Measurement of ground-neutral currents in three phase transformers using a genetically evolved shaping filter

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Abstract. Measuring the current in the neutral-grounding resistor is needed for monitoring resistance-grounded three phase transformers. This current is limited to hundreds of amperes in case of a fault, and are almost negligible otherwise. The current transformer that senses the current must be rated for the fault conditions, thus it is difficult to obtain a precise measurement of the current when there is not a ground-fault in the system.

In this paper we propose a computer-based method for filtering the output of the current transformer and improving its accuracy for small currents. This processing is complicated, as the amount of noise is very high, and this noise is strongly correlated with the useful signal. We propose to use Kalman filtering, based on a model of the system, and augment the state of this model with a shaping filter, whose frequency response, when fed with white Gaussian noise, reproduces our measurements of the ambient noise. In particular, since the Power Spectral Density (PSD) of the noise changes with time, we propose to use a possibilistic description of the PSD of the noise, and search for a model whose PSD is between the soft margins defined by the possibilistic model. We will use a state-space based representation and a genetic algorithm, guided by a fuzzy fitness function, for evolving the shaping filter that best matches the ambient noise. The proposed method has been evaluated with field data captured at a 130KV substation transformer at La Corredoria (Asturias, Spain).

1   Introduction: Ground-neutral monitoring systems

Neutral-grounding resistors (NGR) provide current for ground-fault detection and selective coordination. Its failure is dangerous, as it converts a resistance-grounded system into an ungrounded system. While this is not a common problem in Europe, the probability of the NGR becoming open is higher in those parts of the world where these elements are subject to higher thermal stresses and continuous monitoring of the NGR is needed. Else, there is no indication that the system has become ungrounded, and the risk of transient overvoltages or the presence of voltages on otherwise safe conductors exists.

In Figure 1 a 130KV tree phase substation transformer and its NGR (yellow circle in the left part of the figure) are shown. The NGR is connected to the neutral of the transformer (center part of the same figure) and to earth (right part). This particular resistor is designed for limiting the current to 500A in case of a fault. A suitable current transformer (CT) measures the current through the NGR and its output is constantly monitored.

Fig. 1. La Corredoria substation transformer and its NGR (left part). Detail of the connection between the NGR and the neutral (center) and ground (right), showing the current transformer used in this paper.

The absence of current at the CT means that either the transformer is balanced or the NGR is open, thus the monitoring of this current does not prevent by itself the risks mentioned before. A NGR monitoring system involves more elements [8, 11], as shown in Figure 2 (this system will be further explained in Section 2). In this schema, a resistor is used for measuring the voltage at the neutral of the transformer; this value, divided by the current through the NGR, is the ohmic value of the NGR, as desired.
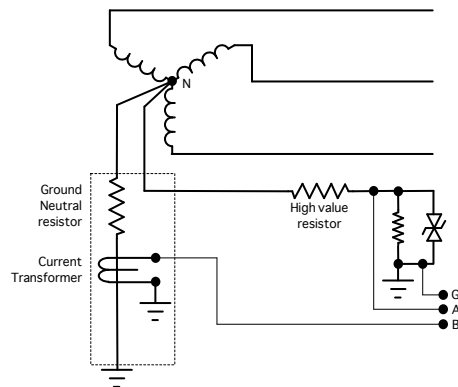

Fig. 2. Schema of a NGR monitoring system.

There is still a caveat with this scheme: in case the system is perfectly balanced, no current flows by the NGR and the voltage at the neutral is also null, thus we cannot tell whether the NGR is open or not. There are two possibilities for overcoming this problem:

– We can inject DC or low frequency current into the NGR path and measure the increase in the voltage of the neutral, or
– We can amplify the output of the current transformer and measure the residual current. We expect that this residual current has components in the first odd harmonics, because of the stray capacities of the line, and also at very high frequencies ($> 1KHz$) because of the use of Power Line Carrier [3], thus it is potentially usable for detecting open NGRs.

In this study we will explore the second option. The main difficulty of this approach is in the level of amplification need, which will be affected by the important ambient noise near a substation transformer. That is to say, we need to separate the measurement of the actual current from those stray currents induced in the transmission line. The problem is, these stray currents are strongly correlated with the signal, and their Power Spectral Density (PSD) also varies with time within certain limits.

We have decided to make a Kalman filtering of the signal, taking into account the spectrum of the noise by mean of a shaping filter [5] that is designed to produce a signal with the same PSD as the ambient noise. This shaping filter will be used to augment the state of the system, as we will explain in Section 2. For obtaining the model of such a shaping filter we propose to use Genetic Algorithms (see Section 3). It is remarked that the novelty of this approach, apart from the definition of specialized genetic operators and the representation of the individuals, is in the fact that the desired PSD is not completely known, thus the GA will be guided by a fuzzy fitness function. In Section 4 we show the results of the application of this system to real-world data. The paper finishes with the concluding remarks in Section 5.

2 Description of the measurement system

The measuring system that will be analyzed in this paper is shown in the left part of Figure 3. The output of the current transformer is connected to a resistive load R_L (the burden resistor of the current transformer has not been shown in the schema). Alas, if the cable is not perfectly shielded then there will be stray currents flowing through it; we have represented them as a coupled coil L_{mutual} in the figure. When studying the noise, we replace the current transformer by a resistance of high value, and measure the voltage dropped in R_L. The voltage drop in R_L depends on the sum of both currents: the current in the secondary of the CT and the stray currents we have mentioned.

According to our own experimentation, at low frequencies, the current in the NGR is comprised of the first odd harmonics (50Hz, 150Hz, 250Hz and 350Hz), while at high frequencies the contribution of the DLC produces a more complex

profile. In this work we limit ourselves to low frequencies, and propose that the current y_C by the NGR is modeled by the state space discrete system

$$x_C(kh + h) = \Phi_C x_N(kh) \tag{1}$$
$$y_C(xk) = C_C x_C(kh) \tag{2}$$

where Φ_C will have all its poles at the unit circle, at frequencies that match the mentioned harmonics. Observe that such a system is unstable and the input is not needed.

Secondly, we want to devise a similar model for the noise, and combine it with the preceding model as shown in the right part of Figure 3. The noise model is a shaping filter whose input is white Gaussian noise and whose output mimics the frequency behavior of the noise, measured as described before. The model of the shaping filter will be expressed in observable canonical form, because of reasons that will be made clear in Section 3:

$$x_N(kh + h) = \Phi_N x_N(kh) + \Gamma_N v(k) \tag{3}$$
$$y_N(xk) = C_N x_N(kh) \tag{4}$$

where $v(k)$ is Gaussian white noise with mean 0 and variance 1, and $C_N = (1, \ldots, 0)$.
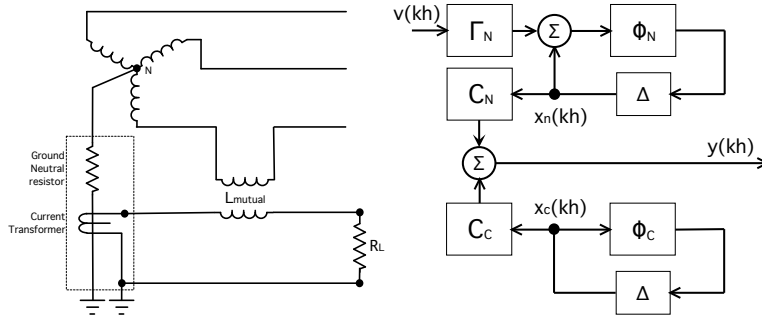

Fig. 3. Block diagram of the augmented system

The augmented system (see Figure 3) is

$$x(kh + h) = \Phi x(kh) + e(k) \tag{5}$$
$$y(xk) = C_N x_N(kh) \tag{6}$$

where $x = (x_C^T | x_N^T)^T$, and $e(k)$ is white noise with covariance R_1

$$R_1 = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \Gamma_N \Gamma_N^T \end{array} \right) \tag{7}$$

and the combination of the two state space matrices we have proposed is

$$\Phi = \begin{pmatrix} \Phi_C & 0 \\ \hline 0 & \Phi_N \end{pmatrix} = \left(\begin{array}{c|c} \begin{matrix} \cos 2\pi f_1 h & \sin 2\pi f_1 h & & \\ -\sin 2\pi f_1 h & \cos 2\pi f_1 h & & 0 \\ & & \vdots & \\ & 0 & & \cos 2\pi f_4 h & \sin 2\pi f_4 h \\ & & & -\sin 2\pi f_4 h & \cos 2\pi f_4 h \end{matrix} & 0 \\ \hline & 0 & \begin{matrix} c_1 \; 1 \; 0 \cdots 0 \; 0 \\ c_2 \; 0 \; 1 \cdots 0 \; 0 \\ \vdots \\ c_8 \; 0 \; 0 \cdots 0 \; 0 \end{matrix} \end{array} \right) \tag{8}$$

$$C = (C_C \mid C_N) = (C_C \mid 1 \ldots 0) \tag{9}$$

thus particularizing the Kalman filter [5] to this case, we obtain the equations

$$\hat{x}(k+1 \mid k) = \Phi \hat{x}(k \mid k-1) + K(k)(Y(k) - C\hat{x}(k \mid k-1)) \tag{10}$$

$$K(k) = \Phi P(k) C^T (CP(k)C^T)^{-1} \tag{11}$$

$$P(k+1) = \Phi P(k)\Phi^T + R_1 - \Phi P(k)C^T(CP(k)C^T)^{-1}CP(k)\Phi^T. \tag{12}$$

3 Genetic optimization of a shaping filter

This section describes the use of Genetic Algorithms for evolving the three matrices Φ_N, Γ_N and C_N described before. Observe that we need to restrict our search to stable systems, i.e. to those matrices Φ_N whose eigenvalues are in the unit circle. We want that the PSD of this system, when the input $v(k)$ is Gaussian white noise, is similar to the PSD of the ambient noise.

There are two challenges in this search: (1) the PSD is time-varying, and (2) it is not trivial to determine the complex eigenvalues of a general matrix [9]. We have solved them as follows:

1. We have divided the spectrum of frequencies between 50 and 500Hz in 10 bands, and computed five confidence intervals at levels 0.50, 0.25, 0.10, 0.05, 0.01 for the energy of the noise for each band. These five intervals are regarded as α-cuts of fuzzy values describing our knowledge about the PSD of the noise.
2. We do not generate random matrices and discard those corresponding to unstable processes. We represent instead Φ_N by means of the roots of its characteristic polynomial, and use the observable canonical form of the system.

In the following paragraphs we discuss both subjects: the computation of the PSD and the representation of the matrices that comprise the model of the noise.

3.1 Fuzzy PSD

Let $y_N(kh)$ the output of the noise model, when the input is $v(kh)$, white Gaussian noise. Let us assume a zero-th order hold ($y_N(t)$ constant in $t \in [kh, kh+h)$) and let the autocorrelation of this time series be

$$E(y_N(t) \cdot y_N(t + \tau)) = \psi_{y_N}(\tau). \tag{13}$$

Then, the PSD of the noise is the Fourier transform of the autocorrelation,

$$\Psi_x(\omega) = \int_{-\infty}^{\infty} \psi_{y_N}(\tau) e^{-j\omega\tau} d\tau \tag{14}$$

and the energy of the band $[\omega_k - \Delta/2, \omega_k + \Delta/2]$ is

$$\text{energy}(\omega_k) = \int_{\omega_k - \Delta/2}^{\omega_k + \Delta/2} \Psi_x(\omega) d\omega. \tag{15}$$

We have defined 10 bands of width 50Hz each, at frequencies $\omega_k = 100k\pi$, $k = 1, \ldots, 10$. We want to obtain 10 fuzzy sets describing the expected energy at each band, and its dispersion. First, the energy at each band ω is measured N independent times, and the values $\text{energy}_i(\omega)$, $i = 1 \ldots N$ are obtained. Second, following the interpretation in $[1, 10, 2]$, we compute the intervals

$$I_\alpha(\omega) = [I_\alpha^-(\omega), I_\alpha^+(\omega)] \tag{16}$$

where $I_\alpha(\omega)$ is the smallest interval for which

$$\#\{i \mid I_\alpha^-(\omega) \le \text{energy}_i(\omega) \le I_\alpha^+(\omega)\} \ge N(1 - \alpha). \tag{17}$$

Lastly, we define the fuzzy PSD of the noise as the fuzzy set with membership function

$$\widetilde{\text{PSD}}_\omega(x) = \sup\{\alpha \mid x \in I_\alpha(\omega)\}. \tag{18}$$

3.2 Representation of the state, input and output matrices

We have mentioned that the state matrix will be codified by mean of its poles, or roots of the characteristic polynomial of Φ_N, which is

$$\sum_{i=0}^{8} c_i \lambda^i = \prod_{i=1}^{8} (\lambda - p_i) \tag{19}$$

where p_i are the complex poles of the system. It is remarked that the poles must be conjugated in pairs or be real numbers for the c_i to be real numbers.

The matrix Γ_N is codified as a vector of 8 real numbers. C has only one term different than zero, and this value is not represented in the genetic chain but solved so that the average energy of the individual is the same as the modal point of the average energy of the noise.

3.3 Generation of the initial population

The individuals are generated at random, but taking into account that the complex poles are paired with their conjugates, so the state matrix of the noise is real. The probability of appearance of each pole is not uniform, but biased towards unstable poles of the form $\cos 100k\pi \pm j \sin 100k\pi$.

3.4 Genetic Operators

The genetic operators are, for the most part, standard two-point crossover and arithmetic mutation [7]. The particularities of both are:

Crossover Since complex poles must be accompanied by their conjugates, we represent both together and prevent the point between them from being an split in the crossover. We also do an arithmetic combination of the cut points, in order to introduce genetic diversity.

Mutation If a pole is mutated, then its conjugate must be modified in accordance. The mutation is defined as the convex combination between the value and a randomly generated parameter.

3.5 Fitness function

The fitness is the degree of compatibility between the fuzzy PSD of the individual (obtained by simulation of the candidate model in 10 random sequences, as explained in Section 3.1) and the fuzzy PSD of the ambient noise,

$$\sum_k \log \frac{\widetilde{\text{PSD}_{\text{model}}}(100k\pi)}{\widetilde{\text{PSD}_{\text{noise}}}(100k\pi)} \tag{20}$$

where the "log" and "quotient" operators are the extensions of the conventional operators to fuzzy aritmetic [4].

3.6 Generational scheme

The generational scheme is steady state, with a tournament of size 5, where the offspring of the two winners replaces the last two individuals in the tournament. The probabilities of mutation and crossover are 0.05 and 1. In the tournament we have used the uniform dominance defined in [6].

4 Numerical results

In this section we will compare first the use of crisp and fuzzy fitness functions for finding the model of the ambient noise, then we will apply it to estimate the current in the NGR of a 130KV transformer, as mentioned.

4.1 Crisp and fuzzy fitness functions

In the left part of Figure 4 we have superimposed some experimental measurements of the PSD of the ambient noise, taken at the same point but at different times. Observe that there is a significant dispersion of the energy of each harmonic, mainly at high frequencies. In this paper we claim that it is preferable to fit a model to a fuzzy description of this data than fitting a model to the average values of the same data.

To illustrate this result, we have learned 30 crisp models and 30 fuzzy models, following the methodology explained in the preceding section. The fitness of a crisp model was computed with the same equation (20) than the fuzzy model, but replacing the fuzzy PSD by the average of the PSDs. The 60 models were tested against an independent set of 10 PSDs of the actual noise, measured at a later time. The results are shown in the right part of Figure 4 and in Table 1.

	Train			Test		
	Best	Average	Worst	Best	Average	Worst
Crisp-GA		5.073		3.430	5.135	9.230
Fuzzy-GA	2.538	4.248	8.133	2.510	4.312	8.477

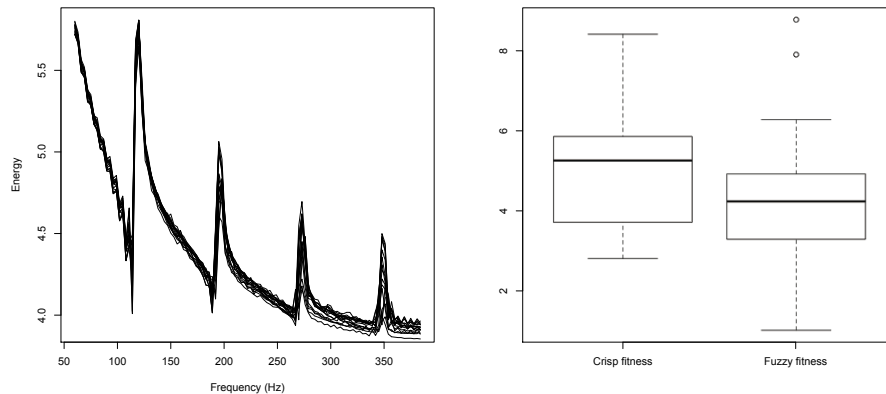**Table 1.** Train and test error (30 repetitions) of crisp and fuzzy approaches

Fig. 4. Left: Dispersion of the ambient PSD. Right: Boxplots of the average error model with crisp and fuzzy fitness functions.

The boxplots show the dispersion of the average error of the model, and the table shows the mean values of the best, average and worst models. The

p-value of a Wilcoxon test between the two samples of test errors ("Average" column) is 0.0399, thus there is a significant difference between crisp and fuzzy methodologies, at a confidence level of 95%.

4.2 Real-world data

We have sampled the output of the current transformer (CT) in the NGR, and the results are displayed in the red curve in the left part of Figure 5. Observe that there is a high amount of high-frequency noise, that makes hard to perceive the actual shape of the current; we have filtered out these components with a low-pass filter before applying the Kalman filtering. We have superimposed (black trace) a capture of the ambient noise taken elsewhere, that is not synchronized with this current. In the right part of the same figure we have displayed again the preprocessed output of the CT, along with the Kalman filtering of this signal (red curve with dots); observe that this filtered signal is a nearly periodical curve comprising the first four odd harmonics, as expected.
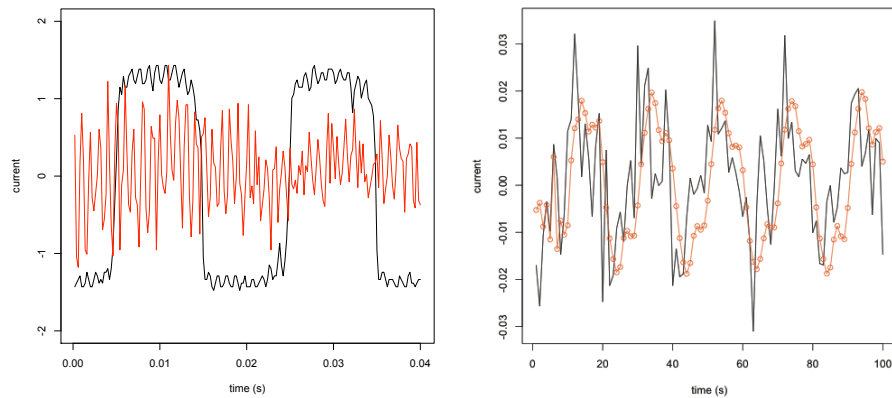

Fig. 5. Left: signal (red) and noise (black) in the CT. Right: Kalman filtering of the CT output (red line with circles)

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5 Concluding remarks and future work

In this paper we have proposed a method for obtaining the state-space equations describing a shaping filter that mimics the ambient noise at a substation transformer. This model is intended to augment the state of a model of the current flowing through the NGR of the transformer, in order to obtain a model whose

input is white noise and therefore suitable for Kalman filtering. We have shown that the use of a possibilistic representation of the dispersion of the PSD of the noise with time can be exploited by a fuzzy fitness function-driven Genetic Algorithm, producing models with improved generalization capabilities.

In future works we intend to extend this study to high frequencies, that have been filtered out in this paper. Frequencies above 1KHz are seldom considered in the determination of the NGR continuity, however we think that the use of the power line for digital transmission causes a measurable effect that could be captured with different sensors (i.e. Rogowski coils) and compared to the measurement in the neutral of the transformer, allowing a finer monitoring of the health of the NGR.

Acknowledgements

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