

## Taximeter verification using imprecise data from GPS

José Villar, Adolfo Otero, José Otero\*, Luciano Sánchez

Computer Science Department, Oviedo University, Spain

### ARTICLE INFO

#### Article history:

Received 20 July 2007

Received in revised form

29 May 2008

Accepted 11 July 2008

Available online 19 October 2008

#### Keywords:

Genetic fuzzy systems

Fuzzy fitness function

GPS

Metrology

Vague data

### ABSTRACT

Until recently, local governments in Spain were using machines with rolling cylinders for testing and verification of taximeters. However, the tyres condition can lead to errors in the process and the mechanical construction of the test equipment is not compatible with certain vehicles. Thus, a new measurement device should be designed.

In our opinion, the verification of a taximeter will not be reliable unless measurements taken on an actual taxi run are used. Global positioning system (GPS) sensors are intuitively well suited for this process, because they provide the position and the speed with independence from those car devices that are under test. Nevertheless, since GPS measurements are inherently imprecise, GPS-based sensors are difficult to homologate. In this paper we will show how these legal problems can be solved. We propose a method for computing an upper bound of the length of the trajectory, taking into account the vagueness of the GPS data. The uncertainty in the GPS data will be modelled by fuzzy techniques. The upper bound will be computed using a multiobjective evolutionary algorithm. The accuracy of the measurements will be improved further by combining it with restrictions based on the dynamic behavior of the vehicles.

© 2008 Elsevier Ltd. All rights reserved.

### 1. Introduction

Taxi fares in Spain are revised once a year, and the taximeters must be recalibrated and verified by a Technical Inspection (TI) through a certified station. The tariff depends on two variables: the speed of the taxi and the distance travelled. In particular, while the speed of the taxi is lower than a threshold, the user is being charged for the elapsed time. Otherwise, he/she is charged for the distance.

Since 1990, the Metrology and Models group at Oviedo University has been responsible for the design of the equipment and devices needed for the verification of taxis in Asturias, Spain. Currently, the verification of the taximeters is being carried out using of a machine with rollers. The drive wheels are placed on the rollers, whose speed is regularly sampled. The test lasts a few minutes, while the driver must be assisted by a TI technician. The total distance in the simulated run is computed by multiplying the circumference of the roller by the number of turns, and the linear speed is estimated from the angular speed of the rollers.

However, the use of a machine with rollers presents some drawbacks:

1. The rollers have a relatively small radius, and the tyres do not deform the same over the rollers than over a flat surface. The

difference between the actual and the theoretical radius means that tyres appear to be smaller than they are for the system. Moreover, this error depends on the tyres condition and the weight of the vehicle.

2. A TI employee can only verify one taximeter at a time, and this task lasts between 15 and 30 min. The price charged for the test could be reduced if an unattended procedure is devised, for which this employee would not be needed.
3. Problems have been detected when verifying a taximeter in a car with electronic driving aids (such as ESP, TCS, etc.). In these cars, the signals that feed the taximeter are taken from the electronic control unit. But, when the car is placed in the rollers, two wheels are moving and the other two are locked. For certain brands of vehicles, the electronic control unit is not prepared for such an abnormal driving condition, and the unit does not produce information about the speed of the moving wheels.

The third problem is the most daunting, because it means that the verification of certain vehicles is not possible. In these cases, length and time fares are verified in two different tests. A chronometer is used for verifying the time fare, and the distance fare is being checked through an actual run of the cab, in a circuit with a known length. Nevertheless, not all TI stations own this kind of facilities. Moreover, we have concerns with this procedure, because the speed of the taxi is metered with the instruments of the vehicle being tested.

\* Corresponding author. Tel.: +34 98 5182500; fax: +34 98 5103382.

E-mail addresses: [villarjose@uniovi.es](mailto:villarjose@uniovi.es) (J. Villar), [otero@lsi.uniovi.es](mailto:otero@lsi.uniovi.es) (A. Otero), [jotero@lsi.uniovi.es](mailto:jotero@lsi.uniovi.es), [jotero@uniovi.es](mailto:jotero@uniovi.es) (J. Otero), [luciano@uniovi.es](mailto:luciano@uniovi.es) (L. Sánchez).

In this situation, ITVASA, the company responsible of the TI in Asturias, Spain, asked the Metrology and Models group at Oviedo University for the design and development of a system capable of the verification of taximeters task with the following requirements: (a) the measures should not depend on the condition of the tyres of the vehicle, (b) it should be possible to verify the time fare and the length fare in an unique test, (c) the test should be unattended, (d) the accuracy of the device should comply with the regulations and (e) any vehicle could be tested, even those with electronic aids.

We have introduced a new portable system that fulfills all these requirements. Our system uses a global positioning system (GPS) receiver to sample the position and the speed of the taxi at regular intervals. The new test is less expensive, because the technician is no longer needed and the cabin where the rollers are installed would be freed for other uses. Due to cost reasons, we also want to use a cheap, consumer grade, GPS. We are aware that higher quality GPS receivers are more accurate, however, each station must acquire between 10 and 20 devices (otherwise the queuing time would not be acceptable) and the cost of deploying the new solution has to be amortized in a few years.

Additionally, there are legal problems that difficult the use of GPS measurements. It is well known that GPS data are inherently imprecise. Moreover, the tolerance is neither a prior knowledge nor constant, but it varies with each measurement. Unless we are able to bound the tolerance in our estimation of the length of the trajectory and the speed of the vehicle, we will not be able to legally reject a taximeter. This is the same problem that happens, for instance, when speed penalties are applied by a highway radar: we cannot penalize a vehicle whose speed is higher than the limit unless we also know (a) the tolerance of the radar and (b) that the measured speed surpasses the limit by more than that tolerance. In any other case, we must assume that the driver (and, conversely, the taxi owner) has not committed an offence.

Therefore, it is difficult to homologate a GPS-based device, because we cannot assess its absolute accuracy, e.g. the tolerance may be 5% for a certain route and 11% for a different route. To solve this problem, in this paper we propose a device that not only produces an estimation of the length of the trajectory, but it also computes an upper bound of this length. In other words, our system accounts the uncertainty of the measures, detects those cases where the accuracy of the test is not well under 10% and invalidates the test, if needed. This way, our system guaranties a minimum accuracy and therefore it can be homologated.

### 1.1. Legal constraints and statistical decisions

Let us suppose we have a measurement device which, given a taximeter with an unknown error  $e$ , produces an estimation  $\hat{e}$  of its error. We will assume that the device is unbiased, i.e.  $E(\hat{e} - e) = 0$ . Therefore, we could define the trivial decision rule that follows:

$$D_0(\hat{e}) = \begin{cases} \text{Accept} & \text{if } \hat{e} \leq 0, \\ \text{Reject} & \text{otherwise.} \end{cases} \quad (1)$$

However, any measurement device will have a tolerance  $\varepsilon$ : this means that  $\hat{e} - \varepsilon \leq e \leq \hat{e} + \varepsilon$  with a very high probability and, conversely, that the probability  $p(|\hat{e} - e| > \varepsilon)$  is near zero. This tolerance has legal implications. Suppose, for instance, that we reject a taximeter because we estimate that its error is  $\hat{e} = 5\%$ , and the tolerance of our device is  $\varepsilon = 7\%$ , which is higher than this error. The taxi owner could argue that there is a chance that the true error of the taximeter is less than or equal to 0, and have our rejection revoked. In short, we cannot reject a taxi unless the

estimation of the error is higher than the tolerance, thus we are sure that the taximeter is incorrect with a high probability.

In Spain, the maximum deviation between the charged fare and the true fare must not be higher than 10%, and therefore we cannot homologate a device with a tolerance higher than this value. It is remarked that the tolerance of a GPS device depends on many factors (geometry or constellation of the satellites, shape of the trajectory, speed, etc.) As we have mentioned in the preceding section, we cannot certify that all measurements taken with the certain GPS device will be more accurate than 10%. However, we will show in this paper that we can determine whether the tolerance of a particular measure has been within the legal margins. This is the main objective of this paper.

We have also mentioned that the legal problem is similar to that of using a radar for measuring the speed of a vehicle. But, the legal assumption of innocence, that most drivers would be glad to accept if accused of surpassing the speed limit, benefits the taxi owner.

The legal decision rule is

$$D(\hat{e}) = \begin{cases} \text{Accept} & \text{if } \hat{e} \leq 10, \\ \text{Reject} & \text{otherwise} \end{cases} \quad (2)$$

which, from a statistical point of view, is not fair. Let  $p(x)$  be the probability of the error of a taximeter being  $x$ . On the one hand, we can reject a correct taximeter with a probability

$$P(\text{Reject} | e \leq 0) = \frac{p(\hat{e} > 10 \cap e \leq 0)}{p(e \leq 0)}. \quad (3)$$

On the other hand, we will pass an incorrect taximeter with probability

$$P(\text{Accept} | e > 0) = \frac{p(\hat{e} \leq 10 \cap e > 0)}{p(e > 0)}. \quad (4)$$

For  $D$  to be fair, we need that both errors are the same. However, they are not. If the tolerance of the device is lower than 10%, then  $p(\hat{e} > 10 \cap e \leq 0)$  is near zero, thus  $P(\text{Reject} | e \leq 0)$  is negligible, but  $P(\text{Accept} | e > 0) \geq p(0 < e < 10 - \varepsilon) / p(e > 0)$ , which is rather high. According to our own experience, Spanish taxi drivers calibrate their taximeters to obtain their maximum legal advantage, i.e. it is by far more frequent that a taxi has an error near 10 than an error lower than 0.

### 1.2. Lower upper bound of a trajectory

In order to obtain the highest number of valid measurements (i.e. those for which the tolerance is lower than 10%) we are interested in knowing the shortest trajectory whose length is known to be longer than the actual path, given a set of imprecise coordinates of the vehicle. Therefore, in this work, it is proposed to calculate the lower upper bound (from now on called LUB) of all possible trajectories that are compatible with the measurements given by the GPS.

Due to the imprecise nature of input data, a new method for establishing the LUB is presented. In doing so, input data are represented as fuzzy data. These data are filtered in order to produce the smallest subset of coordinates that induces a multipolygonal that covers the input data as much as possible. Finally, that filtered data are fed to a deterministic algorithm for computing the upper bound of length of the trajectory.

The filtering process is the most complex part of the procedure. It involves solving a multicriteria optimization problem, for which we will use the genetic algorithm NSGA-II (Deb et al., 2000; Deb and Goel, 2001).

### 1.3. Summary

The structure of this work is as follows. In the next section, we describe how GPS measurements are obtained. We also explain the imprecise nature of GPS measurements, and how they can be interpreted as fuzzy data. Then, a description of the proposal is set out in Section 3, where the filtering process and the issues regarding LUB computation are detailed. Deterministic (Section 3.2) and randomized (Section 3.3) algorithms for computing the LUB are given in the same section. In Section 4 details about the genetic algorithm used to filter the data are given. In Section 5 numerical results are shown. Finally, conclusions and future work are presented.

## 2. The vague nature of the GPS measurements

The term GPS (Hofmann-Wellenhof et al., 2004) refers to a set of devices (satellites and receiver) working together to get a fix (the position) of the receiver. The receiver can receive some signals from the satellites and compute a set of measurements: longitude, latitude, altitude, number of satellites in use, time, etc. Each signal received from a satellite contains information about the time that the signal takes from the satellite to the receiver.

So it can be thought that using signals from four satellites (three for geographical coordinates and one for time correction) could be enough to achieve a fix. A fix computed with that information, however, is very inaccurate: there are some errors in GPS technology that make it necessary to receive signals from more than four satellites. Some of the sources of these errors are: perturbations of the satellites signals when crossing the atmosphere, satellite ephemerids deviation, satellite clock errors, receiver errors and multipath (signals are not received directly from the satellite).

As a rule of thumb, the higher the number of satellites the better the accuracy. But even with a high number of satellites in use (12–16) the geometry or constellation of the satellites must be taken into account to estimate the fix accuracy. This is done using DOP (dilution of precision), a measurement of the probability of the effects of the constellation on the fix accuracy (Langley, 1999); a higher value of DOP indicates a weaker geometry of satellites. DOP has four components: PDOP (3D or spherical DOP), HDOP (latitude and longitude DOP), VDOP (vertical DOP) and TDOP (time DOP).

Under certain conditions, GPS measurement errors follow a bidimensional Gaussian distribution. When many satellites are available that distribution can be regarded as circular (van Diggelen, 2007). Because of this, consumer grade GPS gives an indication of their precision through a magnitude called circular error probable (CEP). Given a probability threshold, the CEP indicates the radius of a circle. This circle is approximately centered on the position where the receiver was when it registered the measurement. If the threshold is 50% the CEP can be calculated from the standard errors of the estimated coordinates with (Langley, 1991; Strang and Borre, 1997)

$$\text{CEP} = 0.56\sigma_x + 0.62\sigma_y, \quad (5)$$

where  $\sigma_x$  and  $\sigma_y$  are the horizontal components of the standard deviations of the measurements. The CEP at 95% probability is also known as R95 and can be obtained multiplying by 2.08 the 50% probability CEP. In Fig. 1, a real example showing how the CEP could vary between consecutive measurements is displayed.

Consumer grade GPS does not send information related to the standard errors. HDOP values are available from the standard NMEA protocol used in most of the GPS receivers and this magnitude accounts for the impact of constellation geometry in

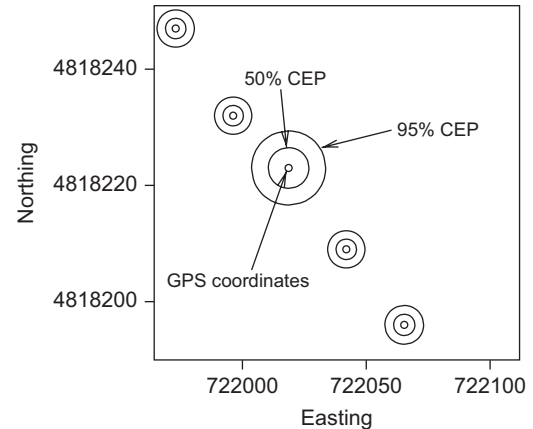


Fig. 1. Real GPS measurements, the CEP value changes from one measurement to another.

horizontal accuracy. Thus, an empirical estimation of CEP/HDOP relationship must be carried out (Dussault et al., 2001; Cressie, 1991). From the definition of CEP, this is easily done from a sample of GPS coordinates taken in a given location. For each HDOP value, the subset of GPS obtained under that value is extracted from the whole data. Then the smallest circle that covers 50% of the points is the CEP at that probability. The procedure for the 95% CEP is analogous.

### 2.1. A fuzzy representation of GPS data

In the context of imprecise probabilities, a fuzzy set could be seen as a set of tolerances. Each tolerance is assigned a confidence rate, and the lower the tolerance the lower the confidence rate (Goodman and Nguyen, 1985). In particular, given an incomplete set of confidence intervals of a random variable, it is possible to generate a random fuzzy variable for which  $\alpha$ -cuts are confidence intervals of rate  $(1 - \alpha)$  (Couso et al., 2001). We will use this representation and perform a multilevel calculation of the LUB.

In the case of the measurements obtained from a GPS unit, two confidence intervals are given at 50% and at 95%. Using the procedure explained in this section, a value of CEP could be calculated for each probability value (Manning and Harvey, 1994). It can easily be seen that the higher the probability value the higher the CEP value. The physical meaning of this fact is simple for the GPS measurements: the higher the confidence rate needed for determining the real position from which the GPS measurement was taken, the higher the CEP value.

## 3. Calculation of a LUB using fuzzy data

The GPS measurements are sampled at equally spaced time intervals. Each measurement is a fuzzy set, as stated before, whose  $\alpha$ -cuts are circles centered on the GPS coordinates. Therefore, each circle is a confidence interval for the true coordinates of the taxi when the measurement was taken. In Fig. 2 some simulated GPS measurements and trajectories are shown. The position where the measurement was taken is on the real trajectory—continuous line—and it can be inside or outside the respective circle of radius CEP. The trajectory using the GPS coordinates is drawn using a dashed line. A trajectory totally compatible with the GPS measurements is also drawn as a dotted line. Notice that the lengths of these trajectories are different, but all of them are compatible with the measurements of the GPS. Thus, to know the accuracy of the measure, we want to compute the LUB of the lengths of all the paths which have all of their

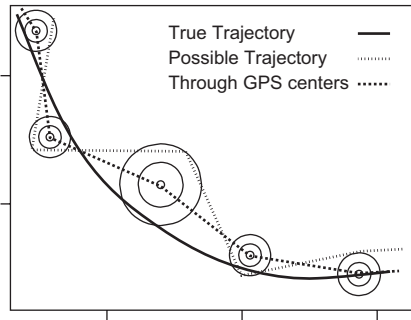


Fig. 2. Simulated example where the differences between the true trajectory and the trajectory through the GPS coordinates are shown.

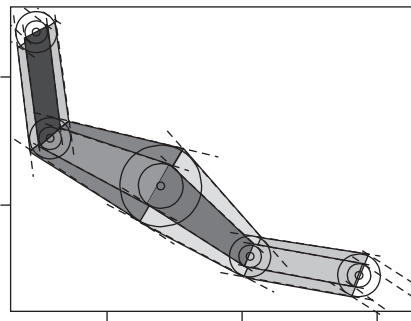


Fig. 3. Simulated example showing how a polygonal that covers the fuzzy input data from GPS could be built.

vertices inside the circles (for each confidence level). This bound is the largest trajectory compatible with the GPS measurements. Observe that the LUB is infinite unless we introduce some constraints. The main assumption in this paper is that *changes in the direction of the vehicle between two samples are small*, thus we can approximate the trajectory between two consecutive samples by a straight line.

We will define a polygonal chain that covers the fuzzy data, by finding the outside tangents to the CEP for each  $\alpha$ -cut, and then computing the cross points of the corresponding tangents between two consecutive fuzzy points (see Fig. 3). Observe that

1. The longest segment contained in each polygon of four sides—a quadrilateral—is one of its diagonals.
2. The longest path contained in two adjacent quadrilaterals always comprises two of these diagonals.
3. The longest path contained in three adjacent quadrilaterals is also composed of diagonals, but they might not be the longest diagonals of each quadrilateral.

The LUB is defined by a subset of the list of vertexes. Later in this section we explain how to obtain this subset. We introduce first some preprocessing algorithms that will improve the accuracy of the final measurement.

### 3.1. Preprocessing the data

The accuracy of the measure can be improved if some collinear points are joined, and some of the worst GPS fixes are discarded. In case the sampling rate is high enough, our hypothesis (straight trajectory between samples) still holds after performing these two changes, and we can safely preprocess the data before computing the estimation of the length and the LUB.

Given an level value  $\alpha$ , the fuzzy input data are represented as a circle centered on the coordinates, and whose radius is the CEP at probability  $(1 - \alpha)$ . We have mentioned that, for each  $\alpha$ -cut value, the data comprise a set of circles, and also that the two outside tangents of two consecutive circles, and the cross points of the tangents to every three consecutive circles define a polygonal chain. The objective of the preprocessing is to obtain a reduced polygonal chain of fuzzy sets which still contains the LUB for each level  $\alpha$ . If crisp data were used, polygonal chain simplification has been studied in Estkowski and Mitchell (2002) and Hershberger and Snoeyink (1992). For fuzzy data, to our knowledge, the most similar work in the available literature is that presented in Anile et al. (2000), where fuzzy data from a geographical data base are used to reconstruct 3D images by means of fuzzy B-splines (de Boor, 1972).

Our process of computing the LUB is a three step procedure: the first step is filtering by collinearity, then filtering by a multiobjective algorithm, and finally using a deterministic algorithm for determining the largest polyline in a polygonal chain; this is done for each  $\alpha$ -cut. These filtering stages are described in the subsections that follow, and the deterministic algorithm is explained in Section 3.2.

#### 3.1.1. Filtering by collinearity

For each  $\alpha$ -cut a polygonal chain is obtained, as explained before. If for three consecutive circles, both pairs of outside tangents are parallel, then the quadrilateral defined by the first and third circles, and the intermediate circle could be filtered. Further removals of points are limited by the sampling rate, i.e. if too many points are filtered out, we cannot assume the trajectory is straight between the first and the last one.

#### 3.1.2. Filtering spurious data

Spurious data are input points where the error is abnormally high. We can remove those points where the fix was not accurate enough, provided that we do not discard a significant number of points, i.e. a fraction  $1 - \alpha$  of the vertices of the reduced polygonal must be in the unfiltered path, for each level  $\alpha$ . In Fig. 4 the process is illustrated for a given  $\alpha$ -value.

Filtering points reduce the area of the polygonal chain, so the LUB will be smaller too, and this is not a desired effect. We do not want to filter representative points. Maximizing the percentage of covered data, while filtering the outliers, are objectives that counteract one another. We will use a multicriteria genetic algorithm to optimize the filtering, as we will explain in Section 4.

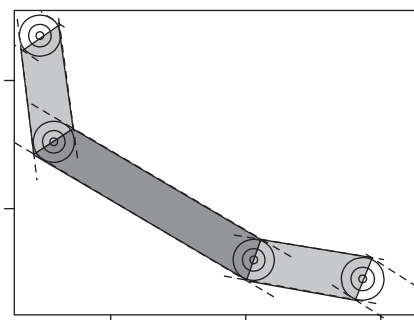


Fig. 4. The simplifying process over a synthetic example: filtering a point reduces the area of the polygonal chain, but the remaining points must resemble the trajectory.



### 3.1.3. Dynamic behavior of the vehicle

The filtering process can be further improved if we assume that the vehicle has inertia and some trajectories contained in the polygonal chain are not feasible. For example, think of a vehicle moving at a certain speed. There exists a maximum angle the vehicle could turn without risking its security. Moreover, for each speed value there exists a maximum angle, lower than that given by security reasons, which is comfortable for the vehicle passengers.

Therefore, we can introduce a second hypothesis in our analysis: *driving a taxi must be comfortable*. The maximum angle of turn is a function of the speed of the vehicle: the larger the speed, the shorter the maximum angle of turn. In determining the angle of turn at each point, the one before and one after fuzzy points are used. For an  $\alpha$ -cut value the fuzzy points are circles. By means of the tangents the polygonal chain for the three fuzzy points could be defined. The largest trajectory included in such polygonal chain is one of the four possible polylines that goes through the vertexes of the polygonal chain. Once the largest trajectory for this three points polygonal chain has been found, if the angle that the segments of the largest trajectory define is larger than the maximum angle of turn at current speed, the trajectory is not considered as a candidate for its length being the LUB.

### 3.2. Deterministic longest path estimation

Once the data are preprocessed, we evaluate its LUB with a deterministic algorithm, that we explain in this section.

For each  $\alpha$ -cut of the chain, we get a polygonal set constructed with trapezoids, as can be seen in Fig. 5. The motion direction is indicated by a thin dashed arrow. Each trapezoid vertex is denoted with a pair of integers, those at the left of the arrow are zero at first, those at the right have one at first. The other number is the step in the motion sequence. The longest path at each step  $i$  goes through  $(0, i)$  vertex or  $(1, i)$  vertex. The set of vertexes that defines the longest path, can be computed by exhaustive exploration of all possible combinations, but this is very expensive in terms of computational cost and proved impracticable in a realistic trajectory with 100 points, for instance. This problem has been studied in the area of Computational geometry and is related with longest path with forbidden pairs (Berman and Schnitger, 1992), that is NPO PB-complete.

Because of this and given that in a realistic trajectory the changes of direction and the changes in distance between left and right vertex are limited due to the dynamics of the taxi, the

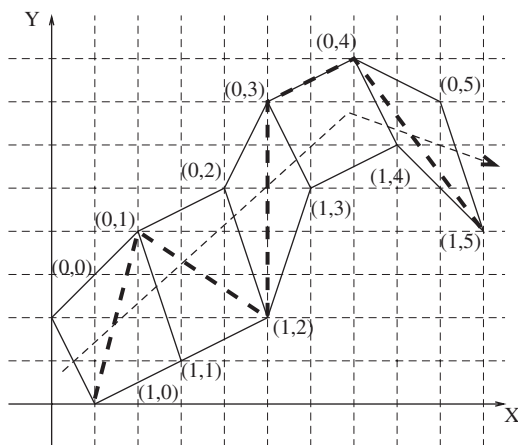


Fig. 5. Example of longest path estimation following the algorithm explained in Section 3.2.

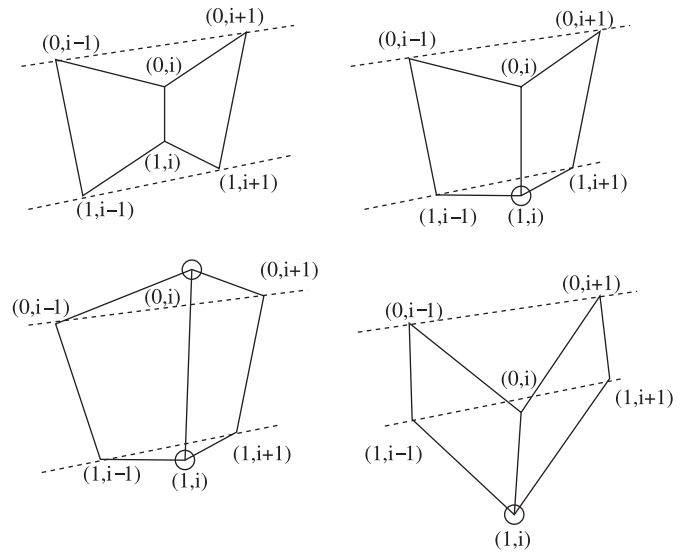


Fig. 6. Possible relative positions of vertexes and lines between prior and next vertexes. This is useful in the determination of convex and non convex vertexes.

geometry of the road and GPS behavior, we use a heuristic that is linear in time with the number of vertex. The heuristic is based on the selection of convex vertexes: when a vehicle turns, the longest path goes through the exterior of the trajectory curvature. The convexity of a vertex is analyzed using the straight lines that rely on previous and following vertexes, the possible relative positions of the central vertex can be seen in Fig. 6, where convex vertexes are marked with a small circle and the lines that pass through vertexes  $(0, i - 1)$ ,  $(0, i + 1)$  and  $(1, i - 1)$ ,  $(1, i + 1)$  are drawn. From left to right and top to bottom, if both vertexes are between the lines, both are concave. If only one is outside the lines, it must be convex. If both are outside the lines, both may be convex (left) or one may be concave and the other is convex. In both cases, if the farthest one from the nearest line is chosen, then it is convex.

The heuristic is as follows: the first segment of the longest path goes from a convex vertex in step 1 to the vertex at step 0 that gives the maximum segment length. From step 1 to the one before the last, the path goes through:

- If there is only a convex vertex, through this vertex.
- If there are two convex vertexes, through the farthest one.
- If there are no convex vertex, through the farthest one.

The last segment ends in the farthest vertex from the previous one.

In Fig. 5 the path computed with this heuristic is marked with a thick dashed line. The first segment goes from  $(1, 0)$  to  $(0, 1)$  because  $(0, 1)$  is convex and the distance to  $(0, 0)$  is shorter. Then the longest path continues to  $(1, 2)$  because it is the only convex. The same happens with  $(0, 3)$  and  $(0, 4)$ . Finally, the path ends in  $(1, 5)$  because it is farther from  $(0, 4)$  than  $(0, 5)$ .

### 3.3. Randomized longest path estimation

There is an alternate implementation for the method proposed in the preceding section. Let us suppose that we superimpose a grid on the chain of circles (see Fig. 7) and compute the whole set of lengths obtained by the selection of one point of the grid from each fix. In order to compute the maximum length from the obtained trajectories, a backtracking algorithm is impracticable due to the high number of points present in real trajectories.

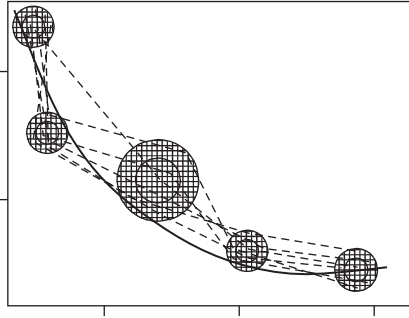


Fig. 7. Discretization of GPS measurements and exhaustive exploration of the lengths of all the trajectories obtained from all the possible combinations of discretized points. Only a fraction of the possible trajectories is shown.

However, we can uniformly sample a number of trajectories and compute a Monte Carlo estimate of the probability distribution of the lengths that arise. The mean value of this distribution would be our estimation of the length, and we can also produce a confidence interval for the mean, from which to calculate our tolerance.

To build a sample trajectory, we have implemented the procedure that follows: for each fix, with a probability 0.95, a point of the grid which is inside the CEP at that level is randomly selected (thus with a probability 0.05 the point is selected from outside the corresponding CEP). The length of the polygonal that joins all these points is stored and the whole procedure repeated a high number of times. The sample distribution of these lengths is finally used to compute the sample mean (which is an estimator of the length that the taximeter should have produced) and a confidence interval for it.

Unfortunately, there are some issues with this method. In particular, the range of the sample distribution of the lengths is bounded by the maximum sample length, which is general is not the maximum length of a compatible path. As a matter of fact, the maximum of the sample would be more reliable estimator of the LUB for our purposes. Since there are not clear improvements in speed neither in accuracy with respect to the method in the preceding section, Monte Carlo analysis is not considered in this work.

#### 3.4. Predetermined trajectories and maps

A trivial improvement of the accuracy could be obtained if we restricted ourselves to a known route, and adjust the GPS coordinates so that each position is replaced by its nearest point in the center of the road. However, this method has been rejected by the experts in the certification agency that will homologate the device. The reason given was that all the measurements have to be reproducible. There will be points with known latitude and longitude where the certification agency will position the devices and check that the tolerance of the measurements is within the range. This kind of certification is not compatible with dynamic changes in the coordinates, because the nearest point of the road may well be out of the circle defined by the CEP, making the determination of the LUB useless.

However, nothing prevents using the map as a constrain, i.e. we know that the car is in the intersection between the road and the confidence interval produced by the GPS. The coordinates of the whole road and not only its center are needed, though. This method has not been implemented either, because we have not measured the width of the road, neither have access to a cartography of the route with legal validity. This last point is important, because any error in the maps would void all the

measurements taken with the devices. Nevertheless, observe that, even if we decided to include this information in the future, it would not alter the computations we present in this paper.

## 4. Genetic filtering of the fuzzy data

In this section the details of the codification and operators used in the genetic algorithm which has been used to filter the data are given. A multiobjective genetic algorithm has been used in the filtering process. Specifically, the multiobjective genetic algorithm used in this work is the well known NSGA-II (Deb et al., 2000; Deb and Goel, 2001). This algorithm is outlined in Fig. 8.

### 4.1. Codification of an individual

Each individual is a subset of the chain of fuzzy points, codified as an array of Booleans. That is to say: input data are a series of timely ordered fuzzy points. Each one of the fuzzy input points has a Boolean value associated for each individual. When the Boolean value for a fuzzy point is set to *true* then that fuzzy point is included by the individual. When the Boolean value for a fuzzy point is set to *false* that fuzzy point is filtered out.

To generate an individual, a probability threshold  $p$  is given, and each fuzzy point in the vector of input fuzzy data is included with probability  $p$ . The origin and the end of the taxicab run must always be included.

### 4.2. Multiobjective fuzzy fitness function

Observe that both the area of the polygonal chain and the percentage of covered fuzzy data by the polygonal chain are fuzzy functions. We want to minimize the area of the defined polygonal chain and to maximize the percentage of covered fuzzy data. Following Sánchez and Couso (2007), we will not defuzzify the objectives, but use a fuzzy valued fitness function to assess the quality of the filtering.

Genetic algorithm can solve fuzzy valued optimization problems. For instance, we can define a total order between the fuzzy values of the fitness function (Abbasbandy and Asady, 2006; Mitchell, 2006; Tran and Duckstein, 2002; Sheen, 2006; Sun and Wu, 2006). In particular, we need to sort fuzzy numbers, which some authors think it is inconsistent with most definitions of total order between fuzzy sets (Yeh and Deng, 2004; Wang et al., 2005). The use of a weaker (partial) order can also be made compatible with a tournament-based selection in conventional genetic algorithms. In this context, in Jahanshahloo et al. (2004) and Wang et al. (2005) an interval representation is used, and in Ganesan and Veeramani (2006) another partial order relation is proposed, restricted to trapezoid membership functions. This last

```

 $R_t = P_t \cup Q_t$ 
 $F = \text{fast-non-dominated-sort}(R_t)$ 
 $P_{t+1} = \phi, \quad i = 1;$ 
while  $|P_{t+1}| + |F_i| > N$ 
  crowding-distance-assignment( $F_i$ )
   $P_{t+1} = P_{t+1} \cup F_i$ 
   $i = i + 1$ 
end while
Sort( $F_i, \prec_n$ )
 $P_{t+1} = P_{t+1} \cup F_i[1 : (N - |P_{t+1}|)]$ 
 $Q_{t+1} = \text{make-new-population}(P_{t+1})$ 
 $t = t + 1$ 

```

Fig. 8. Pseudocode of the NSGA-II algorithm.

solution cannot be applied to our problem, either, as there is neither knowledge nor restrictions about the membership functions type and certainty distributions. Also, different approaches for evaluating the Pareto dominance using fuzzy fitness functions have been proposed. In Youssef and Khan (2000) the use of fuzzy rules for determining the dominance of one individual with regard to another is proposed. Similar works are documented in Trebi-Ollennu and White (1997) and Kiyota et al. (2000). In this work we have decided to use our own implementation of the NSGA-2 algorithm for fuzzy data, which is described in Sánchez et al. (2007) and Sánchez et al. (submitted). We have used an imprecise probabilities based ranking, in combination with the definitions of nondominated sorting and crowding distance explained therein.

#### 4.3. Genetic operators

The definitions of the crossover and mutation must reduce the number of vertexes in the population.

- **Crossover.** Given two parents  $A$  and  $B$ , the offspring are two new chains  $C$  and  $D$  such that  $A \cap B \subseteq C$  and  $A \cap B \subseteq D$ ; a vertex  $v \in A - B$  has a probability  $p^+$  of being in  $C$ , and a vertex in  $B - A$  has a probability  $p^-$  of being in  $C$ , where  $p^-$  is much lower than  $p^+$ .  $D$  set is constructed in the same way.
- **Mutation.** This operator is defined as the random removing of a point of the chain, different from the first or last one. It is important to notice that neither the first nor the last fuzzy points will be included in the genetic operations because both trajectory ends must be included for all individuals.

## 5. Experiments and results

In this section we describe the experiments that we have devised for justifying the claims in this paper. Our experimental design has three objectives:

1. Assessing the theoretical accuracy of the proposed method.
2. Obtaining the actual tolerance of the roller machine.
3. Comparing the accuracies of this method and the roller machine in practical cases.

Therefore, three sets of experiments have been conceived:

1. **Simulated paths:** We have generated synthetic data, and added random noise to it mimicking the properties of a typical GPS sensor. Using this noisy data we made estimates of the percentage of error between our LUB and the true length of each path.
2. **Tolerance of the roller machine:** Even though the roller machine has a theoretical null error, in practice the pressure of the tyres and the level of wear cause a dispersion of the measurements of the same order of magnitude as the GPS sensor.
3. **Compared accuracies:** A real-world example is used to compare the measurements taken with our system and the former method: we have measured a circuit with an ISO-9002 certified odometer and then used it in our own testing procedure. The same vehicle was also tested in the roller machine, and all the results are compared and discussed.

Each one of these categories will be analyzed in detail in the sections that follow.

### 5.1. Simulated paths

We have evaluated our algorithm in realistic paths that cover most of the situations found when the TI test of a taxi is carried out. These synthetic paths simulate several turns, accelerations, decelerations and changes in CEP. GPS longitude/latitude coordinates were translated to Universal Transverse Mercator northing/easting coordinates in order to make distance calculations between GPS fixes easier (Snyder, 1982). With this system, points on the Earth's surface are projected onto an equally spaced planar metric grid, therefore the distance between fixes is the usual Euclidean one. The trajectory is sampled once each second.

Since we need to know the true length of the path, and the differences between the actual position and the GPS data, some random noise is added to each point of the trajectory. At each location, a random number from 4 to 8 is taken as the CEP at 95% probability. The uncertainty in the GPS measurements is simulated using the following procedure: with a probability of 0.95, a point is selected whose distance to the real one is shorter than the CEP, and with probability of 0.05 an outlier is introduced. Part of the generated data is shown in Fig. 9. GPS measurements are represented with circles (actually ellipsoids in Fig. 9, due to scaling issues) with a radius equal to 95% CEP and the original trajectory with a continuous line. As can be seen, 95% of the circles intersect the trajectory, but none of their centers are in the actual trajectory.

We perform two experiments with two trajectories of 120 points each in order to test if the tolerance computed from the LUB is lower than the legal margin or not. If the true length of the trajectory is known, the tolerance is

$$\varepsilon = (\text{LUB} - \text{length})/\text{length}. \quad (6)$$

Several tests were also carried out to assess the genetic filtering. This was launched with the parameters shown in Table 1. Firstly, and for both trajectories, 10 runs of each of the multiobjective algorithm were done, without restrictions based on the dynamic behavior of the vehicle. Finally, a second set of 10 runs were done with the multiobjective algorithm using the same parameters but assuming that the angular velocity of the vehicle had a speed dependent bound.

#### 5.1.1. Case study I

The true length of the first trajectory is 3228.6 m. The distance through the GPS fixes is 3238.5 m. Taximeters which mark more than  $3228.6 + 10\% = 3551.4$  will be rejected. Observe that, in practical circumstances we do not know the actual length of the path and we will reject those taxis that charge more than  $3238.5 + 10\% = 3562.4$ .

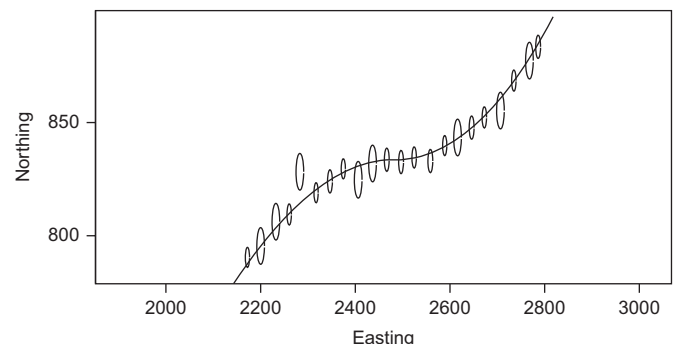


Fig. 9. Example of GPS generated data along with the true trajectory. As can be seen, some of the fixes CEP do not intersect the true trajectory. The most part of the points of the trajectory are inside the CEP at 95%.

**Table 1**  
Parameters used for NSGA-II algorithm

Parameter	Value
Number of generations	1000
Number of individuals	100
Number of populations	1
Minimum percentage of fuzzy points covered by each individual	0.85
Probability $p^+$ for genetic crossover	0.5
Probability $p^-$ for genetic crossover	0.1
Crossover probability	0.7
Mutation probability	0.1

**Table 2**  
Results from 10 runs of NSGA-II without dynamic analysis

Dataset	True length	Measured length	LUB estimation Best	LUB estimation Mean
1	3228.57	3238.521	3456.61	3499.48
2	2741.30	2696.487	3126.99	3192.51

The mean LUB for this trajectory (first row of table in Fig. 2) is 3499.5. Then, the expected tolerance is 0.084 (0.081, if we used the measured values to compute the tolerance). This result means that taximeters charging more than 3551.4 can be safely rejected.

If we filter the data taking into account the dynamics of the vehicle (see Table 3) the expected tolerance is 0.005 (0.002 if we used the measured length) which is much lower than the legal margin.

### 5.1.2. Case study II

The length of the second trajectory is 2741.3 m. This trajectory has stronger turns than the first, and we will see that the LUB will be less tight, given the deterministic procedure explained in Section 3.2. The distance through the GPS fixes is 2696.5 m. If we repeat the analysis done in the preceding case (see second row of Table 2), then the mean LUB is 3127.0. The expected tolerance is 0.141 (0.160), thus we cannot reject a taximeter on the basis of a test carried in this route (see also Table 3).

The use of dynamic restrictions in the genetic filtering causes a significant improvement of the tolerance, which is now 0.021 (0.038) thus we could legally reject taxis that charge more than  $2741.3 + 10\% = 3015.4$  (2966.2) using the same input data.

Observe that, given these results, we recommend the TI station to choose circuits with smooth turns, since they will surely produce a low rate of null verifications. In general terms, the use of convoluted paths is not advised with the system we propose here.

## 5.2. Tolerance of the roller machine

The GPS measurements are inherently imprecise. As a matter of fact, that imprecision is not too high, as the simulation in the preceding section has shown. Even though sometimes we could not demonstrate that the accuracy of the measurements was lower than 10%, the actual accuracies were around 0.3% in the first test case, and 1.6% in the second. Nevertheless, the roller machine has a theoretically null error, because the number of turns of the cylinders can be precisely counted, and from a point of view of a metrology expert, this procedure has more sense and reportedly should be preferred, if applicable.

However, in practice, tyre pressure and wear influence the measurements. The dependence is not immediate, though. The

**Table 3**  
Results from 10 runs of NSGA-II with dynamic analysis

Dataset	True length	Measured length	LUB estimation Best	LUB estimation Mean
1	3228.57	3238.521	3242.64	3243.45
2	2741.30	2696.487	2765.63	2798.01

**Table 4**  
Vehicle's odometer measurements for a roller machine measurement of 1000 m

Press or condition	Min	Med	Max	Wear
Mean	1006.525	1027.437	1009.578	1030.420
Std. deviation	14.29073	3.661429	6.815601	5.170147

In each column the mean and standard deviation at each pressure or condition are shown. The behavior of the system is complex: when the tyre pressure is high, the diameter of the tyre increases and the odometer measurements are lower. If the pressure is too low, the deformation of the tyre increases the perimeter and the measurements are lower also. The effect of the wear is simpler; it always decreases the perimeter of the tyre.

effect of the tyre pressure on the measures taken with the roller machine is not easy to model, because the number of turns of the cylinder for each linear meter depends on both the effective diameter of the wheel and the tyre slip, although conversely the level of wear is directly related to the measured distance.

For studying these dependences, we have designed a small set of experiments. In the first place, we measured the errors introduced by the different pressures in the tyres. Five taxi models were used, and the pressures were:

- the maximum allowed (according to the manufacturer specifications);
- the recommended pressure;
- a very low pressure (0.5 bar below the recommended).

In the second place, for analyzing the dispersion of results for different levels of wear, one last round of experiments was performed with a set of wheels worn to the limit, and inflated to standard pressure. It is also remarked that all the experiments had to be done at very low speed to avoid the electronic traction control system activation.

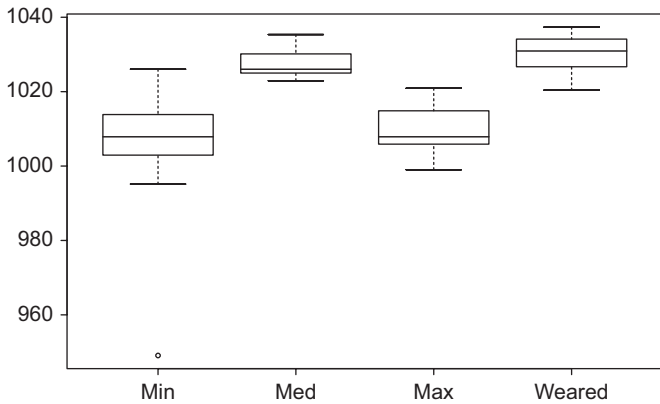
Each round of experiments consisted on 10 runs. Each run began with the vehicle's odometer set to zero and ended when the roller machine reached 1000 m. In Table 4 the mean and standard deviation of the odometer measurements is shown. In Fig. 10 the same data are shown graphically using boxplots.

It should be observed (Table 4) that the dispersion of the measurements is much higher than expected. There have been differences of 30 m in a 1000 m run, or 3%, which is well above the theoretical tolerance of the GPS in the simulated path. We have not tested the combined effect of wear plus different inflating pressures, but the actual dispersion could be even higher. In the next section we compare both methods (rollers and GPS) on an actual path, and confirm that the tolerance of the GPS sensor is not, in practical cases, worse than that of the rollers.

### 5.3. Real-world measurements

When a taximeter is being verified, the taxi owner is sent for a run. He/she must carry the target taximeter, and the GPS-based

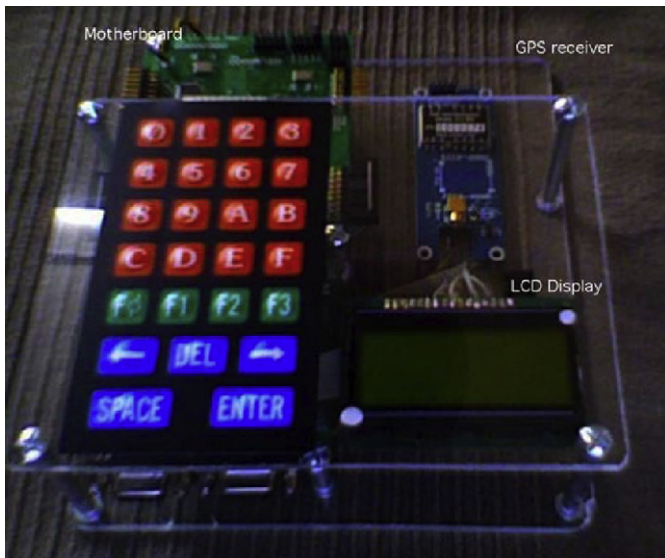




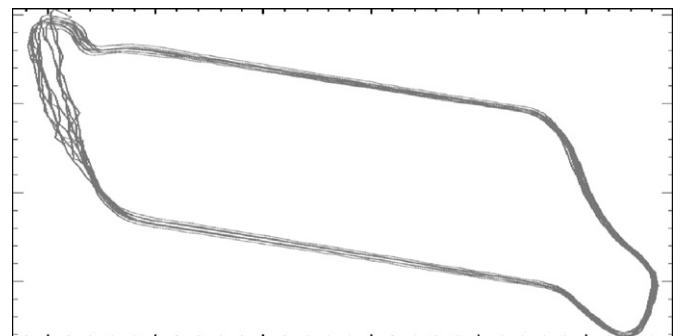
**Fig. 10.** Boxplots of the distance, as measured by the odometer, when the rollers indicate 1000 m. Note how the pressure of the tyre or the level of wear affects to the distance reported by the taximeter.



**Fig. 12.** Aerial picture of the circuit used in the tests. In the left of the image, a portion of the road is shown between the building and an area covered with trees, where GPS measurements are difficult.



**Fig. 11.** Prototype of the data logger that is being developed for the verification of taximeters. The design is based on low cost automotive microcontrollers and a consumer-grade GPS receiver.



**Fig. 13.** Path registered by the GPS, 10 laps to the circuit depicted in the preceding figure. There are large differences between the laps, mostly in the left part of the circuit.

**Table 5**

Mean and standard deviation of the 10 measurements of the real path using MOSA and NSGA-II filtering, unfiltered GPS coordinates and equivalent rolling cylinder machine run

	MOSA	NSGA-II	GPS	Rolling machine
Mean	1118.872	1121.494	1103.635	1106.667
Standard deviation	7.353936	7.665415	16.49395	5.532274

The measured length using a certified odometer was 1093 m.

datalogger device. (A photograph of the prototype which is currently under development is shown in Fig. 11.) Once the taxi comes back to the TI, the GPS measurements are loaded into the computer where the LUB computing will be carried out. Only bidimensional measurements are to be taken into account as the height is not relevant: the trajectories range between 3 and 5 km, and the differences in altitude are a few meters. Each measurement includes the coordinates, the calculated CEP value for the desired  $\alpha$ -cuts and the number of available satellites.

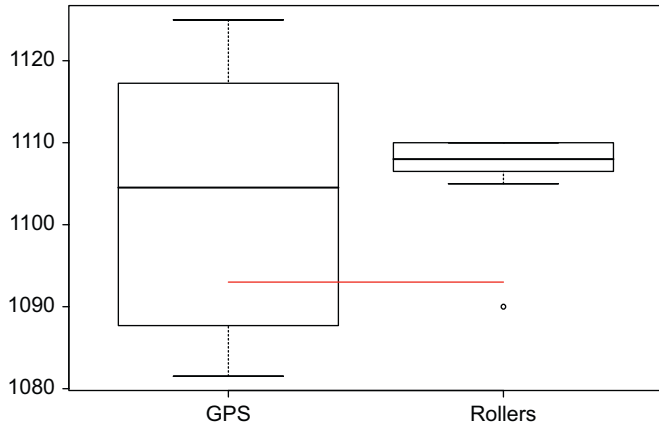
In this section we have used a circuit surrounding the Campus de Viesques' buildings (Gijón, Asturias, Spain). The length of the circuit was measured using an ISO-9002 certified odometer. In Fig. 12, a picture of the circuit can be seen. The circuit comprises two long and two short straight paths, and four sections with close turns. The measured length was 1093 m.

A direct comparison of the rolling machine against the other methods is not possible, so an indirect procedure was employed. We equipped a car with an odometer that reported the distance travelled. Then we placed the same car in the rolling machine, and travelled 1093 m, according to the same instrument. Lastly, we

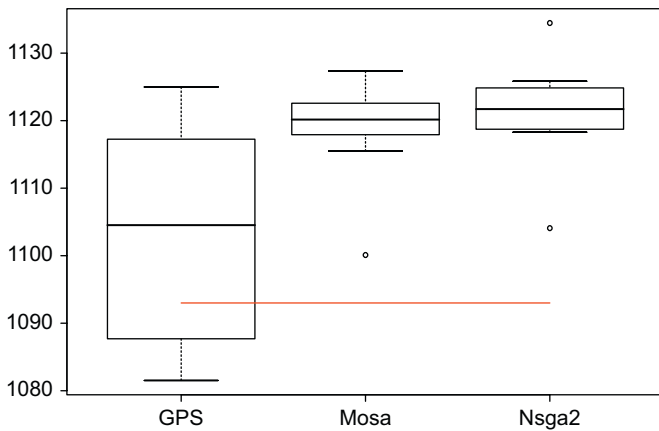
read the measurement produced by the rolling machine. This procedure was repeated 10 times.

In Fig. 13, the route recorded by the GPS in the analyzed laps can be seen. There are large differences between the centers of the measurements between successive laps, especially in the left part of the circuit. This uncontrollable behavior results in a variation of the measured length of the same path, as shown in Table 5. This table collects the mean and standard deviations of the obtained measures, using each method discussed in the paper: MOSA and NSGA-II dynamical filtering, raw GPS coordinates, and cylinders. It is remarked that the variability of the results in the cylinders does not take into account the variability induced by changes in pressure of the tyres neither their wear that will be discussed later in this section.

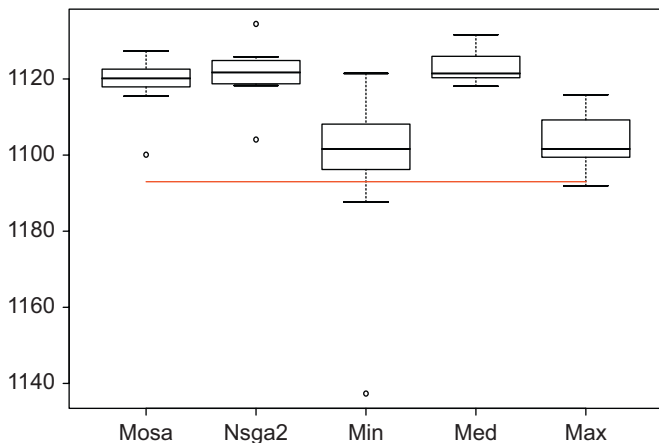
The same results are graphically shown in Figs. 14 and 15, where data collected in 10 runs is summarized by means of boxplots. We have placed an horizontal line at the true length (the value that was measured with the certified odometer) in both



**Fig. 14.** Boxplots of 10 measurements of the circuit using unfiltered GPS coordinates, and the equivalent measurements using rolling cylinders. The measured length using a certified odometer was 1093 m (the horizontal line). Raw GPS measurements had a high variability and were not a suitable alternative for the cylinders.



**Fig. 15.** Boxplots of the 10 measurements of the real path using unfiltered GPS coordinates and filtered GPS coordinates with MOSA and NSGA-II. The measured length using a certified odometer was 1093 m, the horizontal line is placed at that length. Note that the measurements done with filtered GPS coordinates never fall under the actual value of the length.



**Fig. 16.** Boxplots of the 10 measurements of the real path using MOSA and NSGA-II filtered GPS coordinates, and equivalent measurements using rolling cylinders. The measured length using a certified odometer was 1093 m, the horizontal line is placed at that length. Note that filtered GPS measurements are comparable to those made with recommended tyre pressure.

figures. Observe in Fig. 14 that the unfiltered GPS coordinates produce a large dispersion of measurements, and the dashed line is crossed with a probability which is not negligible. However, in Fig. 15 the effect of the evolutionary filtering is clear, as both boxplots, depicting the filtered results, are well above the dashed line. That is to say, the filtering produces a good approximation of the LUB, which is never below the actual length. From the two filtering techniques, MOSA exhibits less variability and a mean lower mean value than NSGA-II, closer to the actual length.

Lastly, in Fig. 16, a comparison between the filtered GPS measurements and the rolling machine is done. Observe that the accuracy of the filtered GPS is not only comparable but better than that of the cylinders if the tyre pressure is too low or high. It is also remarked that the GPS-based verification does not depend on the condition of the tyres.

## 6. Conclusions

There are legal issues concerning the use of GPS devices for verifying taximeters. However, in our opinion a GPS is the measuring device that best balances cost and accuracy for a TI.

To homologate a GPS for this application, we need to guarantee that the tolerance of the measurements is lower than the legal 10% margin. We cannot assert that this tolerance holds in absolute terms, but in this paper we have defined how to compute the upper bound of any trajectory length compatible with GPS data, which effectively is a computation of the upper tolerance of the device, for a particular route. This calculations must be repeated each time a taxi is verified, because the obtained margins depend on the GPS signal reception, the satellite configuration and the shape of the path. We have also found that, the stronger the turns in the calibration trajectory, the less accurate the measurement is. Therefore, we recommend to avoid convoluted paths in the GPS-based verification of taximeters.

## Acknowledgement

This work was funded by Spanish M. of Education, under the Grant TIN-2005-08386-C05-05 and ITVASA.

## References

Abbasbandy, S., Asady, B., 2006. Ranking fuzzy numbers by sign distance. *Information Sciences* 176, 2405–2416.

Anile, A.M., Falcidieno, B., Gallo, G., Spagnuolo, M., Spinello, S., 2000. Modeling uncertain data with fuzzy B-splines. *Fuzzy Sets and Systems* 113, 397–410.

Berman, P., Schnitger, G., 1992. On the complexity of approximating the independent set problem. *Information and Computation* 96, 77–94.

de Boor, C., 1972. On calculating with B-splines. *Journal of Approximation Theory* 6, 50–62.

Couso, I., Montes, S., Gil, P., 2001. The necessity of the strong alpha-cuts of a fuzzy set. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 9 (2), 249–262.

Cressie, N.A.C., 1991. *Statistics for Spatial Data*. Wiley, New York ISBN 0-471-84336-9.

Deb, K., Goel, T., 2001. In: Zitzler, E., Deb, K., Thiele, L., Coello Coello, C.A., Corne, D. (Eds.), *Controlled Elitist Non-dominated Sorting Genetic Algorithms for Better Convergence*. First International Conference on Evolutionary Multi-Criterion Optimization. *Lecture Notes in Computer Science*, vol. 1993. Springer, Berlin, pp. 67–81.

Deb, K., Agrawal, S., Pratap, A., Meyarivan, T., 2000. In: Schoenauer, E.M., Deb, K., Rudolph, G., Yao, X., Lutton, E., Merelo, J.J., Schwefel, H.-P. (Eds.), *A Fast Elitist Non-dominated Sorting Genetic Algorithm for Multi-objective Optimization: NSGA-II*. Proceedings of the Parallel Problem Solving from Nature VI Conference. *Lecture Notes in Computer Science*. Springer, Berlin, pp. 849–858.

van Diggelen, F., 2007. GNSS accuracy: lies, damn lies, and statistics. *GPS World* 9 (1), 41–45.

Dussault, C., Courtois, R., Ouellet, J.P., Huot, J., 2001. Influence of satellite geometry and differential correction on GPS location accuracy. *Wildlife Society Bulletin* 29 (1), 171–179.

Estkowski, R., Mitchell, J.S.B., 2002. Simplifying a polygonal subdivision while keeping it simple. In: SCG '01: Proceedings of the Seventeenth Annual

- Symposium on Computational Geometry. ACM Press, New York, NY, USA, pp. 40–49, ISBN 1-58113-357-X.
- Ganesan, K., Veeramani, P., 2006. Fuzzy linear programming with trapezoid fuzzy numbers. *Annals of Operation Research* 143, 305–315.
- Goodman, I.R., Nguyen, H.T., 1985. *Uncertainty Models for Knowledge-Based Systems*. North-Holland, Amsterdam.
- Hershberger, J., Snoeyink, J., 1992. Speeding up the Douglas-Peucker line simplification algorithm. In: *Proceedings of the 5th International Symposium on Spatial Data Handling*, Charleston, USA, pp. 134–143.
- Hofmann-Wellenhof, B., Lichtenegger, H., Collins, J., 2004. *GPS Theory and Practice*. Springer, Berlin ISBN 3211835342.
- Jahanshahloo, G.R., Soleimani-damaneh, M., Nasbaradi, E., 2004. Measure of efficiency in DEA with fuzzy input–output levels: a methodology for assessing, ranking and imposing of weight restrictions. *Applied Mathematics and Computation* 156, 175–187.
- Kiyota, T., Tsuji, Y., Kondo, E., 2000. New multiobjective fuzzy optimization method and its application. In: *American Control Conference ACS'2000*, vol. 6, Chicago, IL, USA, pp. 4224–4228.
- Langley, R.B., 1991. The mathematics of GPS. *GPS World* 2 (7), 45–59.
- Langley, R.B., 1999. Dilution of precision. *GPS World* 10 (5), 52–59.
- Manning, J., Harvey, B., 1994. Status of the Australian geocentric datum. *Australian Surveyor* 39 (1), 28–33.
- Mitchell, H.B., 2006. Ranking type-2 fuzzy numbers. *IEEE Transactions on Fuzzy Systems* 14 (2), 287–294.
- Sánchez, L., Couso, I., 2007. Advocating the use of imprecisely observed data in genetic fuzzy systems. *IEEE Transactions on Fuzzy Systems* 15 (4), 551–562.
- Sánchez, L., Couso, I., Casillas, J., 2007. Modeling vague data with genetic fuzzy systems under a combination of crisp and imprecise criteria. In: *First IEEE Symposium on Computational Intelligence in Multi-Criteria Decision-Making (MCDM 2007)*, Honolulu, Hawaii, USA.
- Sánchez, L., Couso, I., Casillas, J. Genetic Learning of Fuzzy Rules based on Low Quality Data. *Fuzzy Sets and Systems*, submitted for publication.
- Sheen, J.N., 2006. Fuzzy evaluation of cogeneration alternatives on a petrochemical industry. *Computers and Mathematics with Applications* 49, 741–755.
- Snyder, J.P., 1982. *Map Projections Used by the U.S. Geological Survey*, second ed. Geological Survey Bulletin, vol. 1532. U.S. Government Printing Office, Washington, DC, 313pp.
- Strang, G., Borre, K., 1997. *Random Variables and Covariance Matrices, in Linear Algebra, Geodesy, and GPS*. Wellesley-Cambridge Press, Wellesley, MA.
- Sun, H., Wu, J., 2006. A new approach for ranking fuzzy numbers based on fuzzy simulation analysis method. *Applied Mathematics and Computation* 174, 755–767.
- Tran, L., Duckstein, L., 2002. Comparisons of fuzzy numbers using a fuzzy distance measure. *Fuzzy Sets and Systems* 130, 321–341.
- Trebi-Ollennu, A., White, B.A., 1997. Multiobjective fuzzy genetic algorithm optimization approach to nonlinear control system design. *IEEE Proceedings on Control Theory and Applications* 2 (144), 137–142.
- Wang, Y.-M., Greatbanks, R., Yang, J.-B., 2005. Interval efficiency assessment using data envelopment analysis. *Fuzzy Sets and Systems* 153, 347–370.
- Yeh, C.-H., Deng, H., 2004. A practical approach to fuzzy utilities comparison in fuzzy multicriteria analysis. *International Journal of Approximate Reasoning* 35, 179–194.
- Youssef, H., Sait, S.M., Khan, A., 2000. Fuzzy simulated evolution algorithm for topology design on campus networks. In: *2000 Congress on Evolutionary Computation*, vol. 1, Piscataway, New Jersey, USA, pp. 180–187.