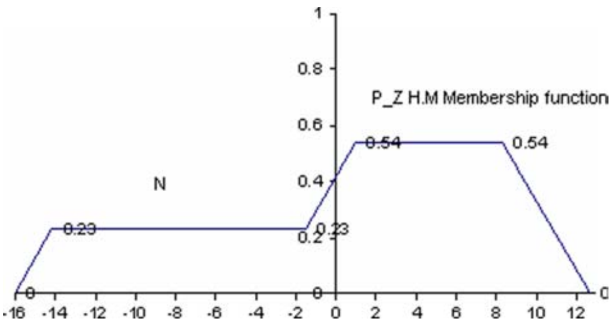

Fig. 10 Aggregated membership function using root mean square RMS

Table 2 Defuzzified outputs

Type of rules	Centroid defuzzified output
Using original rules	0.33
Rule reduction using harmonic mean (HM)	−1.55
Rule reduction using root mean square (RMS)	0.1

output = min (0.26, 0.54) = 0.26 (N)

R_{1-2} and R_{3-4} trigger the negative membership function with the membership degree of 0.26 of N and 0.54 of the RMS of the positive and zero membership function. The new membership function is shown in Fig. 9 and the final aggregated membership function is shown in Fig. 10.

The final output is calculated using centroid defuzzification method for all the three methods and the results are tabulated in Table 2.

The defuzzified output of the system using RMS rule reduction is similar to the existing method. The defuzzified output is lower when HM is used for reducing rules; this is due to the inequality constraint given earlier. HM and GM can also be used to reduce rules, but the defuzzified results lie between −1.55 and 0.1.

4.2 Simulation results

To start the simulation, crisp initial conditions used are $x_1(0) = 1$ and $x_2(0) = -4$ dps. Four cycles of simulation are conducted using the matrix difference equations given above (1,2) for the discrete steps $0 \leq k \leq 3$. Each simulation cycle will result in membership functions for the two input variables. Each simulation cycle after $k = 0$ will begin with the previous values of x_1 and x_2 as the input conditions to the next cycle of the recursive difference equations. Table 3 shows the simulation results using original rules and rule reduction using GM and HM.

Simulations can be carried out similarly using AM and RMS value for rule reduction. The results are shown only for first cycle of iteration. Using AM the first iteration values are $x_1(0) = 1, x_2(0) = -4$, and $u(0) = -4$. Similarly using RMS the first iteration values are $x_1(0) = 1, x_2(0) = -4$, and $u(0) = 4$.

5 Discussion

This nonlinear system is simulatd to compare the performance of the system using original rules and the minimum rules in controlling the balance of the inverted pendulum. Tables 2 and 3 show the defuzzified output of the system using original rules and rule reduction using HM, GM, AM and RMS . When the rules are reduced using HM it is clear from Table 3 that the pendulum oscillates at −2.66 deviation angle, whereas using GM the pendulum oscillates between −3 and +3.3 deviation angle. Using AM, the pendulum starts oscillating at −4.0 deviation angle, and the deviation angle increases when the rules are reduced using RMS, where the pendulum starts oscillating from + 4.0. This is due to the inequality constraint of the aggregation operators mentioned earlier in the paper.

A 3D control surface shown in Figs. 11 and 12 are plotted to visualize the controller of the two sets of rules. The control surface shows the control output (vertical axis) corresponding to some combinations of values of the two input

Table 3 Simulation results for the inverted pendulum

Iteration	Exisiting method			Harmonic mean			Geometric mean		
	x_1	x_2	u	x_1	x_2	u	x_1	x_2	u
First	1	−4	−2	1	−4	−2.66	1	−4	−3
Second	−3	−1	−9.6	−3	−0.34	−2.66	−3	0	−8
Third	−4	5.6	0.0	−3.34	−0.68	−2.66	−3	5	0
Fourth	1.6	1.6	5.28	−4.02	−1.36	−2.66	2	2	3.3
Fifth	3.2	−2.08	1.12	−5.38	−2.72	−2.66	4	0.71	3.3

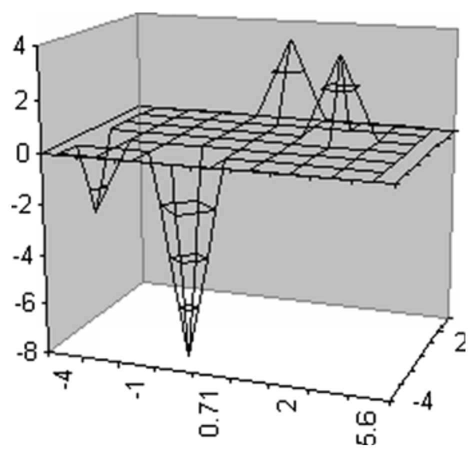

Fig. 11 Control surface for the system using geometric mean (GM) rule reduction

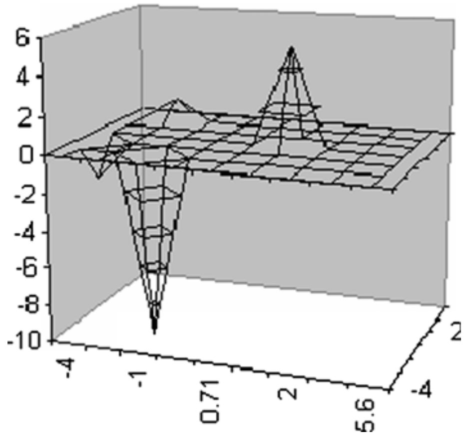

Fig. 12 Control surface for the existing system with original rules

state variables x_1 and x_2. We find the control surface of the reduced set using GM is similar to the control surface using original rules.

There are many ways of reducing rules, but in this paper, we present an easy and simple approach of rule reduction and still maintaining the same defuzzified output as that of the original rules. Moreover, the computation time is obviously shorter by using the reduced rules and it alleviates the complexity of implementation for the rule base.

6 Conclusion

In this paper, we interpret and demonstrate the applicability of using mean operators to reduce rules base controller. The comparison is also made between the reduced set and the original set rules. It is concluded that the expensive computation time will be reduced by using the minimum rules. As a result, this approach can provide a low-cost and robust means of design of the fuzzy rule-based controller. Limitations are obvious in case of incomplete knowledge of the rules. Further experimentation is required to explore the effects of using mean operators instead of t-norm and max and different defuzzification methods to reach a more effective defuzzified output.

References

1. Sudkamp T, Knapp A, Knapp J (2000) A greedy approach to rule reduction in fuzzy models. Systems, man, and cybernetics, 2000 IEEE international conference, vol 5, 8–11 October, pp 3716–3721
2. Zadeh LA (1973) Outline of a new approach to the analysis of complex systems and decision process. IEEE Trans Syst Man Cybern SMC-3:28–44
3. Takagi T, Sugeno M (1985) Fuzzy identification of systems and its applications to modeling and control. IEEE Trans Syst Man Cybern 15:329–346
4. Yager RR, Filev DP (1994) Essentials of fuzzy modeling and control. John Wiley, New York
5. Hung C-C, Fernandez B (1993) Minimizing rules of fuzzy logic system by using a systematic approach. Fuzzy Systems, 1993. Second IEEE international conference, vol 1, 28 March–1 April pp 38–44
6. Roychowdhury S, Wang BH (1998) Measuring inconsistency in fuzzy rules. Fuzzy systems Proceedings, 1998. IEEE world congress on computational intelligence. The 1998 IEEE international conferenceon vol 2, 4–9 May, pp 1020–1025
7. Klose A, Nürnberger A (1999) Applying Boolean Transformations to Fuzzy rule bases. In: Proceeding of 7th European congress on intelligent techniques and soft computing (EUFIT'99)
8. Ross TJ (1997) Fuzzy logic with engineering applications. McGraw Hill, New York