Modelling of Heat Flux in Building using Soft computing

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Abstract. Improving the detection of thermal insulation failures in buildings includes the development of models for heating process and fabric gain -heat flux through exterior walls in the building-. Thermal insulation standards are now contractual obligations in new buildings, the energy efficiency in the case of buildings constructed before the regulations adopted is still an open issue, and the assumption is that it will be based on heat flux and conductivity measurement. A three-step procedure is proposed in this study that begins by considering the local building and heating system regulations as well as the specific features of the climate zone. Firstly, the dynamic thermal performance of different variables is specifically modelled. Secondly, an exploratory projection pursuit method called Cooperative Maximum-Likelihood Hebbian Learning is used to extract the relevant features. Finally, a supervised neural model and identification techniques are applied, in order to detect the heat flux through exterior walls in the building. The reliability of the proposed method is validated for a winter zone, associated to several cities in Spain.

KEYWORDS: Computational Intelligence, Soft computing Systems, Identification Systems, Artificial Neural Networks, Non-linear Systems.

1. Introduction

The identification of thermal insulation failures (TIF) could significantly increase building energy efficiency and substantially contribute to reductions in energy consumption and in the carbon footprints of domestic heating systems. Conventional methods can be greatly improved through the application of learning techniques to detect the TIF when a building is in operation through heat flux model - heat flux through exterior walls in the building-.

Nevertheless, predicting the thermal dynamics of a building is a complex task. The dynamic thermal performance of a building has mainly been used to estimate its power requirements. As an example, the difficulties of obtaining a black-box model for a generic building are documented in [1]. Furthermore, [2] cites examples of the errors associated with different kinds of techniques while providing possible solutions. Also, in order to determine the thermal insulation failures local building regulations need to be analysed in order to profile the premises and the legal specifications for the physical parameters.

This research represents a step forward in the development of techniques to improve dynamic thermal efficiency in existing buildings through a modelling of heat flux in the building. Although this may appear simple at first sight, noise due to occupancy and lighting profiles can introduce distortions and complicate detection. A three-step procedure for testing and validating the model is proposed: firstly, the dynamic thermal behaviour of a specific configuration is calculated using HTB2 software [3]. The outcome of the HTB2 should then be post-processed to obtain a suitable dataset. Subsequently, the dataset is analysed using an exploratory projection pursuit (EPP) [4, 5] called Cooperative Maximum-Likelihood Hebbian Learning (CMLHL) [6, 7], extract the dataset structure and key relationships between the variables. A model is then produced, at the modelling stage, to estimate the heat flux through exterior walls in the building at a specific configuration.

This paper is organised as follows. The following Sub-Section 1.1 details the problem description. Section 2 introduces the unsupervised connectionist techniques for analysing the datasets in order to extract their relevant internal structures. Section 3 deals with classical identification techniques used in the system modelling. Section 4 describes the case of study details and the multi-step procedure. Finally, the conclusions are set out and comments are made on future lines of work.

1.1. Spanish regulations and the problem description

In 2007, several regulations on buildings and construction were approved in Spain. Firstly, the minimum pre-requisites for energy efficiency with which buildings must comply are given in the European Directive 2002/91/CE [8]. Project and specifications, the constructing conditions and the basic requirements in Spain are specified in the CTE (Código Técnico de Edificación [Building Regulations]) [9, 10, 11]. One of the basic requirements is document HE1 that considers the energy consumption limitation in buildings [9] and its updates.

The Spanish regulation on heating systems in buildings is the RITE (Reglamento de las Instalaciones Térmicas en los Edificios) [12], which establishes the minimum requirements and parameters for heating systems and thermal comfort. Finally, the procedure for certifying energy efficiency in new buildings is detailed in [13]. Energy efficiency is calculated as the ratio of combustible consumption needed to satisfy the energy demand of the building. The energy efficiency in the case of buildings constructed before the CTE approval is still an open issue, and the assumption is that it will be based on heat flux and conductivity measurement.

2. Analysis of the Internal Structure of the data set

2.1. Principal Component Analisis

Principal Component Analysis (PCA) originated in work by Pearson [14], and independently by Hotelling [15] describing multivariate data set variations in terms of uncorrelated variables, each of which is a linear combination of the original variables. Its main goal is to derive new variables, in decreasing order of importance, which are linear combinations of the original variables and are uncorrelated with each other.

2.2. A Neural Implementation of Exploratory Projection Pursuit

The standard statistical method of EPP [4, 5], provides a linear projection of a data set, but it projects the data onto a set of basic vectors which best reveal the interesting structure in data; interestingness is usually defined in terms of how far the distribution is from the Gaussian distribution [16].

One neural implementation of EPP is Maximum Likelihood Hebbian Learning (MLHL) [5, 17]. It identifies interestingness by maximising the probability of the residuals under specific probability density functions that are non-Gaussian.

An extended version of this model is the Cooperative Maximum Likelihood Hebbian Learning (CMLHL) [6] model. CMLHL is based on MLHL [5, 17] adding lateral connections [18, 6] which have been derived from the Rectified Gaussian Distribution [16]. The resultant net can find the independent factors of a data set but does so in a way that captures some type of global ordering in the data set.

Considering an N-dimensional input vector (x), and an M-dimensional output vector (y), with W_{ij} being the weight (linking input j to output i), then CMLHL can be expressed [18, 17] as:

1. Feed-forward step:

$$y_i = \sum_{j=1}^{\mathbf{N}} W_{ij} x_j, \forall i \quad (1)$$

2. Lateral activation passing:

$$y_i(t+1) = \left[y_i(t) + \tau(b - Ay) \right]^+ \quad (2)$$

3. Feedback step:

$$e_j = x_j - \sum_{i=1}^{M} W_{ij} y_i, \forall j \quad (3)$$

4. Weight change:

$$\Delta W_{ij} = \eta . y_i . sign(e_j) | e_j |^{p-1} \quad (4)$$

Where: η is the learning rate, τ is the "strength" of the lateral connections, b the bias parameter, p a parameter related to the energy function [5, 6, 17] and A a symmetric matrix used to modify the response to the data [6]. The effect of this matrix is based on the relation between the distances separating the output neurons.

3. System Modelling Using Identification Algorithms

System identification (SI) aims to obtain mathematic models to estimate one or more behaviours from a physical process whose dynamic equations are unknowns. Classic SI refers to the parametrical literature [19], which has its origin from the linear system analysis.

The SI procedure includes several steps [19, 20, 21, 22]: the selection of the models and their structure, the learning methods [23, 24], the identification and optimization criteria and the validation method. The validation ensures that the selected model meets the necessary conditions for estimation and prediction. Typically, validation is carried out using three different methods: the residual analysis $\varepsilon(t,\hat{\theta}(t))$ -by means of a correlation test between inputs, their residuals and their combinations-; the final prediction error (FPE) estimated as explained by Akaike [25] and finally a graphical comparison between desired outputs and the outcome of the models through simulation, with one (or k) steps ahead.

3.1. The ANN in the identification process

The use of ANN in the process of identification requires the selection of several parameters: the number of layers, the number of neurons per layer and the activation functions. The methods by which the parameters are set up are fully documented in the literature [26, 27]. It was found that ANN with two layers using non-linear functions in the hidden layer are universal approximators or predictors [28, 29].

The number of neurons per layer is also a relevant design parameter. It should be analyzed in order to avoid over fitting [30, 31]. Each algorithm will introduce some restrictions in the weight matrix. The most widely used training algorithms in system identification are the Lenvenberg-Marquardt method [32], recursive Gauss-Newton method [19], the batch and recursive versions of the back-propagation algorithm [33].

Several well-known model structures are used when merging system identification with ANN. If the ARX (Autoregressive with external input) model is used as the regression vector θ, the model structure is called NNARX (neural network for ARX model), as can be seen in Eq. (5). Likewise, NNFIR for neural network FIR (Finite Impulse Response) Eq. (6), NNARMAX for neural network ARMAX (Autoregressive Moving Average with external input) Eq. (7) and NNOE for neural network OE (Output Error) Eq. (8), are also extensively used. In the same way, it is possible to use an estimator for the one-step ahead prediction of the output $\hat{y}_1(t \mid \theta)$, i.e., the NNARX and the NNFIR –using Eq. (9)–, or the NNARMAX and the NNOE –using Eq. (10)–. The polynomial degree values -n_a, n_b, n_c, n_d, n_f and n_k- are given as parameters.

$$\varphi(t) = \left[y(t-1) \ldots y(t-n_a) u(t-n_k) \ldots u(t-n_b-n_k+1) \right]^T . \tag{5}$$

$$\varphi(t) = \left[u(t-n_k) \ldots u(t-n_b-n_k+1) \right]^T . \tag{6}$$

$$\varphi(t) = \left[y(t-1) \ldots y(t-n_a) u(t-n_k) \ldots u(t-n_b-n_k+1) e(t-1) \ldots e(t-n_c) \right]^T . \tag{7}$$

$$\varphi(t) = \left[\hat{y}(t-1 \mid \theta) \ldots \hat{y}(t-n_a \mid \theta) u(t-n_k) \ldots u(t-n_b-n_k+1) \right]^T . \tag{8}$$

$$\hat{y}(t \mid \theta) = \hat{y}(t \mid t-1, \theta) = g(\varphi(t), \theta) . \tag{9}$$

$$\hat{y}(t \mid \theta) = g(\varphi(t), \theta) . \tag{10}$$

4. A multi-step method for modelling of heat flux in buildings

A novel three-step method is proposed to detect the heat flux through exterior walls in the building. Firstly, the building is parameterised and its dynamic thermal performance in normal operation is obtained by means of simulation. Then, the data gathered is processed using CMLHL as a dimensionality reduction technique to choose the most relevant features in order to determine the heat flux. The second step outcome is a data set which is finally used to train and validate the heat flux model.

4.1. Thermal dynamics data gathering by means of simulation

In order to simulate the thermal behaviour of a building, the following variables and data sets should be gathered: building topology; climate zone according to the specific regulations; building materials that comply with local regulations for the chosen climate zone; meteorological data for the climate zone and the simulated time period: such as solar radiation, outdoor temperature, wind speed, etc., and realistic profiles for heating, lighting, small power devices, occupancy and ventilation.

In this study, the country where the system is applied is Spain. In Spain, the Spanish regulations establish five winter/summer zones, from E1 (more severe climate zones) to A3 (gentler climate zone).

Having defined and/or gathered these data sets, then the chosen simulation tool is applied to obtain the output data. In our case, the simulation software used is HTB2 [3]. The typical values that each variable could take for an E winter climate zone of maximum severity in Spain -i.e. the cities of León, Burgos or Soria among others- are shown in Table 1.

Table 1. Typical values of each variable in a E winter climate zone city in Spain

Variable (Units)	Range of values	Transmittance level (W/m^2K)
Fabric gain -heat flux- (w), $y_1(t)$.	0 to -7,100	-External cavity wall: 0.54 -Double glazing: 2.90 -Floor/ceiling: 1.96 -Party wall between buildings: 0.96 -Others party wall: 1.05 -Internal partition: 2.57
Heater gain (W), $u_1(t)$.	0 to 4,500	
Occasional gain –small power, occupancy and lighting gain- (W), $u_2(t)$.	0 to 5,500	
Ventilation gain (w), $u_3(t)$.	0 to -5,500	
Exterior air temperature in February (ºC), $u_4(t)$.	1 to 7	
Air temperature of the house (ºC), $u_5(t)$.	14 to 24	

4.2. Selection of the relevant features

As detailed in Section 2, PCA (Fig. 1.a.) and CMLHL (Fig. 1.b.), which were both applied to this real-life problem, are instrumental in identifying the internal structure of the data. In this procedure, the data set gathered in the previous step is analysed. The objective is to find the relationships between the input variables with respect to the heat flux. CMLHL (Fig. 1.b.) allows to detect the relations of dependence and to choose the more relevant features. The outcome of this step is a new data set with the features for which a relationship with the heat flux is found.

4.3. System identification applied to model normal building operation

Once the relevant variables and their transformations have been extracted from the thermal dynamics data, then a model to fit the normal building operation should be obtained in order to identify bias in the heat flux through exterior walls in the building. The different model learning methods used in this study were implemented in Matlab© [34]. The experiment followed the identification procedure detailed in Section 3: the model structures were analyzed in order to obtain the models that best suited the dataset. The Akaike Information Criterion (AIC) is used to obtain the best degree of the model and its delay for each model structure. A total of seventy techniques were carried out to obtain the models, among which are the following: the frequency response analysis; the finite impulse response method (FIR); Black-box techniques: up to 31 different combinations of model structure and optimization techniques are considered -such as the least-squares method and QR factorization of ARX models [19, 21]. The nonlinear model structures synthesis: up to thirty four different combinations of model structures and optimization techniques are considered -such as the Lenvenberg-Marquardt method and the recursive Gauss-Newton method for the NNARX, NNFIR, NNARMAX and NNOE models [19, 32, 33].

Three different residual analyses based on cross correlation are carried out: residual analysis between the residual $\hat{R}_{\varepsilon}^{N}(\tau)$, between the residual and the input $\hat{R}_{\varepsilon u}^{N}(\tau)$ and the non-linear residual correlation $\hat{R}_{\varepsilon^2 u^2}^{N}(\tau)$.

The heating process exhibits nonlinear behaviour between output and inputs. Due to this reason, the linear modelling techniques do not behave properly except in the linear behaviour zones of the process. Consequently, the heating process has been modelled using soft-computing techniques, specifically an ANN.

5. Experimentation and results

In order to validate the proposal some realistic situations have been considered. A building in the E winter zone, in the city of Avila city is used as the actual building. This building was parameterised and the HTB2 simulation tool was used to gather the data set. This initial data set has been analysed in order to select the features that best described the relationships with the heat flux. As may be seen in Fig. 1, PCA (Fig. 1.a) and CMLHL (Fig. 1.b), both methods have identified the occasional gain as the most relevant variable but in the CMLHL projections (Fig. 1.b.) can be noticed more structured clusters than in the PCA projections.

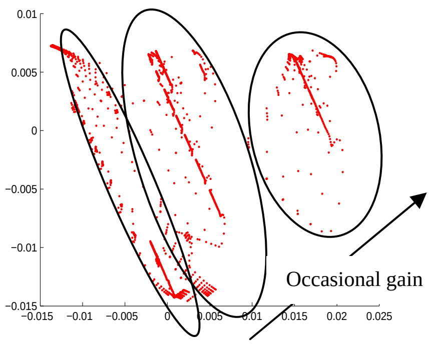

Fig. 1.a. PCA projections.

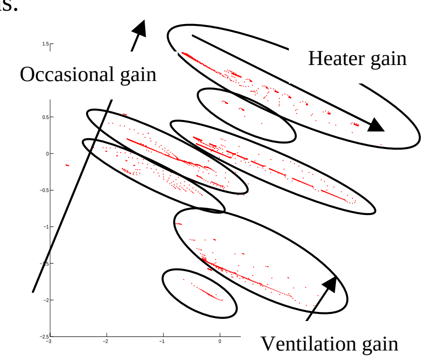

Fig. 1.b. CMLHL projections.

Fig. 1. PCA projections (Fig. 1.a.) and CMLHL projection (Fig. 1.b.) after 20000 iterations using a learning rate of 0.05, 3 output neurons p=0.3 and τ=0.3.

Having analysed the results obtained with the CMLHL model (Fig. 1.b.) it is concluded that CMLHL has identified four relevant variables and seven clusters order by occasional gain. Inside each cluster there are further classifications by the heater gain, the ventilation gain and to a lesser degree the exterior air temperature. Then, the heat flux and the dataset may be said to have an interesting internal structure. When the dataset is considered sufficiently informative, the third step in the process begins. This step performs an accurate and efficient optimization of the heating system model to detect heat flux model in the building, through the application of several conventional modelling systems.

Thus, an ANN was used to monitor the thermal dynamics of the building. The objective was to find the best suite of polynomial model orders [n_a n_{b1} n_{b2} n_{b3} n_{b4} n_c n_d n_f n_{k1} n_{k2} n_{k3} n_{k4}]. Using the data set from the previous stage and the Optimal Brain Surgeon (OBS) [30, 31] network pruning strategy to remove superfluous weights, the best suite model was found from the residual analysis. Fig. 2 shows the time responses of the heat flux - $y_1(t)$ - and of the estimated heat flux - $\hat{y}_1(t\,|\,m)$ - for the NNARX model described in Eq. (5). The x-axis shows the number of samples used in the estimation and validation of the model and the y-axis represents the normalized output variable range: which is the normalized heat flux of the house. The estimation and validation data sets include 2000 and 1126 samples, respectively, and have a sampling rate of 1 sample/minute. Fig. 3 indicates the final neural network structure chosen for modelling the heat flux.

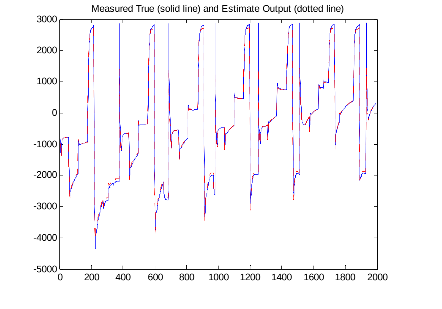

Fig. 2.a. NNARX model for the training data set.

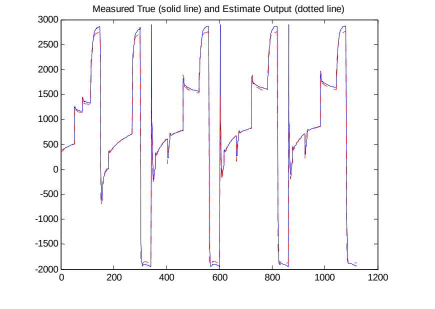

Fig. 2.b. NNARX model for the validation data set.

Fig. 2. Output response of NNARX model: The actual output (solid line) is graphically presented with one-step-ahead prediction (dotted line). In Fig. 2.a. the real measure can be compared with the estimated data, while in Fig. 2.b. the real measure is compared with the validation data.

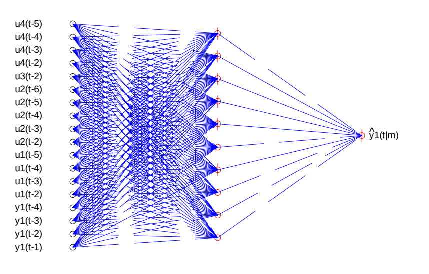

Fig. 3.a. Initial fully connected structure of the NNARX model.

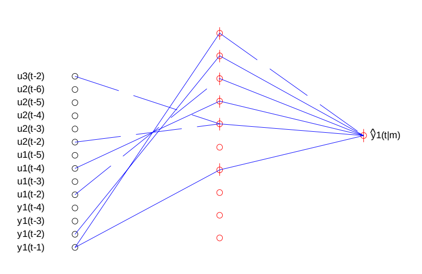

Fig. 3.b. Optimal architecture of the NNARX model, with the network pruned.

Fig. 3. Optimal architecture of the NNARX model, with the network pruned, for the heat flux through exterior walls in the building -output y_1(t)- (Fig. 3.b.). Positive weights are represented in solid lines, while a dashed line represents a negative weight. A vertical line through the neuron represents a bias. The initial fully connected structure is shown in the Fig. 3.a.

From Fig. **2**, it can be concluded that the pruned network NNARX model is able to simulate and predict the behaviour of the heat flux through exterior walls in the building –as a consequence of the heating process- and it is capable of modelling more than 91,4% of the actual measurements. The model thus obtained is ANN model, NNARX regressor, the order of the polynomials of the initial fully connected structure are $n_a=4$, $n_{b1}=4$, $n_{b2}=5$, $n_{b3}=1$, $n_{b4}=4$, $n_{k1}=2$, $n_{k2}=2$, $n_{k3}=2$, $n_{k4}=2$. The model was obtained using the regularized criterion. This model was optimised by CMLHL analysis, residual analysis and the pruned network, using OBS. The model structure has 10 hidden hyperbolic tangent units and 1 linear output unit. The network is estimated using the Lenvenberg-Marquardt method, and the model order is decided on the basis of the best AIC criterion of the ARX model.

6. Conclusions and future work

Effective thermal insulation is an essential component of energy efficient heating systems in buildings. Thus, the possibility of improving the detection of thermal insulation failures represents a fresh challenge for building energy management.

The new methodology proposed in this study to predict the heat flux through exterior walls in the building can be used to determine the normal operating conditions of thermal insulation in buildings in Spain, which is a mandatory test in the evaluation of insulation in buildings.

Future work will create a quality standard of the heat flux process, based on the type of insulation that can detect faults of insulation in buildings. Also, modelling the ventilation and infiltration in the process of heating, in order to develop generic methods, so that it can allow adequate ventilation in intelligent buildings, but efficiently, that is at the lowest possible cost.

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