Genetic Lateral and Amplitude Tuning of Membership Functions for Fuzzy Systems

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***Abstract*— In this work, we extend the genetic lateral tuning of membership functions [1] based on the linguistic 2-tuples representation [2], in order to also perform a tuning of the support amplitude of the membership functions. To do so, we present a new symbolic representation which extends the linguistic 2-tuples representation model with a parameter β to represent the amplitude variation of the support of its associated membership function.**

I. Introduction

The Linguistic Fuzzy Modeling (LFM) pretends to model systems building linguistic models with a good trade-off between *interpretability* and *accuracy*. However, in this kind of modeling the *accuracy* and the *interpretability* of the obtained model are contradictory properties directly depending on the learning process and/or the model structure. To overcome this problem, many different possibilities have been considered in the specialized literature [3], improving the accuracy of the LFM and maintaining the interpretability to a high degree.

One of the most used approaches to improve the accuracy of the fuzzy rule-based systems (FRBSs) is the tuning of MFs, which consists of the variation of the different parameters that identify the membership functions associated to the labels composing the Data Base (DB), 3 parameters when triangular membership functions [4] are considered. In the case of problems presenting a large number of variables this leads to tuning models with too many parameters, which could affect to the good performance of the optimization method considered.

In [1], a new linguistic rule representation model was presented for the genetic tuning of the DB. This approach is based on the linguistic 2-tuples representation [2], which allows the lateral displacement of the labels considering an unique parameter per label. In this way, two main objectives were achieved:

- to obtain linguistic labels containing a set of samples with a better covering degree (accuracy improvements) maintaining their original shapes, and
- to reduce the search space respect to the classical tuning in order to easily obtain optimal models.

However, the amplitude of the support of the membership functions is fixed through this tuning process. This amplitude determines the specificity of a label and involves a potential accuracy improvement, since it could determine the best covering region of such label, although it involves a slight lost of interpretability.

In this work, we extend the 2-tuples representation model to also perform a tuning of the support amplitude of the membership functions, with the main aim of improving the system accuracy and trying to maintain part of the interpretability as much as possible respect to the lateral tuning. To do so, we present a new symbolic representation with three values (s, α, β) respectively representing a label, the lateral displacement and the amplitude variation of the support of its associated membership function. The tuning method consists of the optimization of the two parameters α and β for each label considered in the Rule Base (RB). It also involves a search space reduction respect to the classical tuning that helps to the evolutionary search technique to obtain more precise Knowledge Bases.

The next section presents the proposed lateral and amplitude tuning of membership functions and the new model for rule representation. Section III proposes the evolutionary tuning method considered in this work. Section IV shows an experimental study of the method behavior applied on a real-world estimation problem. Finally, section V points out some concluding remarks.

II. Lateral and Amplitude Tuning

In this section, we will introduce the lateral tuning of membership functions. Then, the extension of the lateral tuning to also perform the amplitude tuning will be described, presenting the new rule representation and two different tuning approaches (global approach and local approach).

A. Preliminaries: The Lateral Tuning

In [1], a new model of tuning of FRBSs was proposed considering the linguistic 2-tuples representation scheme introduced in [2], which allows the lateral displacement of the support of a label and maintains the interpretability associated to the obtained linguistic FRBSs. This proposal also introduces a new model for rule representation based on the symbolic translation concept.

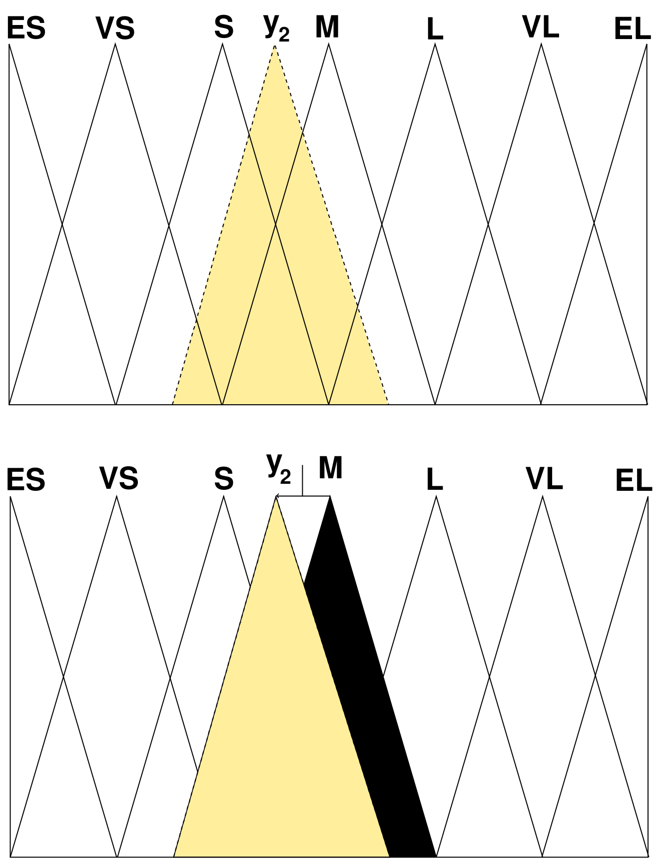

Fig. 1. Lateral Displacement of the Linguistic Label M

Figure 1 shows the lateral displacement of the label M. The new label "y_2" is located between B and M, being enough smaller than M but closer to M.

The symbolic translation of a linguistic term is a number within the interval [-0.5, 0.5) that expresses the domain of a label when it is moving between its two lateral labels. Formally, we have the pair,

$$(s_i, \alpha_i), \;\; s_i \in S, \;\; \alpha_i \in [-0.5, 0.5).$$

Figure 2 depicts the symbolic translation of a label represented by the pair $(S_2, \; -0.3)$, considering a set S with five linguistic terms represented by their ordinal values ({0, 1, 2, 3, 4}).

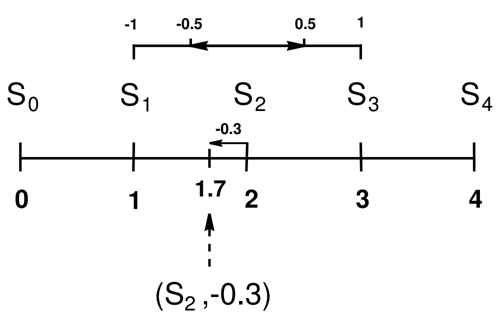

Fig. 2. Symbolic translation of a label

In [2], both the linguistic 2-tuples representation model and the needed elements for linguistic information comparison and aggregation are presented and applied to the Decision Making framework. In the context of the FRBSs, we are going to see its use in the linguistic rule representation. In the next we present this approach considering a simple control problem.

Let us consider a control problem with two input variables, one output variable and a DB defined from experts determining the membership functions for the following labels:

$$\begin{aligned} X_1&: \;\; Error \rightarrow \{N, Z, P\}, \\ X_2&: \;\; \triangledown Error \rightarrow \{N, Z, P\}, \\ Y&: \;\; Power \rightarrow \{L, M, H\} \;\; . \end{aligned}$$

Based on this DB definition, an example of classical rule and linguistic 2-tuples represented rule is:

Classical Rule,
R1: If the **error** is Zero and the $\triangledown$**Error** is Positive then the **Power** is High .

Rule with 2-Tuples Representation,
R1: If the **error** is (Zero, 0.3) and the $\triangledown$**Error** is (Positive, -0.2) then the **Power** is (High, -0.1) .

Analized from the rule interpretability point of view, we could interpret the obtained rule as:

If the **Error** is "higher than Zero" and
the **Error Variation** is "a little smaller than Positive"
then the **Power** is "a bit smaller than High".

B. The Lateral and Amplitude Tuning of Membership Functions

The lateral tuning model tunes the lateral displacements of the support of the membership functions whereas the amplitude of the support of such membership functions remains fixed during all the tuning process. However, This amplitude determines the specificity of a label and involves a potential accuracy improvement, since it could determine the best covering region of such label, althougt the interpretability is lost to some degree.

To adjust the displacements and amplitudes of the membership function supports we propose a new rule representation that considers two parameters, α and β, relatively representing the lateral displacement and the amplitude variation of a label. In this way, each label can be represented by a 3-tuple (s, α, β), where α is a number within the interval [-0.5, 0.5) that expresses the domain of a label when it is moving between its two lateral labels (as in the 2-tuples representation), and β is also a number within the interval [-0.5, 0.5) that allows to increase or reduce the support amplitude of a label until a 50% of its original size. Let us consider a set of labels S representing a fuzzy partition. Formally, we have the triplet,

$$(s_i, \alpha_i, \beta_i), \;\; s_i \in S, \;\; \{\alpha_i, \beta_i\} \in [-0.5, 0.5).$$

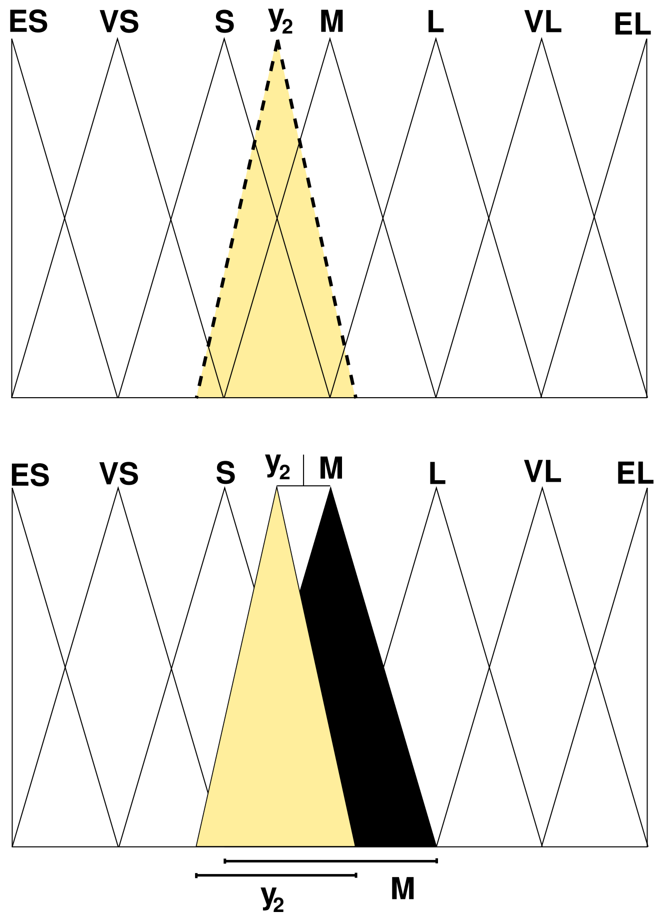

Fig. 3. Lateral Displacement and Amplitude Variation of the Linguistic Label M considering the set of labels S={ES, VS, S, M, L, VL, EL }

Figure 3 depicts the lateral and amplitude variation of the label M considering triangular and symmetrical equidistant membership functions. The new label "y_2" is located between labels S and M, and has a shorter support than the original label M. Let us represent the new label "y_2" as the 3-tuple (M, α, β). The support of this label, Sup_{y_2}, can be computed in the following way:

$$\begin{array}{l} Sup_M = c_M - a_M \\ Sup_{y_2} = Sup_M + \beta * Sup_M \end{array} \quad ,$$

where c_M and a_M are respectively the right and the left extreme of the support of M, and Sup_M is the size of the support of M.

This proposal decreases the tuning complexity, since the 3 or 4 parameters per label (triangular or trapezoidal membership functions) are reduced to 2 parameters. In [1], two different rule representation approaches were proposed for the lateral tuning of membership functions, a global approach and a local approach. We will consider the same two possibilities to perform the proposal tuning, the most interpretable one and the most accurate one:

- Global Tuning of the Semantics. The tuning is applied to the level of linguistic partition. In this way, the pair (X_i, label) takes the same tuning values in all the rules where it is considered. For example, X_i is (High, 0.3, 0.1) will present the same values for those rules in which the pair "X_i is High" was initially considered.
- Local Tuning of the Rules. In this case, the tuning is applied to the level of rule. The pair (X_i, label) is tuned in a different way in each rule, based on the quality measures associated to the tuning method (usually the system error).

Rule k: X_i is (High, 0.2, 0.05)
Rule j: X_i is (High, -0.1, 0.3)

Notice that, since symmetrical triangular membership functions and a FITA (*First Infer, Then Aggregate*) fuzzy inference [5] will be considered in both, the global and the local approach, a tuning of the amplitude of the consequents has no sense, by which the β parameter will be only applied on the antecedents. In this way, considering the same control problem of the previous subsection, an example of a 3-tuples represented rule is (amplitude variation only applied in the antecedents):

Rule with 3-Tuples Representation,
R1: If the **error** is (Zero,0.3,0.1) and the $\triangledown$**Error** is (Positive,-0.2,-0.4) then the **Power** is (High,-0.1) .

On the other hand, the use of the β factor (amplitude) is close to the use of non-linear scaling factors [6], [7] or linguistic modifiers [6], [8]. However there are some differences with these approaches:

- By using non-linear scaling factors or linguistic modifiers an example that is covered by a label can not be uncovered and *vice versa*, which imposes some restrictions to the search.
- Contrary to the non-linear scaling factors or linguistic modifiers, the tuning of the support amplitude keeps the shape of the membership functions (triangular and symmetrical). In this way, from the parameters α and β applied to each label, we could obtain the equivalent triangular membership functions, by which the final tuned FRBS could be represented as a classical Mamdani [9], [10].

The evolutionary lateral tuning method based on this representation model is shown in the next section.

III. Evolutionary Post-Processing Algorithm

The automatic definition of fuzzy systems can be considered as an optimization or search process and nowadays, Evolutionary Algorithms, particularly GAs, are considered as the more known and used global search technique. Moreover, the genetic coding that they use allow them to include prior knowledge and to use it leading the search up. For this reason, Evolutionary Algorithms have been successfully applied to learn fuzzy systems in the last years, giving way to the appearance of the so called Genetic Fuzzy Systems (GFSs) [4], [11].

Evolutionary Algorithms in general and, GAs in particular, has been widely used in the tuning of FRBSs. In this work, we will consider the use of GAs to design the proposed tuning, particular by the genetic model of CHC [12]. In the following, the components needed to design the evolutionary tuning process are explained:

- Evolutionary model of CHC.
- DB codification
- Initial gene pool
- Chromosome evaluation
- Genetic operators

A. Evolutionary model of CHC

The genetic model of CHC makes use of a "Population-based Selection" approach. N parents and their corresponding offspring are combined to select the best N individuals to take part of the next population. The CHC approach makes use of an incest prevention mechanism and a restarting process to provoke diversity in the population, instead of the well known mutation operator.

This incest prevention mechanism will be considered in order to apply the crossover operator, i.e., two parents are crossed if their hamming distance divided by 2 is over a predetermined threshold, L_T. Since, we will consider a real coding scheme, we have to transform each gene considering a Gray Code with a fixed number of bits per gene ($BITSGENE$) determined by the system expert. In this way, the threshold value is initialized as:

$$L_T = (\#Genes * BITSGENE)/4.0,$$

where $\#Genes$ is the number of genes in the chromosome. Following the original CHC scheme, L_T is decremented by one when there is no new individuals in the population in one generation. In order to make this procedure independent of $\#Genes$ and $BITSGENE$, in our case, L_T will be decremented by a $\varphi\%$ of its initial value (being φ determined by the user, usually 10%). The algorithm restarts when L_T is below zero.

A scheme of this algorithm is shown in Figure 4.

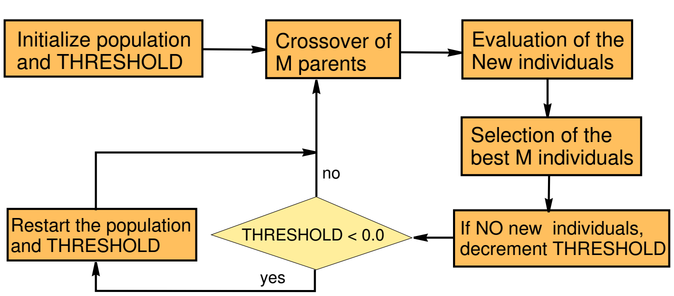

Fig. 4. Scheme of CHC

B. DB Codification and Initial Gene Pool

Taking into account, that two different types of tuning have been proposed (global tuning of the semantics and local tuning of the rules), there are two different kinds of coding methods. In both cases, a real coding scheme is considered, i.e., the real parameters are the GA representation units (genes).

In the following both schemes are presented:

- Global Tuning of the Semantics: Joint of the parameters of the fuzzy partitions, lateral (C^L) and amplitude (C^A) tuning. Let us consider the following number of labels per variable: $(m^1, \ldots, m^n)$, with n being the number of system variables ($n-1$ input variables and 1 output variable). Then, a chromosome has the following form (where each gene is associated to the tuning value of the corresponding label),

$$C_T = (C^L + C^A),$$

$$C^L = (c^L_{11}, \ldots, c^L_{1m^1}, \ldots, c^L_{n1}, \ldots, c^L_{nm^n}),$$
$$C^A = (c^A_{11}, \ldots, c^A_{1m^1}, \ldots, c^A_{(n-1)1}, \ldots, c^A_{(n-1)m^n}).$$

- Local Tuning of the Rules: Joint of the lateral (C^L) and amplitude (C^A) rule parameters. Let us condider that the FRBS has M rules, $(R1, R2, \ldots, RM)$, with n system variables ($n-1$ input variables and 1 output variable). Then, the chromosome structure is,

$$C_T = (C^L + C^A),$$

$$C^L = (c^L_{11}, \ldots, c^L_{1m^1}, \ldots, c^L_{n1}, \ldots, c^L_{nm^n}),$$
$$C^A = (c^A_{11}, \ldots, c^A_{1m^1}, \ldots, c^A_{(n-1)1}, \ldots, c^A_{(n-1)m^n}).$$

To make use of the available information, the initial FRBS obtained from automatic fuzzy rule learning methods is included in the population as an initial solution. To do so, the initial pool is obtained with the first individual having all genes with value '0.0' (no displacement or amplitude variation), and the remaining individuals generated at random in [-0.5, 0.5).

C. Chromosome Evaluation

To evaluate a determined chromosome we will use the well-known Mean Square Error (MSE):

$$\text{MSE} = \frac{1}{2 \cdot N} \sum_{l=1}^{N} (F(x^l) - y^l)^2,$$

with N being the data set size, $F(x^l)$ being the output obtained from the FRBS decoded from the said chromosome when the l-th example is considered and y^l being the known desired output.

D. Genetic Operators

The genetic operators considered in CHC are crossover and restarting approach (no mutation is considered). A description of these operators is presented in the following:

TABLE I

RESULTS OBTAINED BY THE STUDIED METHODS

Methods	#R	MSE$_{tra}$	σ_{tra}	t-test	MSE$_{test}$	σ_{test}	t-test
WM	65	57605	2841	+	57934	4733	+
WM + T	65	18602	1211	+	22666	3386	+
WM + PAL	65	10545	279	+	13973	1688	+
WM + GL	65	23064	1479	+	25654	2611	+
WM + LL	65	3664	390	+	5858	1798	+
WM + GLA	65	17950	1889	+	21212	2686	+
WM + LLA	65	**2747**	**282**	$\star$	**4540**	**788**	$\star$

1) Crossover Operator: The crossover operator is based on the the concept of environments. These kinds of operators show a good behavior as said in [13]. Particularly, we consider the PBLX operator (an operator based on the BLX-α). This operator presents a good cooperation when it is introduced within models forcing the convergence by pressure on the offspring (as the case of CHC).

2) Reinicialización: To get away from local optima, this algorithm uses a restart approach [12]. In this case, the best chromosome is maintained and the remaining are generated at random by adding to each gene of the best chromosome a random number generated within the variation interval [-0.125, 0.125). If the resulting value is minor (major) than -0.5 (0.5) it is replaced by the extreme value -0.5 (0.5). It follows the principles of CHC [12], performing the restart procedure when the threshold value L_T is lower than zero.

IV. EXPERIMENTS AND ANALYSIS OF RESULTS

To evaluate the goodness of the two proposed approaches, local and global tuning, several experiments have been carried out considering a real-world problem [14]. This problem handles four input variables and therefore, it involves *a large search space*. A short description of this problem can be found in the following subsection.

TABLE II

STUDIED METHODS

Ref.	Methods	Description
[15]	**WM**	*Ad-hoc* Data-Driven Method
[16]	**T**	Classical Genetic Tuning
[17]	**PAL**	Tuning of Parameters, Domains and Local Linguistic Modifiers
[1]	**GL**	Global Lateral Tuning
[1]	**LL**	Local Lateral Tuning
—	**GLA**	Global Lateral and Amplitude Tuning
—	**LLA**	Local Lateral and Amplitude Tuning

Table II presents a brief description of the studied methods. The WM method is considered to obtain the initial RB to be tuned. The tuning methods are applied once this initial RB has been obtained. T is a classical membership function parameter tuning algorithm. The PAL method has been compared with tuning methods of the parameters, domain, linguistic modifiers and with any combination of any two of them obtaining the best results [17]. For this reason, we only consider the PAL method (parameters, domains and linguistic edges) in this study.

The initial linguistic partitions to obtain the initial RB are comprised by *five linguistic terms* with triangular-shaped fuzzy sets giving meaning to them (number of labels by which they presented the best behavior). With respect to the fuzzy reasoning method used, we have selected the *minimum t-norm* playing the role of the implication and conjunctive operators, and the *center of gravity weighted by the matching* strategy acting as the defuzzification operator.

Finally, the following values have been considered for the parameters of each method: 50 individuals, 50,000 evaluations and φ=0.1 (0.2 as mutation probability per chromosome, 0.6 as crossover probability and 0.35 for the factor a in the max-min-arithmetical crossover operator for T and PAL).

A. Problem Description: Estimating the Maintenance Costs of Medium Voltage Lines

This problem consist of relating the *maintenance costs of medium voltage line* with the following four variables: *sum of the lengths of all streets in the town*, *total area of the town*, *area that is occupied by buildings*, and *energy supply to the town*. We will deal with estimations of minimum maintenance costs based on a model of the optimal electrical network for a town in a sample of 1,059 towns. A wider description of this problem can be found in [14].

To develop the different experiments in this contribution, we consider a *5-folder cross-validation model*, i.e., 5 random partitions of data with a 20%, and the combination of 4 of them (80%) as training and the remaining one as test. In this way, 5 partitions considering an 80% (847) in training and a 20% (212) in test are considered for the experiments.

B. Results and Analysis

For each one of the 5 data partitions, the tuning methods has been run 6 times, showing for each problem the averaged results of a total of 30 runs. Moreover, a *t-test* (with 95 percent confidence) was applied to the best averaged result in training or test by comparing one by one this result to the averaged results of the remaining methods.

The results obtained by the analyzed methods are shown in Table I, where $\#R$ stands for the number of rules, MSE$_{tra}$ and MSE$_{tst}$ respectively for the averaged error obtained over

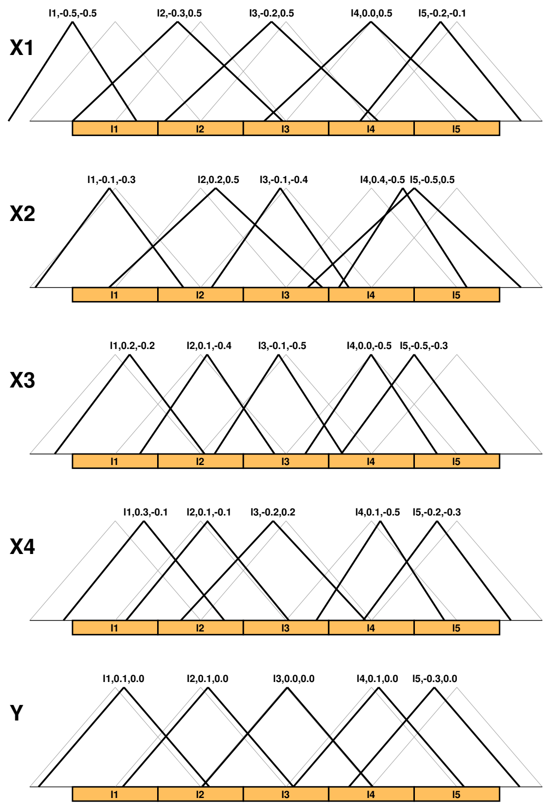

Fig. 5. Initial and tuned DB of a model obtained with WM+GLA (global approach)

the training and test data, σ for the standard deviation and *t-test* represents the following information:

- $\star$ Denotes the best averaged result
- $+$ Denotes a significant worst behavior than the best

Analyzing the results presented in Table I we can point out the following conclusions:

- The lateral and amplitude tuning methods show an important reduction of the MSE respect to the classical methods (specially the WM+LLA method) and reasonable improvements respect to the lateral tuning.
- The best results are obtained by the local approach, presenting a good relationship between the search space complexity and the results obtained, and getting a good trade-off between accuracy and local interpretability. Furthermore, since the lateral and amplitude variations are related to the original global labels, a global interpretation could be done in these terms.
- The WM+GLA method obtains better results than the classical tuning, which does not achieve the WM+GL method.
- The WM+LLA method reduces the typical deviation in both, training and test. It seems to show the robustness of this method.

Figures 5 and 6 respectively depicts the evolved fuzzy linguistic partitions and the RB obtained by the WM+GLA

#R: 65 ***MSE-tra: 19636.934*** ***MSE-tst: 22378.074***

X1	X2	X3	X4	Y
I1 ,-0.50,-0.50	I1 ,-0.06,-0.27	I1 , 0.20,-0.25	I1 , 0.34,-0.12	I1 , 0.15
I1 ,-0.50,-0.50	I1 ,-0.06,-0.27	I1 , 0.20,-0.25	I2 , 0.12,-0.09	I2 , 0.13
I1 ,-0.50,-0.50	I2 , 0.25, 0.50	I1 , 0.20,-0.25	I1 , 0.34,-0.12	I1 , 0.15
I1 ,-0.50,-0.50	I2 , 0.25, 0.50	I1 , 0.20,-0.25	I2 , 0.12,-0.09	I2 , 0.13
I1 ,-0.50,-0.50	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I1 , 0.34,-0.12	I1 , 0.15
I1 ,-0.50,-0.50	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I2 , 0.12,-0.09	I2 , 0.13
I2 ,-0.27, 0.46	I1 ,-0.06,-0.27	I1 , 0.20,-0.25	I1 , 0.34,-0.12	I1 , 0.15
I2 ,-0.27, 0.46	I1 ,-0.06,-0.27	I1 , 0.20,-0.25	I2 , 0.12,-0.09	I2 , 0.13
I2 ,-0.27, 0.46	I1 ,-0.06,-0.27	I2 , 0.06,-0.42	I1 , 0.34,-0.12	I1 , 0.15
I2 ,-0.27, 0.46	I1 ,-0.06,-0.27	I2 , 0.06,-0.42	I2 , 0.12,-0.09	I2 , 0.13
I2 ,-0.27, 0.46	I2 , 0.25, 0.50	I1 , 0.20,-0.25	I1 , 0.34,-0.12	I1 , 0.15
I2 ,-0.27, 0.46	I2 , 0.25, 0.50	I1 , 0.20,-0.25	I2 , 0.12,-0.09	I2 , 0.13
I2 ,-0.27, 0.46	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I1 , 0.34,-0.12	I2 , 0.13
I2 ,-0.27, 0.46	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I2 , 0.12,-0.09	I2 , 0.13
I2 ,-0.27, 0.46	I3 ,-0.06,-0.39	I2 , 0.06,-0.42	I1 , 0.34,-0.12	I2 , 0.13
I2 ,-0.27, 0.46	I3 ,-0.06,-0.39	I2 , 0.06,-0.42	I2 , 0.12,-0.09	I2 , 0.13
I2 ,-0.27, 0.46	I3 ,-0.06,-0.39	I3 ,-0.09,-0.50	I1 , 0.34,-0.12	I3 , 0.02
I2 ,-0.27, 0.46	I3 ,-0.06,-0.39	I3 ,-0.09,-0.50	I2 , 0.12,-0.09	I3 , 0.02
I3 ,-0.15, 0.50	I2 , 0.25, 0.50	I1 , 0.20,-0.25	I1 , 0.34,-0.12	I1 , 0.15
I3 ,-0.15, 0.50	I2 , 0.25, 0.50	I1 , 0.20,-0.25	I2 , 0.12,-0.09	I2 , 0.13
I3 ,-0.15, 0.50	I2 , 0.25, 0.50	I1 , 0.20,-0.25	I3 ,-0.15, 0.16	I2 , 0.13
I3 ,-0.15, 0.50	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I1 , 0.34,-0.12	I2 , 0.13
I3 ,-0.15, 0.50	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I2 , 0.12,-0.09	I2 , 0.13
I3 ,-0.15, 0.50	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I3 ,-0.15, 0.16	I3 , 0.02
I3 ,-0.15, 0.50	I3 ,-0.06,-0.39	I2 , 0.06,-0.42	I1 , 0.34,-0.12	I2 , 0.13
I3 ,-0.15, 0.50	I3 ,-0.06,-0.39	I2 , 0.06,-0.42	I2 , 0.12,-0.09	I3 , 0.02
I3 ,-0.15, 0.50	I3 ,-0.06,-0.39	I2 , 0.06,-0.42	I3 ,-0.15, 0.16	I3 , 0.02
I3 ,-0.15, 0.50	I3 ,-0.06,-0.39	I3 ,-0.09,-0.50	I2 , 0.12,-0.09	I3 , 0.02
I3 ,-0.15, 0.50	I3 ,-0.06,-0.39	I3 ,-0.09,-0.50	I3 ,-0.15, 0.16	I3 , 0.02
I3 ,-0.15, 0.50	I4 , 0.37,-0.50	I3 ,-0.09,-0.50	I2 , 0.12,-0.09	I3 , 0.02
I3 ,-0.15, 0.50	I4 , 0.37,-0.50	I3 ,-0.09,-0.50	I3 ,-0.15, 0.16	I3 , 0.02
I3 ,-0.15, 0.50	I4 , 0.37,-0.50	I4 ,0.00,-0.45	I2 , 0.12,-0.09	I3 , 0.02
I3 ,-0.15, 0.50	I4 , 0.37,-0.50	I4 ,0.00,-0.45	I3 ,-0.15, 0.16	I4 , 0.08
I4 , 0.04, 0.50	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I1 , 0.34,-0.12	I2 , 0.13
I4 , 0.04, 0.50	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I2 , 0.12,-0.09	I2 , 0.13
I4 , 0.04, 0.50	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I3 ,-0.15, 0.16	I3 , 0.02
I4 , 0.04, 0.50	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I4 , 0.10,-0.50	I3 , 0.02
I4 , 0.04, 0.50	I3 ,-0.06,-0.39	I2 , 0.06,-0.42	I1 , 0.34,-0.12	I2 , 0.13
I4 , 0.04, 0.50	I3 ,-0.06,-0.39	I2 , 0.06,-0.42	I2 , 0.12,-0.09	I3 , 0.02
I4 , 0.04, 0.50	I3 ,-0.06,-0.39	I2 , 0.06,-0.42	I3 ,-0.15, 0.16	I3 , 0.02
I4 , 0.04, 0.50	I3 ,-0.06,-0.39	I2 , 0.06,-0.42	I4 , 0.10,-0.50	I3 , 0.02
I4 , 0.04, 0.50	I3 ,-0.06,-0.39	I3 ,-0.09,-0.50	I2 , 0.12,-0.09	I3 , 0.02
I4 , 0.04, 0.50	I3 ,-0.06,-0.39	I3 ,-0.09,-0.50	I3 ,-0.15, 0.16	I4 , 0.08
I4 , 0.04, 0.50	I3 ,-0.06,-0.39	I3 ,-0.09,-0.50	I4 , 0.10,-0.50	I4 , 0.08
I4 , 0.04, 0.50	I4 , 0.37,-0.50	I3 ,-0.09,-0.50	I1 , 0.34,-0.12	I3 , 0.02
I4 , 0.04, 0.50	I4 , 0.37,-0.50	I3 ,-0.09,-0.50	I2 , 0.12,-0.09	I3 , 0.02
I4 , 0.04, 0.50	I4 , 0.37,-0.50	I3 ,-0.09,-0.50	I3 ,-0.15, 0.16	I4 , 0.08
I4 , 0.04, 0.50	I4 , 0.37,-0.50	I3 ,-0.09,-0.50	I4 , 0.10,-0.50	I4 , 0.08
I4 , 0.04, 0.50	I4 , 0.37,-0.50	I4 ,0.00,-0.45	I2 , 0.12,-0.09	I4 , 0.08
I4 , 0.04, 0.50	I4 , 0.37,-0.50	I4 ,0.00,-0.45	I3 ,-0.15, 0.16	I4 , 0.08
I4 , 0.04, 0.50	I4 , 0.37,-0.50	I4 ,0.00,-0.45	I4 , 0.10,-0.50	I5 ,-0.26
I4 , 0.04, 0.50	I5 ,-0.50, 0.50	I4 ,0.00,-0.45	I2 , 0.12,-0.09	I3 , 0.02
I4 , 0.04, 0.50	I5 ,-0.50, 0.50	I4 ,0.00,-0.45	I3 ,-0.15, 0.16	I4 , 0.08
I4 , 0.04, 0.50	I5 ,-0.50, 0.50	I4 ,0.00,-0.45	I4 , 0.10,-0.50	I5 ,-0.26
I4 , 0.04, 0.50	I5 ,-0.50, 0.50	I5 ,-0.50,-0.29	I2 , 0.12,-0.09	I5 ,-0.26
I4 , 0.04, 0.50	I5 ,-0.50, 0.50	I5 ,-0.50,-0.29	I3 ,-0.15, 0.16	I5 ,-0.26
I5 ,-0.23,-0.11	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I2 , 0.12,-0.09	I2 , 0.13
I5 ,-0.23,-0.11	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I4 , 0.10,-0.50	I3 , 0.02
I5 ,-0.23,-0.11	I2 , 0.25, 0.50	I2 , 0.06,-0.42	I5 ,-0.23,-0.27	I4 , 0.08
I5 ,-0.23,-0.11	I2 , 0.25, 0.50	I3 ,-0.09,-0.50	I2 , 0.12,-0.09	I3 , 0.02
I5 ,-0.23,-0.11	I2 , 0.25, 0.50	I3 ,-0.09,-0.50	I4 , 0.10,-0.50	I3 , 0.02
I5 ,-0.23,-0.11	I2 , 0.25, 0.50	I3 ,-0.09,-0.50	I5 ,-0.23,-0.27	I4 , 0.08
I5 ,-0.23,-0.11	I4 , 0.37,-0.50	I3 ,-0.09,-0.50	I2 , 0.12,-0.09	I3 , 0.02
I5 ,-0.23,-0.11	I4 , 0.37,-0.50	I3 ,-0.09,-0.50	I4 , 0.10,-0.50	I4 , 0.08
I5 ,-0.23,-0.11	I4 , 0.37,-0.50	I3 ,-0.09,-0.50	I5 ,-0.23,-0.27	I5 ,-0.26

Fig. 6. RB of a model obtained with WM+GLA

from one of the 30 runs performed.

V. CONCLUSIONES

In this work, we extend the lateral tuning of membership functions proposing a new post-processing method for the lateral and amplitude tuning of membership functions. This approach proposes a new representation model which extend the linguistic 2-tuples representation model with a parameter β to tune the amplitude of the support of the labels.

The linguistic 3-tuples based rule representation together with the proposed evolutionary tuning algorithm, provides a good mechanism to obtain accurate models, although it involves a slight lost of interpretability, specially in the local approach. However, in most of the cases only small variations have been performed on the original membership functions, preserving the interpretability to a reasonable level.

The use of rule selection methods to reduce the number of rules together with the lateral and amplitude tuning is a good further work to obtain more compact models with a similar accuracy.

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REFERENCES

[1] R. Alcalá, J. Casillas, O. Cordón, A. González, and F. Herrera, "A genetic rule weighting and selection process for fuzzy control of heating, ventilating and air conditioning systems," *Engineering Applications of Artificial Intelligence*, vol. 18, no. 3, pp. 279–296, 2005.

[2] F. Herrera and L. Martínez, "A 2-tuple fuzzy linguistic representation model for computing with words," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 6, pp. 746–752, 2000.

[3] J. Casillas, O. Cordón, F. Herrera, and L. Magdalena, Eds., *Accuracy improvements in linguistic fuzzy modeling*. Springer-Verlag, 2003.

[4] O. Cordón, F. Herrera, F. Hoffmann, and L. Magdalena, *GENETIC FUZZY SYSTEMS. Evolutionary tuning and learning of fuzzy knowledge bases*, ser. Advances in Fuzzy Systems - Applications and Theory. World Scientific, 2001, vol. 19.

[5] O. Cordón, F. Herrera, and A. Peregrín, "Applicability of the fuzzy operators in the design of fuzzy logic controllers," *Fuzzy Sets and Systems*, vol. 86, no. 1, pp. 15–41, 1997.

[6] O. Cordón, M. J. del Jesús, and F. Herrera, "Genetic learning of fuzzy rule-based classification systems cooperating with fuzzy reasoning methods," *International Journal of Intelligent Systems*, vol. 13, pp. 1025–1053, 1998.

[7] B. Liu, C. Chen, and J. Tsao, "Design of adaptive fuzzy logic controller based on linguistic-hedge concepts and genetic algorithms," *IEEE Trans. Syst., Man, Cybern. - Part B: Cybernetics*, vol. 31, no. 1, pp. 32–53, 2001.

[8] A. González and R. Pérez, "A study about the inclusion of linguistic hedges in a fuzzy rule learning algorithm," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 7, no. 3, pp. 257–266, 1999.

[9] E. H. Mamdani, "Application of fuzzy algorithms for control of simple dynamic plant," in *Proc. IEEE*, vol. 121, no. 12, 1974, pp. 1585–1588.

[10] E. H. Mamdani and S. Assilian, "An experiment in linguistic synthesis with a fuzzy logic controller," *International Journal of Man-Machine Studies*, vol. 7, pp. 1–13, 1975.

[11] O. Cordón, F. Gomide, F. Herrera, F. Hoffmann, and L. Magdalena, "Ten years of genetic fuzzy systems: Current framework and new trends," *Fuzzy Sets and Systems*, vol. 41, no. 1, pp. 5–31, 2004.

[12] L. J. Eshelman, "The CHC adaptive search algorithm: How to have safe serach when engaging in nontraditional genetic recombination," in *Foundations of genetic Algorithms*, G. Rawlin, Ed. Morgan Kaufman, 1991, vol. 1, pp. 265–283.

[13] F. Herrera, M. Lozano, and A. M. Sánchez, "A taxonomy for the crossover operator for real-coded genetic algorithms: An experimental study," *International Journal of Intelligent Systems*, vol. 18, pp. 309–338, 2003.

[14] O. Cordón, F. Herrera, and L. Sánchez, "Solving electrical distribution problems using hybrid evolutionary data analysis techniques," *Applied Intelligence*, vol. 10, pp. 5–24, 1999.

[15] L. X. Wang and J. M. Mendel, "Generating fuzzy rules by learning from examples," *IEEE Trans. Syst., Man, Cybern.*, vol. 22, no. 6, pp. 1414–1427, 1992.

[16] O. Cordón and F. Herrera, "A three-stage evolutionary process for learning descriptive and approximate fuzzy logic controller knowledge bases from examples," *International Journal of Approximate Reasoning*, vol. 17, no. 4, pp. 369–407, 1997.

[17] J. Casillas, O. Cordón, M. J. del Jesús, and F. Herrera, "Genetic tuning of fuzzy rule deep structures preserving interpretability and its interaction with fuzzy rule set reduction," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 1, pp. 13–29, 2005.